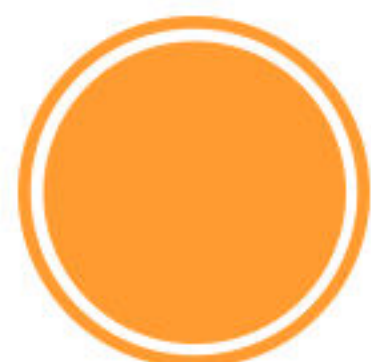


This pdf is collection of **Pathfinder video solutions** (MCQ/BYUs) available on my youtube channel  [Arihant Senior - Ankit Singhvi](#)

Release date:
Sept-2023



Revision Book for **Pathfinder XII**

11 Optics	4 - 55
12 Electrostatics	56 - 153
13 Current	154 - 233
14 Magnetism	234 - 282
15 EMI	283 - 351
16 Modern physics	352 - 381

by Ankit Singhvi

Ankit Singhvi
Dual Degree, IIT Madras

a MCQ b BYU		11 Optics 12 Electrostatics				12 Current 14 Magnetism				15 EMI 16 Modern physics					
	1	11b-6	26	11b-30	51	12a-20	76	12b-9	101	12b-30	126		151	13a-24	176
	2	11b-7	27		52	12a-21	77	12b-10	102	12b-31	127		152		177
	3	11b-8	28	11b-31	53		78	12b-11	103	12b-32	128	12b-54	153		178
11a-1	4	11b-9	29	11b-32	54	12a-22	79	12b-12	104	12b-33	129	13a-1	154	13b-1	179
11a-2	5	11b-10	30	11b-34	55	12a-23	80	12b-13	105	12b-34	130	13a-2	155	13b-2	180
11a-3	6	11b-11	31	12a-1	56	12a-24	81		106	12b-35	131	13a-3	156	13b-3	181
11a-4	7	11b-12	32	12a-2	57	12a-25	82	12b-14	107	12b-36	132	13a-4	157	13b-4	182
11a-5	8	11b-13	33	12a-3	58	12a-26	83	12b-15	108		133	13a-5	158	13b-5	183
11a-6	9		34	12a-4	59	12a-27	84	12b-16	109	12b-37	134	13a-6	159	13b-6	184
11a-7	10	11b-14	35	12a-5	60	12a-28	85		110	12b-38	135	13a-7	160	13b-7	185
11a-8	11	11b-15	36	12a-6	61	12a-29	86	12b-17	111	12b-39	136		161	13b-8	186
11a-9	12	11b-16	37	12a-7	62	12a-30	87	12b-18	112	12b-40	137	13a-8	162	13b-9	187
11a-10	13	11b-17	38	12a-8	63		88	12b-19	113	12b-41	138	13a-9	163	13b-10	188
11a-11	14	11b-18	39	12a-9	64		89	12b-20	114	12b-42	139	13a-10	164	13b-11	189
11a-12	15	11b-19	40	12a-10	65	12b-1	90		115	12b-43	140	13a-11	165	13b-12	190
	16	11b-20	41	12a-11	66	12b-2	91	12b-21	116	12b-44	141	13a-12	166	13b-13	191
	17	11b-21	42	12a-12	67	12b-3	92		117	12b-45	142	13a-13	167	13b-14	192
11a-15	18	11b-22	43	12a-13	68	12b-4	93	12b-22	118	12b-46	143	13a-14	168	13b-15	193
	19	11b-23	44		69	12b-6	94	12b-23	119	12b-47	144	13a-15	169	13b-16	194
	20	11b-24	45	12a-14	70	12b-7	95	12b-24	120	12b-48	145	13a-16	170	13b-17	195
11b-1	21	11b-25	46	12a-15	71	12b-8	96	12b-25	121	12b-49	146	13a-17	171	13b-18	196
11b-2	22	11b-26	47	12a-16	72		97	12b-26	122	12b-50	147	13a-18	172		197
11b-3	23	11b-27	48	12a-17	73		98	12b-27	123	12b-51	148	13a-19	173	13b-19	198
11b-4	24	11b-28	49	12a-18	74		99	12b-28	124	12b-52	149	13a-20	174	13b-20	199
11b-5	25	11b-29	50	12a-19	75		100	12b-29	125	12b-53	150	13a-22	175	13b-21	200

a MCQ b BYU		11 Optics 12 Electrostatics				12 Current 14 Magnetism				15 EMI 16 Modern physics					
13b-22	201	13b-46	226	14b-3	251	14b-23	276	15b-1	301	15b-24	326	15b-44	351	16b-10	376
13b-23	202	13b-47	227		252	14b-24	277	15b-2	302	15b-25	327	16a-1	352	16b-11	377
13b-24	203	13b-48	228	14b-4	253	14b-25	278	15b-3	303	15b-26	328	16a-2	353	16b-12	378
13b-25	204	13b-49	229	14b-5	254	14b-26	279	15b-4	304	15b-27	329	16a-3	354	16b-13	379
13b-26	205	13b-50	230	14b-6	255	14b-27	280	15b-5	305	15b-28	330	16a-4	355	16b-14	380
13b-27	206	13b-51	231	14b-7	256		281	15b-6	306	15b-29	331	16a-5	356	16b-15	381
13b-29	207	13b-52	232	14b-8	257	14b-28	282	15b-7	307	15b-30	332	16a-6	357		
13b-30	208		233	14b-9	258	15a-1	283	15b-8	308	15b-31	333	16a-7	358		
13b-31	209	14a-1	234	14b-10	259	15a-2	284	15b-9	309		334	16a-8	359		
13b-32	210	14a-2	235	14b-11	260	15a-3	285	15b-10	310	15b-32	335	16a-9	360		
13b-33	211	14a-3	236	14b-12	261		286	15b-11	311	15b-33	336	16a-10	361		
13b-34	212	14a-4	237		262	15a-4	287	15b-12	312	15b-34	337		362		
13b-35	213	14a-5	238		263	15a-5	288	15b-13	313		338		363		
13b-36	214	14a-6	239	14b-13	264	15a-6	289	15b-14	314	15b-35	339	16a-13	364		
13b-37	215	14a-7	240	14b-14	265		290	15b-15	315		340		365		
13b-38	216	14a-8	241	14b-15	266	15a-7	291	15b-16	316	15b-36	341		366		
	217	14a-9	242	14b-16	267	15a-8	292	15b-17	317	15b-37	342		367		
	218	14a-10	243	14b-17	268	15a-9	293	15b-18	318		343	16b-1	368		
13b-39	219	14a-11	244	14b-18	269	15a-10	294		319	15b-38	344	16b-2	369		
13b-40	220	14a-12	245	14b-19	270	15a-11	295	15b-19	320	15b-39	345	16b-3	370		
13b-41	221	14a-13	246		271	15a-12	296	15b-20	321		346	16b-4	371		
13b-42	222		247	14b-20	272	15a-13	297	15b-21	322	15b-40	347	16b-5	372		
13b-43	223		248	14b-21	273	15a-14	298	15b-22	323	15b-41	348	16b-6	373		
13b-44	224	14b-1	249		274	15a-15	299	15b-23	324	15b-42	349	16b-7	374		
13b-45	225	14b-2	250	14b-22	275	15a-16	300		325	15b-43	350	16b-8	375		
												16b-9			

by Ankit Singhvi



1. When the Sun is behind dark clouds and there are gaps in the clouds, you will often see sunlight "fanning" out from the gaps as shown in the given photograph.

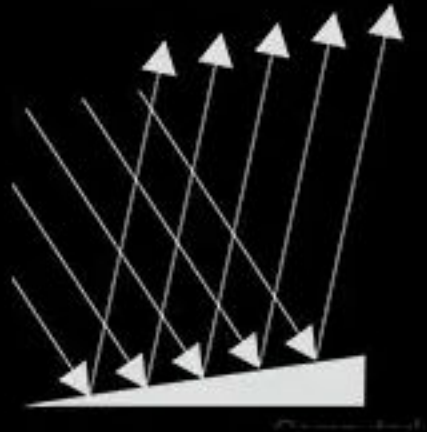
Though the rays are closely parallel but we see them diverging as they approach the earth. Which of the following statements most suitably explains this effect?

- ✓(a) Distant objects appear smaller than nearer objects.
- (b) Sunrays are actually radiated from the sun radially.
- (c) Light bends at sharp corners of opaque object that it encounters.
- (d) The gaps work as light sources at finite distance hence radiate diverging light.

Similar as watching a tall building from the road.



In light rays the effect is more prominent, as the clouds are at several kilometer height. Much much taller than buildings.



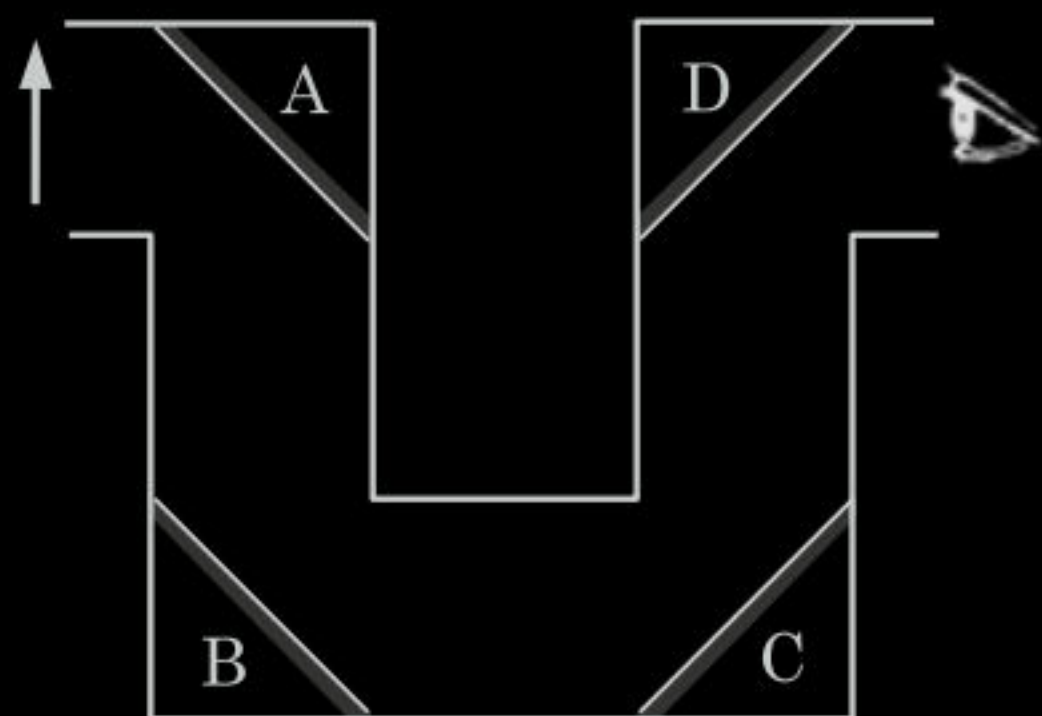
Specular
reflection



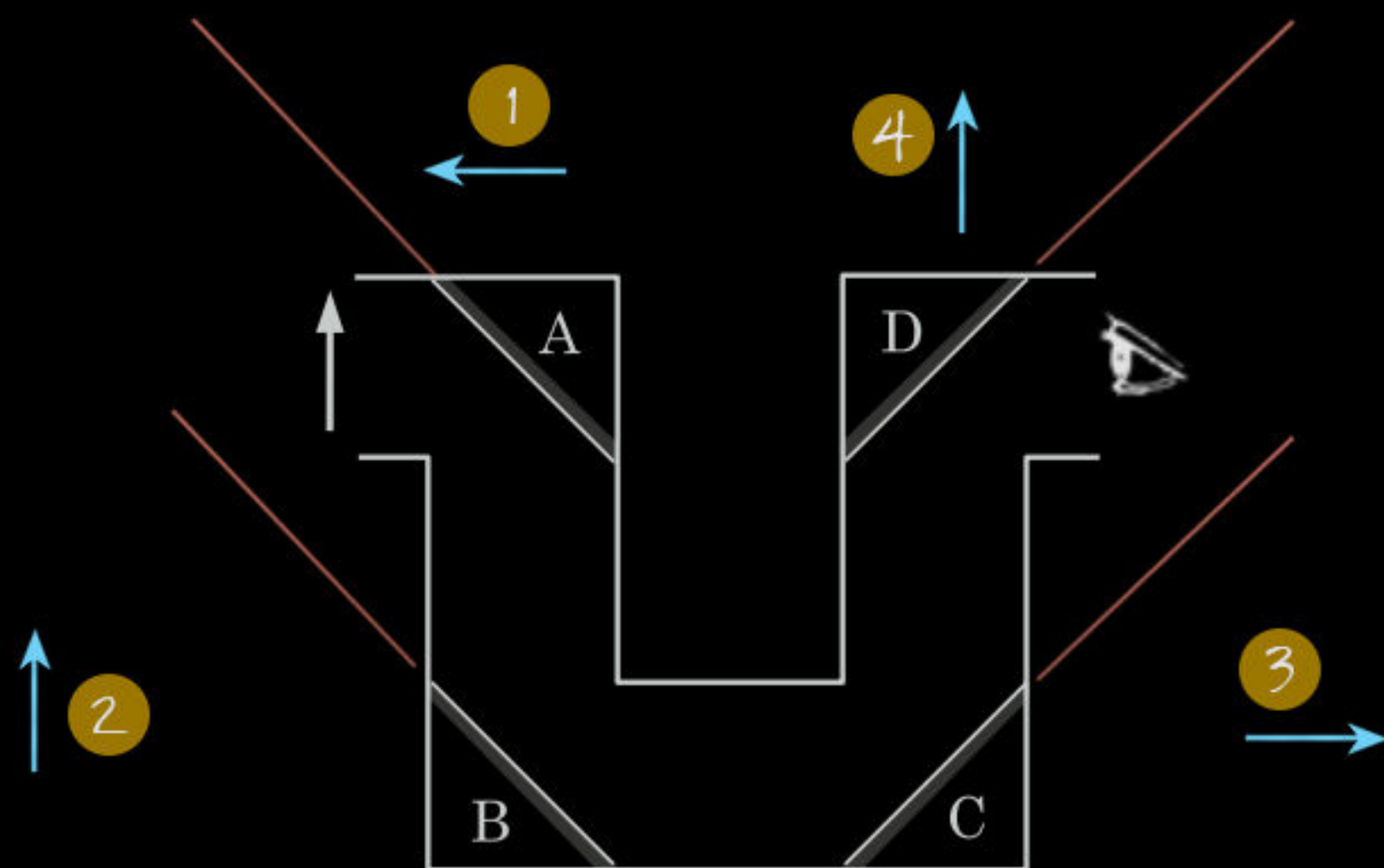
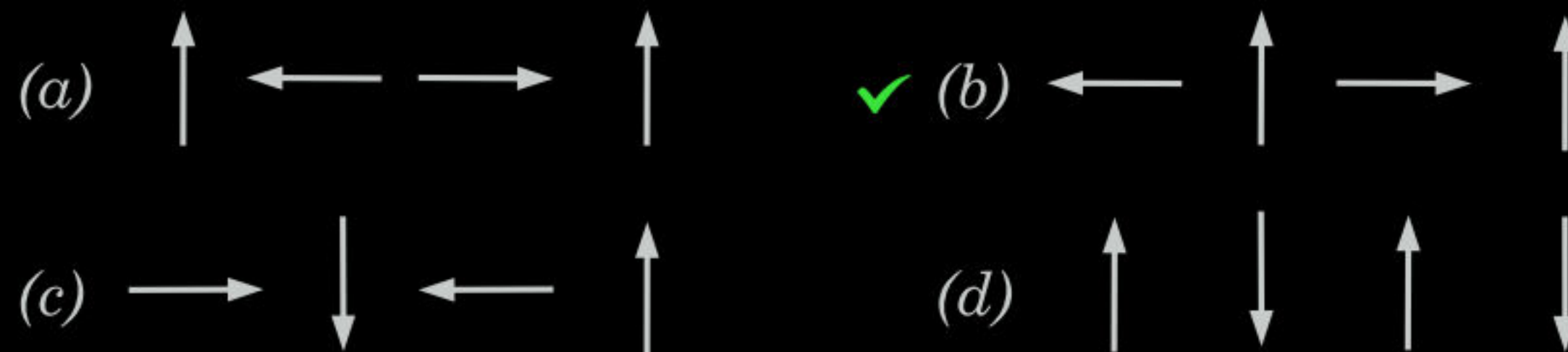
Diffused
reflection

2. Bodies that can be impregnated with water, colours of their surface appear richer after moistening. Which of the following is the most appropriate reason for this effect?

- ✓ (a) Due to moistening, surface irregularities are covered with water film, which reduces diffused reflection of light.
- (b) Due to moistening, surface irregularities are covered with water film, which reduces amount of reflected light.
- (c) Due to moistening, surface irregularities are covered with water film, which increases amount of reflected light.



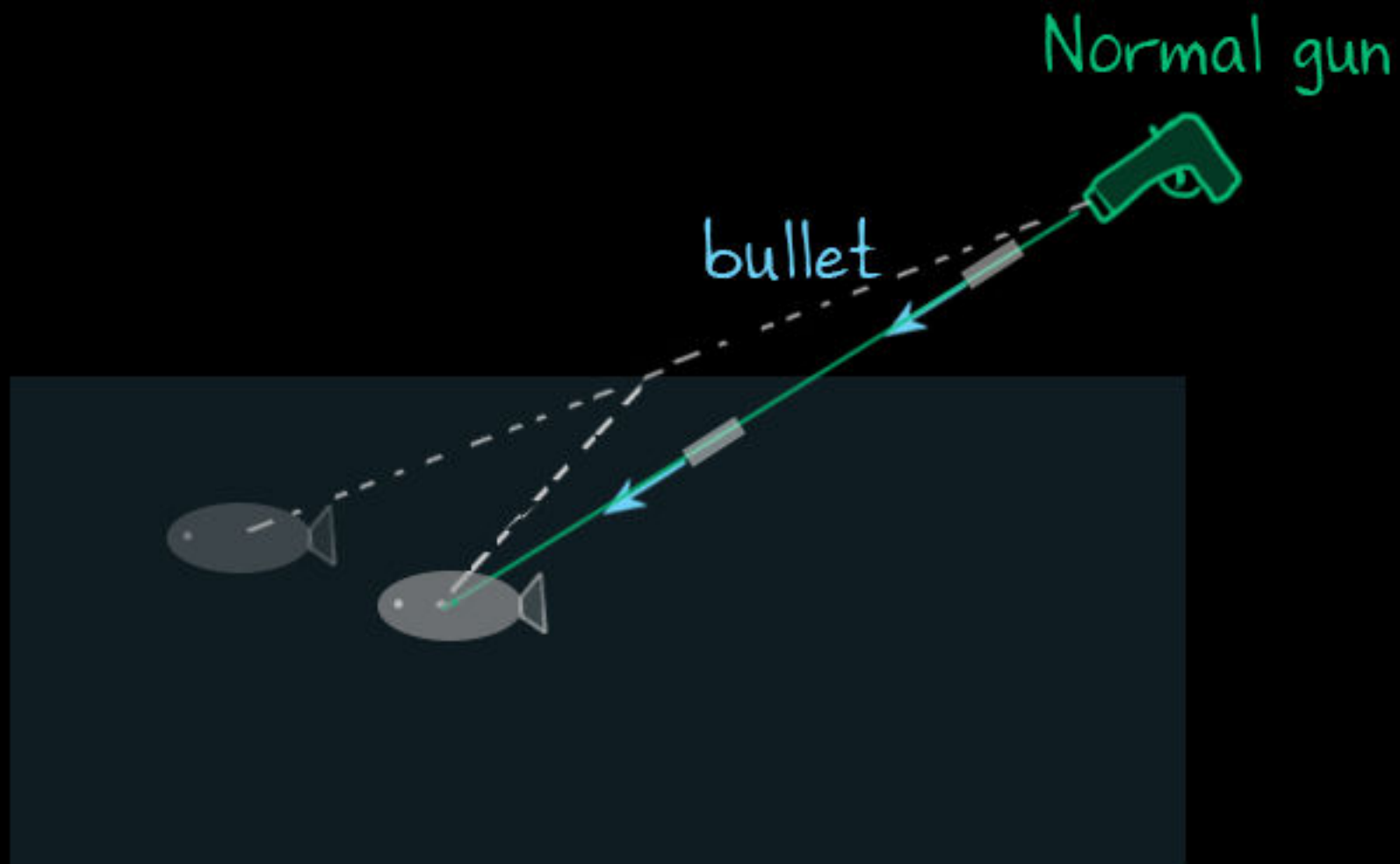
3. An arrow object is viewed through a bent metal tube with the help of four plane mirrors A, B, C and D as shown in the figure. Every mirror is inclined at an angle of 45° with the horizontal. Which of the following represents correct images made by these mirrors in sequence?



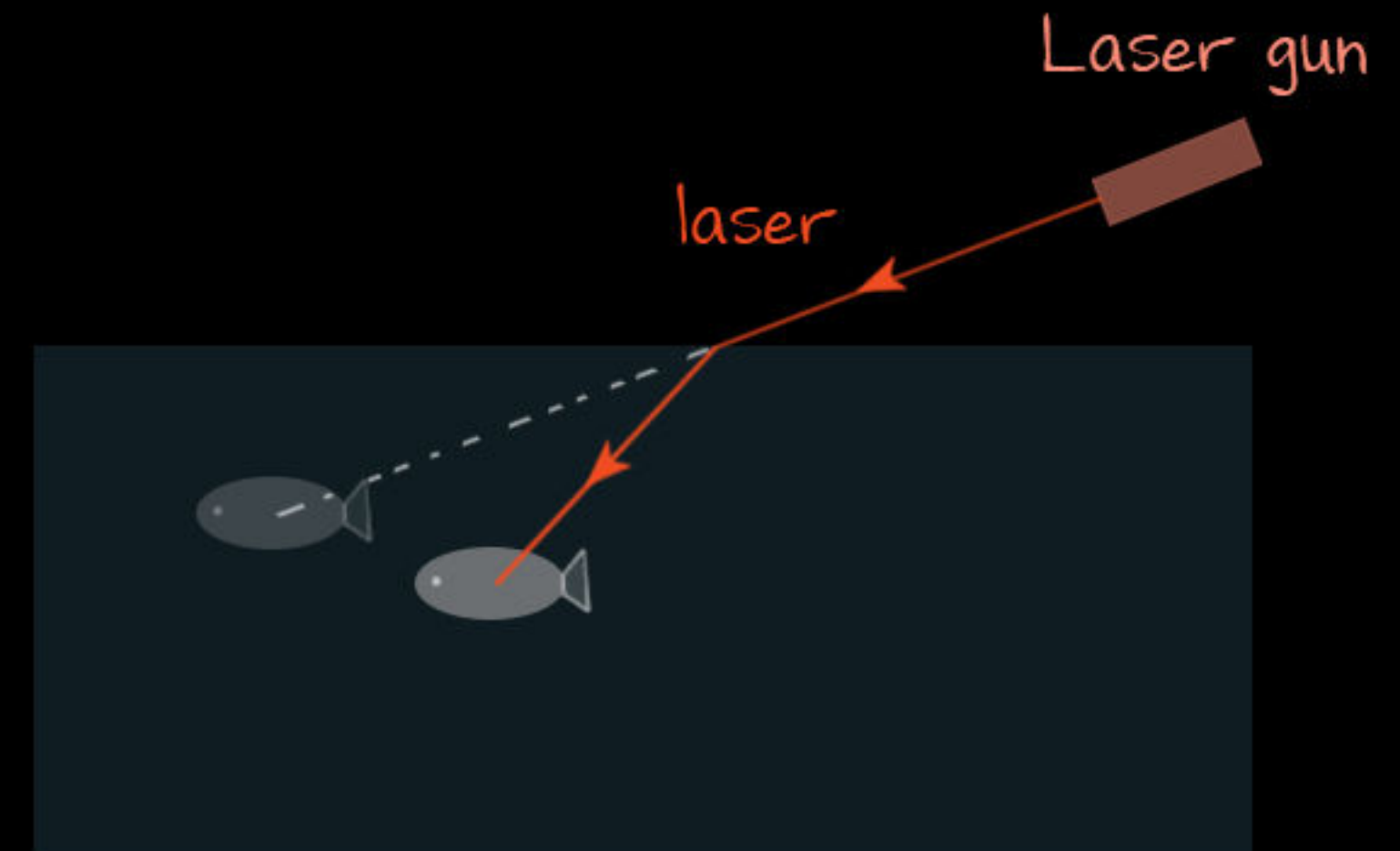
4. While fishing from a dock, you see a fish in the water. For this, you can use either a **bow and arrow**, or a **laser gun**. Which of the following strategy you must follow?

- (a) Aim the arrow as well as the laser gun both at the fish.
✓(b) Aim the arrow below the fish and the laser gun at the fish.
(c) Aim the arrow below the fish and the laser gun above the fish.
(d) Aim the arrow above the fish and the laser gun below the fish.

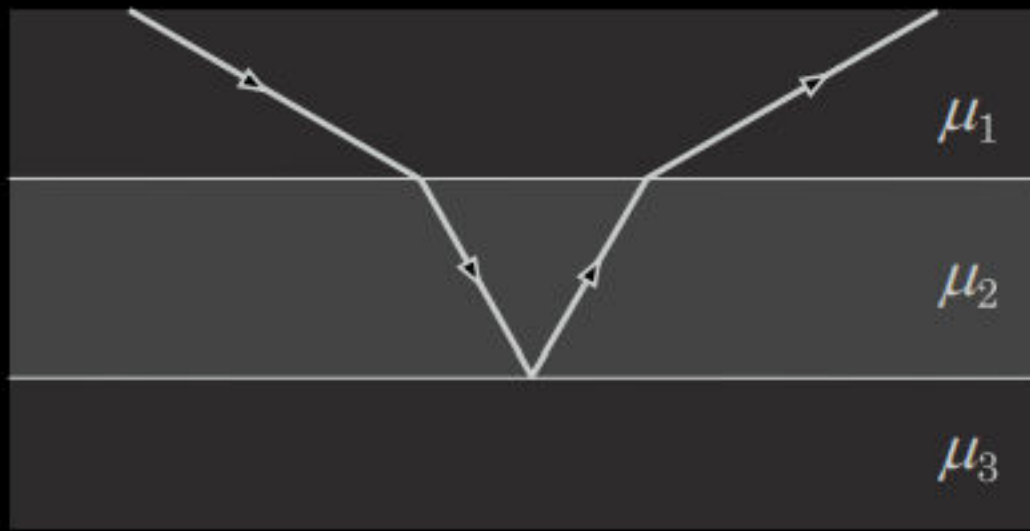
Using heat of laser



Aimed below the fish as 'seen' by the observer.
Bullet travels in straight line.



Aimed at the fish as 'seen' by the observer.
Laser bends along with light.



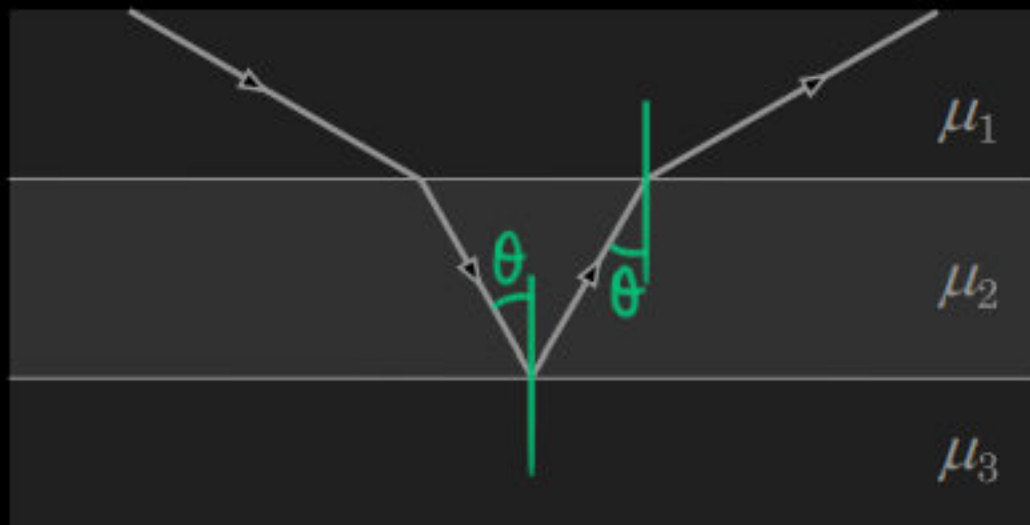
5. Light travelling through three transparent substances follows a path shown in the figure. Arrange their indices of refraction in order from smallest to largest.

✓(a) $\mu_3 < \mu_1 < \mu_2$

(b) $\mu_1 < \mu_3 < \mu_2$

(c) Neither of the above is possible.

(d) More information is required.

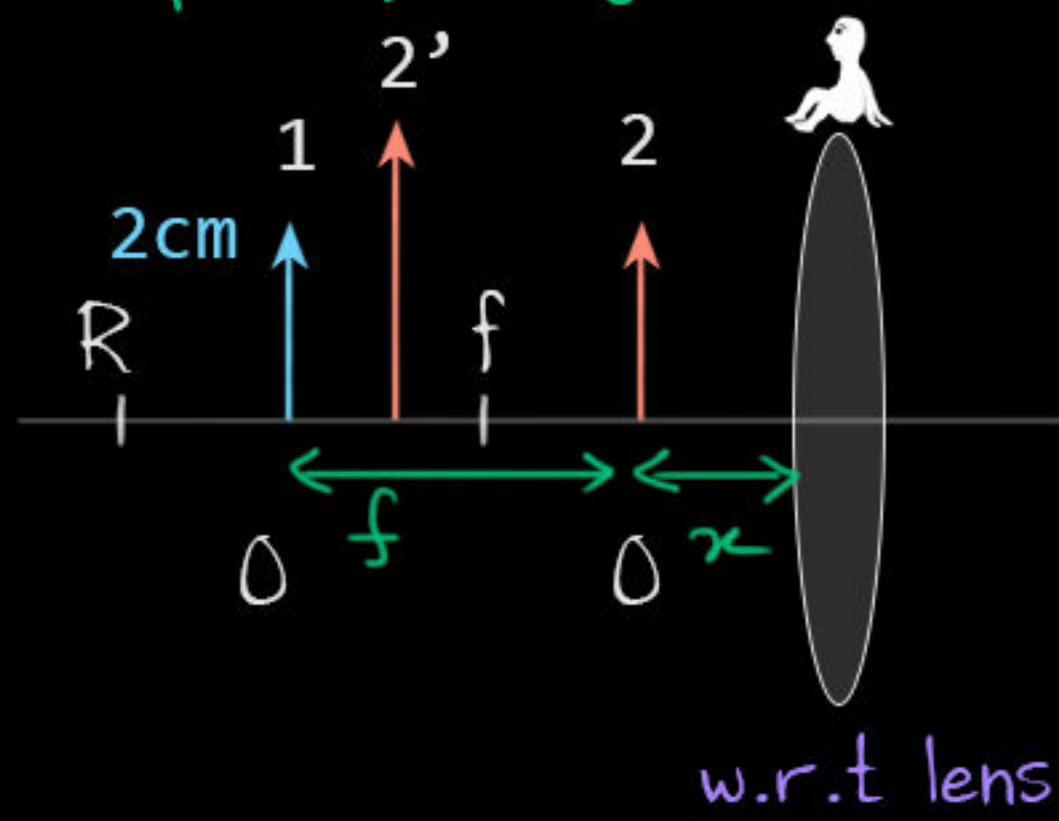


Clearly: $\mu_2 > \mu_1$

Now, with same incidence θ from same medium 2, ray can not enter 3 but enters 1.

∴ 3 is lighter than 1. And $\mu_1 > \mu_3$

Concept: With different object location, only a convex lens can make two equal size images



6. A lens held above a coin placed on a table forms an image of the coin. After the lens is moved vertically a distance equal to its focal length, it forms another image of the coin equal in size to the previous image. If diameter of the coin is 2.0 cm, what is the diameter of the image?

(a) 3.0 cm.
(c) 5.0 cm

✓(b) 4.0 cm
(d) 6.0 cm

$$|m| = \left| \frac{f}{f+u_1} \right| = \left| \frac{f}{f+u_2} \right| = \frac{I}{O}$$

$$\Rightarrow |f+u_1| = |f+u_2|$$

$$\Rightarrow \cancel{f} + (-\cancel{f} - x) = -[f + (-x)]$$

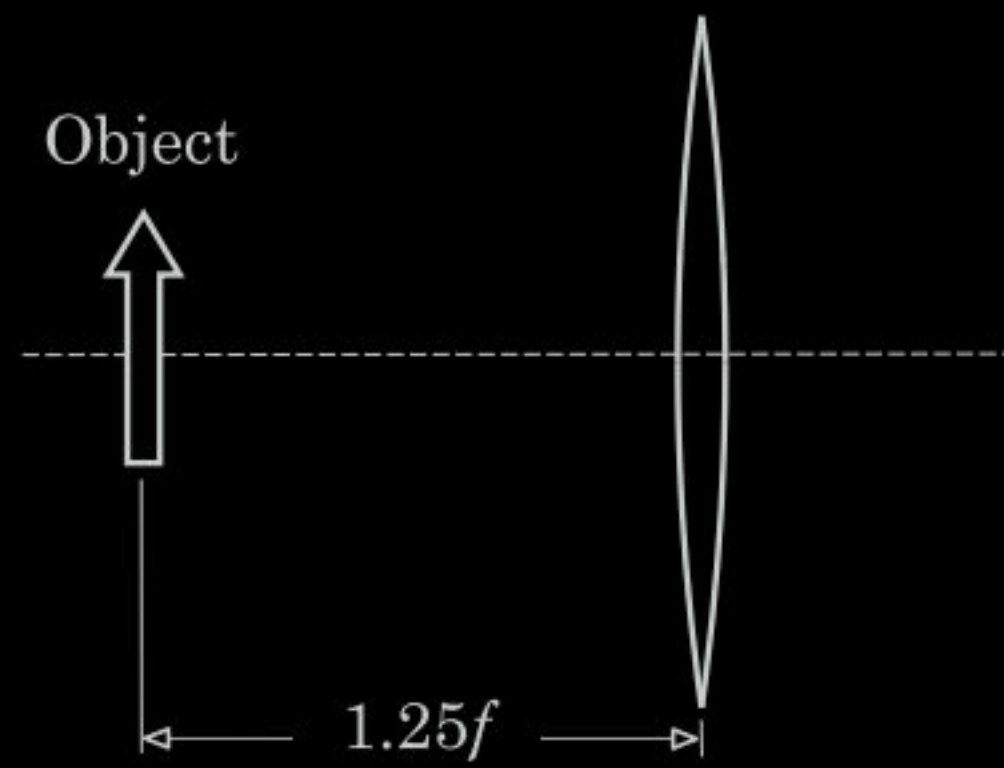
$$-x = -f + x$$

$$\Rightarrow x = f/2$$

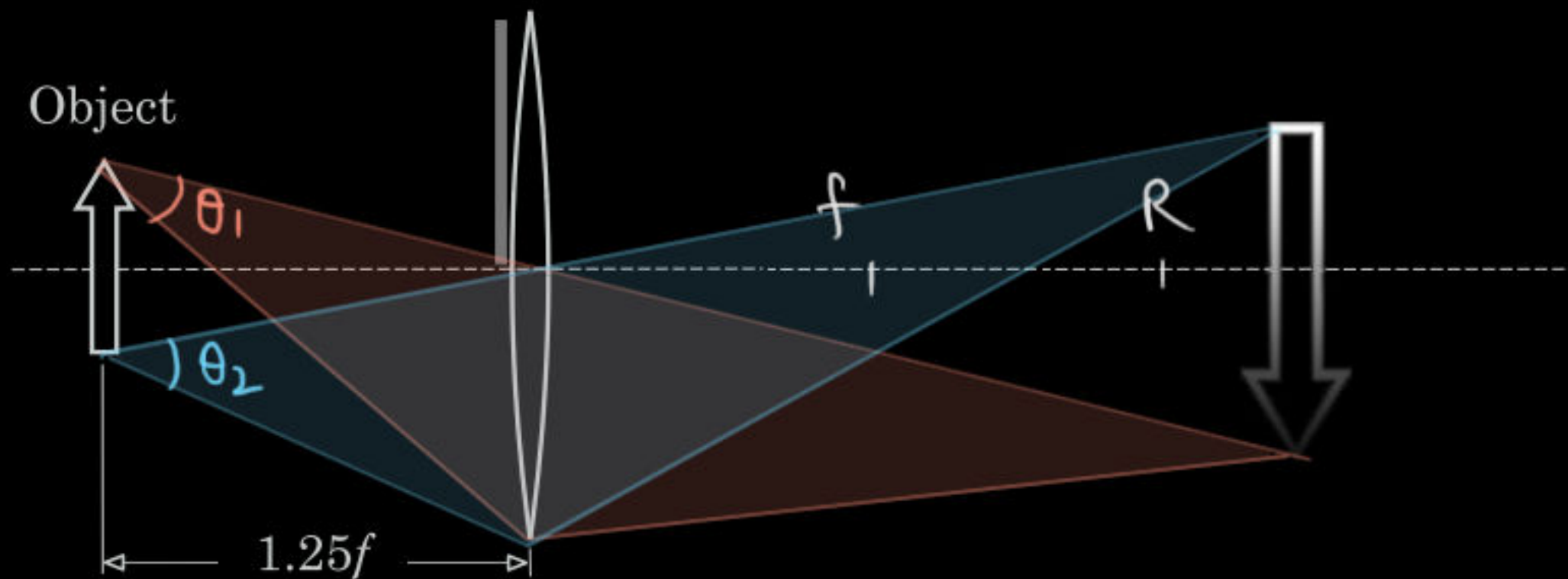
$$\frac{f}{\cancel{f} + (-\cancel{f} - x)} = \frac{I}{2}$$

$$\Rightarrow |I| = \frac{2f}{x} \rightarrow f/2$$

$$= 4 //$$



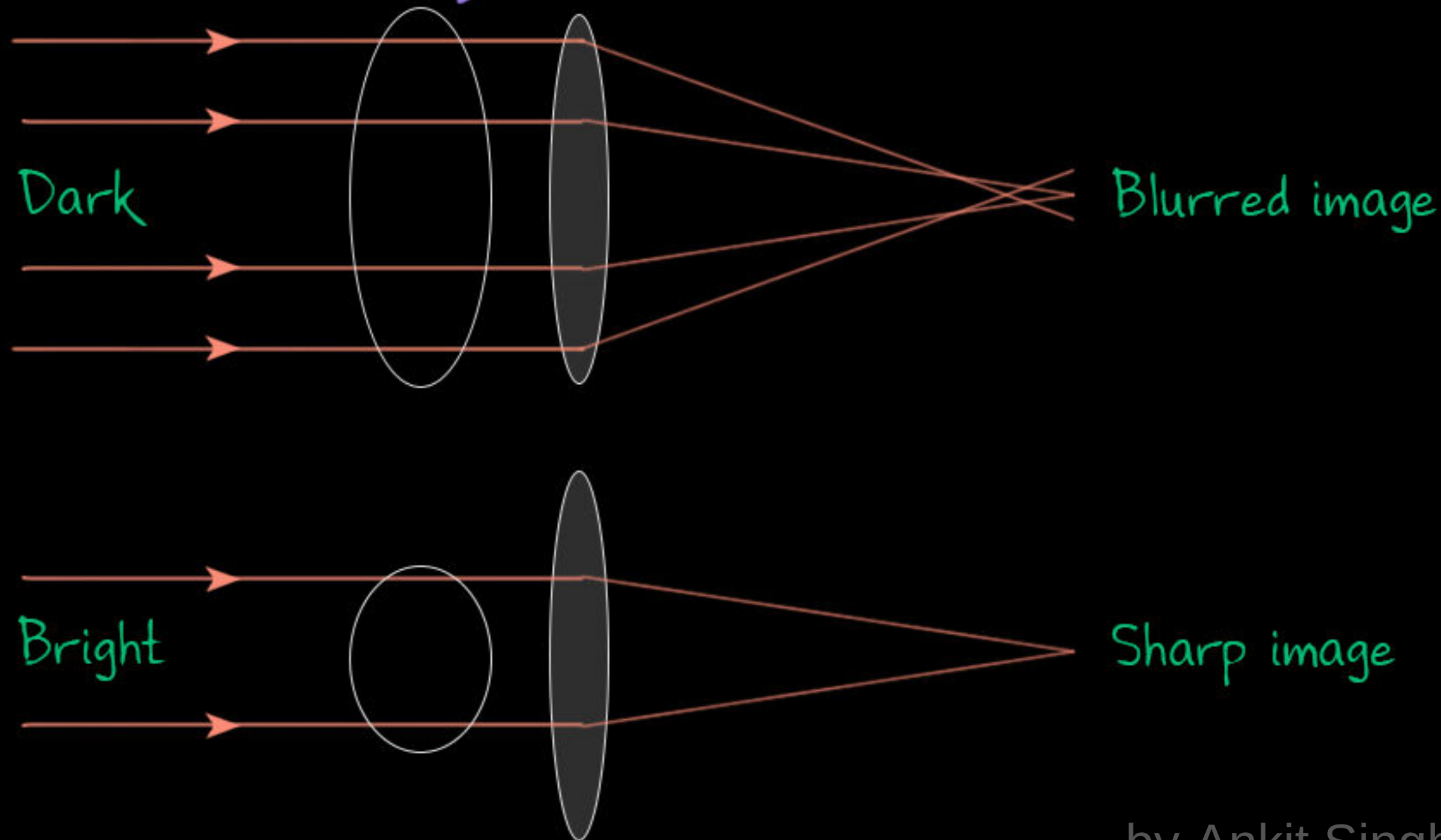
7. A converging lens of focal length f is used to cast an image of an object as shown in the figure. The upper half of the lens is now covered by an opaque coating. Which of the following statements is most appropriate?
- ✓(a) Upper half of the image is brighter than its lower half.
 - (b) Lower half of the image is brighter than its upper half.
 - (c) Entire portion of the image appears with uniformly reduced brightness.
 - (d) Entire portion of the image appears with reduced brightness but to predict intensity distribution more information is required.



Clearly: $\theta_2 > \theta_1$
 $\Rightarrow$ Solid angle: $\Omega_2 > \Omega_1$
 $\Rightarrow$ Upper half of image is brighter than lower half.

Pupil is wide open in dark,
to allow more light inside.

But it results in spherical
aberration, as light is
entering through larger
area of the lens.



8. It is a common belief that reading in reasonably bright light is more comfortable than reading in dim light. Which of the following is the most appropriate reason for this effect?

- (a) Contraction of ciliary muscles reduces astigmatism.
- ✓ (b) Contraction of pupil in bright light reduces spherical aberration.
- (c) In bright light image formed on retina has more intensity than that in dim light.
- (d) This is purely a psychological effect; bright light merely stimulates some more visual neurons that are normally dormant.

We know, for a achromatic compound lens (one convex, one concave):

$$\frac{w_1}{f_1} + \frac{w_2}{f_2} = 0 \quad (1)$$

where, w is dispersive power of each lens given by:

$$w = \frac{\mu_v - \mu_r}{\mu_y - 1} \quad (2)$$

and, f_1, f_2 are focal length for yellow light. Given by:

$$\frac{1}{f} = (\mu_y - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] \quad (3)$$



9. Refractive indices of two varieties of glass A and B depend on wavelength λ of light according to the following equations.

$$\mu_A = \mu_{A0} - \beta_A (\lambda - \lambda_0) \quad \text{and} \quad \mu_B = \mu_{B0} - \beta_B (\lambda - \lambda_0) \quad (4)$$

Here λ_0 is wavelength of yellow light. An achromatic combination of lenses is made from these materials. Radii of curvature of two outer surface of this combination are r_1 and r_2 . Which of the following is a correct expression for radius of curvature of the common surface?

Putting 2x3 in 1:

$$(\mu_{v1} - \mu_{r1}) \left[\frac{1}{r_1} - \frac{1}{-r_2} \right] + (\mu_{v2} - \mu_{r2}) \left[\frac{1}{-r_2} - \frac{1}{r_2} \right] = 0$$

Using 4:

$$-\beta_A (\cancel{\lambda_v} - \cancel{\lambda_r}) \left[\frac{1}{r_1} + \frac{1}{r_2} \right] + \beta_B (\cancel{\lambda_v} - \cancel{\lambda_r}) \left[\frac{1}{r_2} + \frac{1}{r_2} \right] = 0$$

$$\Rightarrow \frac{\beta_B}{r_2} - \frac{\beta_A}{r_2} = \frac{\beta_A}{r_1} - \frac{\beta_B}{r_2}$$

$$\Rightarrow \text{+ve } r_2 = \frac{r_1 r_2 (\beta_B - \beta_A)}{r_2 \beta_A - r_1 \beta_B} //$$

with sign: -ve $r_2 =$

$$\checkmark (a) \frac{r_1 r_2 (\beta_A - \beta_B)}{r_2 \beta_A - r_1 \beta_B}$$

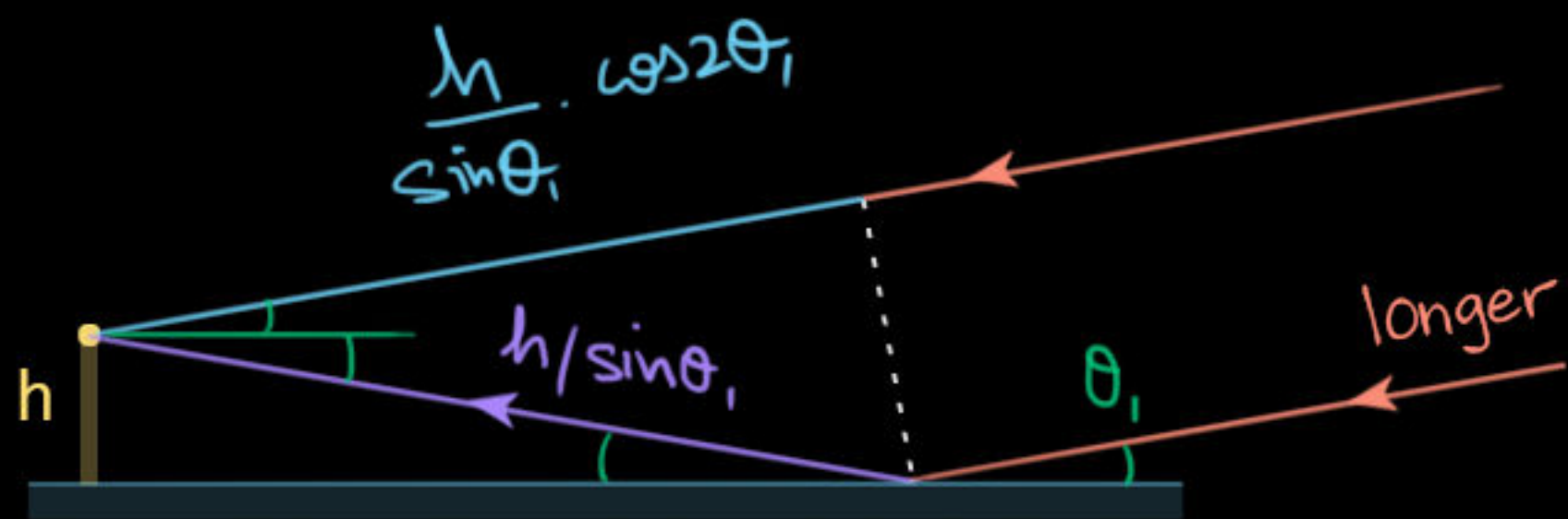
Youtube / Arthant Senior

by Ankit Singhvi

Concept: Between two consecutive maximas or minimas, difference in path difference is λ ①

10. In the middle of a calm lake, a radio receiver is installed on a pole at a height 4.0 m above water level. The receiver is used to track radio signals from a satellite orbiting the earth. As the satellite rises above the horizon, intensity of the signals at the receiver varies periodically. It is maximum, when the satellite is 3° above the horizon and then again when the satellite is 6° above the horizon. Wavelength of the satellite signals is closest to

✓(c) 42 cm



Extra due to reflection from water.

$$\Delta x_1 = \left(\frac{\lambda}{2} + \frac{h}{\sin \theta_1} \right) - \left(\frac{h}{\sin \theta_2} \right) \quad ②$$

Similarly,

$$\Delta x_2 = \left(\frac{\lambda}{2} + \frac{h}{\sin \theta_2} \right) - \left(\frac{h}{\sin \theta_1} \right)$$

$$\text{From 1,2: } \lambda = \Delta x_2 - \Delta x_1$$

$$\Rightarrow \lambda = \frac{h}{\sin \theta_2} (1 - \cos 2\theta_2) - \frac{h}{\sin \theta_1} (1 - \cos 2\theta_1)$$

$$= 2h (\sin \theta_2 - \sin \theta_1)$$

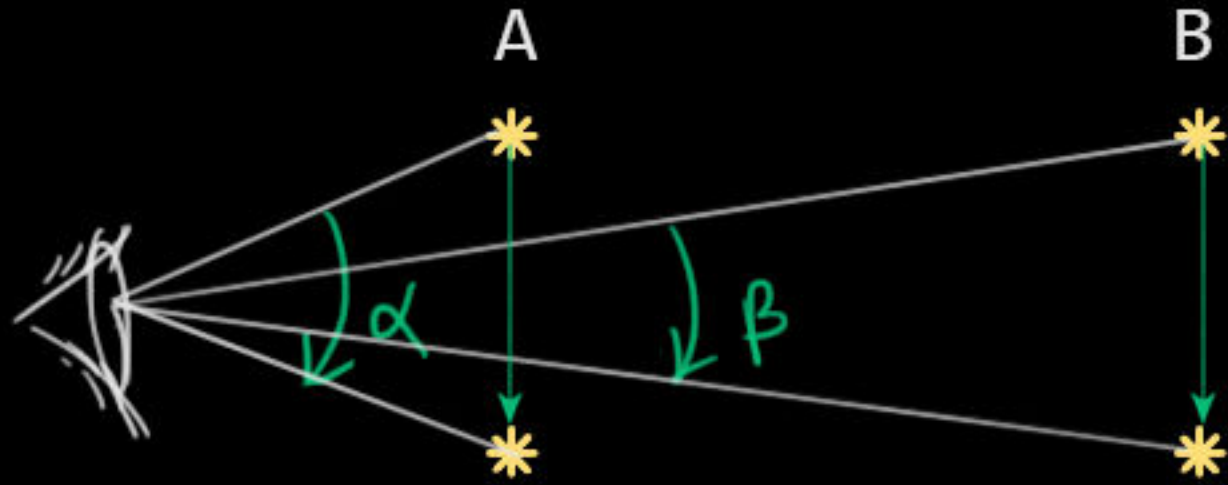
$$= 2 \cdot 4 \left(\frac{6 - 3}{360} \cdot 2\pi \right)$$

$$= 0.42 \text{ m}$$

Note: Putting this value in equation 2, we find that $\Delta x_1 = 3\lambda/2$. So the question probably meant two minimas at 3° and 6° .

Approach: w.r.t eye All objects, near or distant are moving with same velocity.

A w.r.t eye



$$\alpha > \beta$$

⇒ Direction of object A has changed more than B.

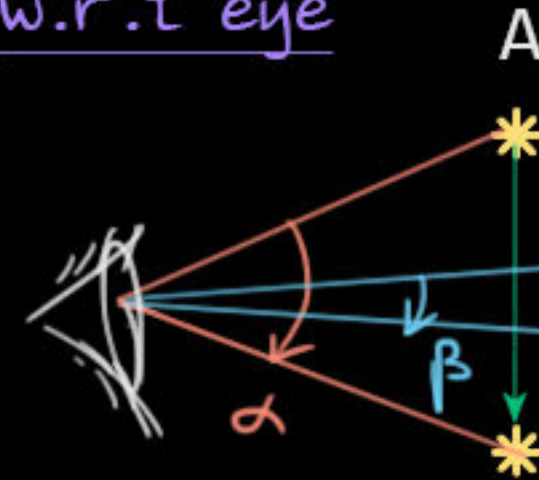
⇒ Eye makes brain think A has traveled more.

11. While travelling by a train or by a bus, if we look outside, we observe some effects. These effects and some facts possibly explaining these effects are given in the first and the second columns of the following table. Suggest suitable matches.

Column I	Column II
(a) Nearby objects appear moving past faster than distant objects.	(p) Human eye senses the direction from which light enters it.
(b) Extremely distant objects appear moving along with us.	(q) It cannot resolve two points if they subtend an angle less than one minute
	(r) Persistence of eye is 0.1 s.

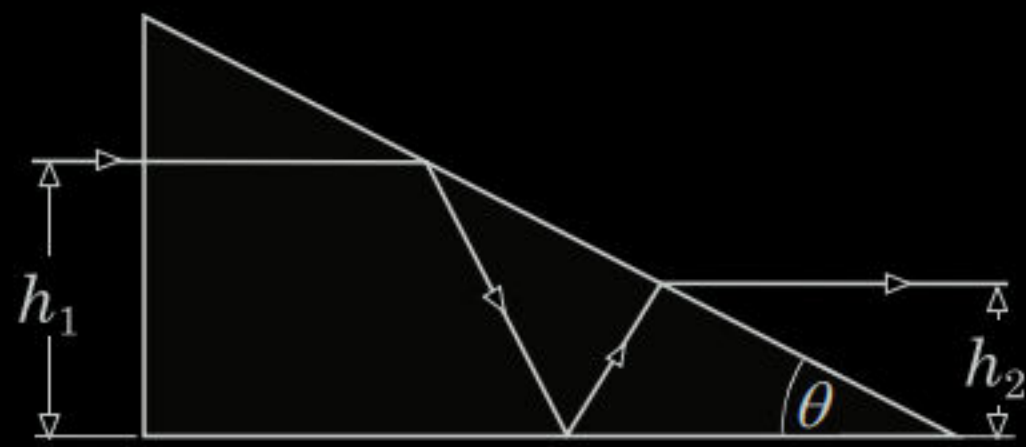
B

w.r.t eye



Relative to A, C has traveled $(\alpha - \beta)$ counterclockwise.

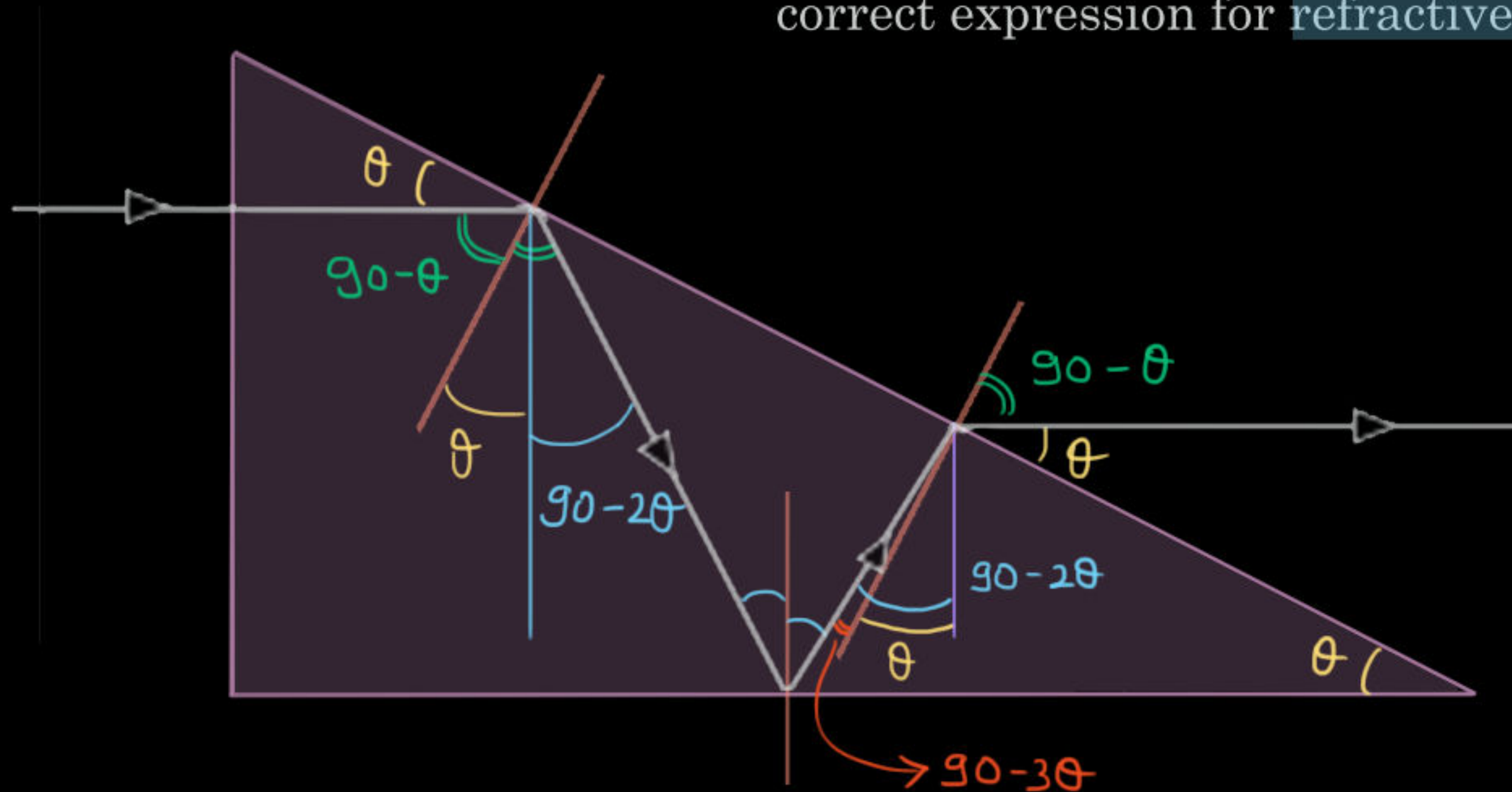
As C is too far and θ is small, eye can not resolve C and C' as two different points. So, C won't appear to move at all. And relative to A it will appear to move up.



Consider the setup shown in the figure, which uses a right-angled prism to transmit a horizontal incident ray. This device is known as **anamorphic de-magnifier**. During its passage through the prism, the ray experiences total internal reflection (TIR) twice and then exits as shown in the figure. The prism is made from a glass of refractive index μ .

12. If the emerging ray propagates horizontally, which of the following is a correct expression for refractive index μ ?

✓(d) $\mu = \frac{\cos \theta}{\cos 3\theta}$

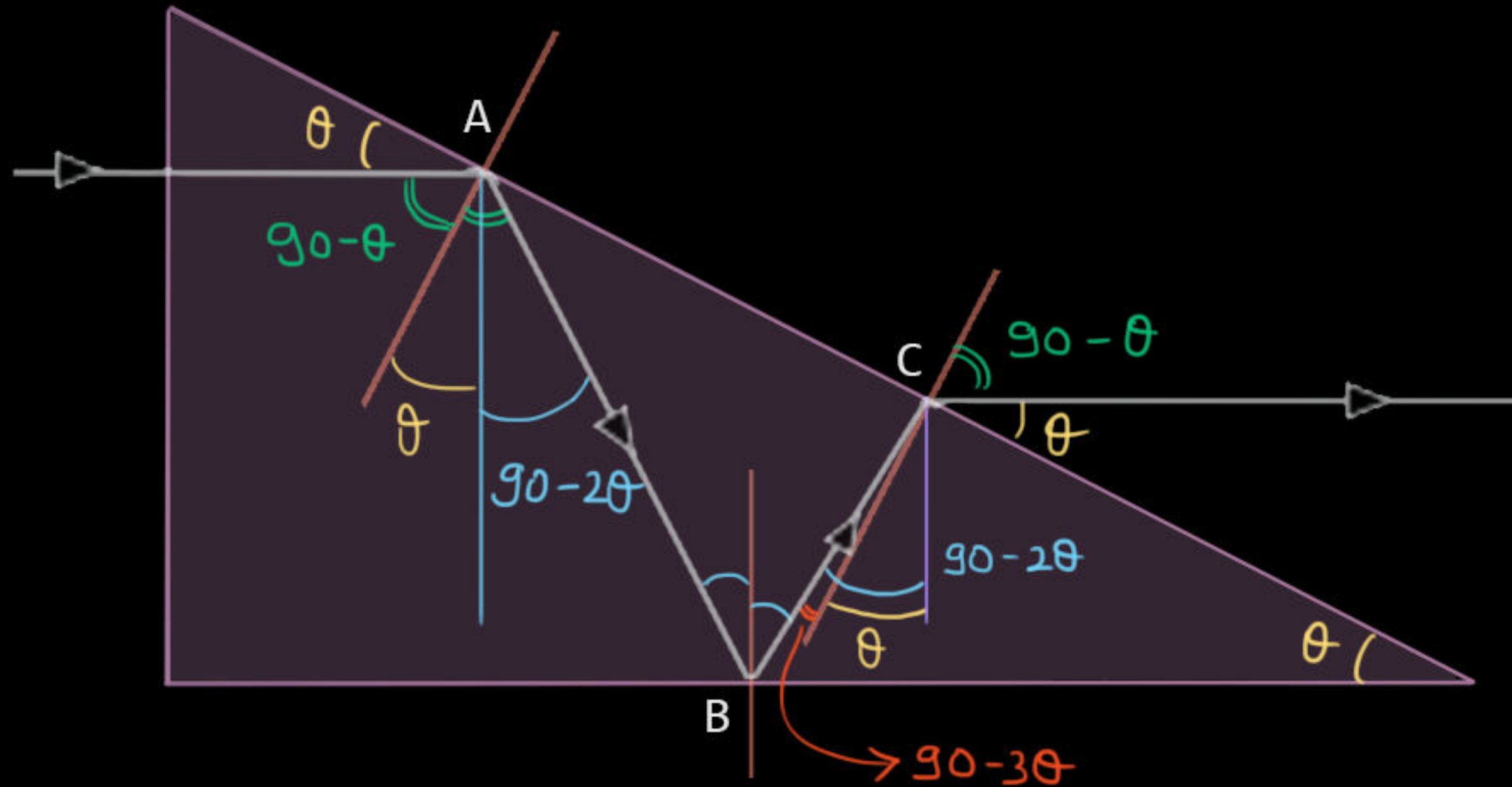


$$\mu \sin(90 - 3\theta) = 1 \cdot \sin(90 - \theta)$$

$$\Rightarrow \mu = \frac{\cos \theta}{\cos 3\theta}$$

13. Range of values of tip angle θ for the device to work as anamorphic de-magnifier is

$$\checkmark (d) \frac{1}{3} \cos^{-1} \left(\frac{1}{\mu} \right) < \theta < \frac{1}{2} \cos^{-1} \left(\frac{1}{\mu} \right)$$



If we take Cosine or Arccos on both sides of an inequality, then sign changes.

For device to work, it must perform TIR at B (thus automatically at A), but not at C.

$$\therefore \angle C < \theta_c < \angle B$$

$$\Rightarrow 90-3\theta < \theta_c < 90-2\theta$$

↓ sine

$$\Rightarrow \cos 3\theta < \frac{1}{\mu} < \cos 2\theta$$

↓ $\cos^{-1}$

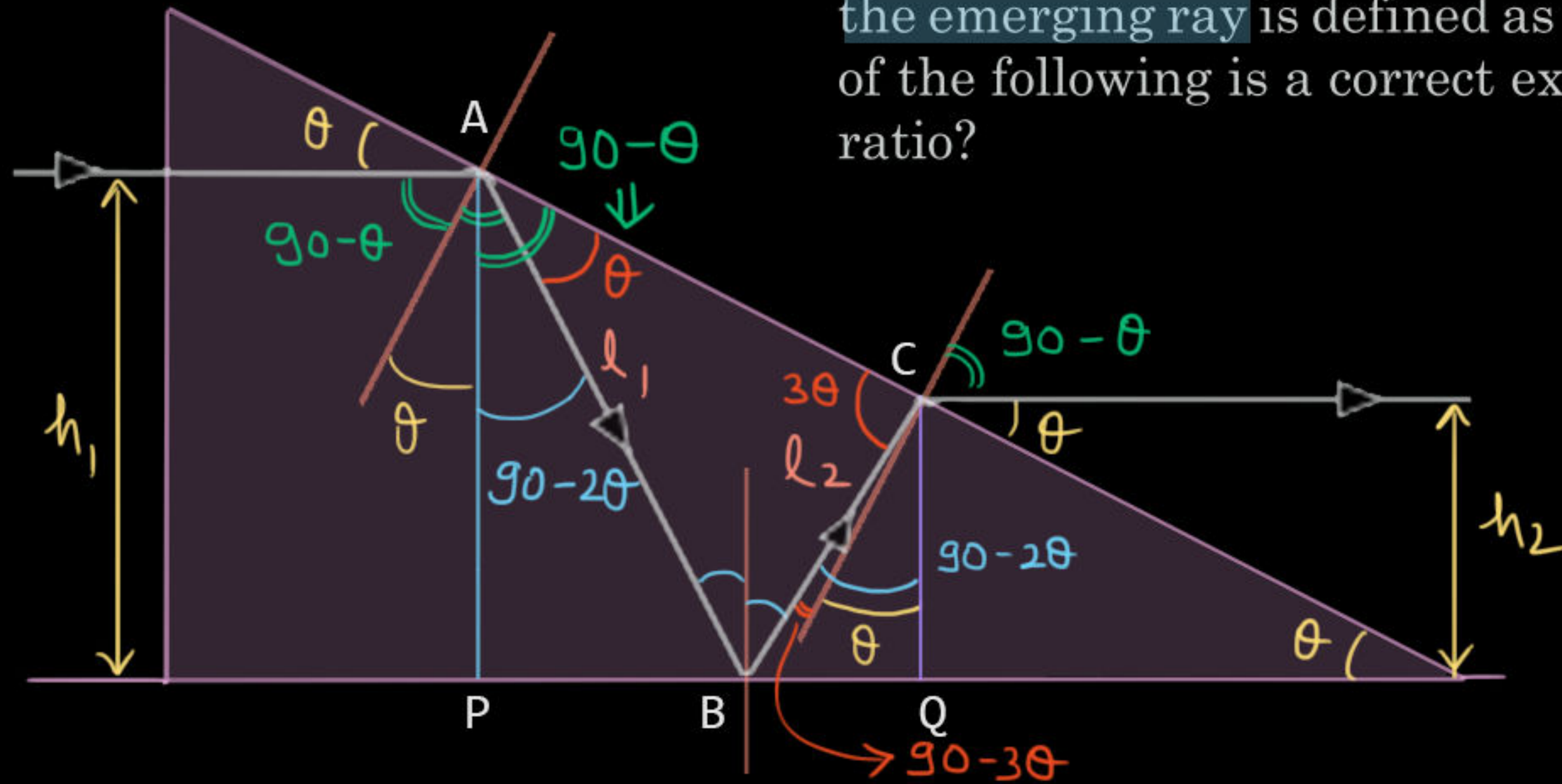
$$\Rightarrow 3\theta > \cos^{-1} \frac{1}{\mu} > 2\theta$$

$$\Rightarrow \frac{1}{3} \cos^{-1} \frac{1}{\mu} < \theta < \frac{1}{2} \cos^{-1} \frac{1}{\mu} //$$

Youtube / Arihant Senior

by Ankit Singhvi

14. Ratio of height of the incoming ray from the base of the prism to that of the emerging ray is defined as the vertical de-magnification ratio. Which of the following is a correct expression for the vertical de-magnification ratio?



$\frac{h_2}{h_1}$ ✓ (a) $\frac{\sin \theta}{\sin 3\theta}$ ✓ (d) $\frac{\mu}{1+2\mu}$

∴ $\triangle APB \sim \triangle CQB$

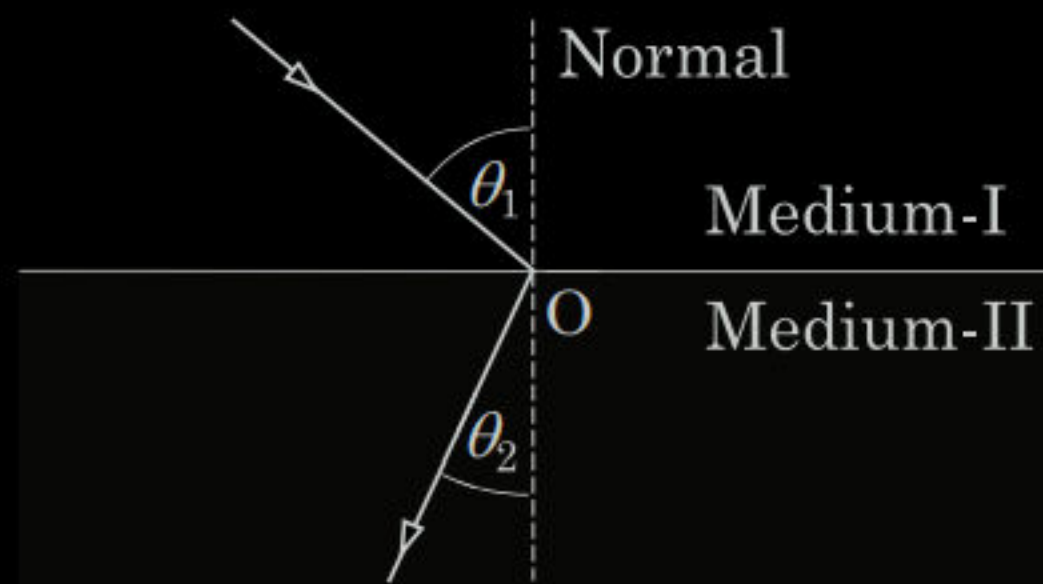
$\frac{h_2}{h_1} = \frac{l_2}{l_1}$ (1)

From 1,2: $\frac{h_2}{h_1} = \frac{\sin \theta}{\sin 3\theta} = \frac{\sin \theta}{3\sin \theta - 4\sin^3 \theta} = \frac{1}{4\cos^2 \theta - 1} = \frac{\mu}{1+2\mu}$

Sine law in $\triangle ABC$

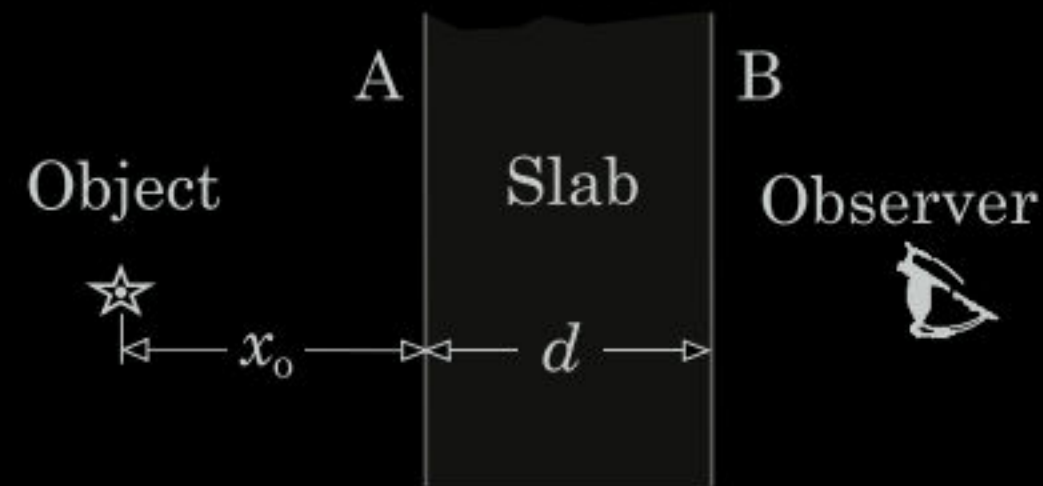
$\frac{l_1}{\sin 3\theta} = \frac{l_2}{\sin \theta}$ (2)

$\mu = \frac{\cos \theta}{\cos 3\theta} = \frac{1}{4\cos^2 \theta - 3} \Rightarrow \cos^2 \theta = \frac{1}{4} \left(\frac{1}{\mu} + 3 \right)$



If Snell's law is found valid for two mediums, whose refractive indices have opposite signs, the incident and the refracted rays must remain on the same side of the normal at the point of incidence. In the figure is shown a ray crossing the interface between two mediums I and II. The ray makes angles θ_1 and θ_2 with normal in the medium-I and II. The refractive indices of the mediums μ_1 and μ_2 have opposite signs.

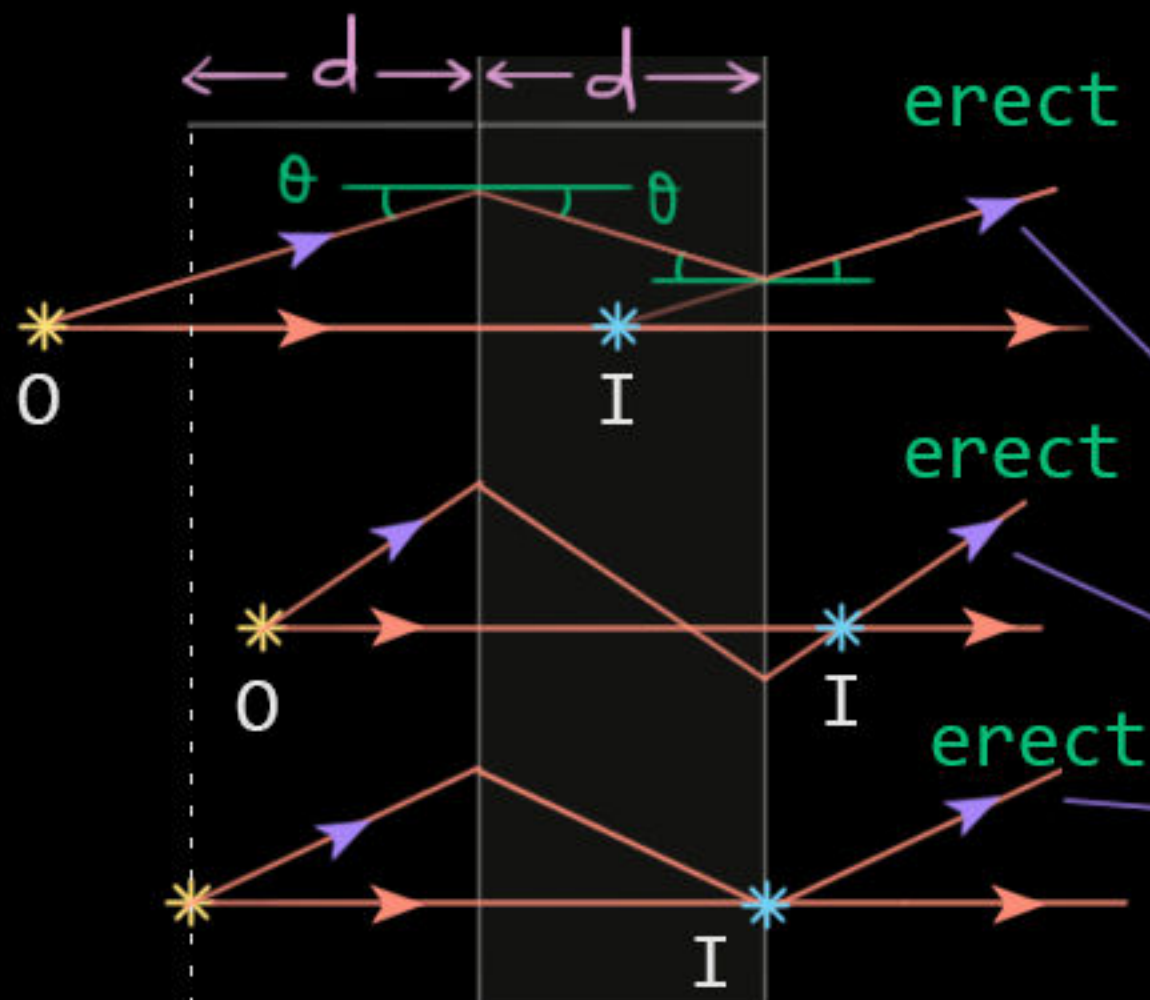
Snell's law is expressed for the above situation as usual by the equation $\mu_1 \sin \theta_1 = \mu_2 \sin \theta_2$. Therefore, to satisfy this equation, the angles θ_1 and θ_2 must also have opposite signs.



15. A self-luminous point object placed at a distance x_0 from a slab of transparent material of negative refractive index is viewed through the slab as shown in the figure. Thickness of the slab is d , refractive index of material of the slab with respect to the outside medium is $\mu = -1$.

For different moduli of values of x_0 and d the observer may find real or virtual image of the object. Which of the following statements is true?

- ✗ (a) If $x_0 > d$, image is virtual and for $x_0 < d$ image is real.
- (b) If $x_0 < d$, image is virtual and for $x_0 > d$ image is real.
- (c) If $x_0 \leq d$, image is virtual and for $x_0 > d$ image is real.
- ✓ (d) If $x_0 > d$, image is virtual and for $x_0 \leq d$ image is real.

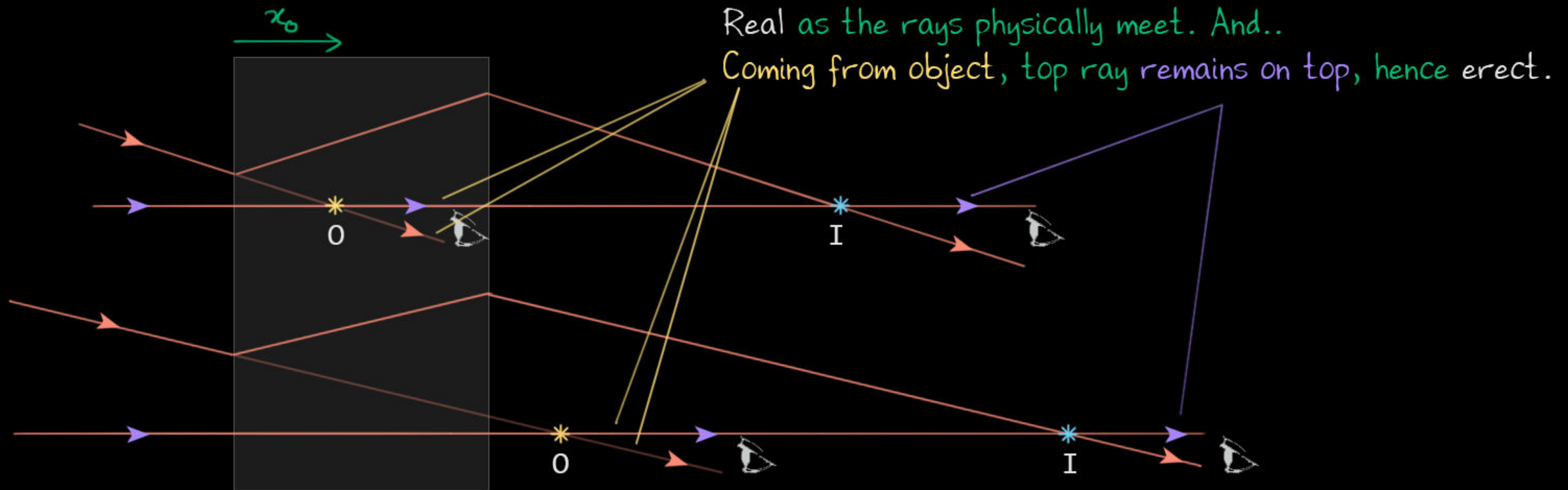


Coming from object, top ray remains on top, hence erect.

If top ray came out at bottom, it would have been inverted.

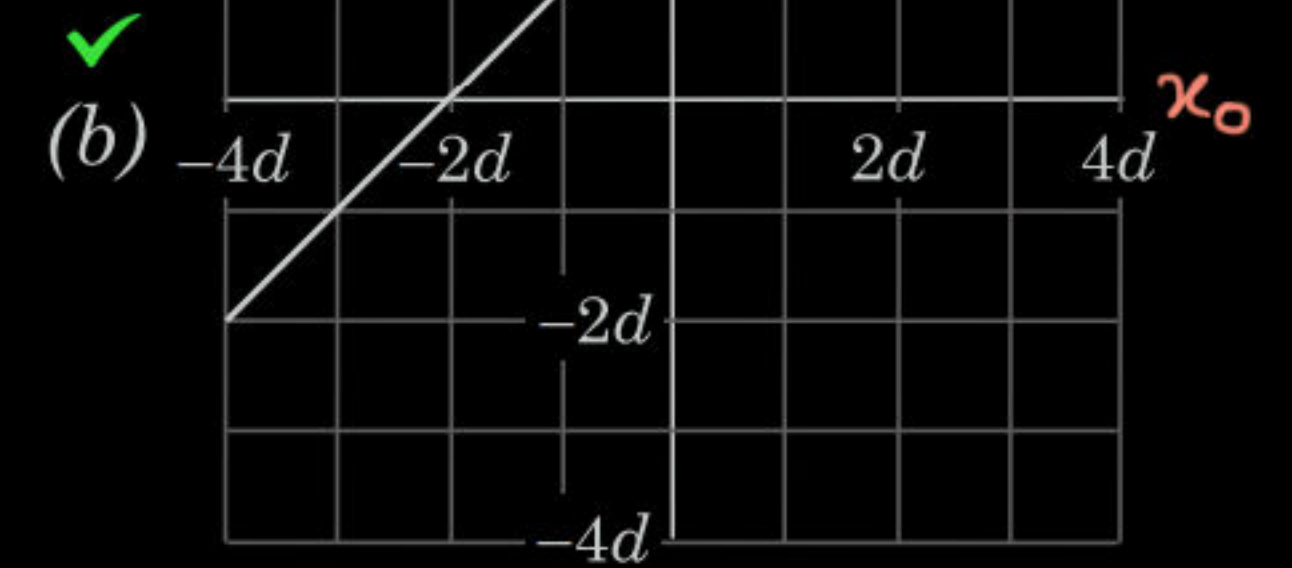
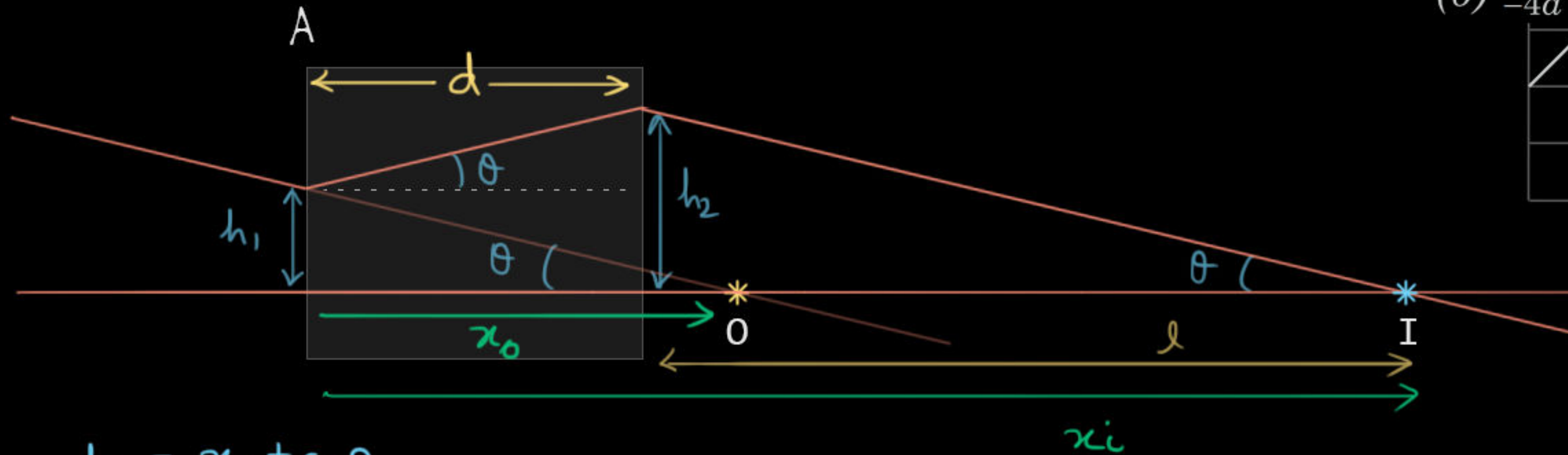
16. Suppose a virtual object instead of a real object is at a distance x_0 from the surface A of the slab. Now which of the following statements is true?

- ✓ (a) For the cases $x_0 < d$ and $x_0 > d$ a real and erect image is formed outside the slab.



17. Distances x_o and x_i of the object and the final image are measured from the face A of the slab. The Cartesian sign conventions are also followed. Which one of the following graphs correctly represents relationship between x_o as abscissa and x_i as ordinate?

Approach: Assume $\theta \Rightarrow$ find $h_1 \Rightarrow h_2 \Rightarrow l \Rightarrow x_i = d + l$



$$h_1 = x_o \tan \theta$$

$$\Rightarrow h_2 = h_1 + d \tan \theta = (x_o + d) \tan \theta$$

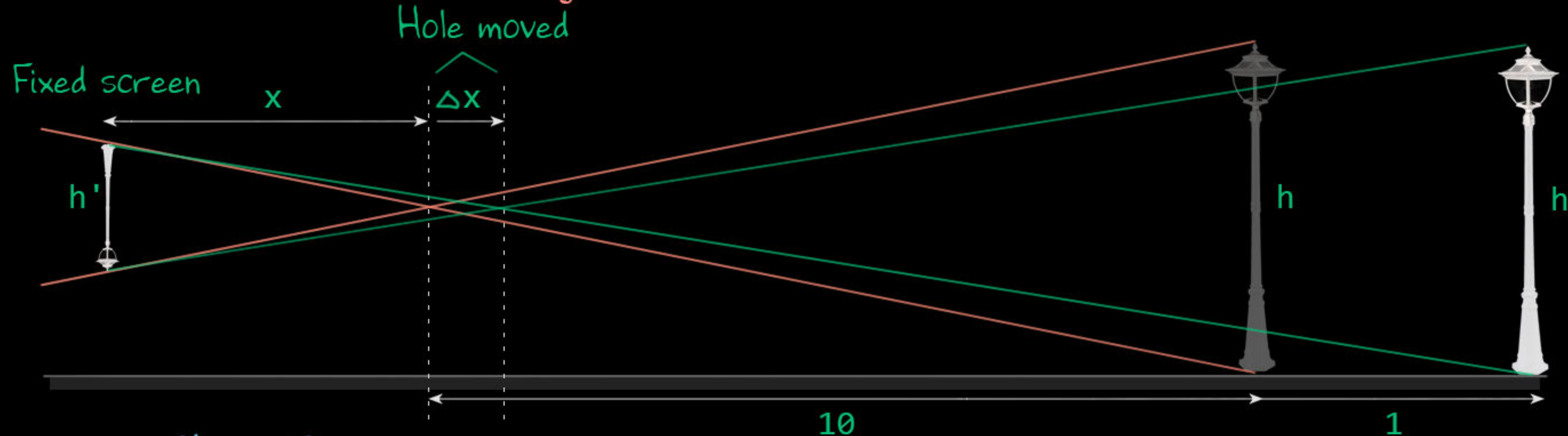
$$\Rightarrow l = \frac{h_2}{\tan \theta} = x_o + d$$

$$\therefore x_i = d + l = x_o + 2d$$

w.r.t camera screen, pole is shifted 1 m farther. Then hole is shifted Δx to maintain same size of image.

1. A pin hole-camera of adjustable length (hole to screen distance), placed 10 m away from a pole casts image of the pole. If it is shifted 1.0 m farther, find percentage change in its length required to keep the size of image unchanged.

Note: We can fix the position of hole and move the screen as well. Then we can solve it w.r.t hole and get the same answer.



Initially: $\frac{x}{h'} = \frac{10}{h}$ ①

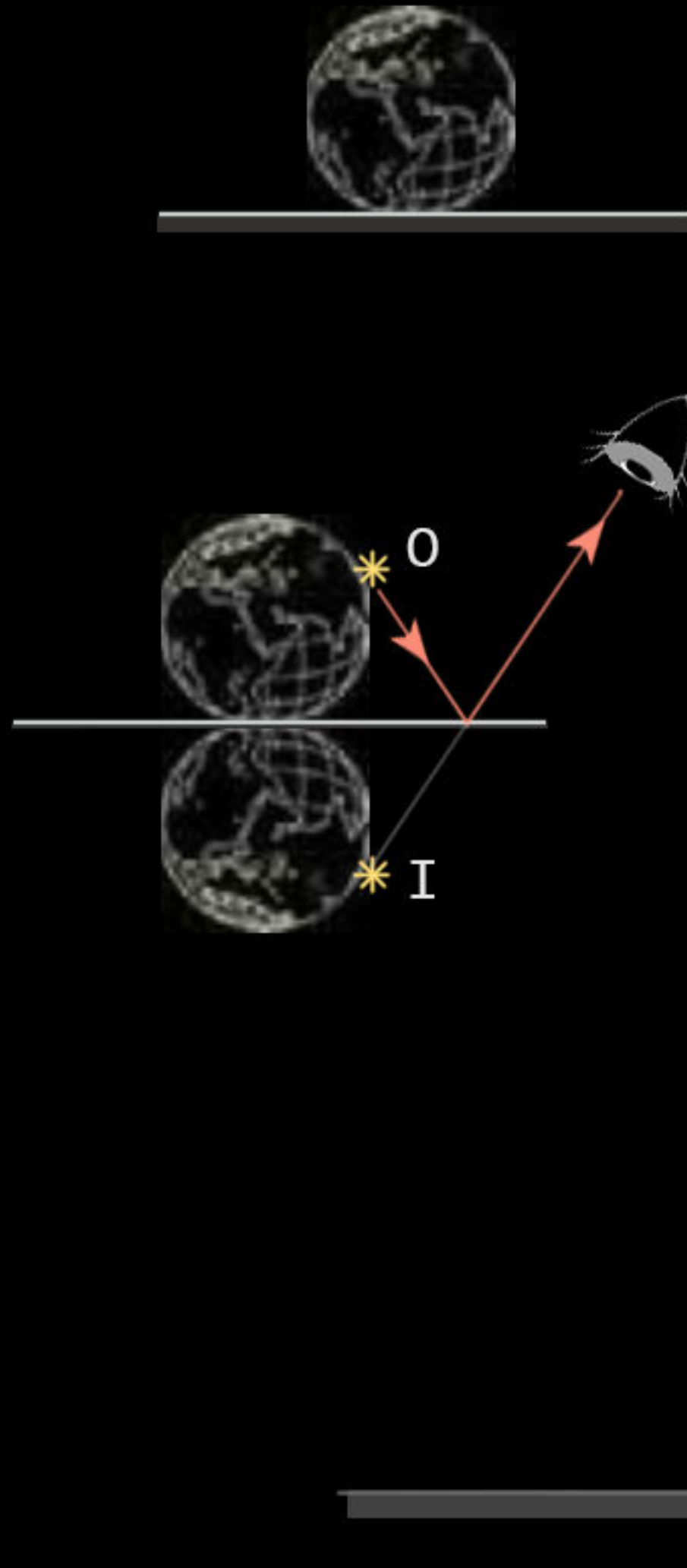
Later: $\frac{x + \Delta x}{h'} = \frac{10 + 1 - \Delta x}{h}$ ②

dividing

$$\frac{x + \Delta x}{x} = \frac{11}{10}$$

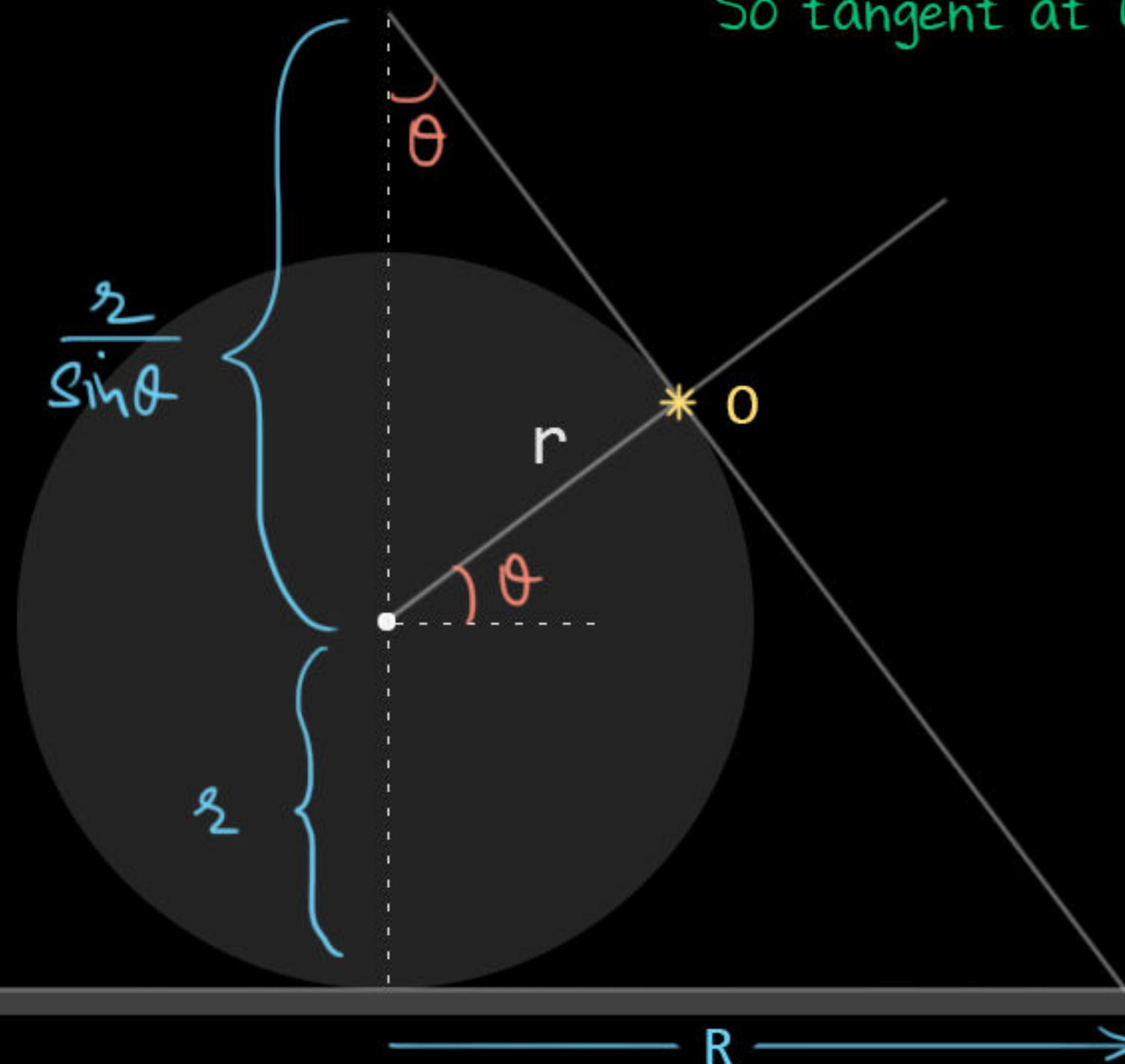
$$\Rightarrow \frac{\Delta x}{x} = \frac{1}{10} = 10\% \text{ increase}$$

2. A globe of radius $r = 10$ cm is placed on a circular plane mirror touching centre of the mirror with its south pole. Find the minimum radius of the mirror so that image of the latitude $\theta = 37^\circ$ N be visible in the mirror.



For image of O (* which is at latitude θ) to be visible in mirror, a ray from O should be able to reach the mirror.

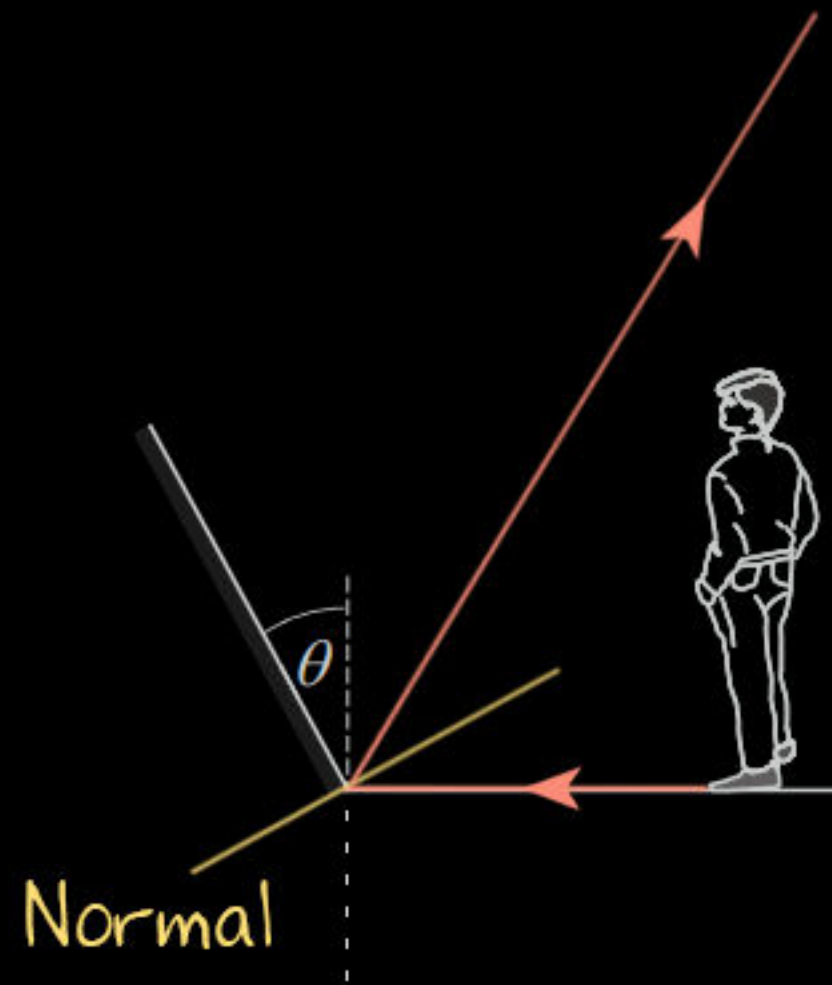
So tangent at O hits the mirror at required R .



$$\therefore R = \left(\frac{r}{\sin \theta} + r \right) \tan \theta$$

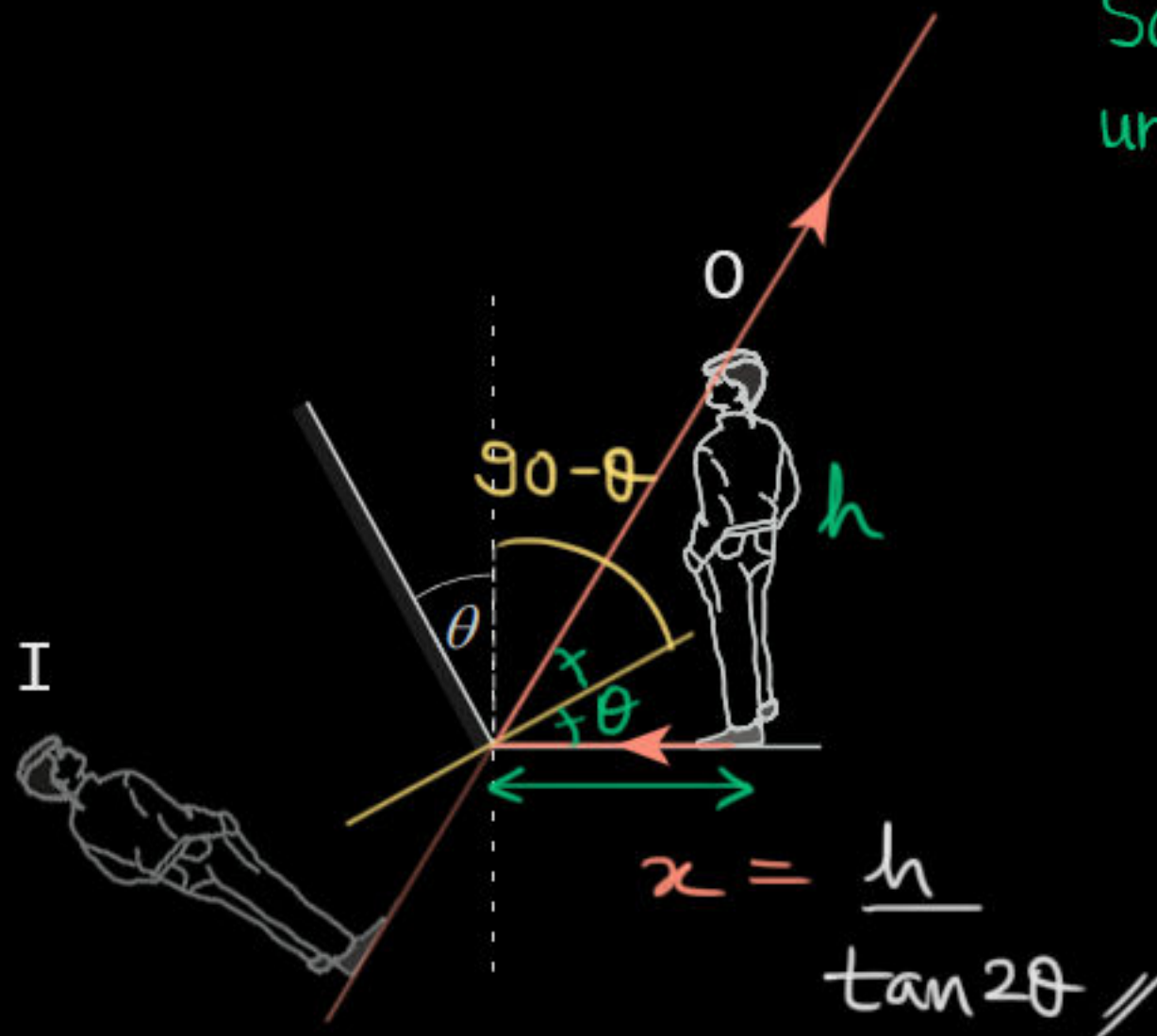
$$= r \left(\frac{1}{\cos \theta} + \tan \theta \right) //$$

3. A large plane mirror with its bottom on the floor is tilted at an angle θ to the vertical (see figure). A boy whose eyes are at a height h above the floor is standing in front of the mirror. At what maximum distance from the mirror should the boy be to see his full image in the mirror?

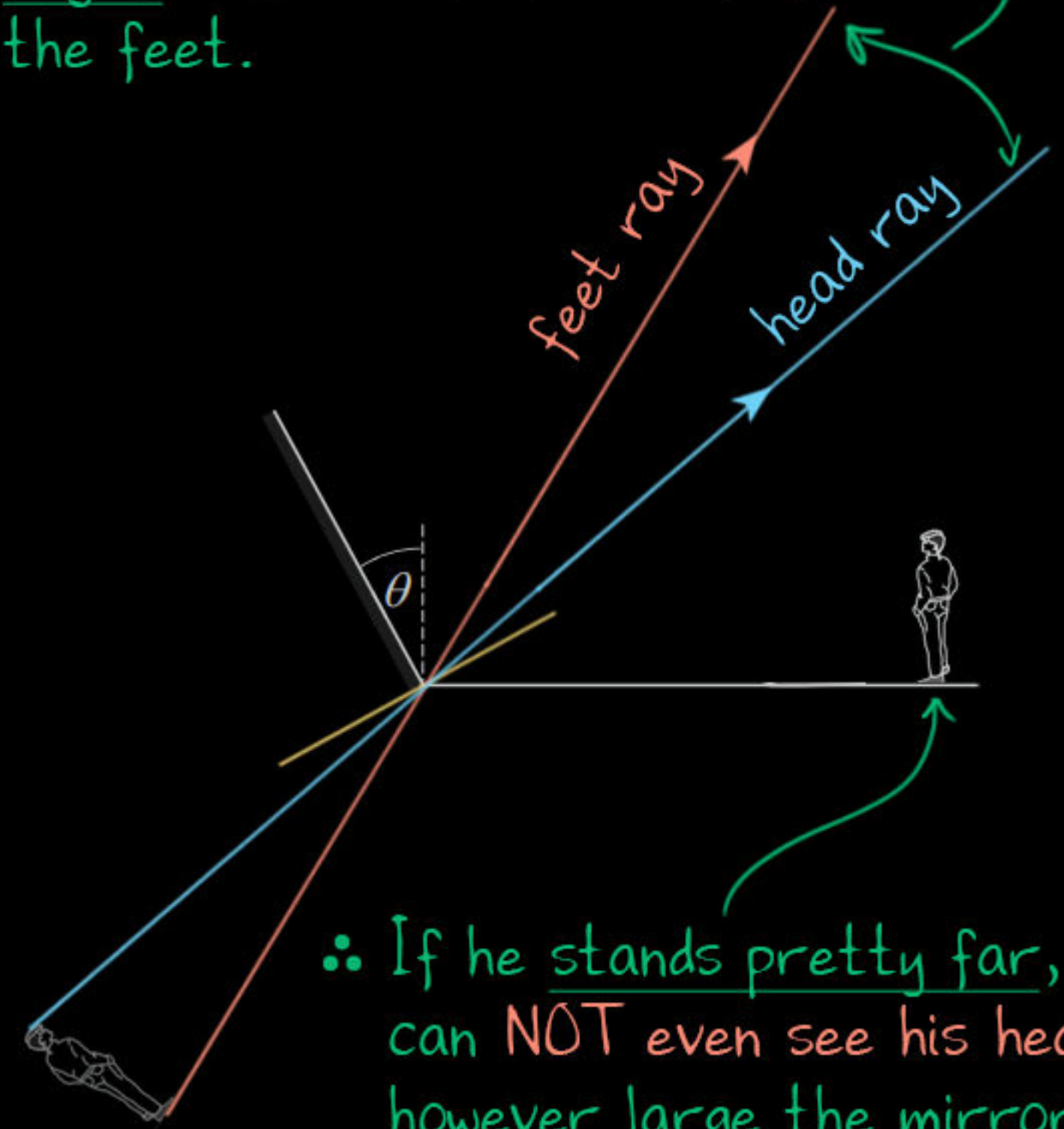


He can NOT see his feet, as the ray from feet is going above his eyes after reflection.

So he moves forward until he can.



Any other observer in this region can see the head but not the feet.



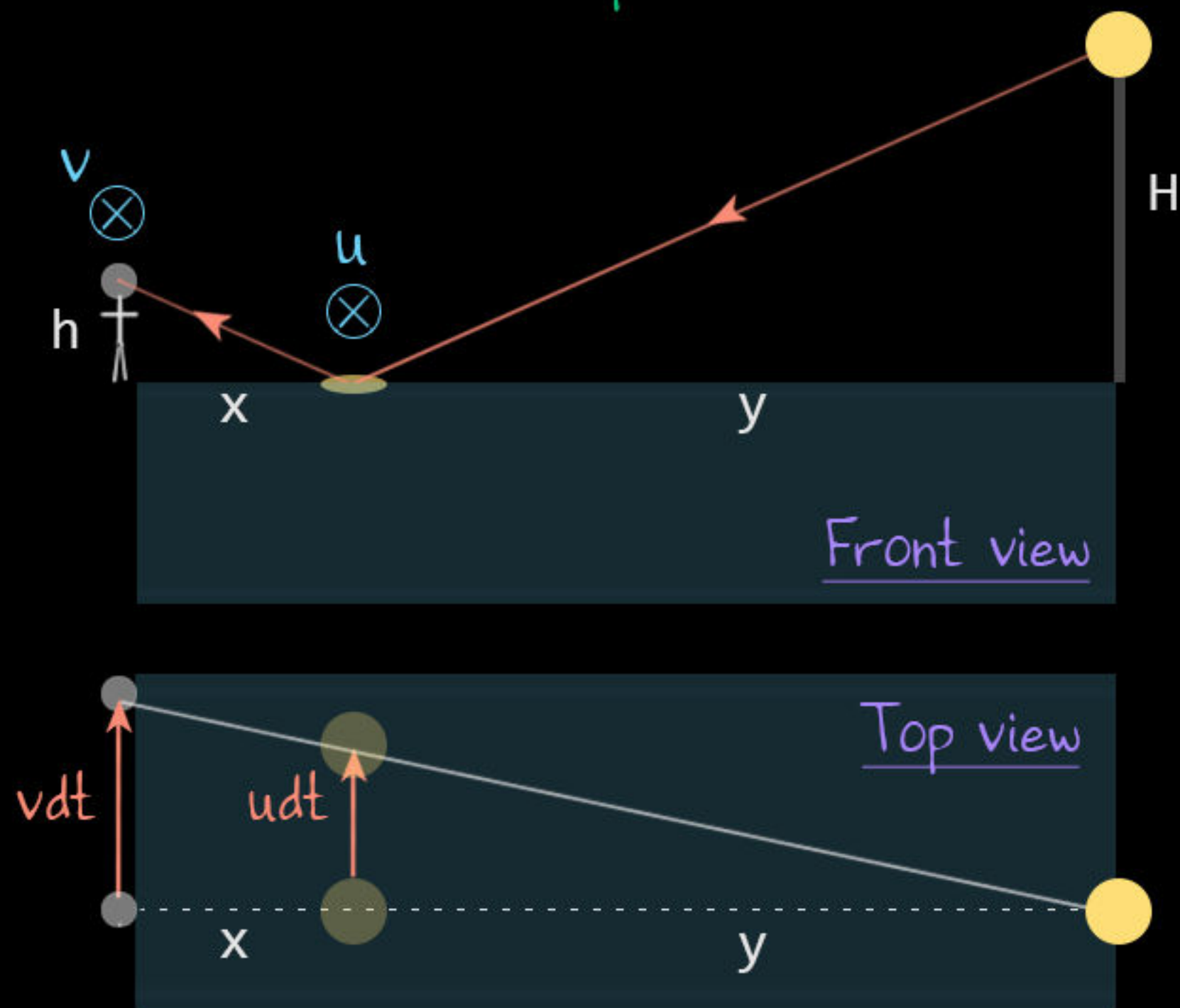
∴ If he stands pretty far, he can NOT even see his head, however large the mirror is.

Youtube / Arihant Senior

It is NOT velocity of flare w.r.t boy.

It is velocity of flare as it appears to move on surface of water to the boy.
It's made clear in the top view.

4. One night, a boy of height $h = 1.8$ m is standing on the bank of a straight canal, on the other bank of which a lamp is installed at a height $H = 5.4$ m on a pole. There are no ripples on the water surface so light emanated from the lamp, appears as a flare (bright spot) after reflection from the water surface. When the boy starts walking along the bank, the flare appears to him moving at a constant speed $u = 2.4$ m/s relative to the ground. Find speed of the boy.

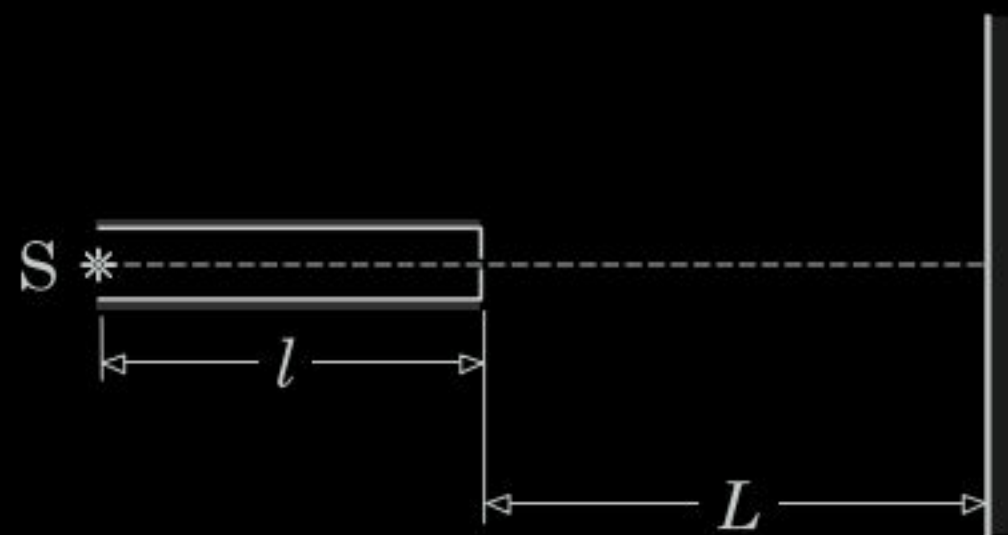


$$\frac{h}{H} = \frac{x}{y}$$

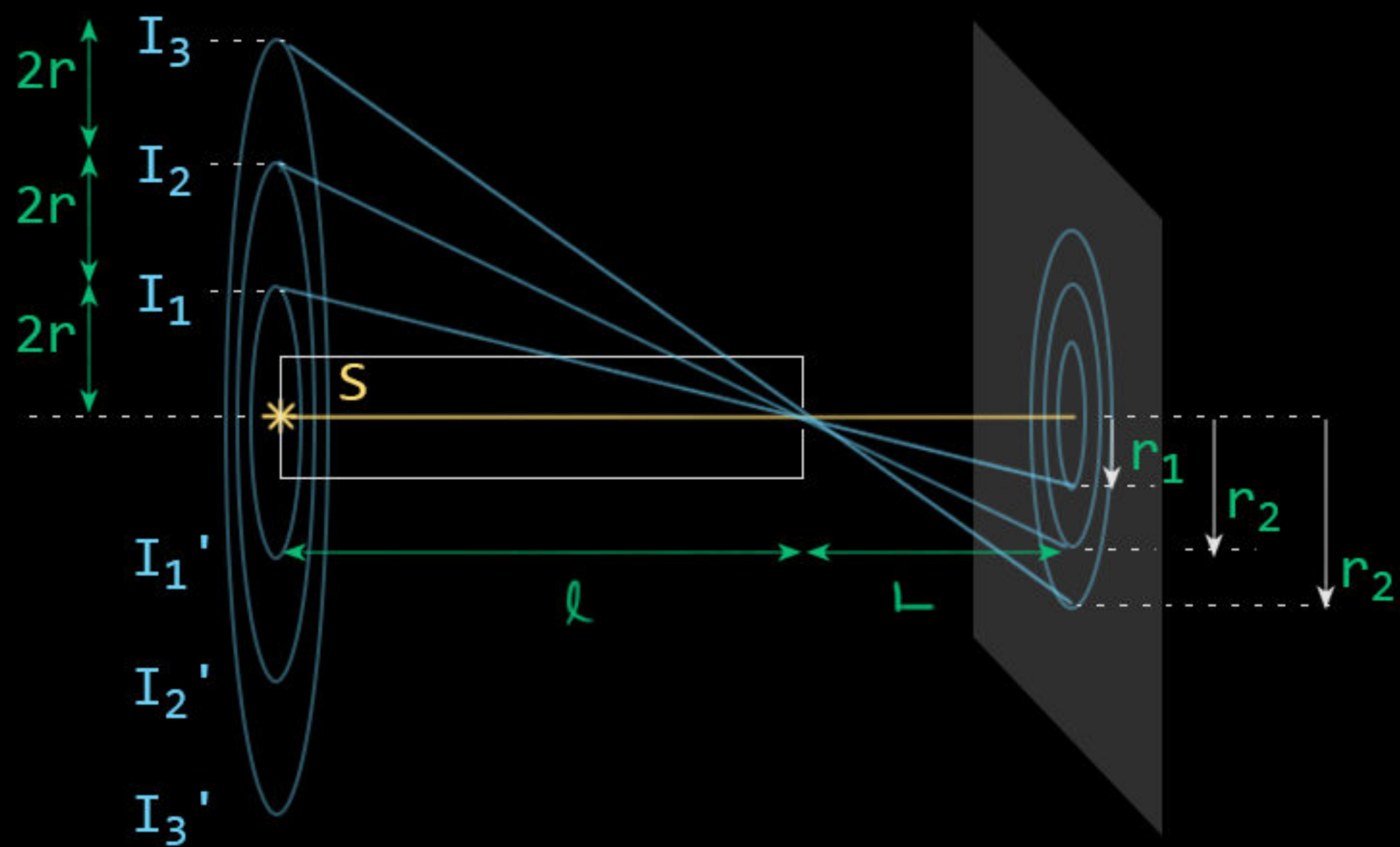
$$\frac{u dt}{v dt} = \frac{y}{x+y}$$

eliminating x/y

$$v = u \left(1 + \frac{H}{h} \right)$$



5. One end of a cylindrical tube of length l and radius r is covered with an opaque disc having a small hole in the centre and a point light source S is placed at centre of the other end. Inner surface of tube is perfectly reflecting. A white screen is placed normal to the axis of the tube at a distance L from the end covered with the disc as shown in the figure. After the source is switched on, what kind of pattern of light distribution, will you obtain on the screen?



I_1 and I_1' are images of S .

I_2 is image of I_1' (from top surface of mirror)

I_2' is image of I_1 (from bottom surface of mirror)

I_3 is image of I_2' (from top surface of mirror)

I_3' is image of I_2 (from bottom surface of mirror)

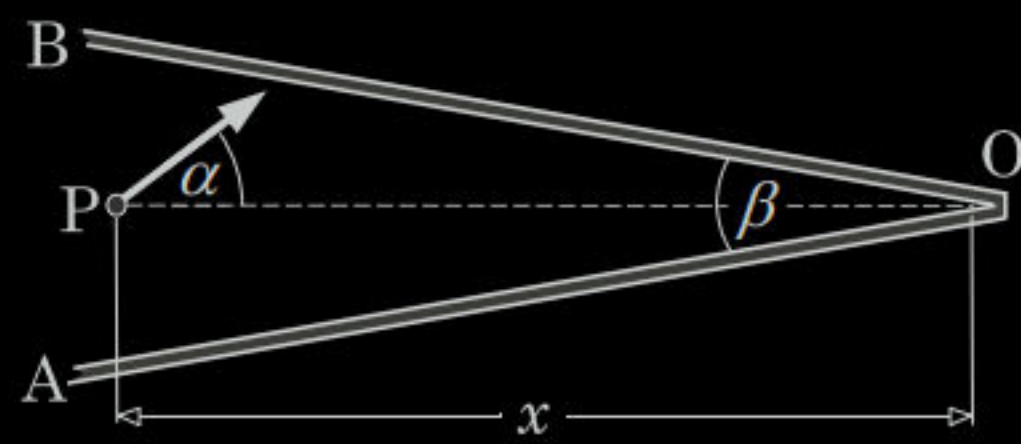
and so on.....

∴ Similar triangles: $\frac{2nr}{l} = \frac{r_n}{L}$

$\Rightarrow r_n = \frac{2nrL}{l}$

Youtube / Arihant Senior

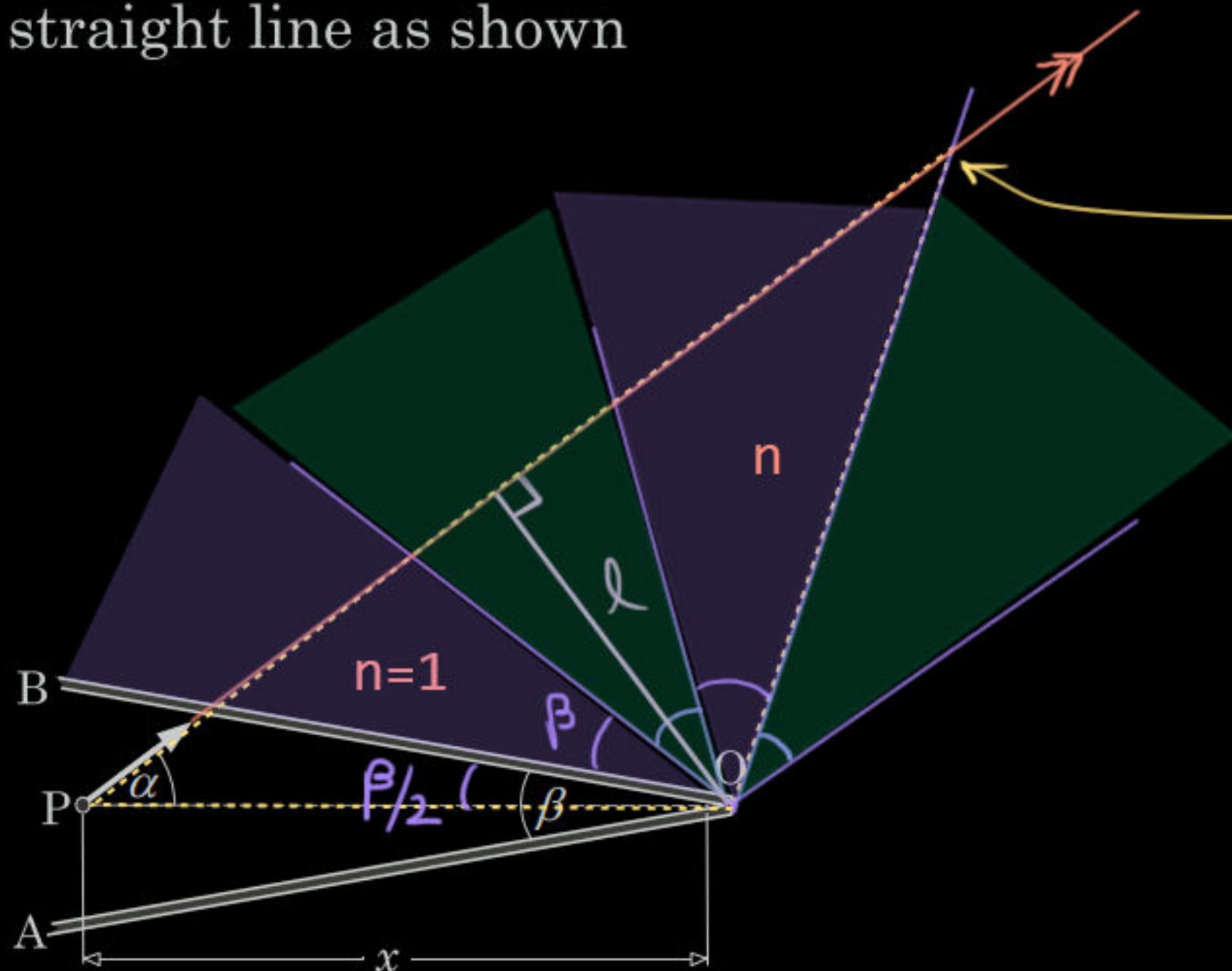
by Ankit Singhvi



6. In free space, a particle is projected from a point P on axis of a fixed rigid cone AOB, at an angle $\alpha = 37^\circ$ with the axis (see figure). Distance of point P from the apex O is $x = 10 \text{ cm}$ and the apex angle of the cone is $\beta = 20^\circ$. All the collisions of the ball with the cone are perfectly elastic.

- (a) Find the distance of closest approach of the ball from the apex.
 (b) How many times will the ball collide with the cone?

Hint: if the cone is consider flipped each time to account for a collision, the path of the ball will appear a straight line as shown



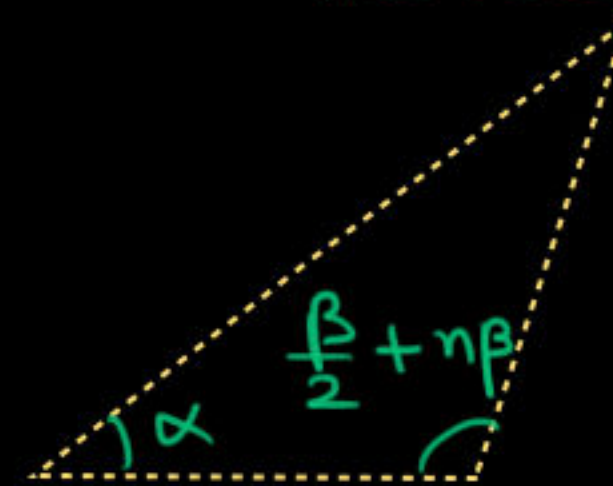
A $\frac{l}{x} = \sin \alpha \Rightarrow l = x \sin \alpha //$

B If ray passes through n^{th} flipped mirror,

Total reflections are: $r = n+1$ 1

This dotted triangle will then be possible if:

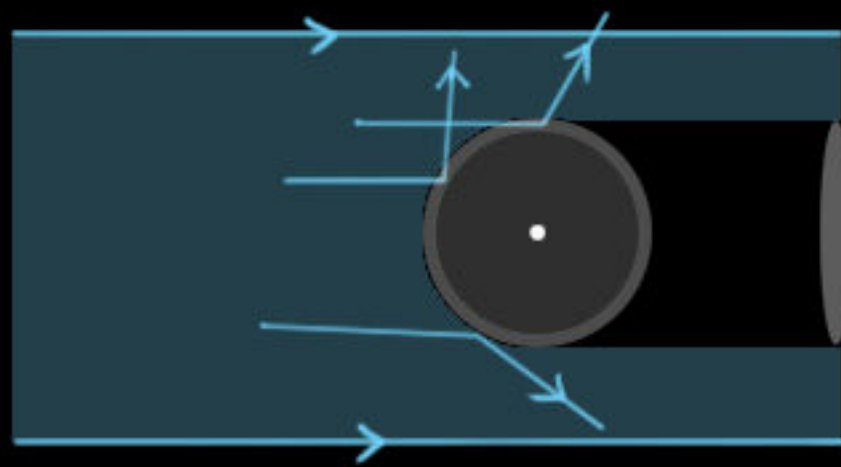
$$\alpha + \left(\frac{\beta}{2} + n\beta \right) < \pi \quad 2$$



From 1,2: $\alpha + \frac{\beta}{2} + (n-1)\beta < \pi$

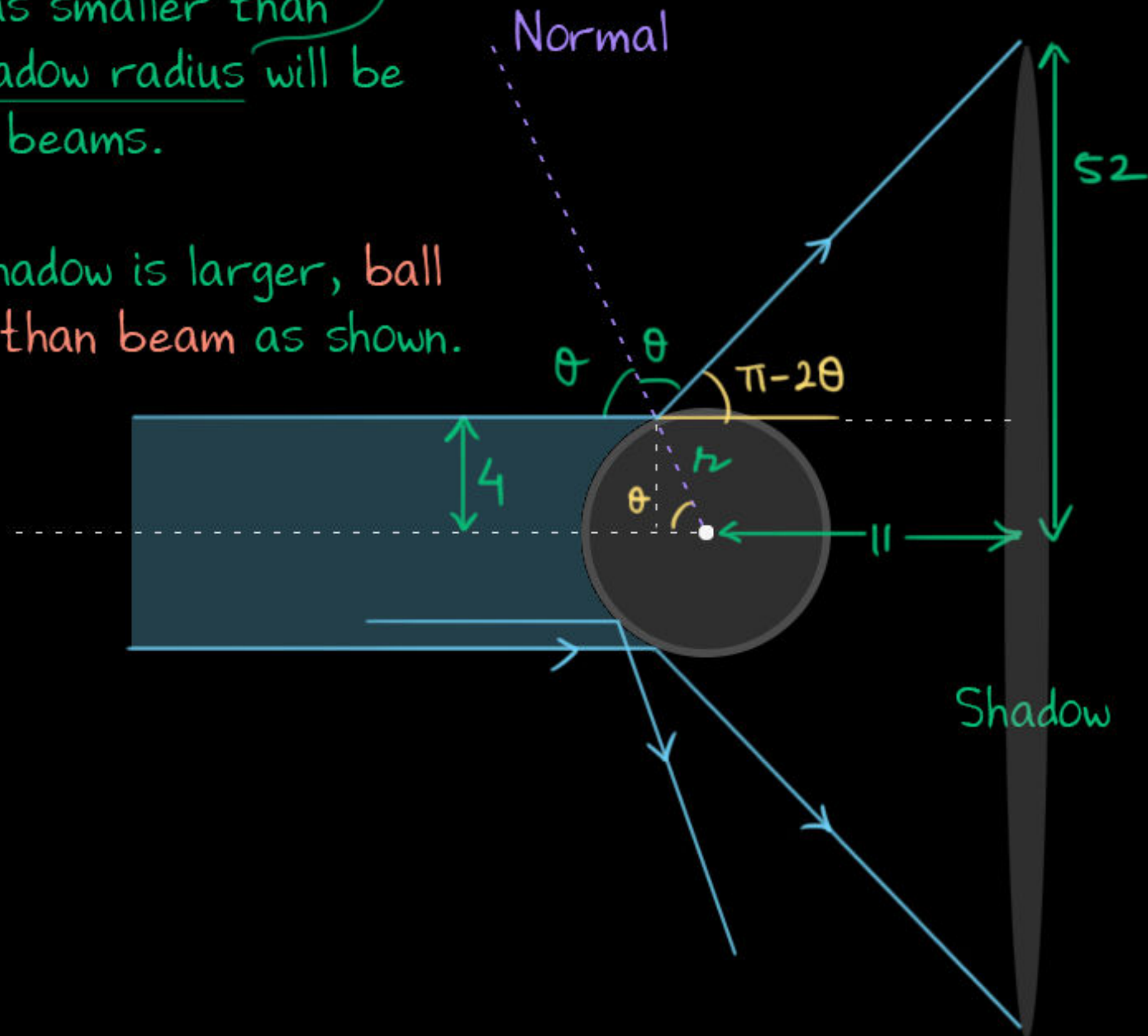
$$\Rightarrow n < \frac{1}{\beta} \left[\pi - \alpha + \beta/2 \right] \Rightarrow n < 7.65$$

$$\therefore n = 7 //$$



If ball was smaller than beam, shadow radius will be less than beams.

But as shadow is larger, ball is larger than beam as shown.



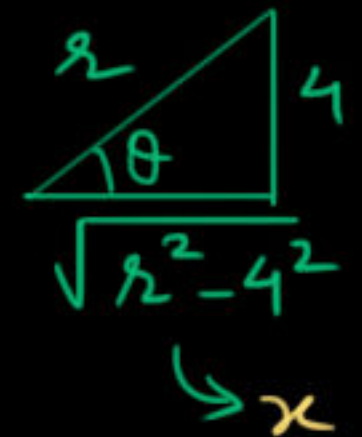
7. Parallel beam of light incident normally on a wall illuminates a round spot of radius 4.0 cm on it. When a ball, surface of which is mirrored is placed with its centre on the axis of the beam and at a distance of 11 cm from the wall, a large part of the wall is illuminated, but at the centre a circular shadow of radius 52 cm is formed. Find radius of the ball.

$$\sin \theta = 4/r \quad (1)$$

$$\tan(\pi - 2\theta) = \frac{52 - 4}{11 + r \cos \theta} \quad (2)$$

Solving 1,2:

$$\frac{2 \tan \theta}{\tan^2 \theta - 1} = \frac{48}{11 + r \cos \theta}$$



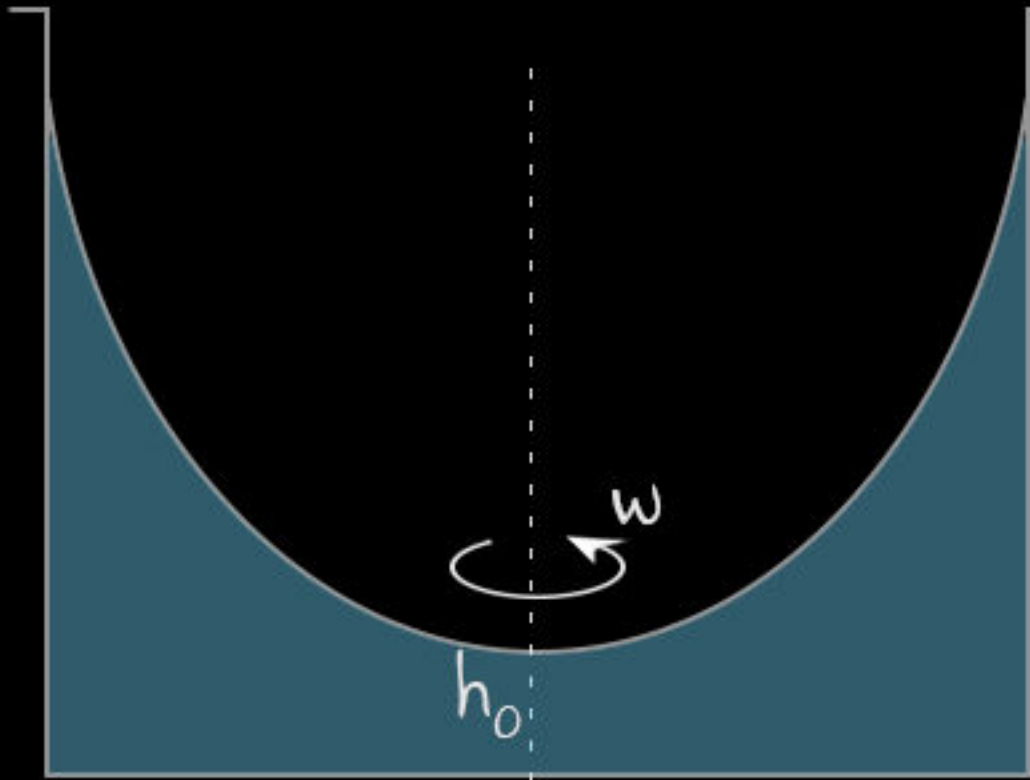
$$\Rightarrow \frac{2 \cdot 4/x}{\frac{16}{x^2} - 1} = \frac{48}{11 + x}$$

$$\Rightarrow 7x^2 + 11x - 96 = 0 \Rightarrow x = \sqrt{r^2 - 4^2} = 3$$

$$\Rightarrow r = 5$$

Youtube / Arihant Senior

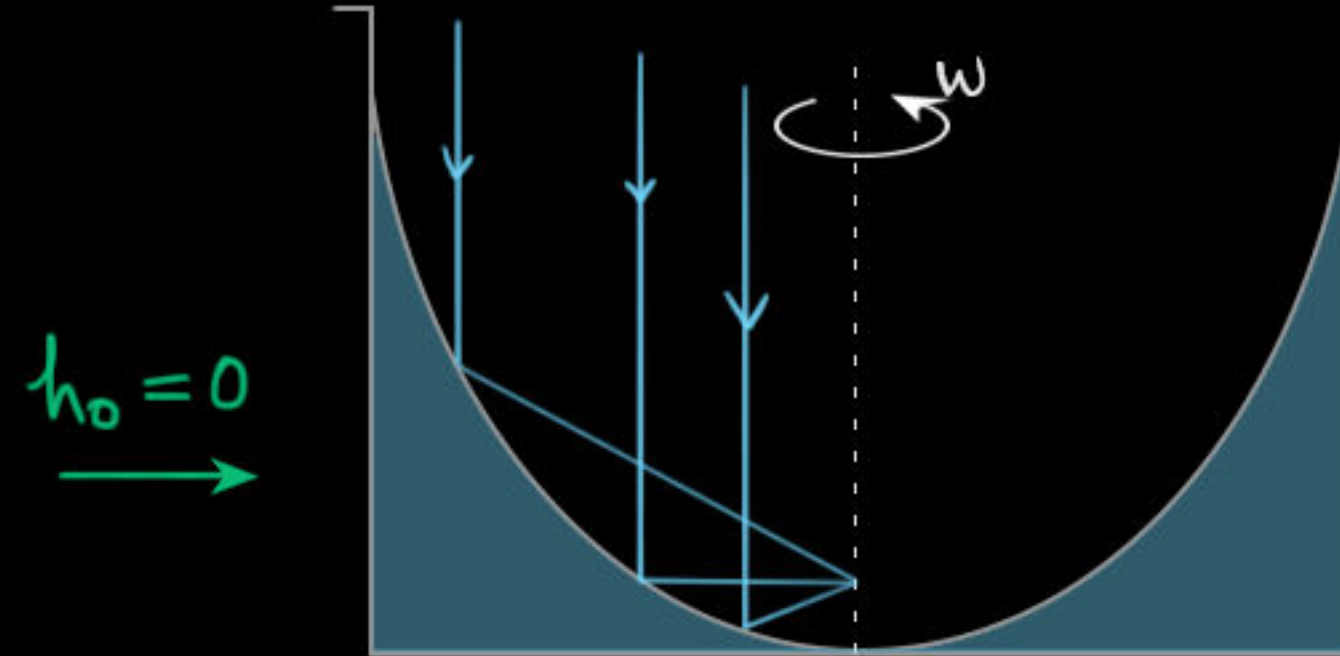
8. If a cylindrical container filled with mercury is rotated with an angular velocity ω about its vertical axis of symmetry, the mercury-air interface takes shape of a paraboloid of revolution. If this surface is used as a mirror, where should a photo film be placed to get a clear picture of a distant star? Acceleration of free fall is g .



$$y = h_0 + \frac{\omega^2 x^2}{2g}$$

A parabola

Proved in Irodov 1.330



$$h_0 = 0$$

$$y = \frac{\omega^2 x^2}{2g}$$

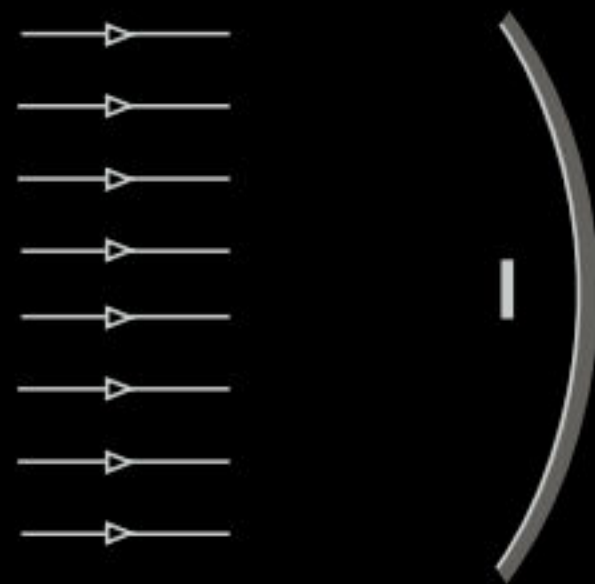
Given

$$x^2 = \frac{2g}{\omega^2} y \quad \therefore a = \frac{g}{2\omega^2}$$

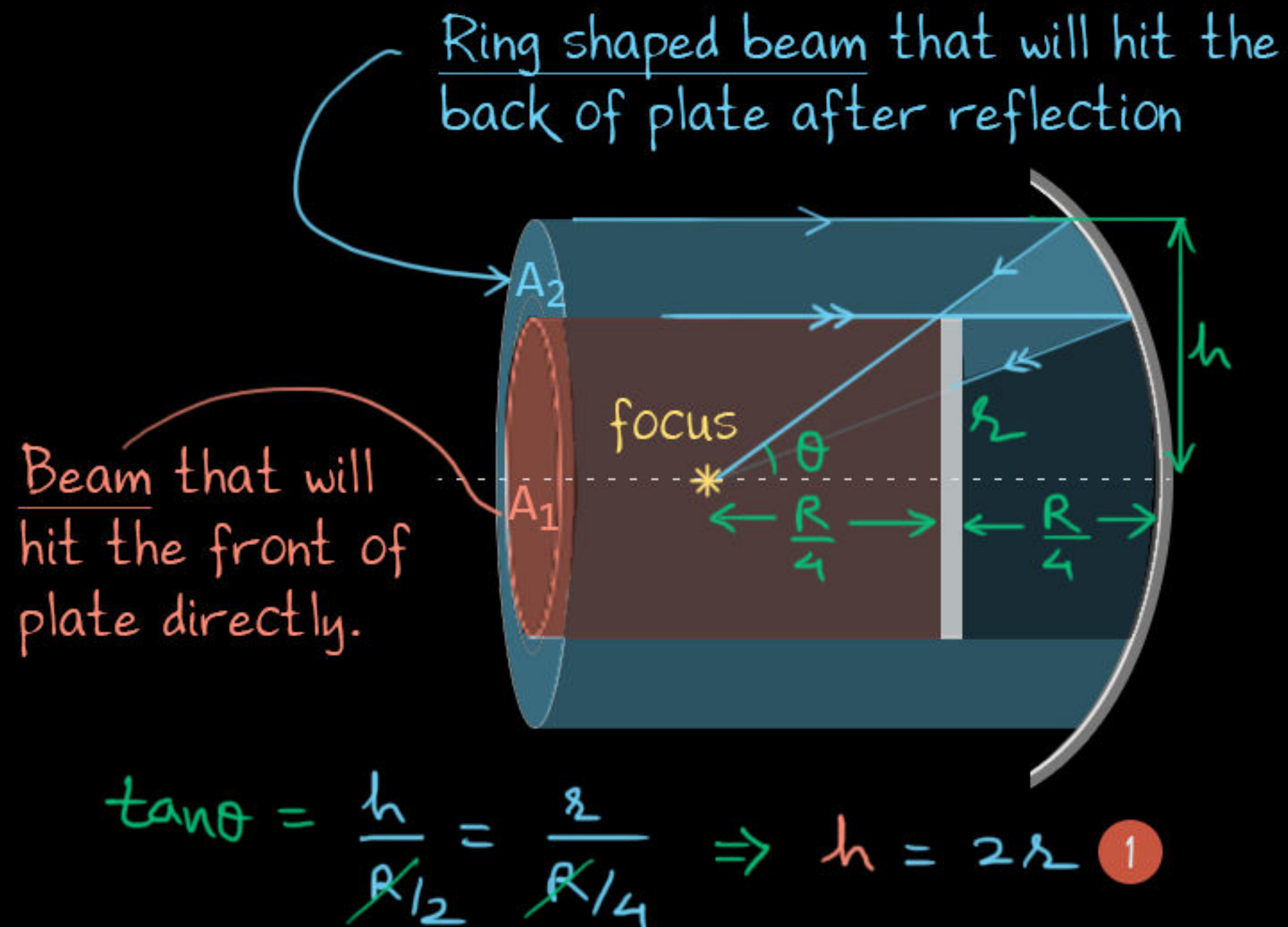
We also know:

- 1 Rays parallel to a parabolic mirror converge at its focus.
- 2 Focus of $x^2 = 4ay$ lies at $(0, a)$

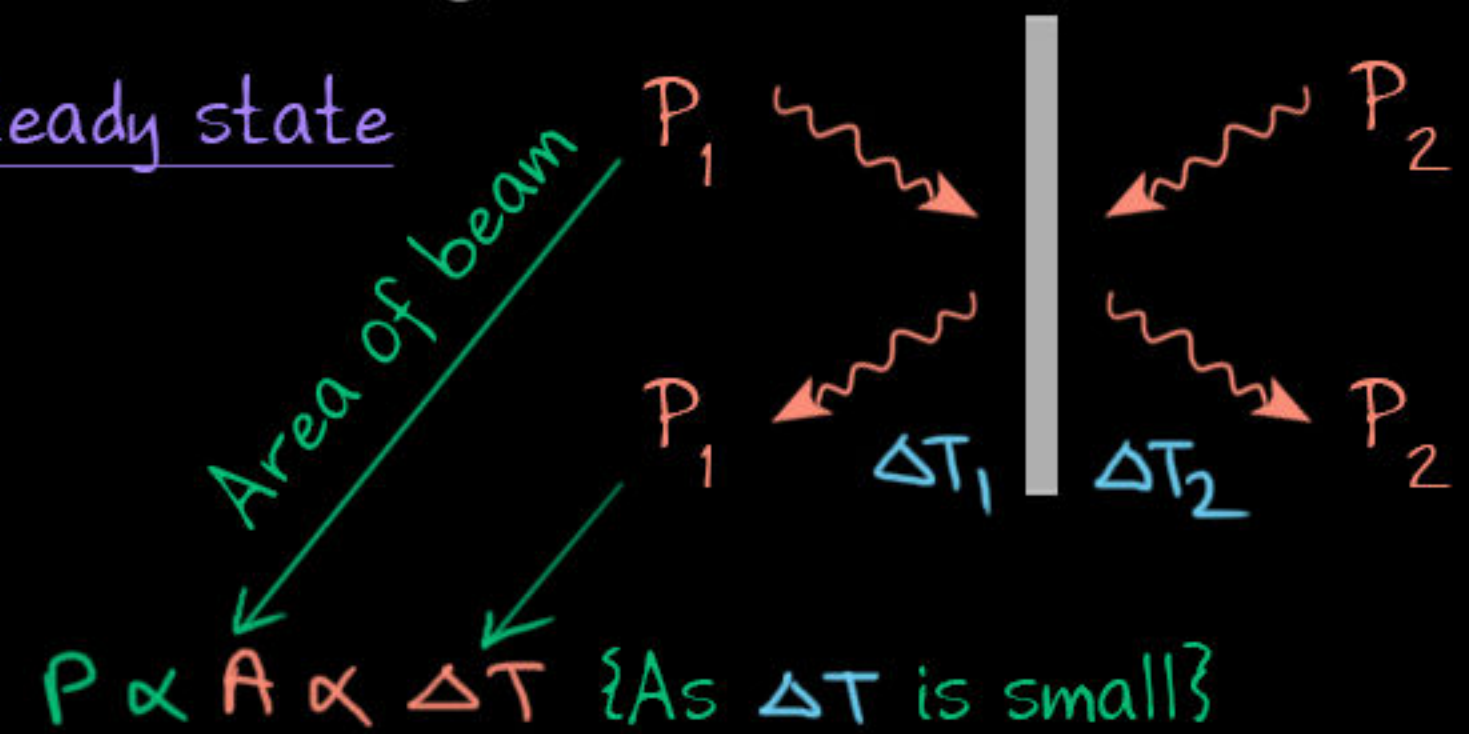
$\Rightarrow$ Film should be placed at $(0, \frac{g}{2\omega^2})$



9. A wide homogeneous beam of light falls on a concave spherical mirror of radius R parallel to the optical axis. A small opaque disc of radius r ($r \ll R$) made of a perfectly heat insulating material is placed at a distance $R/4$ from the pole of the mirror perpendicular to the optical axis. In steady state, both the surfaces of the disc acquire different temperatures slightly higher than the surroundings. Find the ratio $\Delta T_1/\Delta T_2$, where ΔT_1 and ΔT_2 are the temperature differences of the left and the right surfaces of the disc and the surroundings.

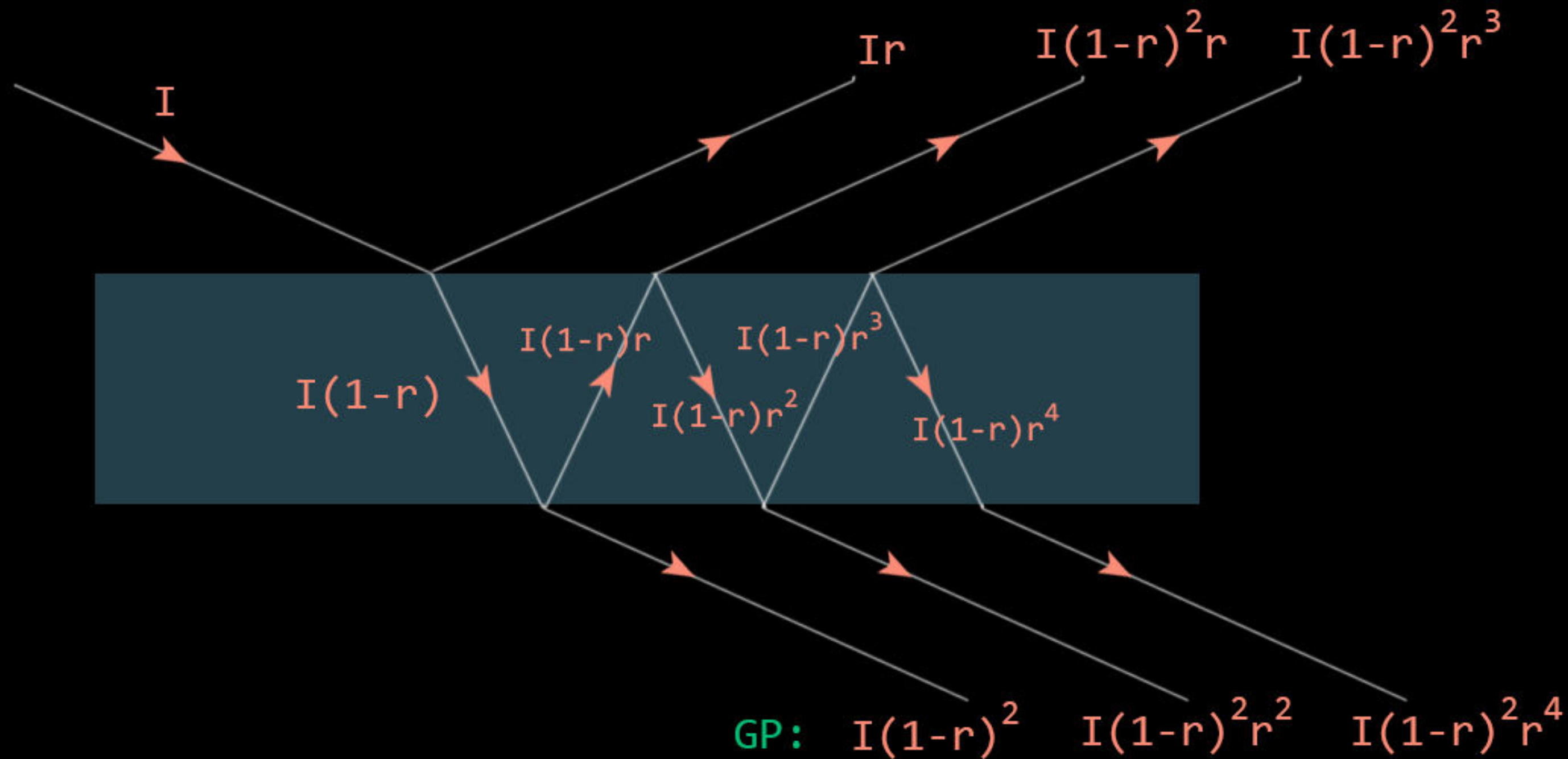


At steady state



$$\therefore \frac{\Delta T_1}{\Delta T_2} = \frac{A_1}{A_2} = \frac{\pi r^2}{\pi h^2 - \pi r^2} \xrightarrow{①} = \frac{r^2}{4r^2 - r^2} = \frac{1}{3}$$

10. If energy reflection coefficient is the same whether light is reflected from glass back into air or from air back into glass and has a value $r = 0.25$. If a narrow beam of light falls obliquely on a large thin glass plate, what fraction of the light energy will be transmitted through the plate?



$$\therefore \text{Fraction transmitted} = \frac{I(1-r)^2 [1 + r^2 + r^4 + \dots]}{I} = (1-r)^2 \frac{1}{1-r^2} = \frac{(1-r)}{(1+r)}$$

by Ankit Singhvi

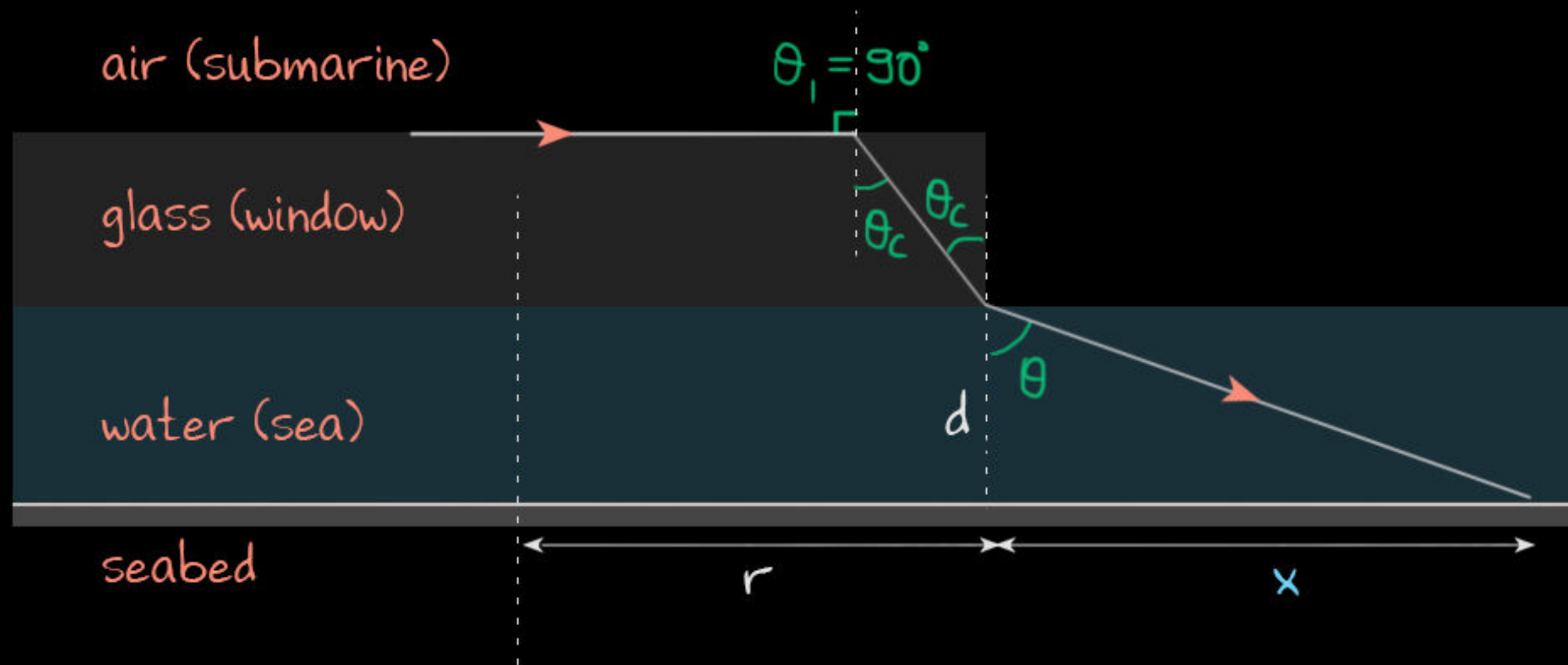
Youtube / Arihant Senior

Concept:

To maximise visible seabed, we need to maximise θ ,

Which is when θ_i is maximum, i.e. $\theta_i = 90^\circ$

11. A research submarine has a circular glass window in its bottom to observe the seabed. Radius of the window is $r = 30$ cm, thickness of the glass is $t = 20$ mm, refractive index of water is $\mu_w = 1.25$ and that of the glass is $\mu_g = 1.5$. If the seabed is $d = 6.0$ m beneath the window, estimate radius of portion of the seabed visible through the window.



$$\mu_a \sin 90^\circ = \mu_g \sin \theta_c = \mu_w \sin \theta$$

$$\mu_w \sin \theta = \sqrt{\mu_w^2 - 1}$$

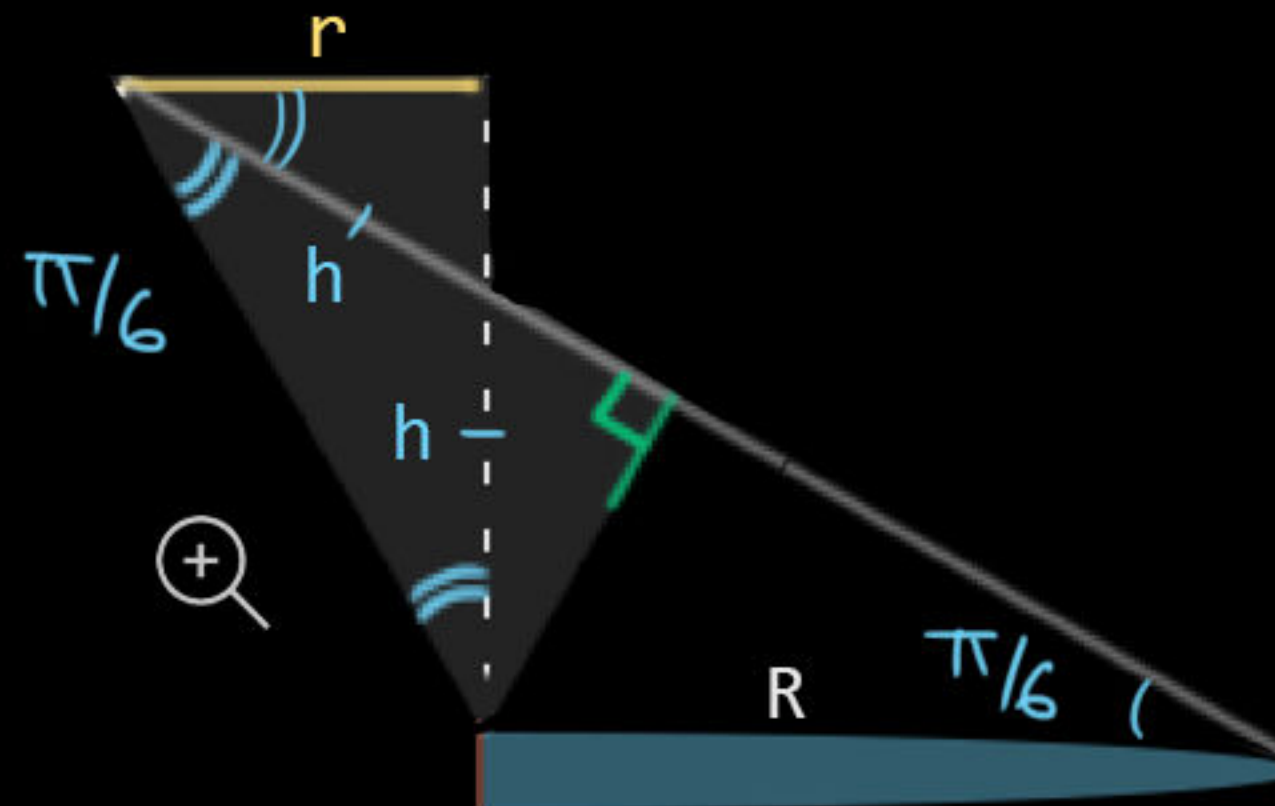
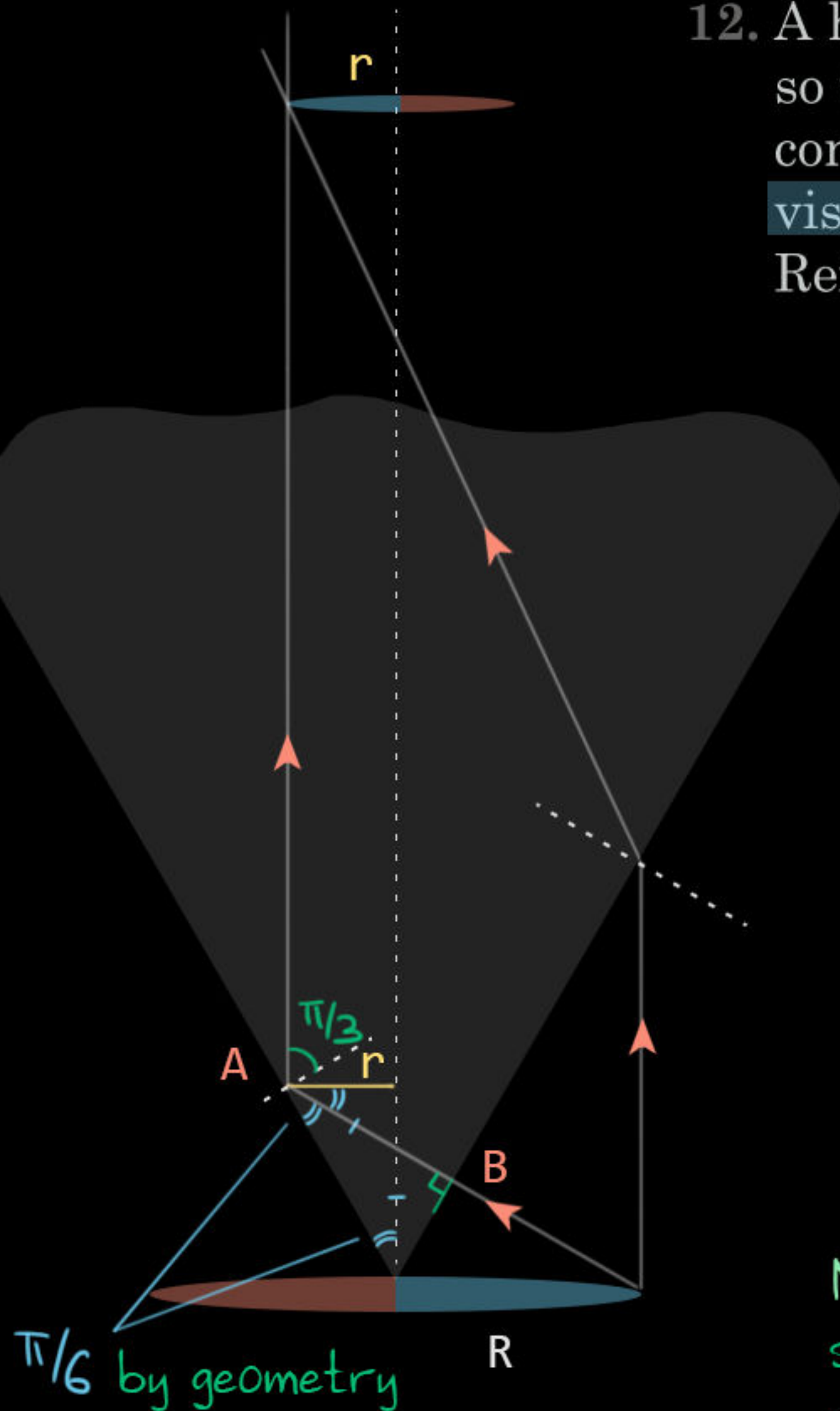
$$\therefore \text{Radius of visible seabed} = r + x = r + d \tan \theta = r + d \sqrt{\mu_w^2 - 1}$$

12. A homogeneous glass cone of half angle 30° is placed inverted on a disc so that its axis is vertical. Radius of the disc is 4.0 cm and the tip of the cone coincides with the centre of the disc. What is the radius of the disc visible through the circular base of the cone from a large height? Refractive index of the glass of the cone is 1.5.

$$\text{At A: } 1.5 \sin \pi/3 = 1 \sin r$$

Greater than 1 $\therefore$ TIR at A

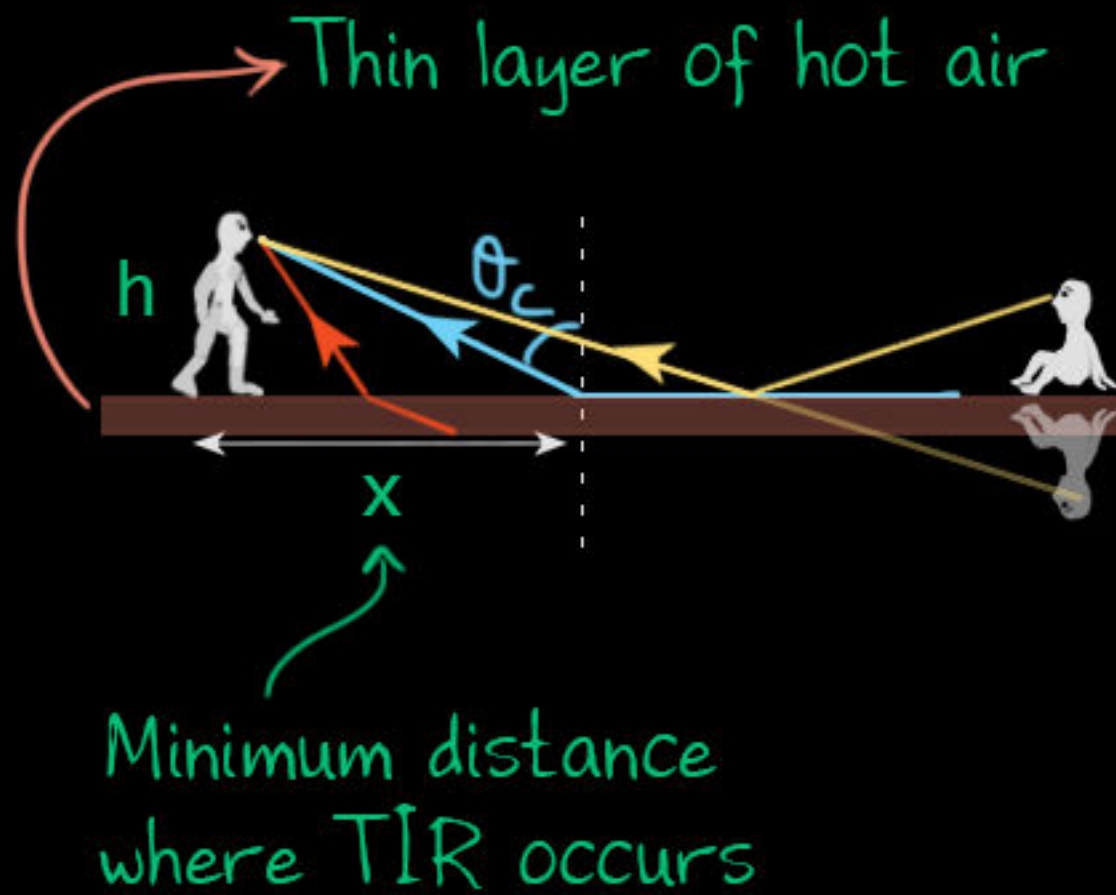
$\Rightarrow 30^\circ$ at B



$$\begin{aligned} \therefore r &= h \cos \pi/6 \\ &= R \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} \\ &= R/2 = 2 \end{aligned}$$

Note: Shown image is in the glass. When viewed from air above the flat surface of cone, next image will only vertically shift, but radius will remain same.

Showing three rays that reach the observer.



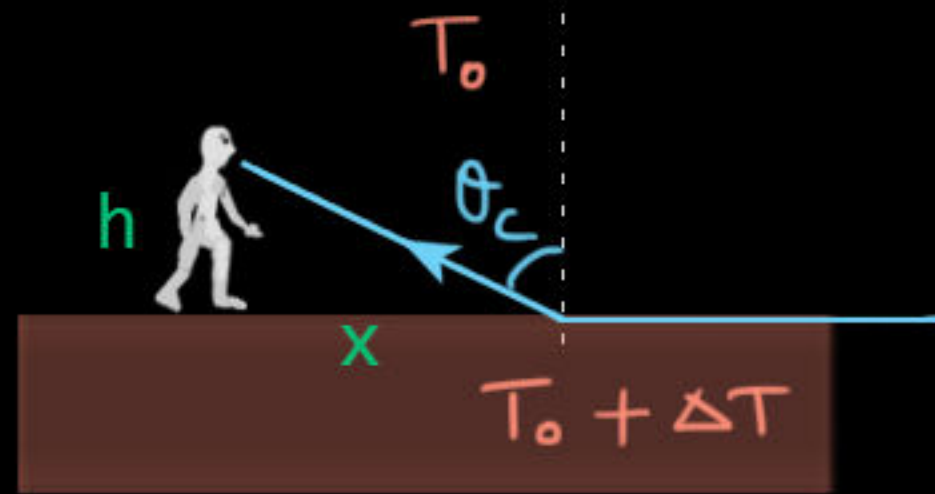
13. Refractive index of air depends on temperature and pressure. Within few metres of height difference near the ground, air pressure remains almost uniform but air temperature changes by appreciable amount. Suppose under such conditions, the refractive index μ depends only on the temperature, and this dependence has the form

$$\mu = 1 + \frac{a}{T}$$

Here a is a constant, value of which at normal pressure is $1/6$ K.

The sun heats up the air above a road and simultaneous random small movements of air make the dry asphalt appear like a puddle. This phenomenon is called **mirage**. Suppose that in a thin layer immediately above the road, temperature of air is $\Delta T = 20$ K more than temperature $T_0 = 300$ K of the air elsewhere. From what minimum horizontal distance x an observer of height $h = 1.75$ m can see a puddle on the road. Also plot a rough graph for relation $x(\Delta T)$.

$$\mu = 1 + \frac{a}{T}$$



$$\left(1 + \frac{a}{T_0}\right) \sin \theta_c = \left(1 + \frac{a}{T_0 + \Delta T}\right) \sin 90^\circ$$

$$x = h \tan \theta_c$$

$$= \frac{h}{\sqrt{\frac{1}{\sin^2 \theta_c} - 1}} = \frac{h}{\sqrt{\frac{(1 + \frac{a}{T_0})^2}{(1 + \frac{a}{T_0 + \Delta T})^2} - 1}} = \frac{h}{\sqrt{(1 + \frac{2a}{T_0})(1 - \frac{2a}{T_0 + \Delta T}) - 1}}$$

$$= \frac{h}{\sqrt{1 + \frac{2a}{T_0} - \frac{2a}{T_0 + \Delta T} - \frac{4a^2}{T_0(T_0 + \Delta T)} - 1}} = h \sqrt{\frac{T_0(T_0 + \Delta T)}{2a \Delta T}} //$$

$\rightarrow \approx 0$

We know:

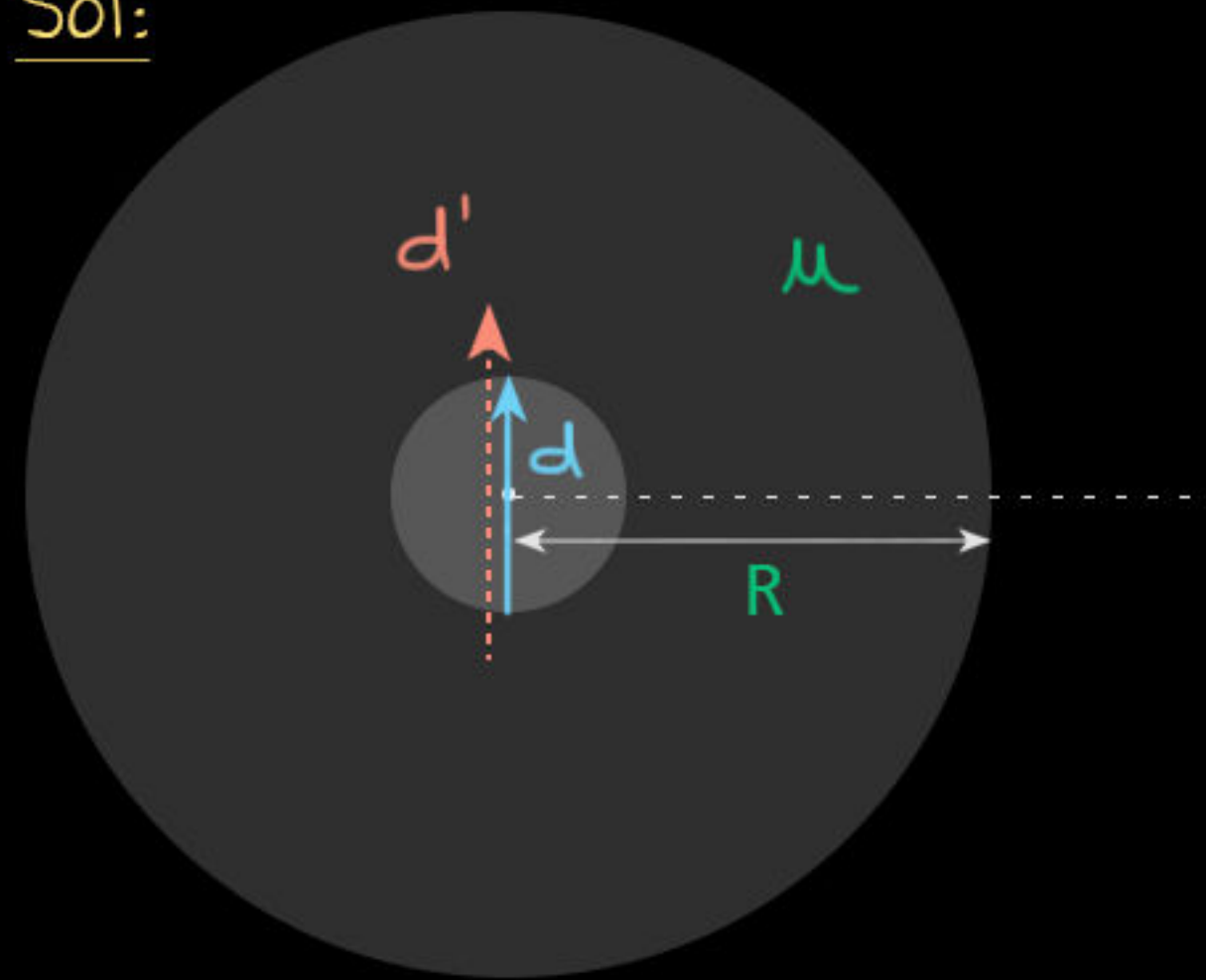


$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad 1$$

$$m = \frac{I}{O} = \frac{\mu_1 v}{\mu_2 u} \quad 2$$

14. A steel wire is coaxially embedded in a glass tube, external diameter of which is much greater than that of the wire. Refractive index of the glass is $4/3$. If diameter of the steel wire appears $8/3$ mm when looked into the glass tube through its curved surface, find the actual diameter of the steel wire?

Sol:



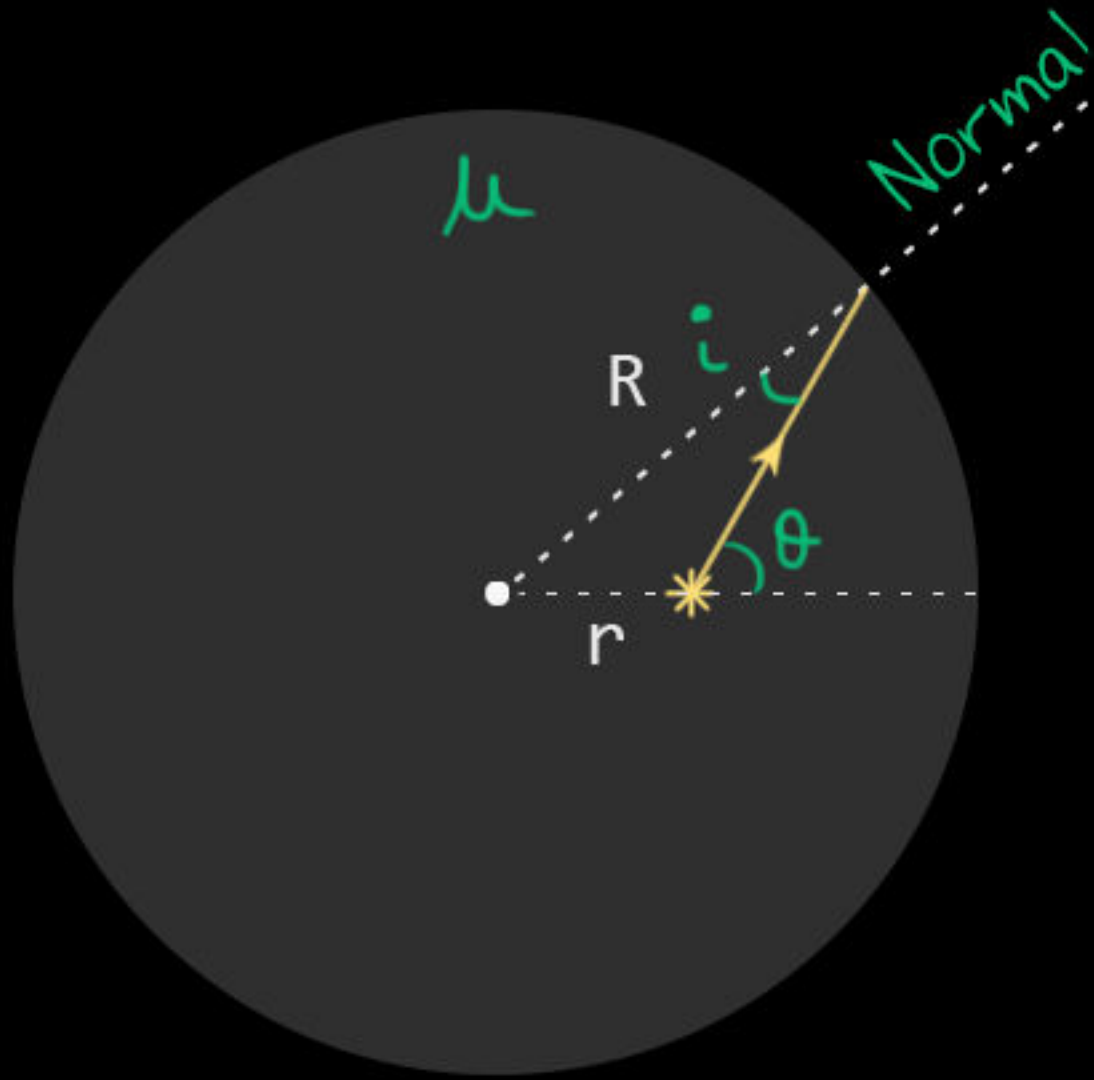
$$1: \frac{1}{v} - \frac{\mu}{(-R)} = \frac{1 - \mu}{(-R)}$$

$$\Rightarrow v = -R$$

$$2: m = \frac{d'}{d} = \frac{\mu(v)}{1(u)} = \frac{\mu(-R)}{(-R)} = \mu$$

$$\Rightarrow d = \frac{d'}{\mu} = \frac{8/3}{4/3} = 2 //$$

15. An isotropic point light source is placed inside a homogeneous sphere of radius R at a distance r from the centre. What should the value of refractive index of the material of the sphere be so that all the light emitted by the source comes out of the sphere?



To trap the light: $i_{\max} < \theta_c$

$$\Rightarrow \sin(i_{\max}) < \sin \theta_c$$

$$\Rightarrow \sin(i_{\max}) < \frac{1}{\mu} \quad (1)$$

Finding $i_{\max}$

$$\frac{r}{\sin i} = \frac{R}{\sin(\pi - \theta)}$$

$$\Rightarrow \sin i = \frac{r \sin \theta}{R}$$

$$\therefore @ \theta = 90^\circ \quad \sin(i_{\max}) = \frac{r}{R} \quad (2)$$

From 1,2:

$$\frac{r}{R} < \frac{1}{\mu}$$

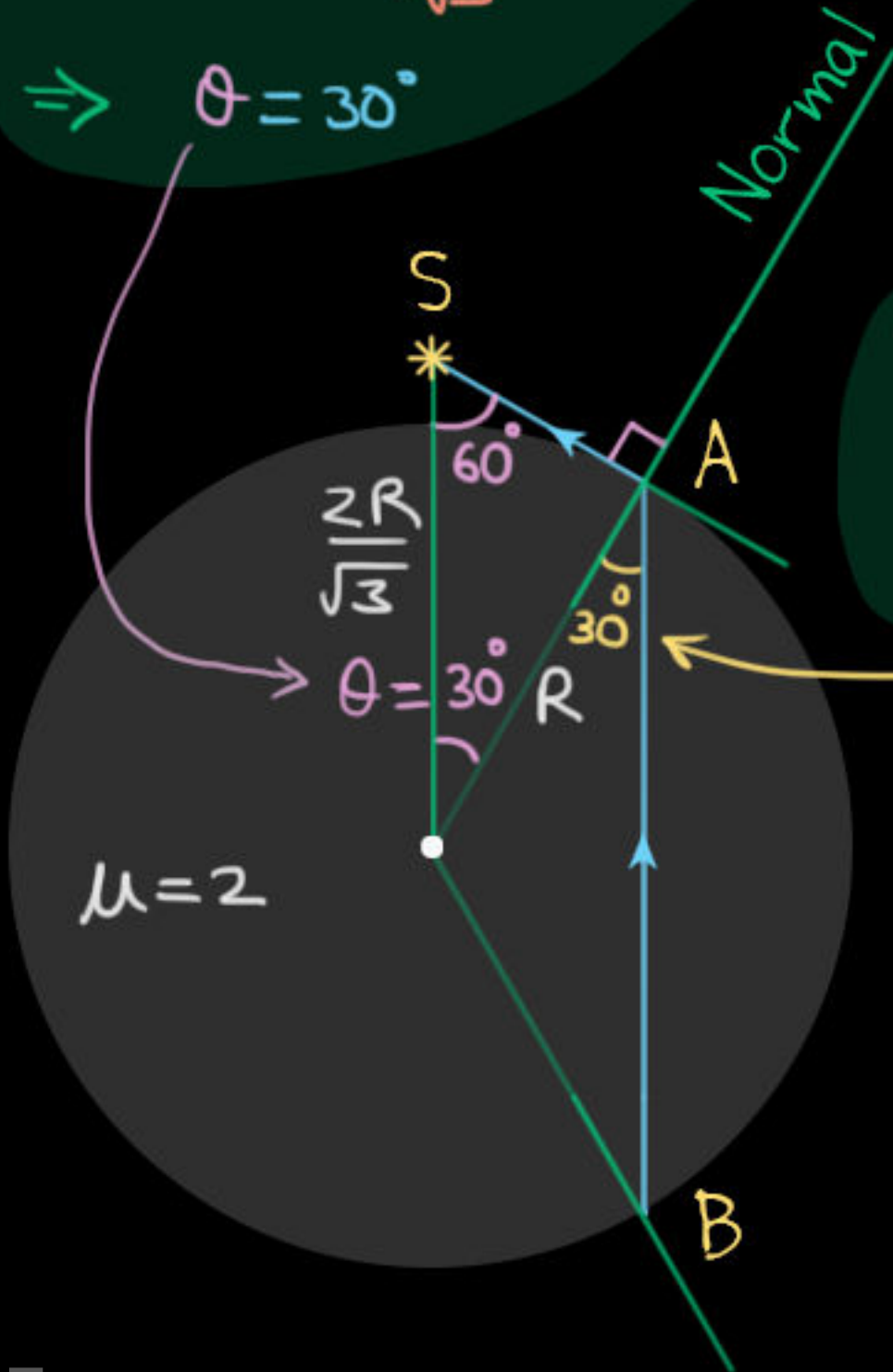
$$\Rightarrow \mu < R/r //$$

Ray coming to spider tangentially from globe be AS.

16. A spider is hanging above the north pole of a transparent globe of diameter d at a distance $d/\sqrt{3}$ from the centre of the globe. Refractive index of the material of the globe is $\mu = 2$. Between which latitudes a fly can sit on the globe so that it cannot be seen by the spider. Treat the spider and the fly both as point objects.

$$\cos \theta = \frac{R}{2R/\sqrt{3}} = \frac{\sqrt{3}}{2}$$

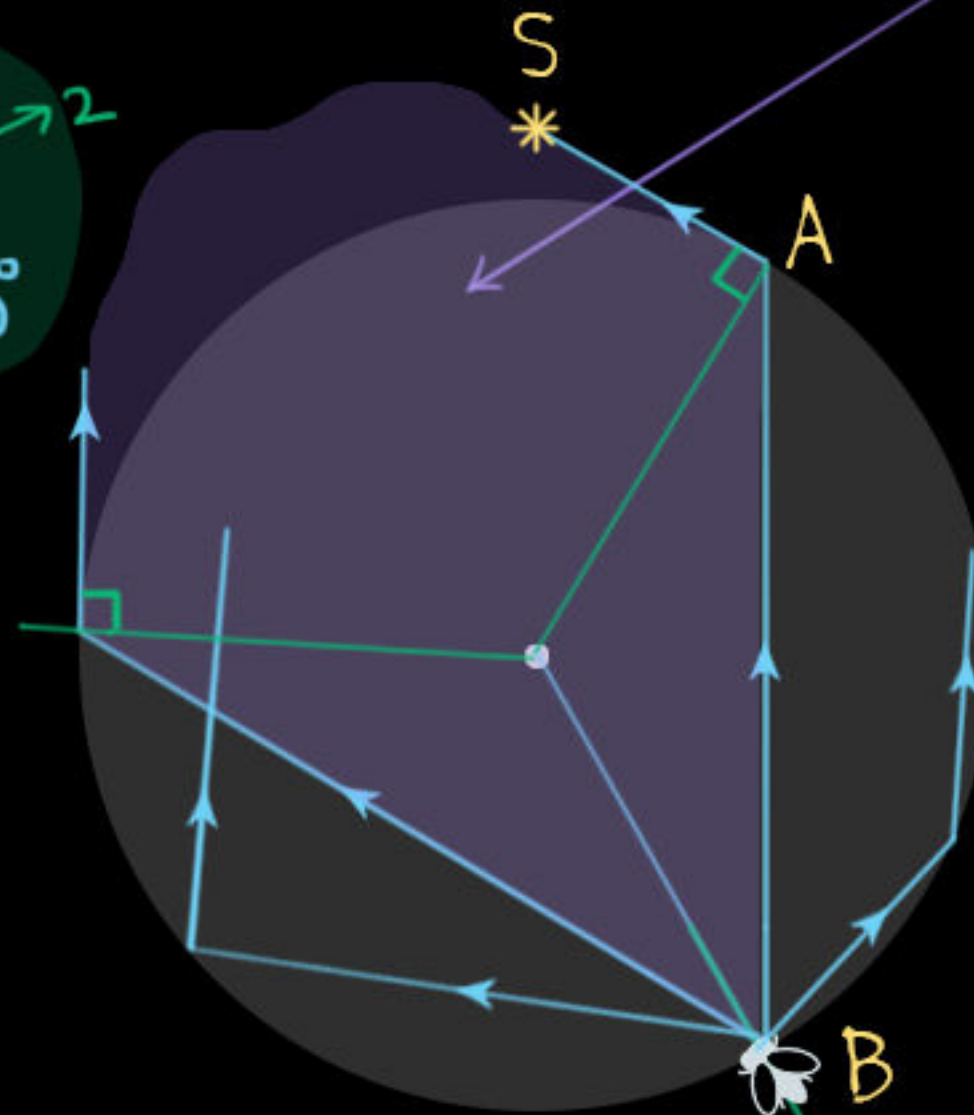
$$\Rightarrow \theta = 30^\circ$$



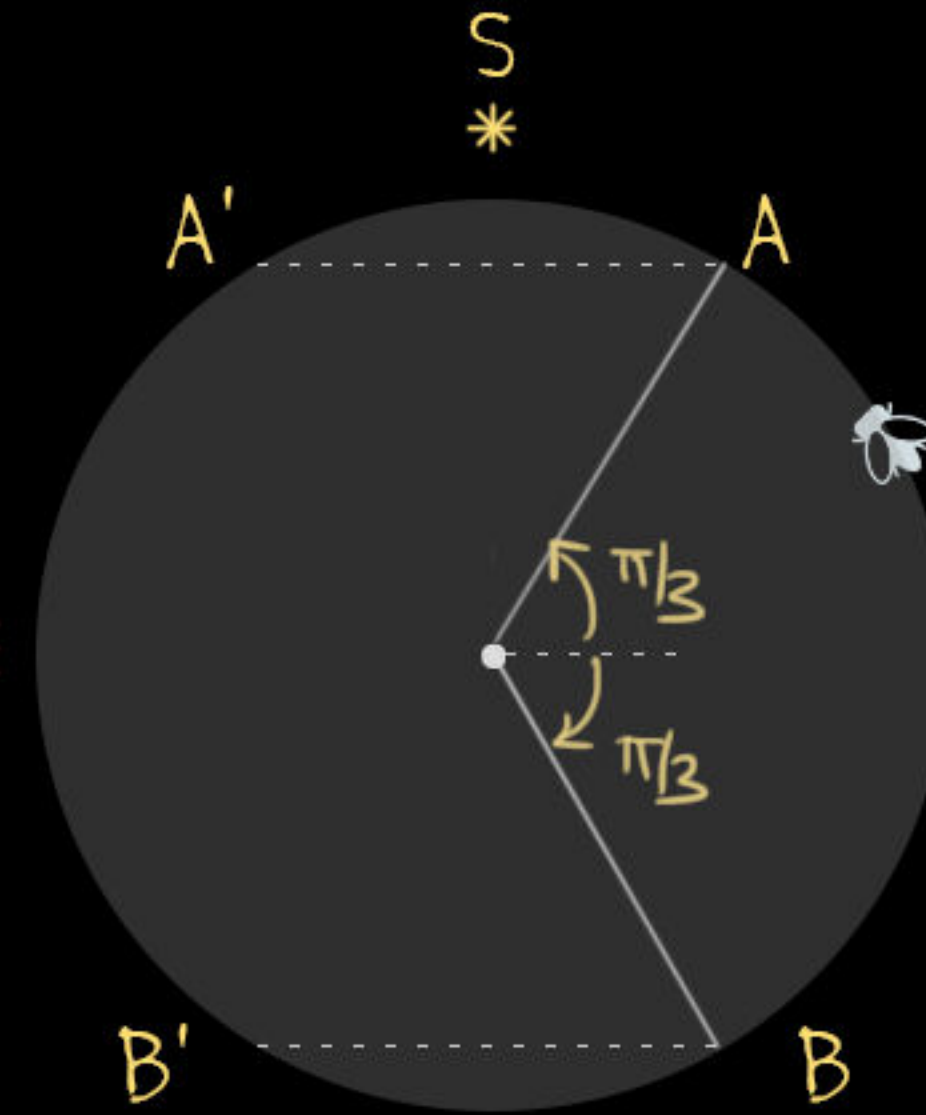
$$\sin \theta_c = \frac{1}{\mu} \rightarrow 2$$

$$\Rightarrow \theta_c = 30^\circ$$

Spider has to be in shaded region to see the fly at B. As other rays will get internally reflected.



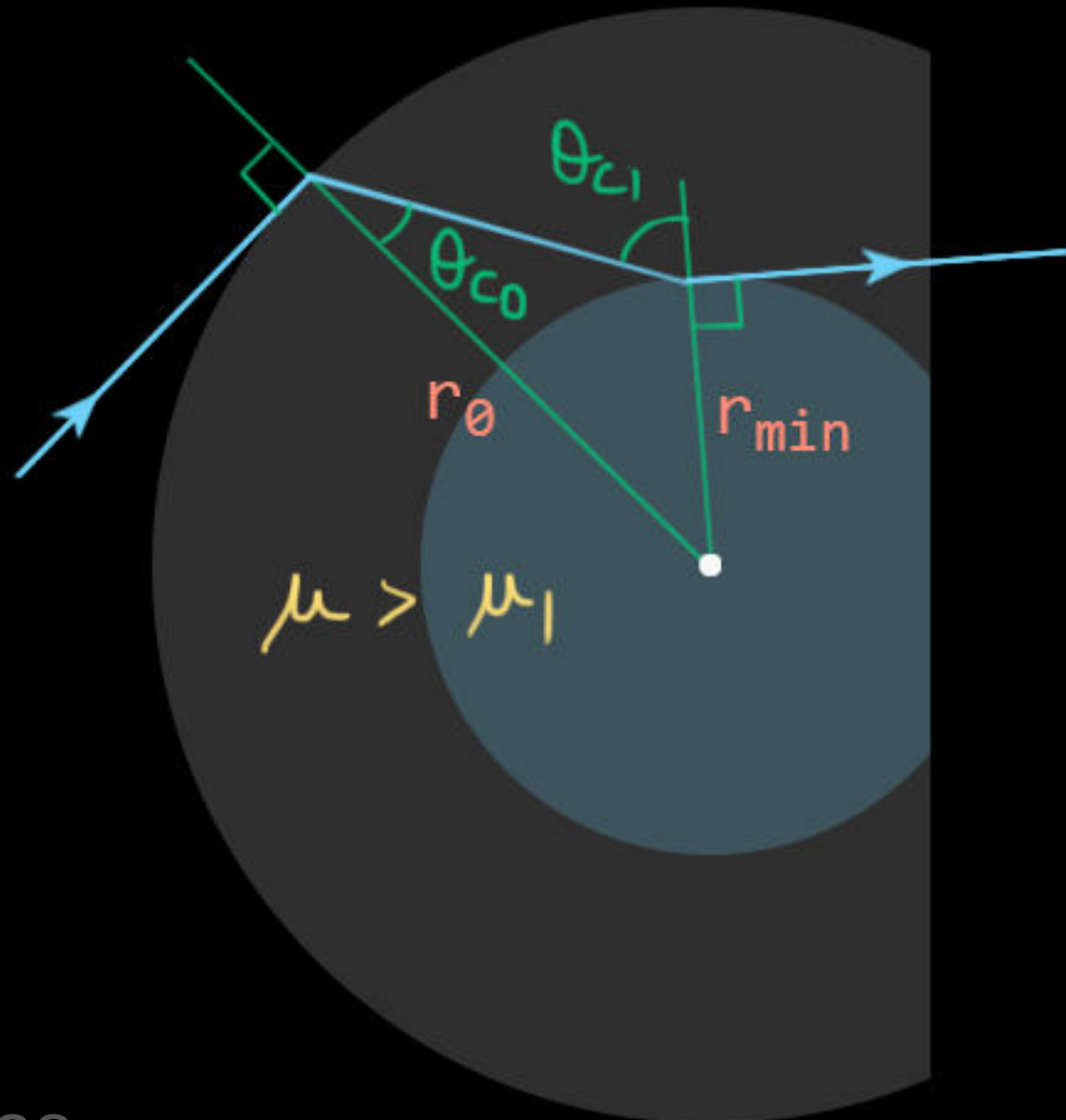
A to A', fly will be directly visible to spider.
B to B', fly will be visible to spider through glass.



∴ Latitude of fly to be unseen goes from A to B. i.e: 60° N to 60° S.

For $r_{\min}$:

- 1 Tangential ray from air must enter glass.
- 2 Inside glass, that ray should just enter the liquid.



17. A capillary of outer radius r_0 made of a glass of refractive index μ is filled with a liquid of refractive index μ_1 ($\mu_1 < \mu$). What should the minimum internal radius of the capillary be so that all the light incident on the capillary enter the liquid?

Sine law

$$\frac{r_0}{\sin(\pi - \theta_{c1})} = \frac{r_{\min}}{\sin \theta_{c0}} \quad (1)$$

Outer surface

$$(1) \sin 90^\circ = \mu \sin \theta_{c0} \quad (2)$$

Inner surface

$$\mu \sin \theta_{c1} = \mu_1 \sin 90^\circ \quad (3)$$

From 1, 2, 3:

$$\frac{r_0}{\mu_1/\mu} = \frac{r_{\min}}{1/\mu}$$

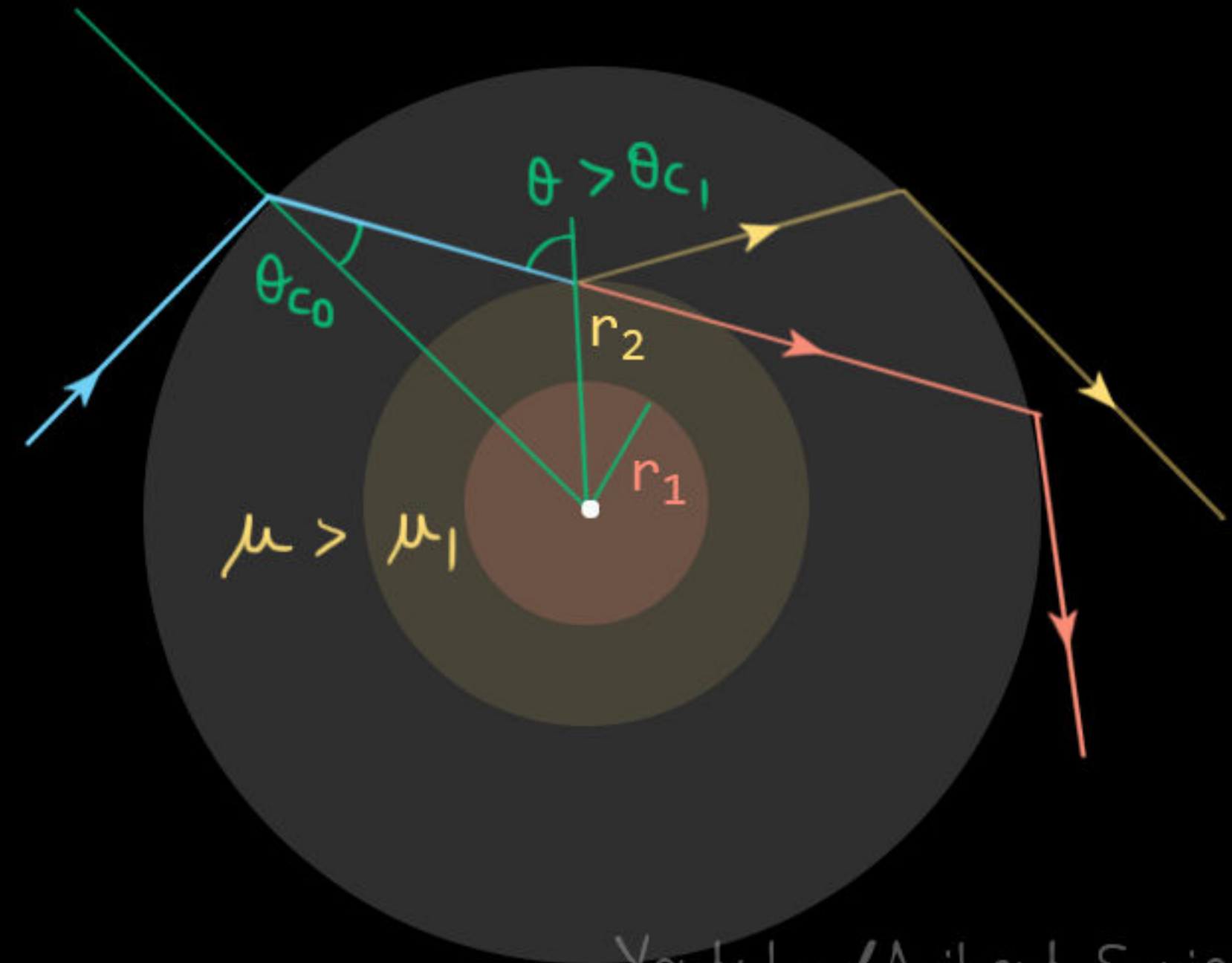
$$\Rightarrow r_{\min} = \frac{r_0}{\mu_1}$$

by Ankit Singhvi

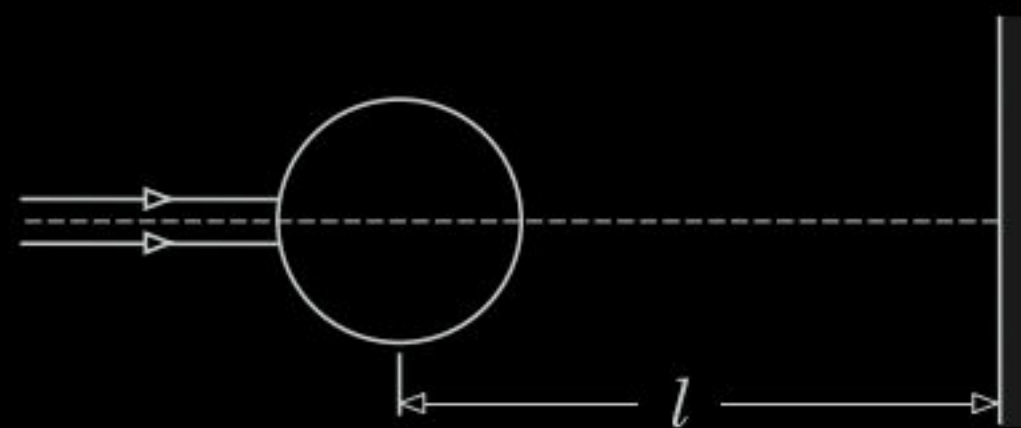
What if we reduce r further?

Ans: Ray will either TIR from liquid surface or completely miss it.

Let: $r_1 < r_2 < r_{\min}$



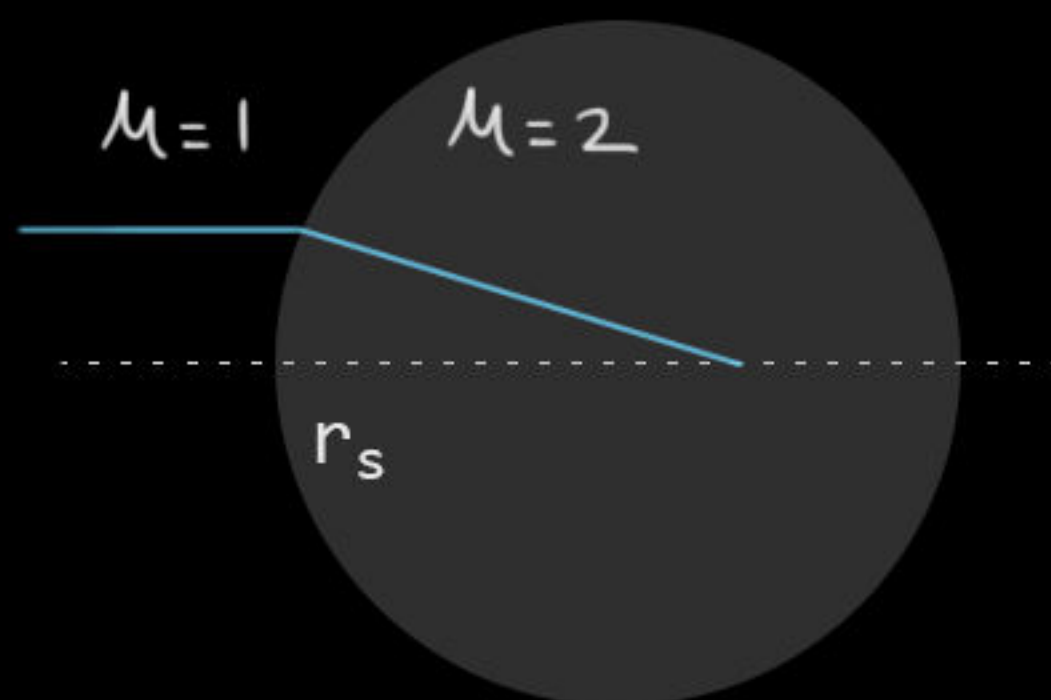
Youtube / Arihant Senior



18. A narrow parallel beam of light when incident normally on a screen, makes a light spot of radius $r_b = 0.5$ cm on the screen. Now a transparent sphere of radius $r_s = 20.0$ cm made from a material of refractive index $\mu = 2.0$ is introduced with its centre on the axis of the beam. If centre of the sphere is at a distance $l = 100$ cm from the screen, what would be radius R of the light spot?

• Its a thick lens, we need to do basic refraction at both surfaces.

First surface

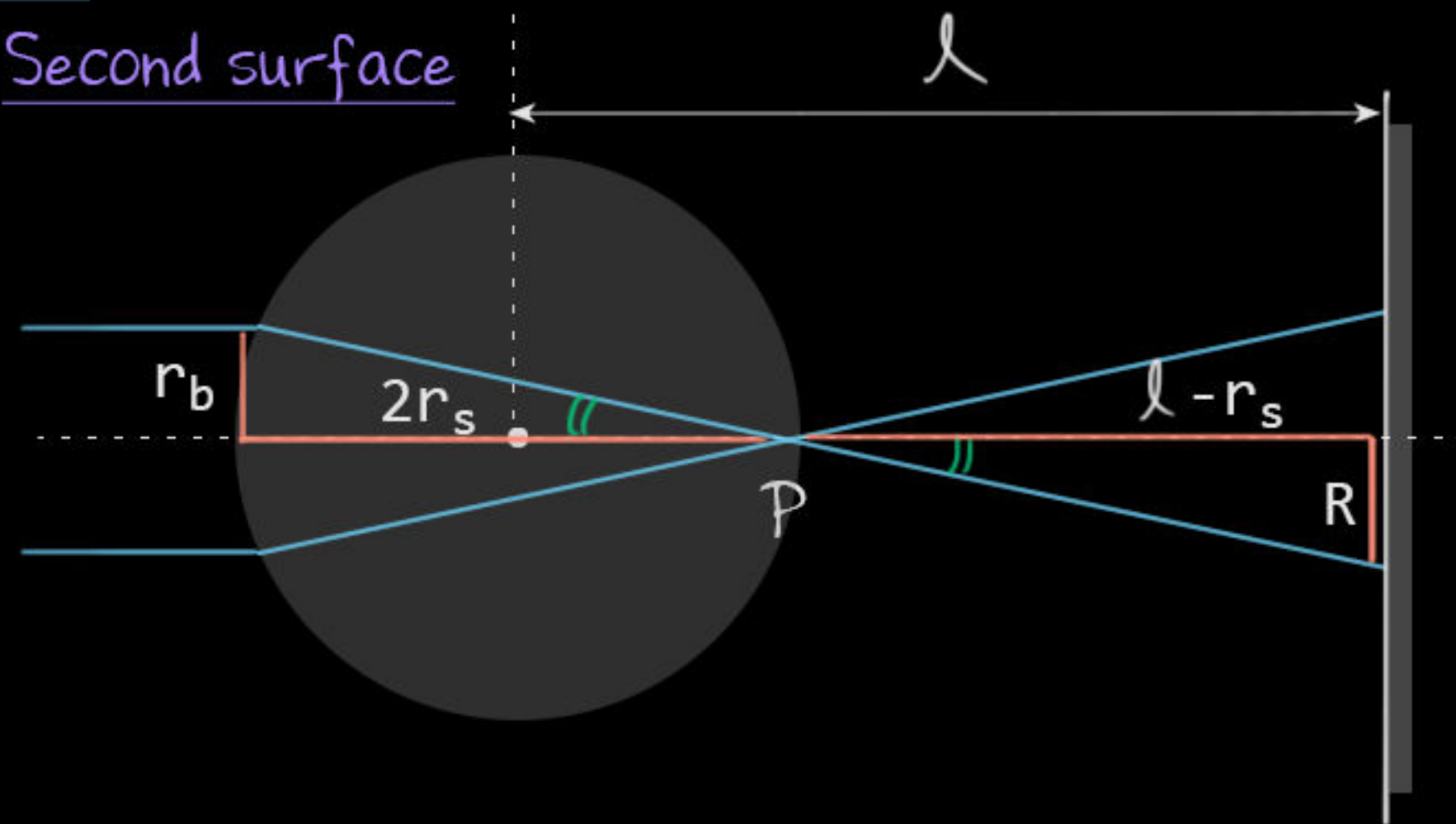


$$\frac{2}{v} - \frac{1}{\infty} = \frac{2-1}{r_s}$$

$$\Rightarrow \frac{2}{v} = \frac{1}{r_s}$$

$$\Rightarrow v = 2r_s$$

Second surface



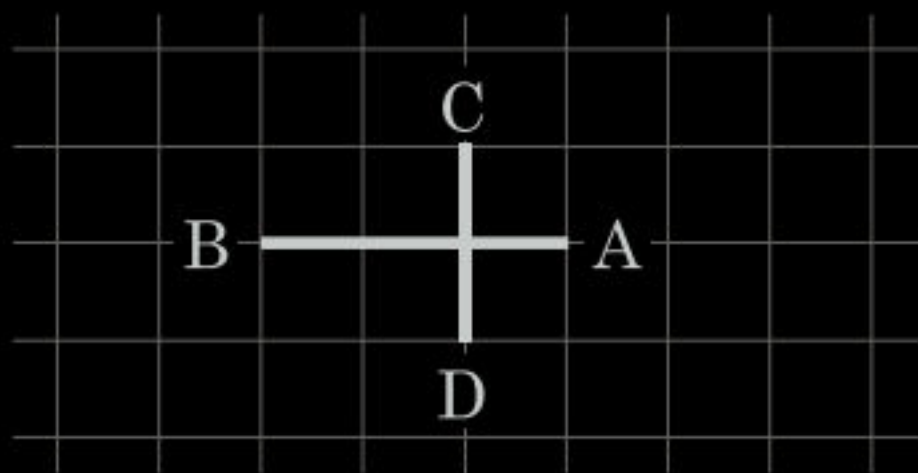
$$\tan \theta \approx \frac{r_b}{2r_s} = \frac{R}{l - r_s}$$

$$\Rightarrow R \approx \frac{r_b(l - r_s)}{2r_s}$$

• Once inside, the paraxial rays (as r_b is quite small) will cross the pole P undeviated.

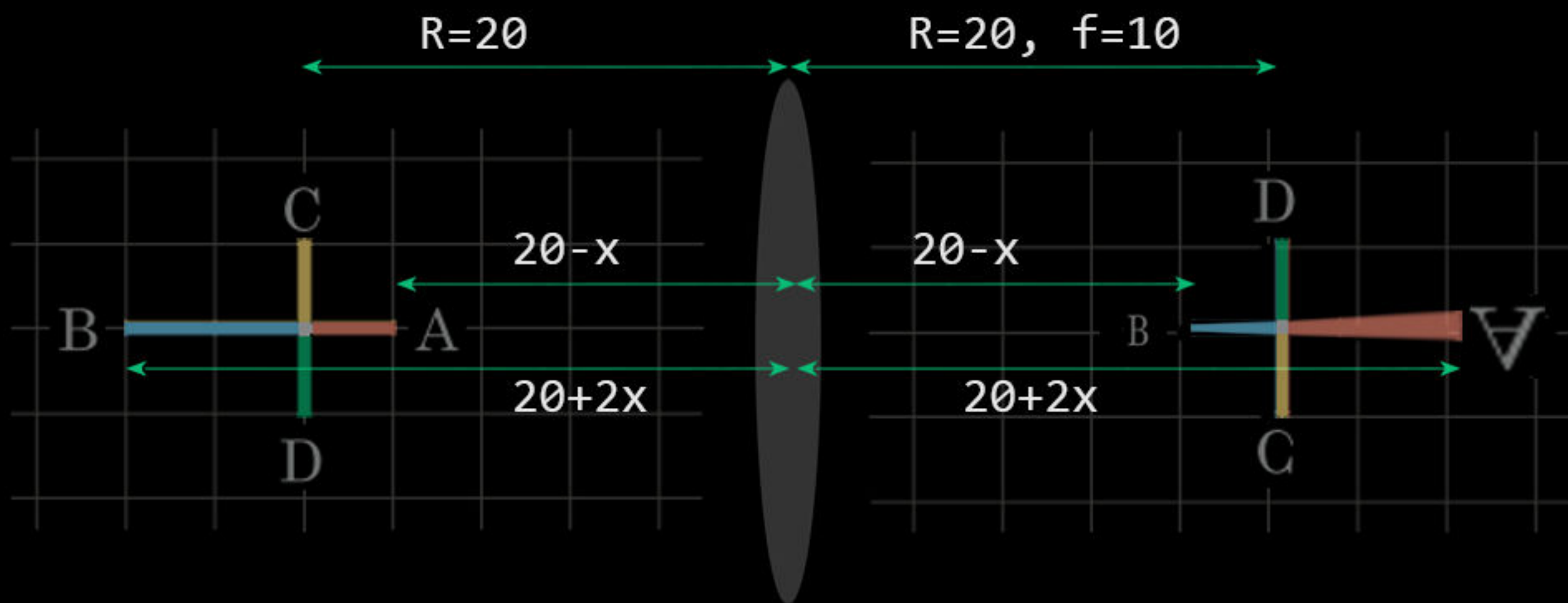
by Ankit Singhvi

Youtube / Arihant Senior



19. Two thin rods AB and CD welded on each other make a frame that is shown on a graph paper. This frame is placed in front of a **converging lens of focal length 10 cm** with the arm AB coinciding with the optical axis of the lens. By shifting the frame on the optical axis, **if an image of exactly the same size as that of the frame** can be obtained, **find length of the rod AB.**

As height of CD is unchanged.
It must lie on R.



$$B: \frac{1}{20-x} - \frac{1}{-(20+2x)} = \frac{1}{10}$$

$$\Rightarrow x = 5$$

$$\therefore AB = 3x = 15 //$$

Youtube / Arihant Senior

by Ankit Singhvi

20. A high-speed movie camera is being used to shoot an oscillating simple pendulum at a speed $n = 24$ frames per second and takes $N = 48$ frames to shoot one complete oscillation. Focal length of the lens of the camera is $f = 50$ mm and length of the pendulum in the image on the film is $l = 10$ mm. Find distance of the pendulum from the camera lens. You may use $\pi^2 = g$.

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$m = \frac{f}{f+u}$$

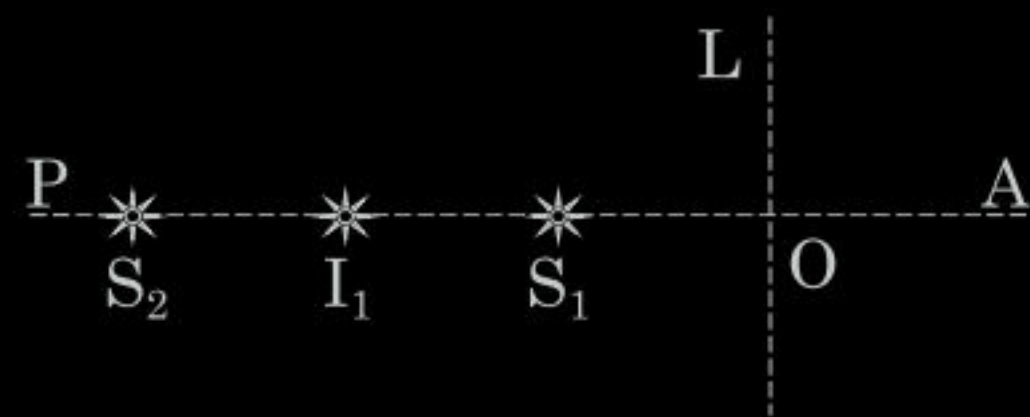
$$\Rightarrow T = 2\pi \sqrt{\frac{l}{\pi^2}}$$

$$\Rightarrow l = 1 \text{ m} = 1000 \text{ mm}$$

$$\Rightarrow -\left(\frac{10}{1000}\right) = \frac{50}{50+u}$$

$$\Rightarrow u = 5050 \text{ mm} \approx 5 \text{ m}$$

As m is $-ve$ for real image.
And its real, as image is
captured on a film.



For paraxial rays,

1 Small deviation: $\delta \approx \frac{h}{f} \propto h$

2 $y = BC \approx AC \approx OC$

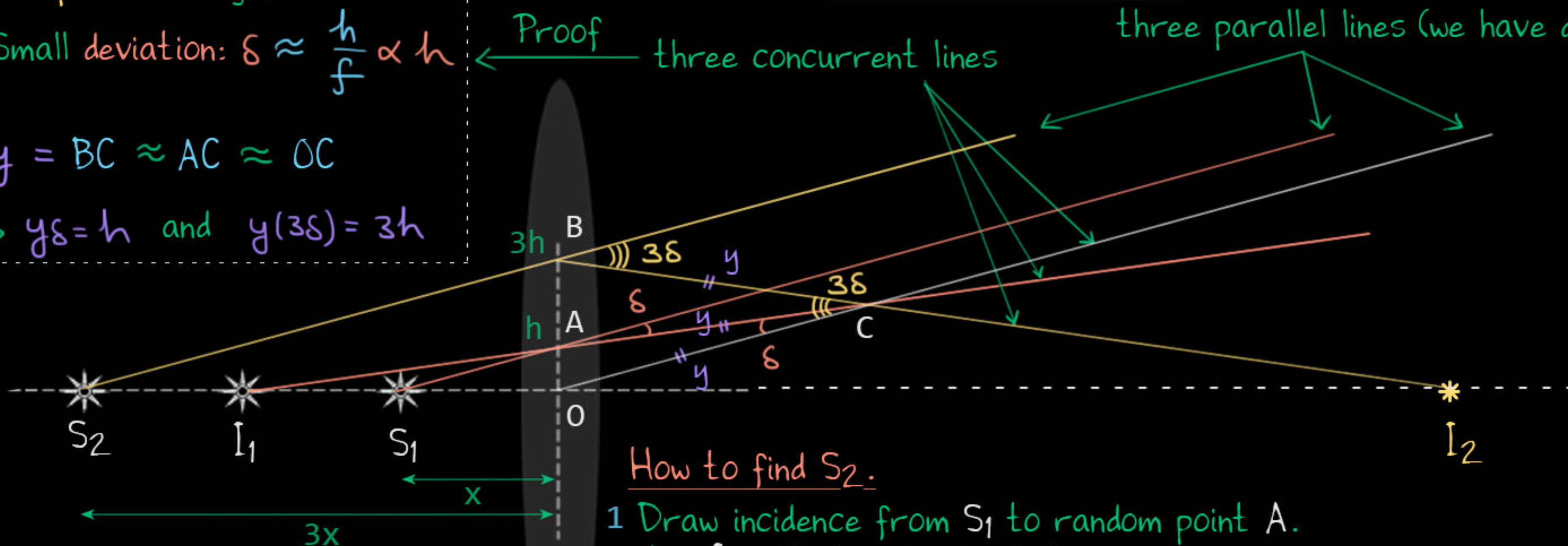
$\Rightarrow y\delta = h$ and $y(3\delta) = 3h$

21. In the arrangement shown, S_1 and S_2 are two point light sources placed on the optical axis PA of a lens L of unknown nature and I_1 is the image of the source S_1 . Distances S_2I_1 , I_1S_1 and S_1O are equal. Draw a ray diagram to obtain image of the source S_2 and with the help of this diagram determine location of the image.

Proof

three concurrent lines

three parallel lines (we have drawn)



How to find S_2 .

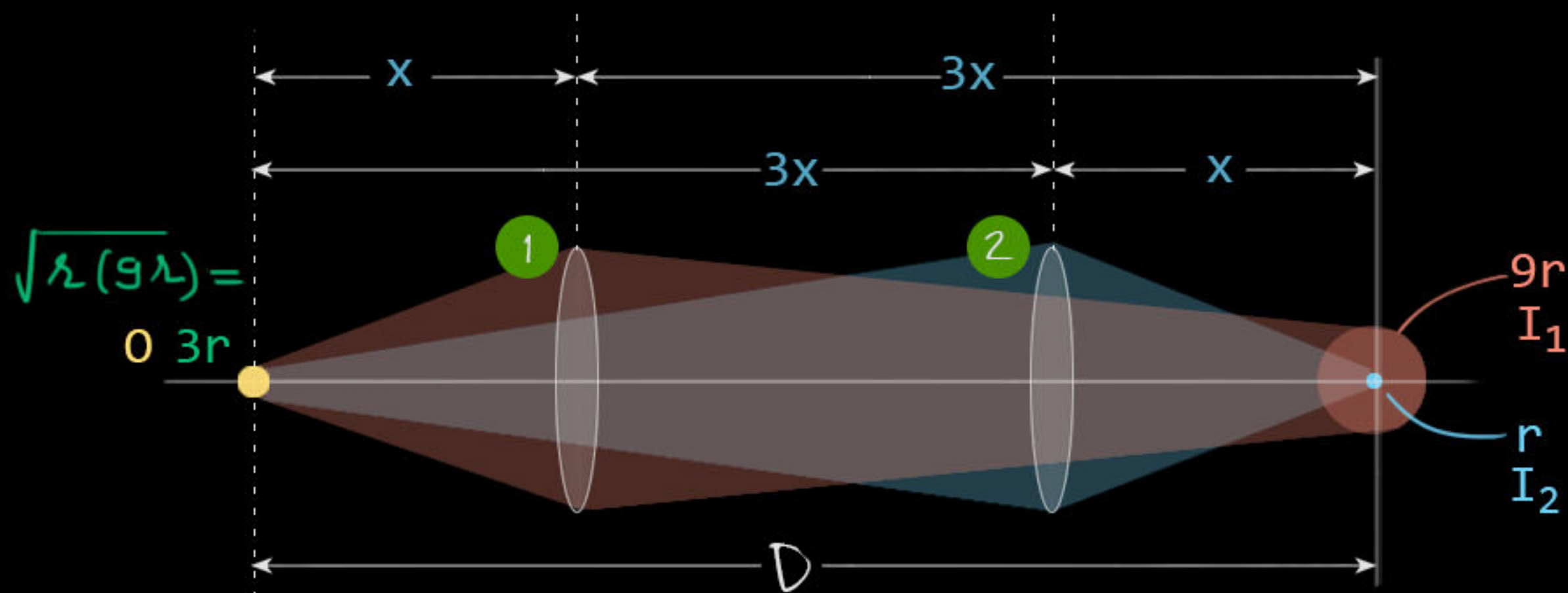
- 1 Draw incidence from S_1 to random point A.
- 2 Join I_1 with A and extend it.
- 3 From O draw a ray parallel to S_1A . Let it meet extended I_1A at C.
- 4 Draw S_2B parallel to S_1A . Let it meet the aperture at B.
- 5 Join BC and extend it to meet the principal axis at required I_2 .

Proven in Irodov 5.36

$$f = \frac{D\sqrt{n}}{(1+\sqrt{n})^2} \quad \eta = \frac{m_1}{m_2}$$

$$0 = \sqrt{I_1, I_2}$$

Let radii of images be r and $9r$.



$$A \quad \eta = \frac{m_1}{m_2} = \frac{I_1}{I_2} = \frac{9x}{x} = 9$$

$$\therefore f = \frac{120\sqrt{9}}{(1+\sqrt{9})^2} = \frac{120(3)}{16} = 22.5 //$$

B Spot Brightness $\propto \frac{\text{Incident power} \propto \Omega \propto \frac{1}{u^2}}{\text{Spot area} \propto r^2} \propto \frac{1}{r^2 u^2}$

$$\therefore \frac{B_1}{B_2} = \frac{r_2^2 u_2^2}{r_1^2 u_1^2} = \frac{r^2 (3x)^2}{(9r)^2 (x)^2} = \frac{1}{9} //$$

Youtube / Arihant Senior

22. In a setup of displacement method experiment, distance between the screen and a light source is 120 cm and the lens used has a small aperture. By moving the lens between the source and the screen, sharp images are obtained on the screen for two different positions of the lens. The ratio of sizes of these two images is 1:9.

(a) What is the focal length of the lens?

(b) Which image is the brighter one? Determine the ratio of the brightness of these two images.

Approach: Focus depends on direction of incident rays.

To make our problem easier let's work with the same focus.

To do that, if we reverse the rays in second case, object becomes image and vice versa.

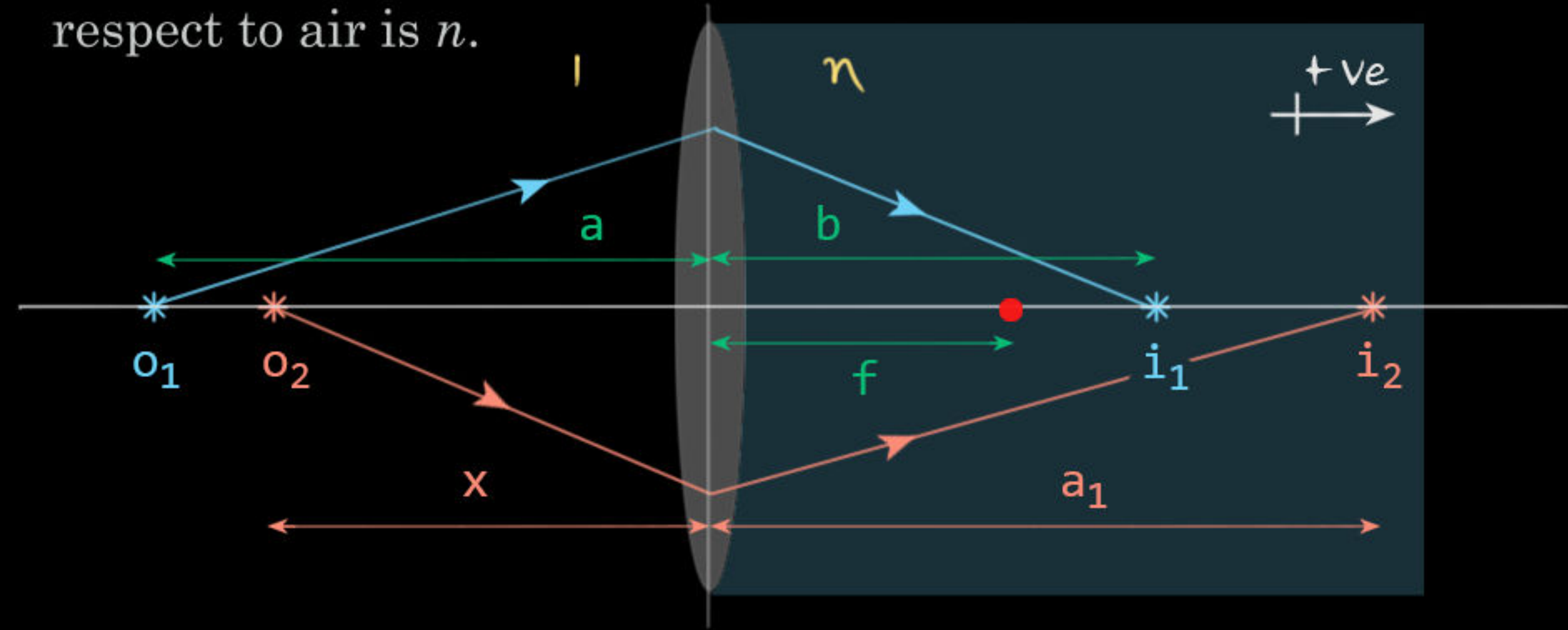
Now we have common focus as shown.

We know: $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{f}$

$$\frac{n}{b} - \frac{1}{(-a)} = \frac{n-1}{f} \quad (1)$$

$$\frac{n}{a_1} - \frac{1}{(-x)} = \frac{n-1}{f} \quad (2)$$

23. A thin glass lens is fixed in the wall of an aquarium filled with water. A real inverted image of an object that is in the air at a distance a from the lens is obtained in the aquarium at a distance b . Where will the image of an object be formed, if the object is located in the aquarium at a distance a_1 ($a_1 > b$) from the lens? Refractive index of water with respect to air is n .



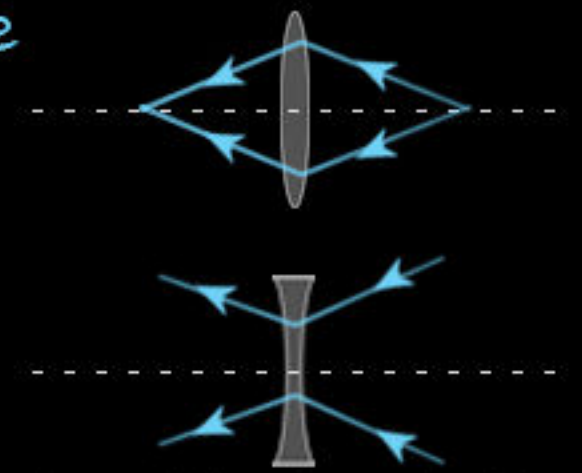
From 1,2:

$$\frac{n}{b} + \frac{1}{a} = \frac{n}{a_1} + \frac{1}{x}$$

$$\Rightarrow x = \frac{na(a_1 - b) + ba_1}{a a_1 b}$$

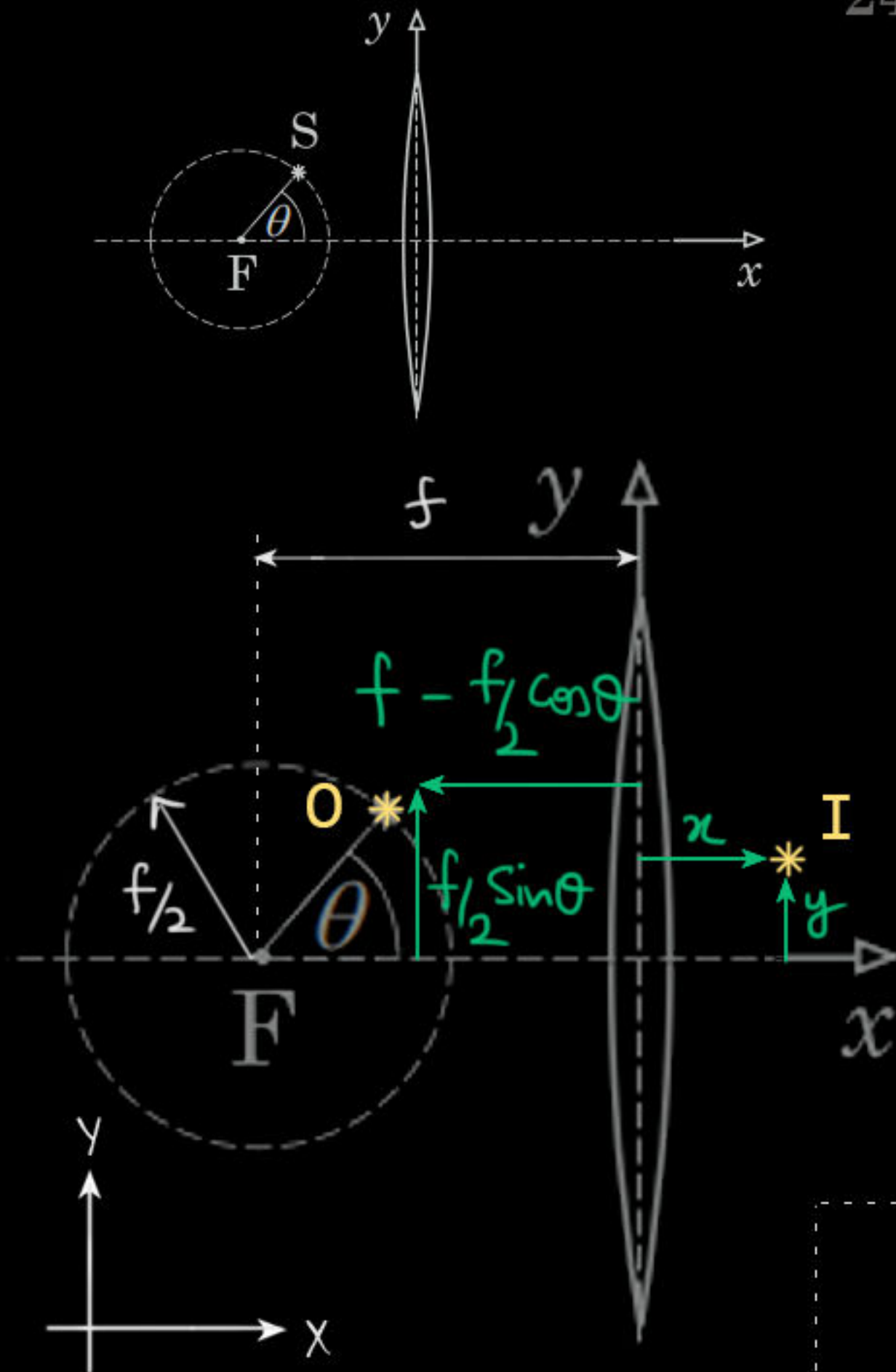
by Ankit Singhvi

Note: We can reverse ray directions, only if rays can be physically traced back. i.e., if both object and image are real (as in this case) or if both are virtual.



Youtube / Arihant Senior

24. A point light source S moves on a circular path, centre of which is on one of the foci F of a thin converging lens of focal length f . Radius of the circular path is $r = f/2$ and its plane is perpendicular to the plane of the lens. In the coordinate system shown, find position coordinates of the image of source in terms of the focal length f and the angular position θ of the source. Also, plot the locus of the image.



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{-(f - \frac{f}{2} \cos \theta)} = \frac{1}{f}$$

$$\Rightarrow x = \frac{f(\cos \theta - 2)}{\cos \theta} \quad (1)$$

$$m = \frac{I}{O} = \frac{f}{f+u}$$

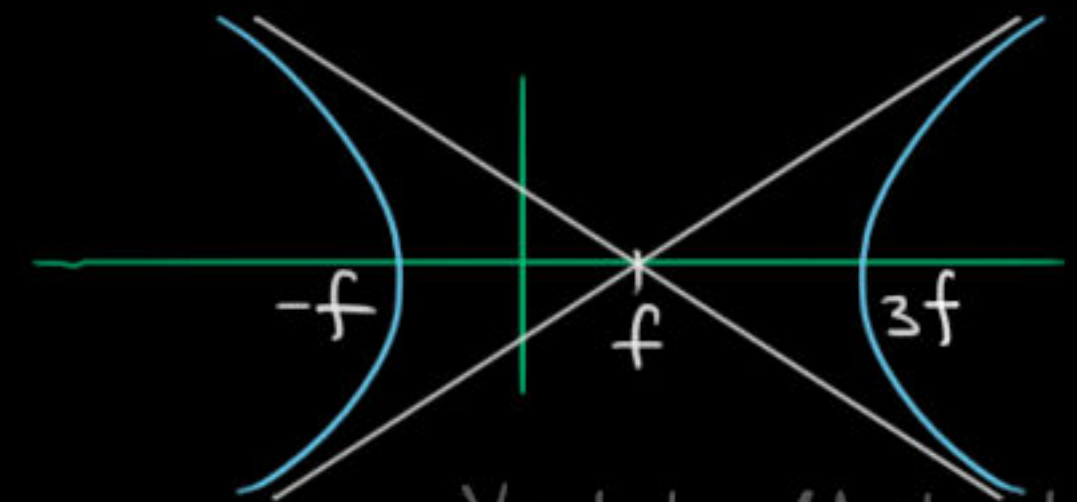
$$\Rightarrow \frac{y}{\frac{f}{2} \sin \theta} = \frac{f}{f + (-f + \frac{f}{2} \cos \theta)}$$

$$\Rightarrow y = f \tan \theta \quad (2)$$

Eliminating θ from 1,2: $\therefore$ Horizontal

$$(x-f)^2 - 4y^2 = 4f^2 \quad +ve$$

$\Rightarrow$ Hyperbola @ center $\{f, 0\}$
by Ankit Singhvi



Youtube / Arihant Senior

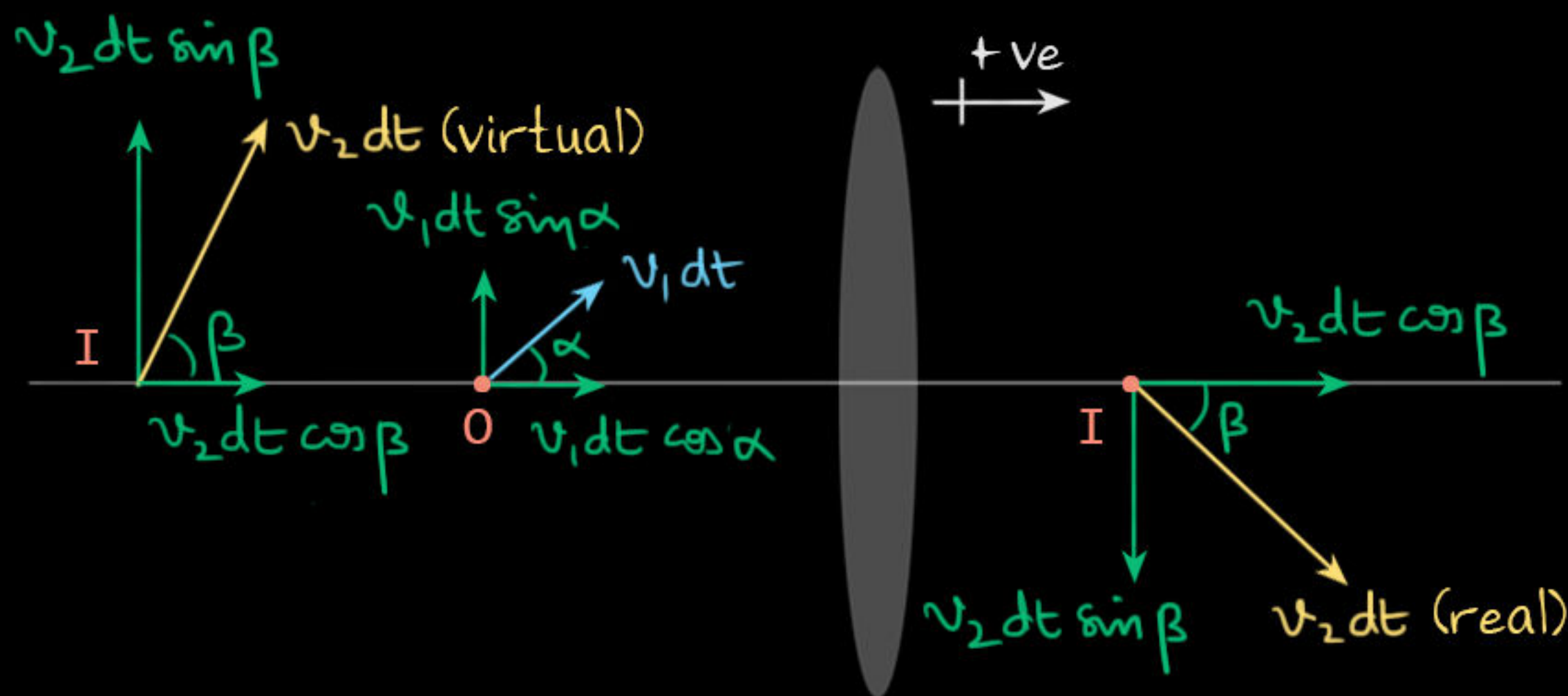
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

↓ differentiate

We know: $\frac{v^2}{u^2} = \frac{dv}{du}$

25. A thin converging lens of focal length f makes image of a moving point source of light. When the source is crossing the optical axis of the lens, its velocity vector makes an angle α with the optical axis and velocity vector of the image makes an angle β with the optical axis. At what distance from the lens, does the source cross the optical axis?

Lets trace the object and image for small time dt as they cross the axis.



$$\frac{v^2}{u^2} = \frac{v_2 dt \cos \beta}{v_1 dt \cos \alpha} \quad (1)$$

$$m = \frac{v}{u} = \frac{\pm v_2 dt \sin \beta}{v_1 dt \sin \alpha} \quad (2)$$

-ve for real

Dividing 1,2: $\frac{v}{u} = \pm \frac{\tan \alpha}{\tan \beta}$

Also, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{u} \left(\pm \frac{\tan \beta}{\tan \alpha} - 1 \right) = \frac{1}{f} \quad \text{+ve for real}$$

$$\Rightarrow |u| = -u = f \left(1 \pm \frac{\tan \beta}{\tan \alpha} \right)$$

distance

Note: If object moves in +X direction, so will the image, regardless of it being real or virtual.

by Ankit Singhvi

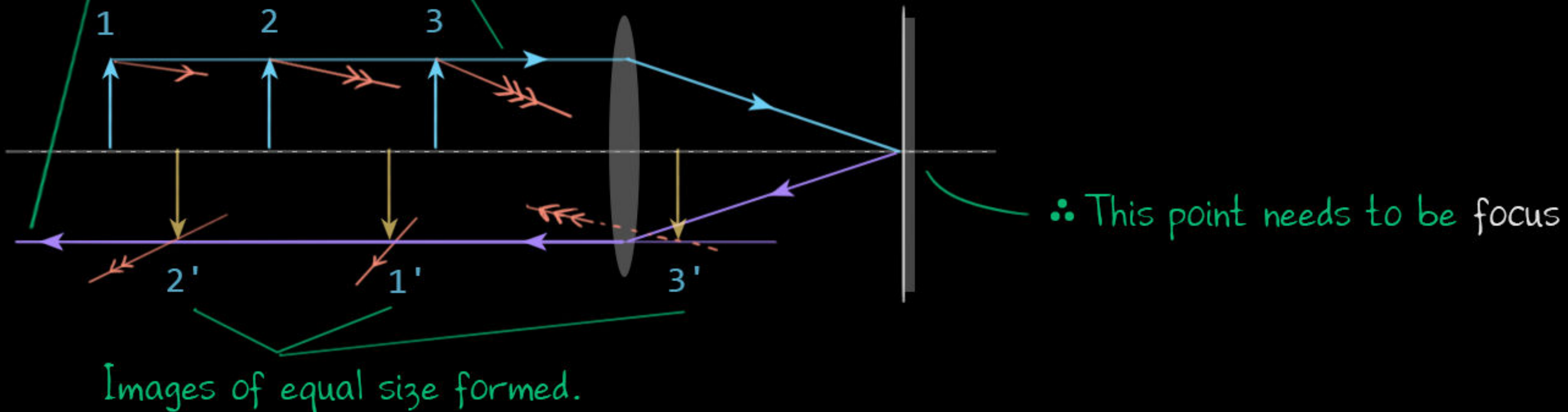
Youtube / Arihant Senior

26. A converging lens of focal length f is placed in front of and parallel to a plane mirror. An object is placed on the optical axis of the lens on the segment not included between the lens and the mirror. What should be the distance between the lens and the mirror so that final magnification is independent of the distance of the object from the lens?

Only common ray possible of all similar sized objects.

must finally result in

Only common ray possible of all similar sized images.
So for different v , image height is still same (which is same as object size).



27. Two converging lenses of focal lengths f_1 and f_2 are placed with their optical axes coinciding. This lens system is used to form image of an object. If it is found that size of the image does not depend on distance between the first lens and the object, find the distance between the lenses.

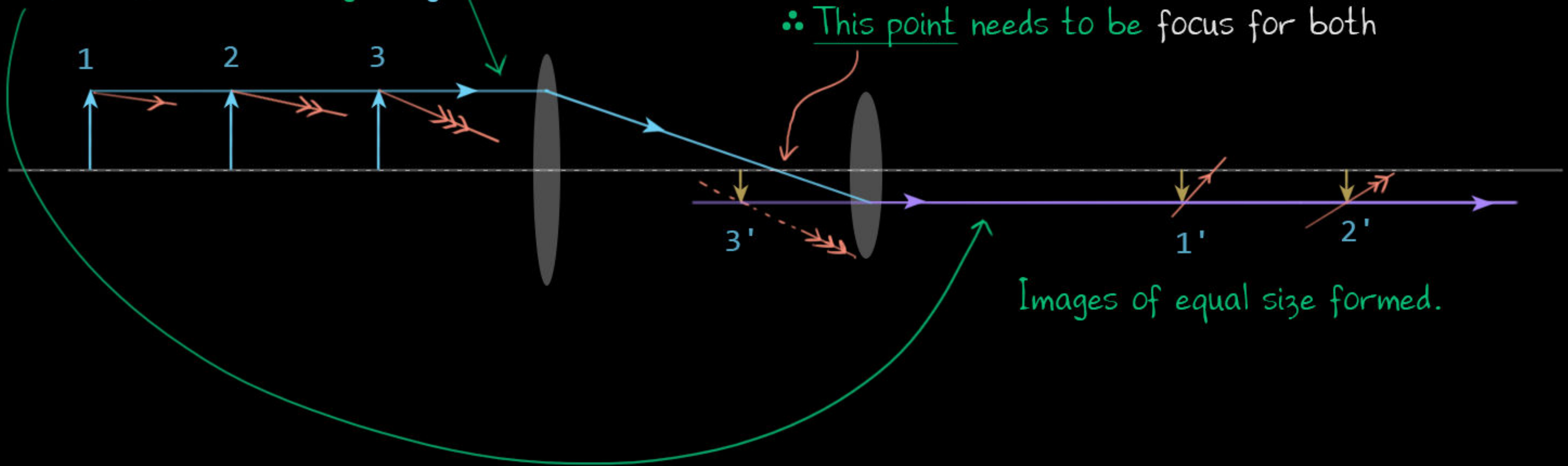
Only common ray possible of all similar sized objects.

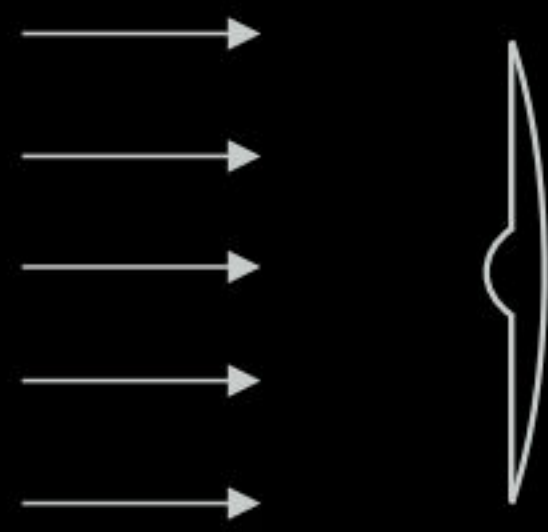


must finally result in



Only common ray possible of all similar sized images.
So for different v , image height is still same

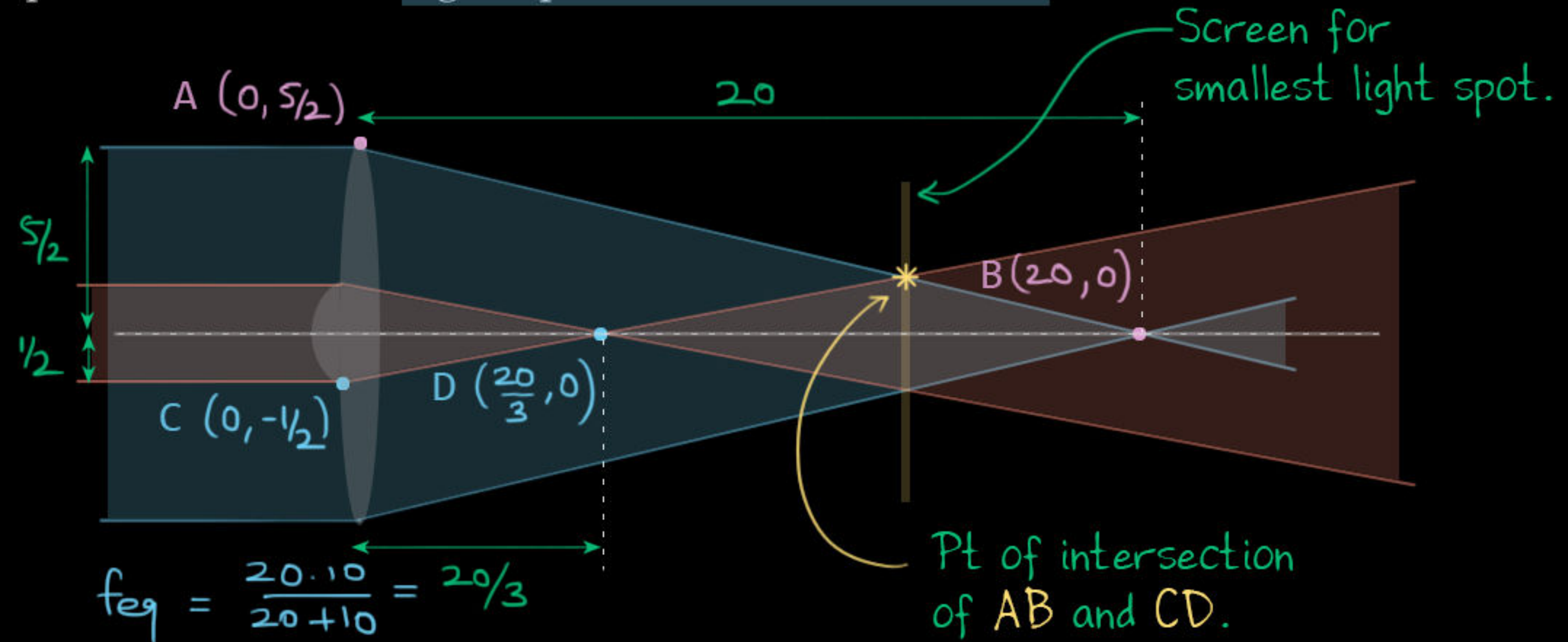




28. A composite lens is placed coaxially in the path of a cylindrical light beam of diameter 5 cm as shown in the figure. The composite lens is made by gluing two plano-convex lenses. Diameters of the first and second lens are 1.0 cm and 5.0 cm and their focal lengths are 10 cm and 20 cm respectively. Where to the right of the lens a screen should be placed to obtain a light spot of smallest diameter on it?

We know: when two thin lenses are kept close:

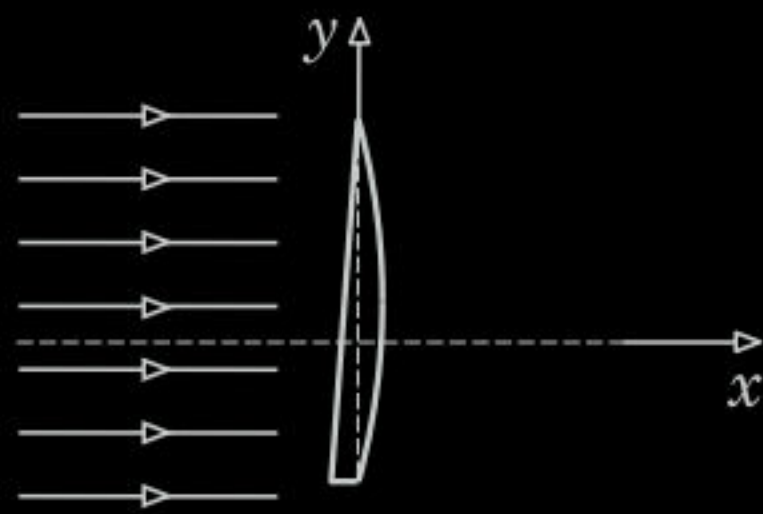
$$f_1 \quad f_2 \quad \equiv \quad f_{eq} = \frac{f_1 \cdot f_2}{f_1 + f_2}$$



Equation of line AB: $\frac{y - 5/2}{x - 0} = \frac{0 - 5/2}{20 - 0}$ ①

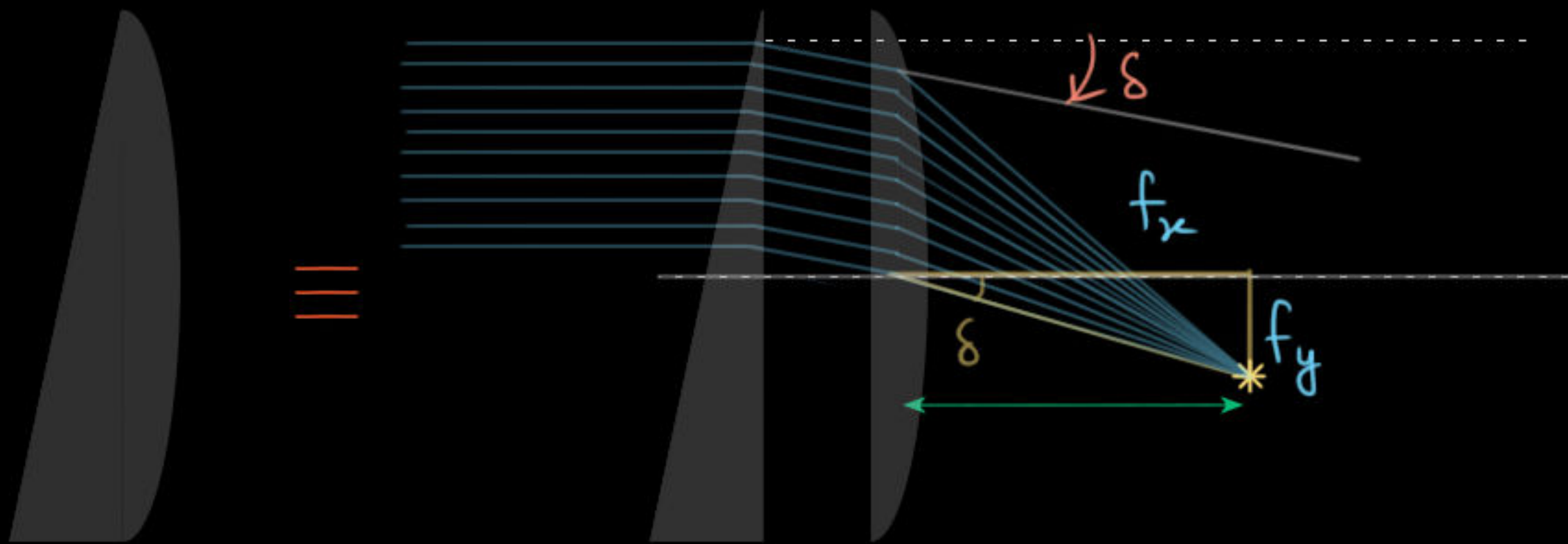
CD: $\frac{y - (-1/2)}{x - 0} = \frac{0 - (-1/2)}{20/3 - 0}$ ②

From 1, 2: $x = 15$



29. Due to manufacturing defect, plane surface of a thin plano-convex lens has been made tilted at a small angle α outwards from its usual place. The spherical surface has radius R and refractive index of the glass used is μ . A parallel beam of light is incident as shown in the figure. Will this defective lens focus the beam and if so, find coordinates of the focus?

Prism 1 deviates the parallel rays by δ .
Then rays will eventually converge on lens 2's focus.



$$\text{Of lens: } \frac{1}{f_x} = (\mu - 1) \left[\frac{1}{\infty} - \frac{1}{(-R)} \right] \Rightarrow f_x = \frac{R}{\mu - 1}$$

Prism deviation
 $\delta = (\mu - 1) \alpha$

$$\tan \delta = \frac{f_y}{f_x}$$

$$\Rightarrow (\mu - 1) \alpha = \frac{f_y}{(R/(\mu - 1))}$$

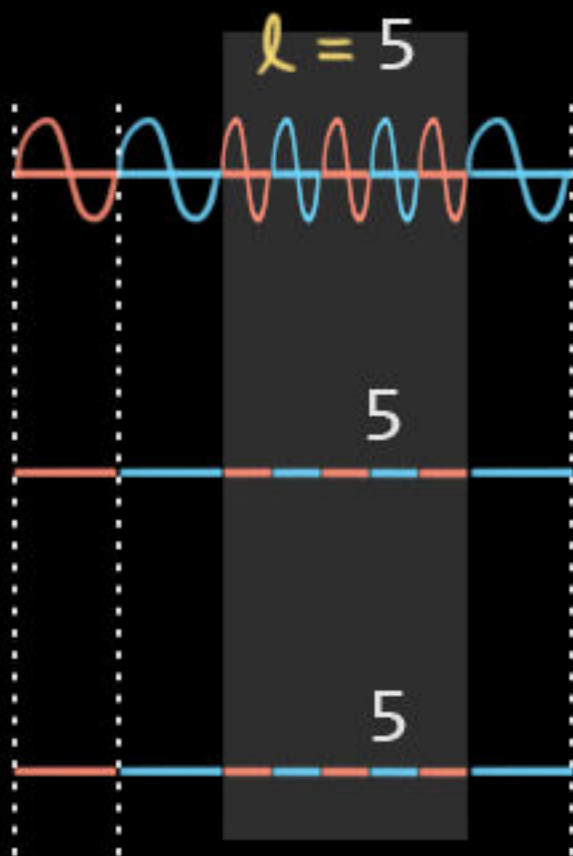
$$\Rightarrow f_y = R \alpha$$

We know, for a ray traveling in a medium, its equivalent path length in vacuum is $= \mu l$

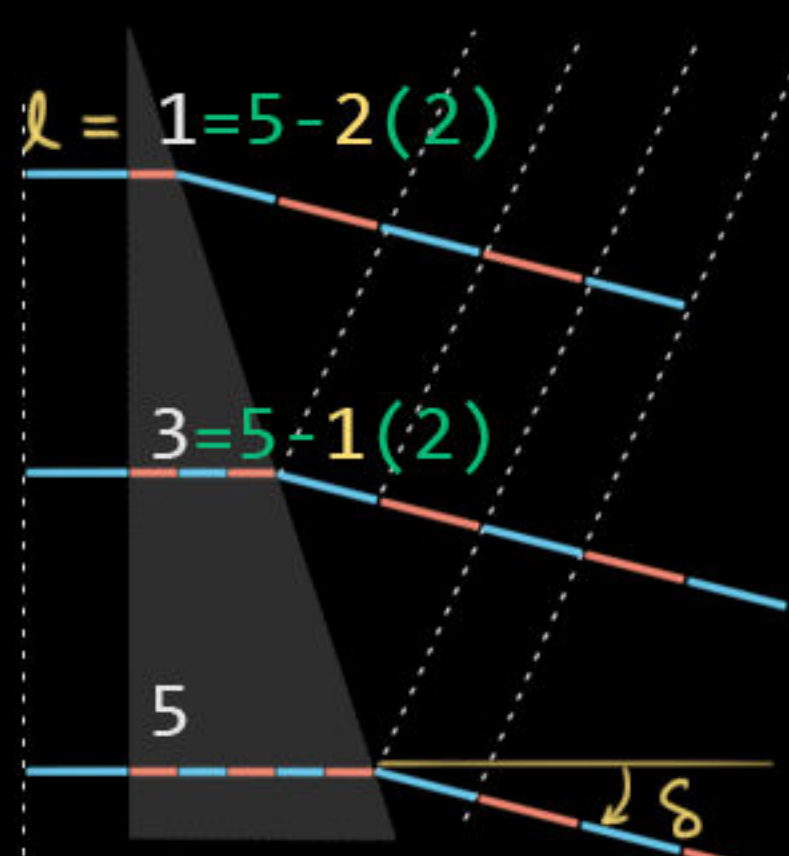
Regular lens

(μ is fixed, l varies)

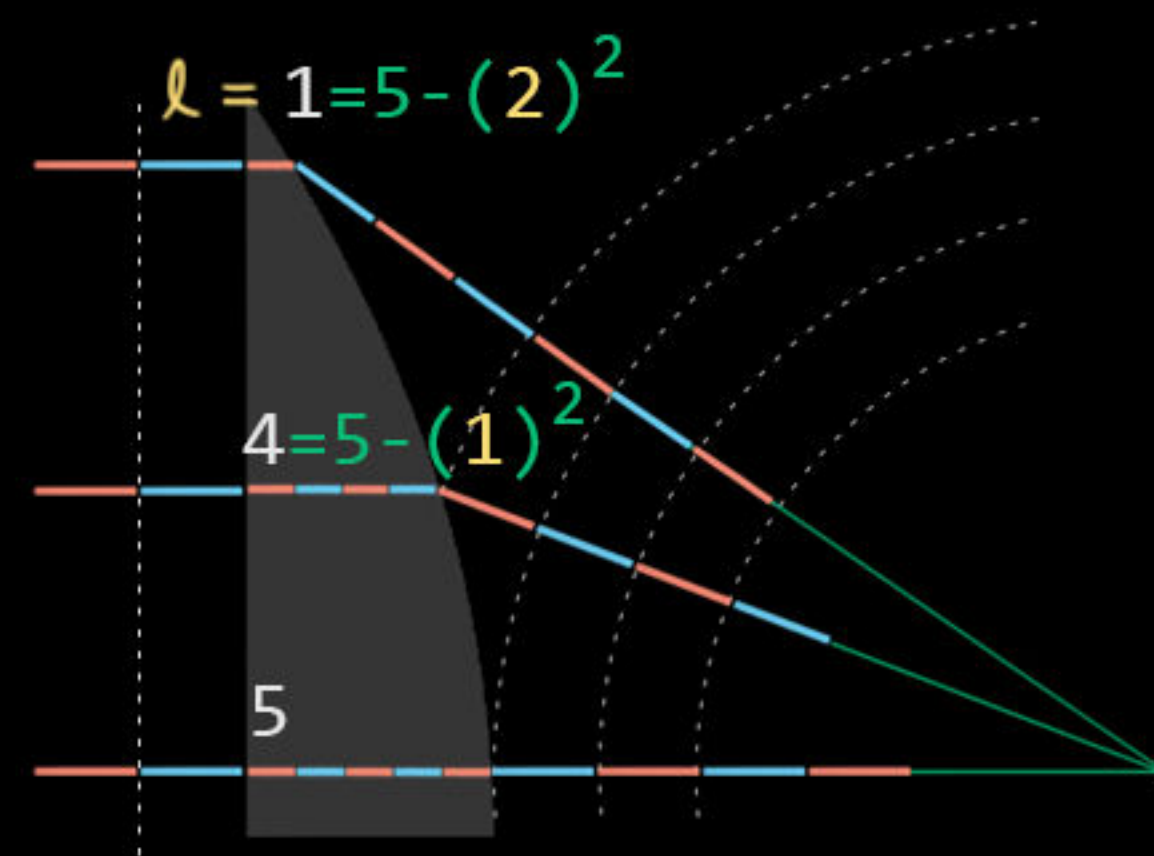
l is constant



l reduces linearly from bottom



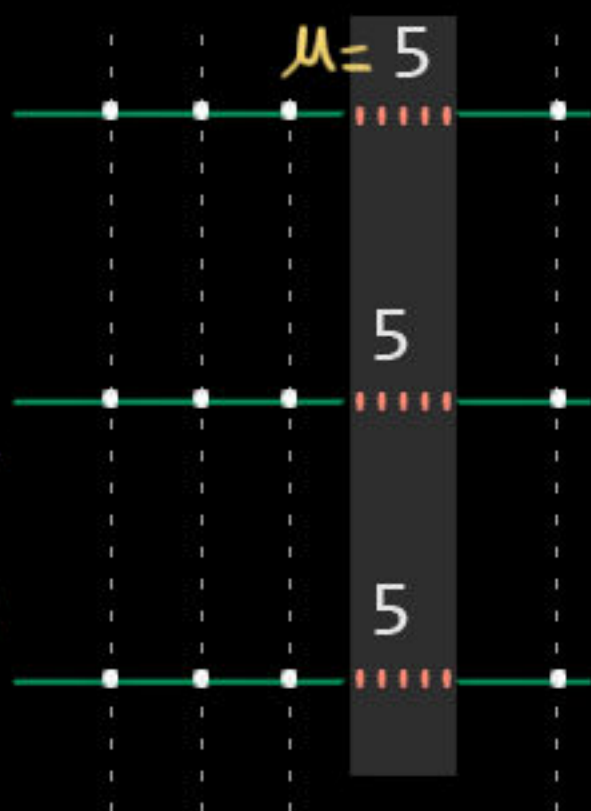
l reduces squarely from bottom



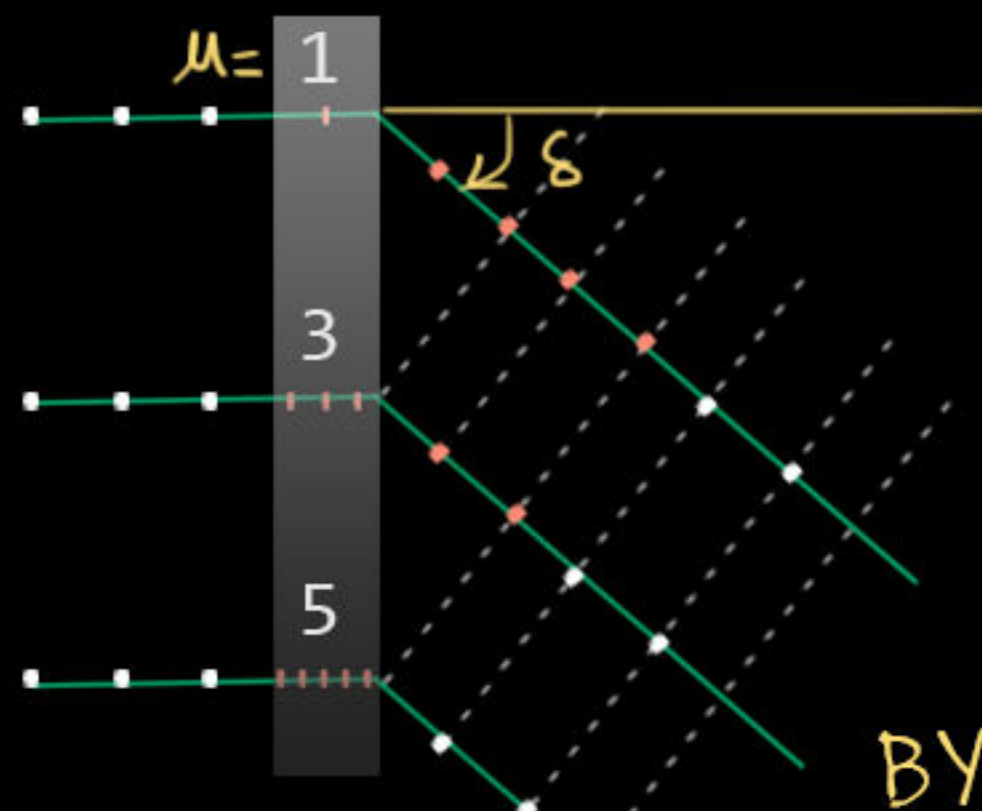
GRIN(Gradient index) lens

(l is fixed, μ varies)

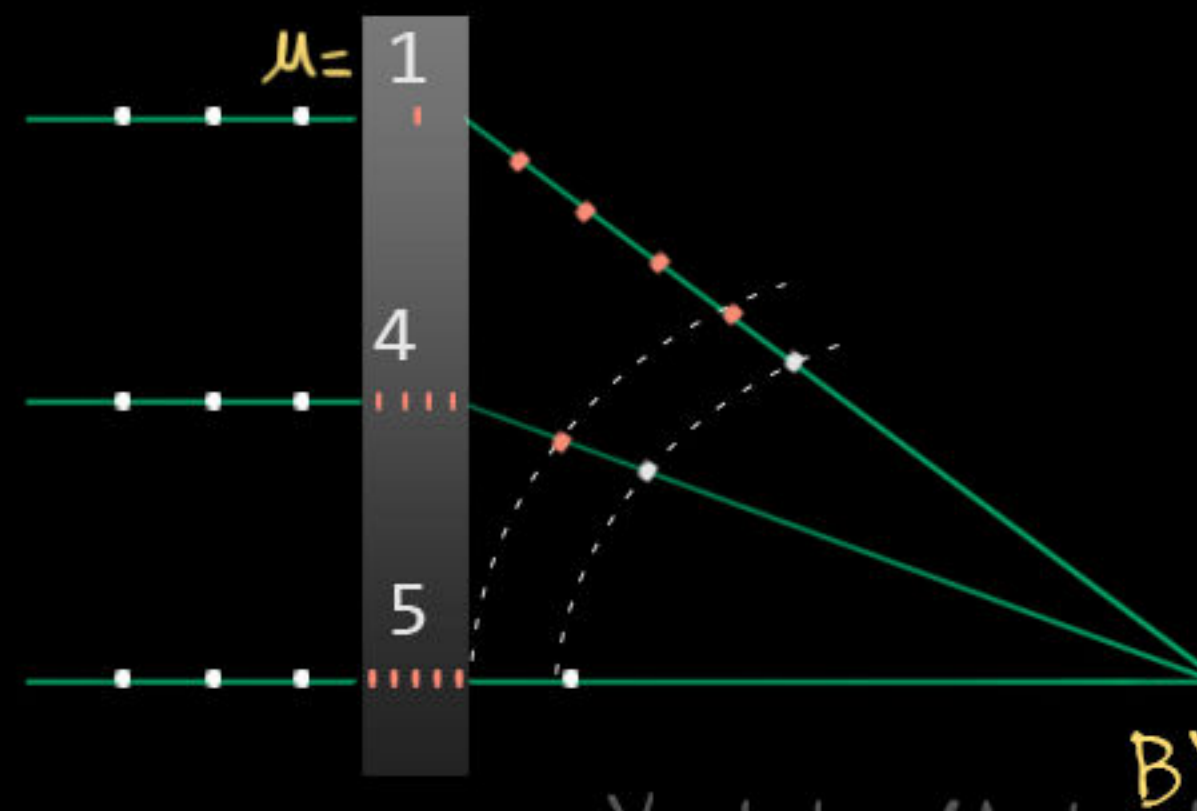
μ is constant



μ reduces linearly from bottom



μ reduces squarely from bottom

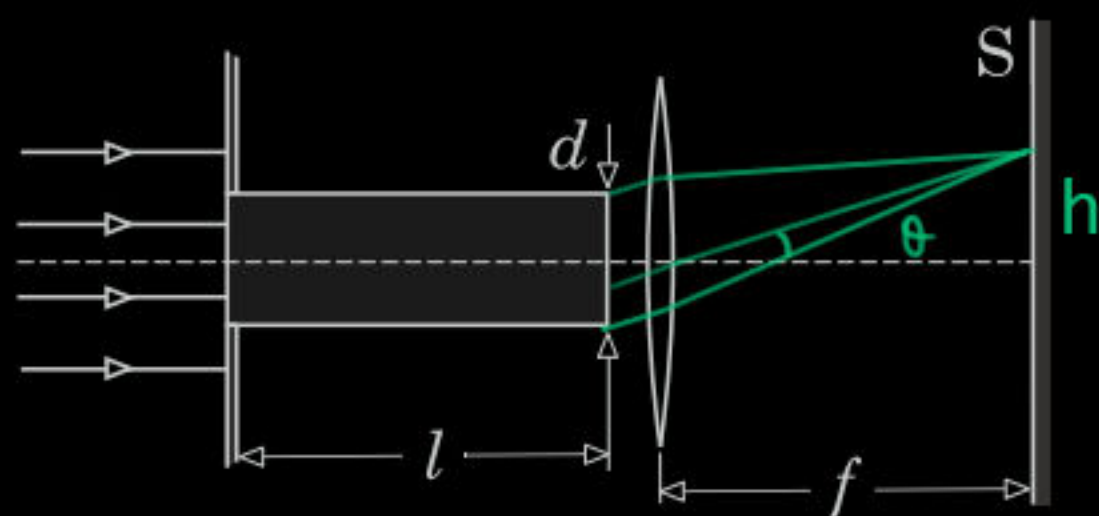


BYU 30

by Ankit Singhvi

BYU 31

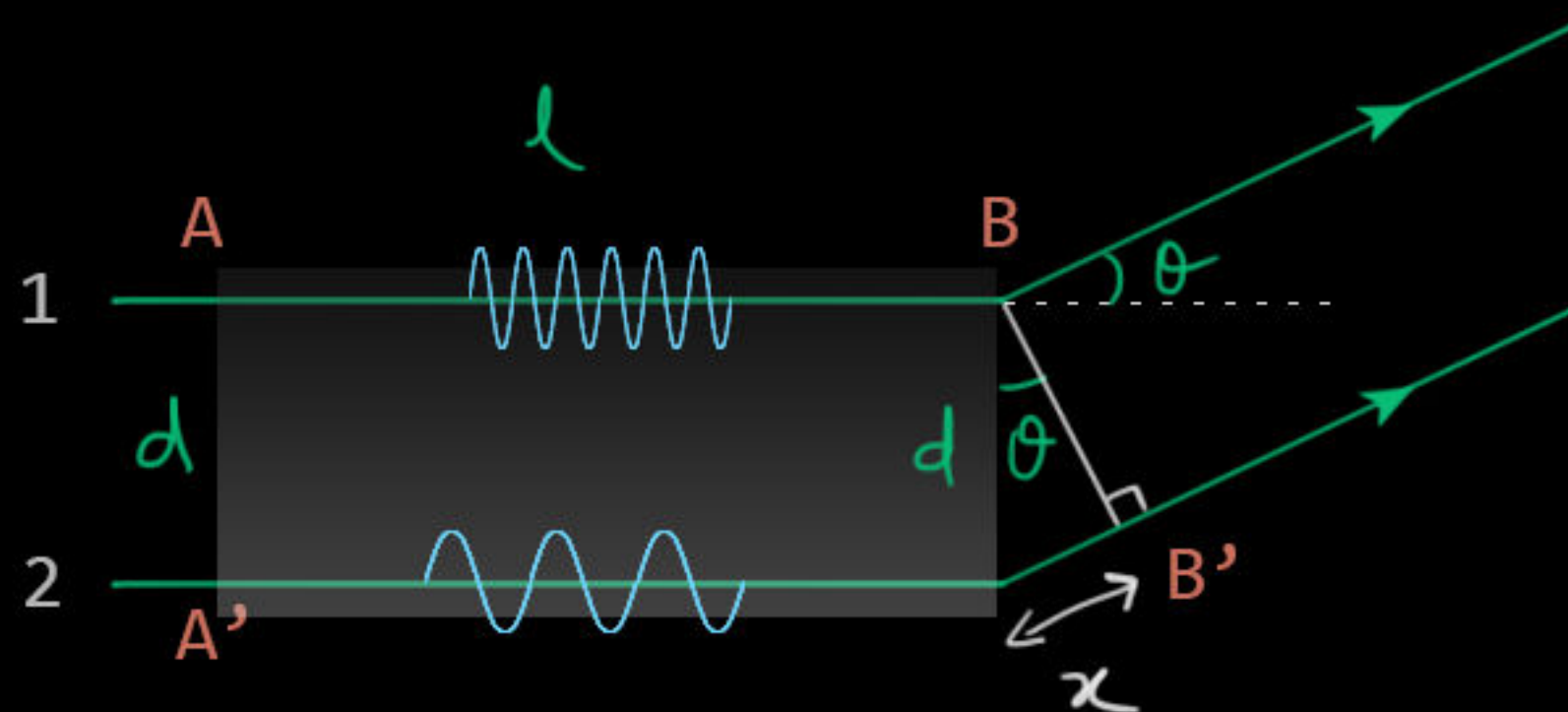
Youtube / Arihant Senior



30. A transparent rectangular slab of length $l = 20$ cm and thickness $d = 4.0$ cm is placed in front of a converging lens of focal length $f = 20$ cm. A screen S is placed in the focal plane of the lens. Refractive index of the material of the slab increases linearly from $\mu_0 = 1.5$ at the bottom face by an amount $\delta\mu = 2.0 \times 10^{-4}$ between the bottom and the top faces. A parallel beam is made to incident on the slab as shown in the figure. What kind of light pattern appears on the screen?

∴ The slab will act as a thin prism and therefore bend the beam up (by let's say θ)

⇒ After passing through lens, beam will converge to a point on focal plane.

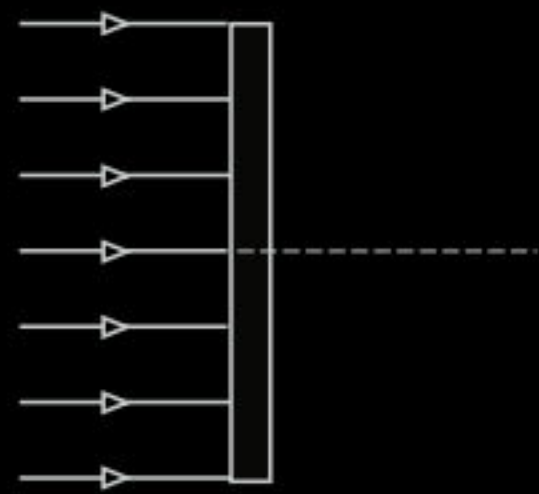


Optical path: $AB = A'B'$

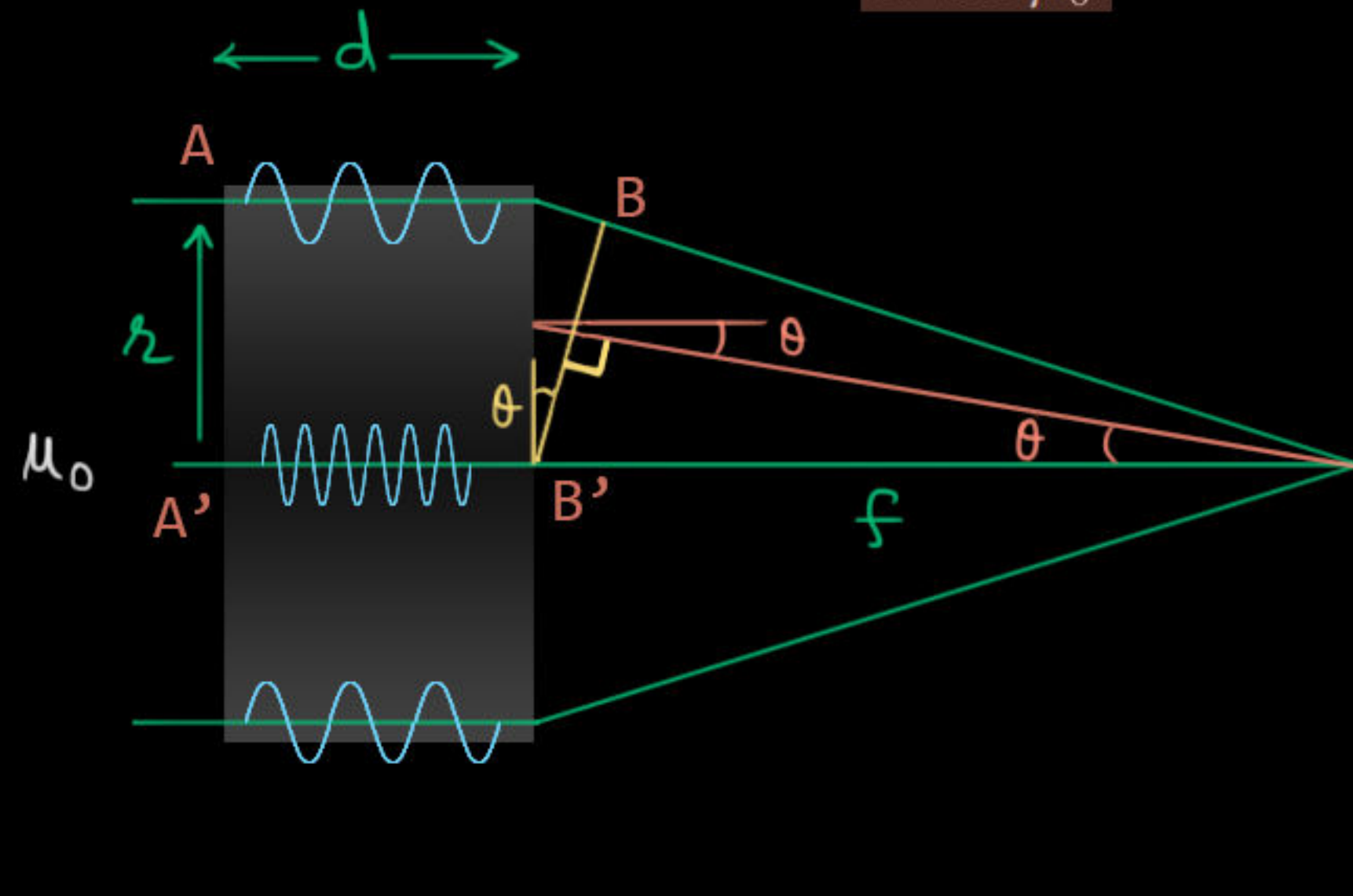
$$\Rightarrow \mu_1 l = \mu_2 l + x$$

$$\Rightarrow \sin \theta = \frac{(\mu_1 - \mu_2) l}{d} = \frac{\delta\mu \cdot l}{d}$$

$$h = f \tan \theta \approx f \sin \theta = \frac{f \cdot \delta\mu \cdot l}{d}$$



31. Refractive index of a material can be adjusted by diffusing impurities; moreover, by adjusting concentration distribution of diffused impurities, a specimen of any desired **gradient of refractive index** can be obtained. Consider a thin disc of uniform **thickness d** made of a transparent material, **refractive index of which is made to vary with distance r from its centre** by suitable diffusion of impurities. If a parallel beam of light incident normally on the disc, **converges at a distance f ($f \gg r$)** after passing through the disc, **find** suitable expression for **radial variation of its refractive index μ** . Assume refractive index at the **centre of the disc** to be μ_0 .

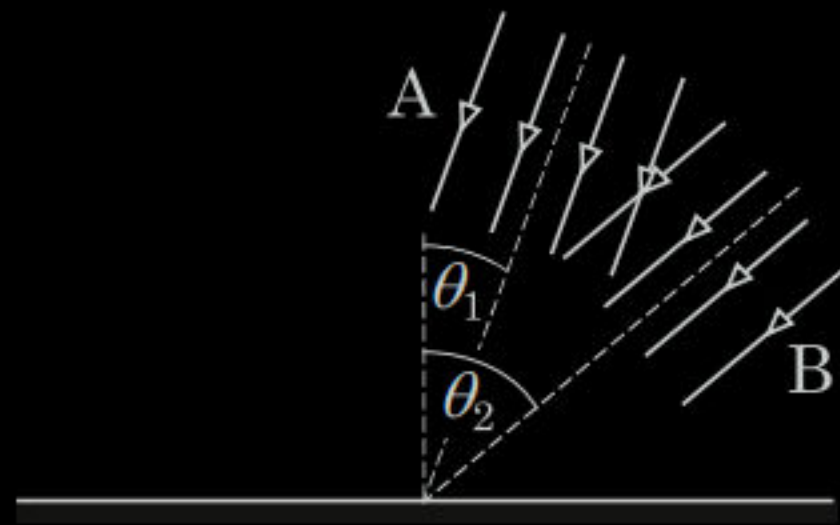


Optical path: $AB = A'B'$

$$\Rightarrow \mu_r d + r \sin \theta = \mu_0 d$$

$$\Rightarrow \mu_r = \mu_0 - r \frac{\sin \theta}{d} \approx \frac{r/2}{f}$$

$$= \mu_0 - \frac{r^2}{2df}$$



32. Two identical beams A and B of plane coherent waves of the same intensity and wavelength λ fall on a plane screen. The directions of the beam propagations make angles θ_1 and θ_2 with the normal to the screen and lie in the same plane as shown in the figure. Find distance between adjacent interference fringes on the screen.

Let A be a maxima.

As we move towards left, path lengths of both beams increase (by let's say a_1 and a_2)

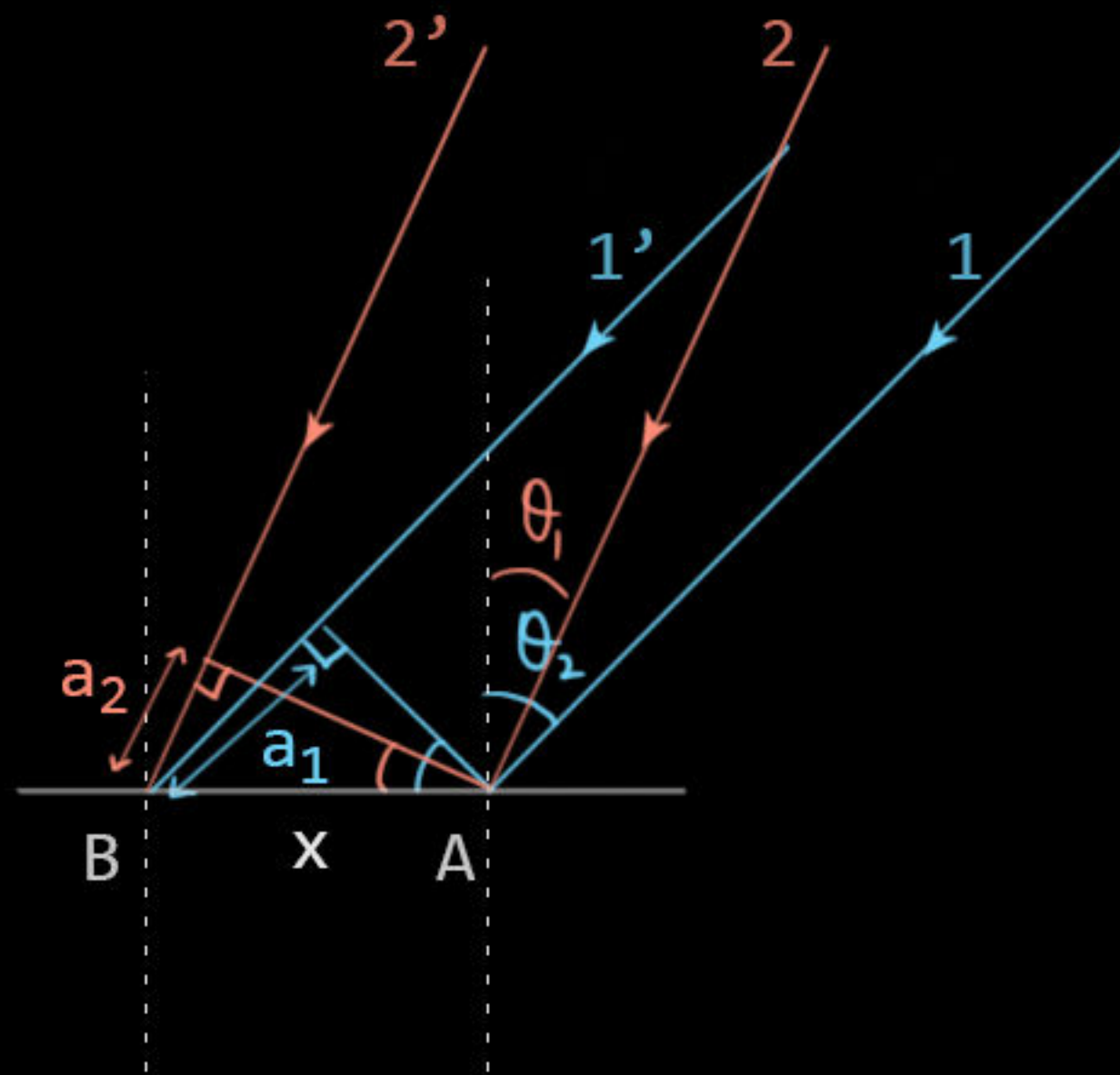
But path length of 1 increases more than 2's.

When that increment is λ , another maxima will be observed.

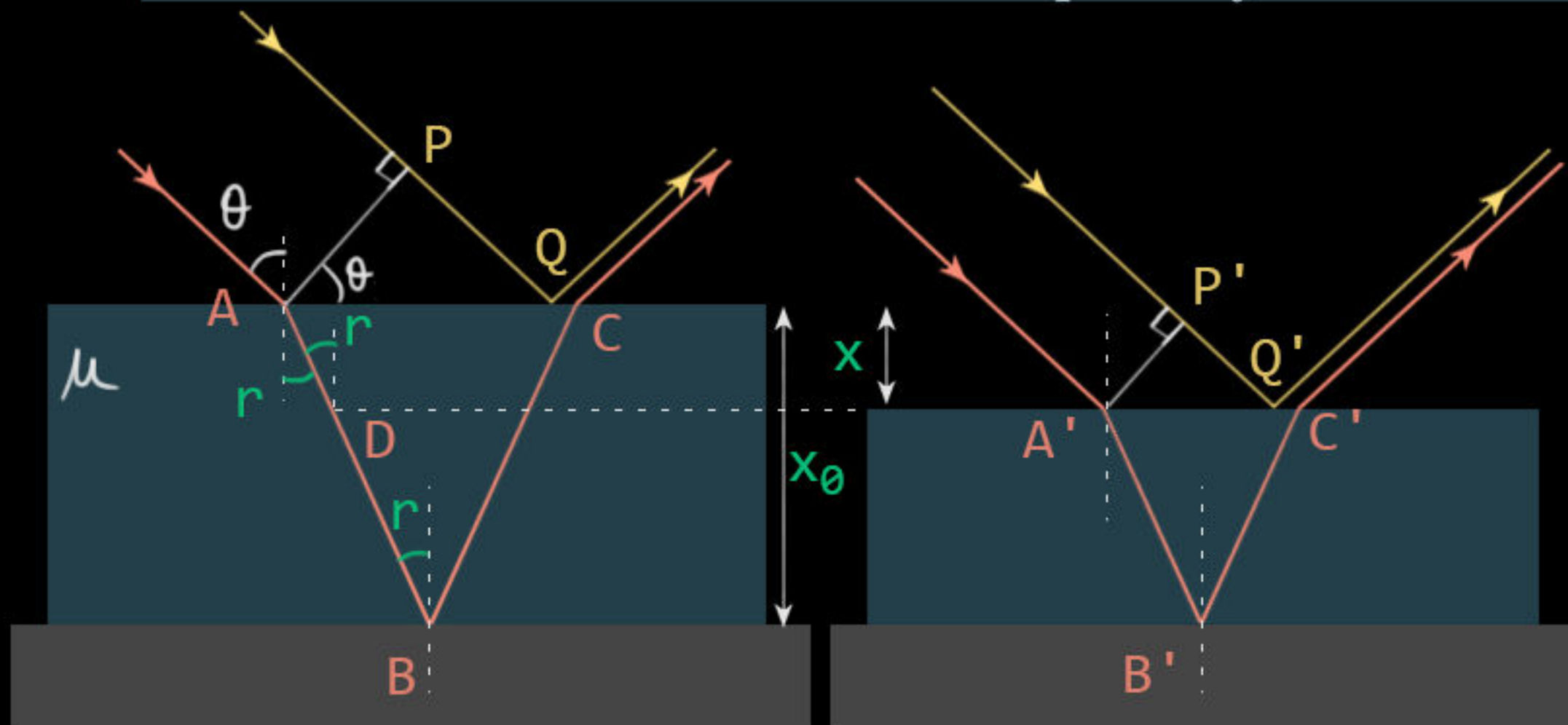
∴ At next maxima: $a_1 - a_2 = \lambda$

$$\Rightarrow x \sin \theta_2 - x \sin \theta_1 = \lambda$$

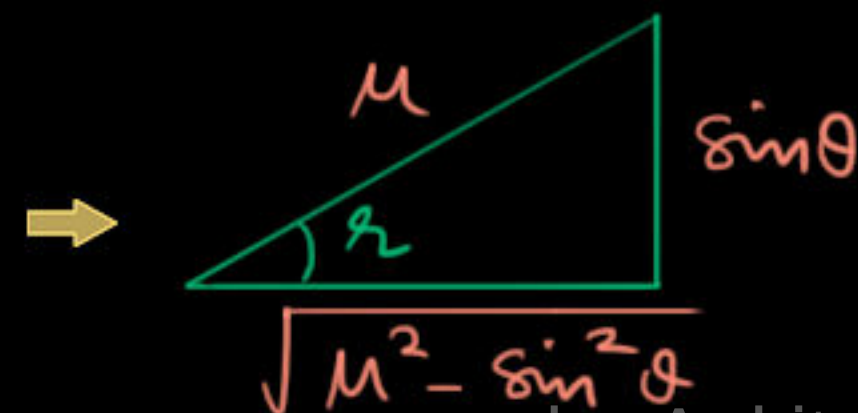
$$\Rightarrow x = \frac{\lambda}{\sin \theta_2 - \sin \theta_1}$$



34. A liquid of refractive index μ is poured on a large horizontal clear glass plate to make a thin layer of it. When a broad light beam is projected on the liquid layer at an angle θ with the vertical, an interference pattern is obtained. Due to evaporation, thickness of the liquid layer decreases making changes in the interference pattern. If in a time interval Δt , intensity of reflected light at a point decreases from a maxima to a minima and then increases to a maxima, find suitable expression for the time rate at which thickness of the liquid layer decreases. i.e $x/\Delta t$



$$\sin \theta = \mu \sin r$$



by Ankit Singhvi

$$ABC - PQ = n\lambda \quad (1)$$

$$A'B'C' - P'Q' = (n-1)\lambda \quad (2)$$

Subtracting:

$$(ABC - A'B'C') - (PQ - P'Q') = \lambda$$

$\underbrace{\mu(2AD)}_{\frac{x}{\cos r}} \quad \underbrace{(AC - A'C') \sin \theta}_{2x_0 \tan r \quad 2(x_0 - x) \tan r} = \lambda$

$$\Rightarrow \frac{2\mu x}{\cos r} - 2x \tan r \sin \theta = \lambda$$

$$\Rightarrow \frac{2\mu x \mu}{\sqrt{\mu^2 - \sin^2 \theta}} - 2x \frac{\sin^2 \theta}{\sqrt{\mu^2 - \sin^2 \theta}} = \lambda$$

$$\Rightarrow 2x \sqrt{\mu^2 - \sin^2 \theta} = \lambda$$

$$\Rightarrow \frac{x}{\Delta t} = \frac{\lambda}{2\sqrt{\mu^2 - \sin^2 \theta} \Delta t}$$

Youtube / Arinani Senior

1. When identical point charges are placed at the vertices of a cube of edge length a , each of them experiences a net force of magnitude F . Now these charges are placed on the vertices of another cube of edge length b . What will magnitude of the net force on any of the charges be? These cubes are simply geometrical constructs and not made of any matter.

(a) $\frac{aF}{b}$

(b) $\frac{bF}{a}$

✓ (c) $\frac{a^2 F}{b^2}$

(d) $\frac{b^2 F}{a^2}$

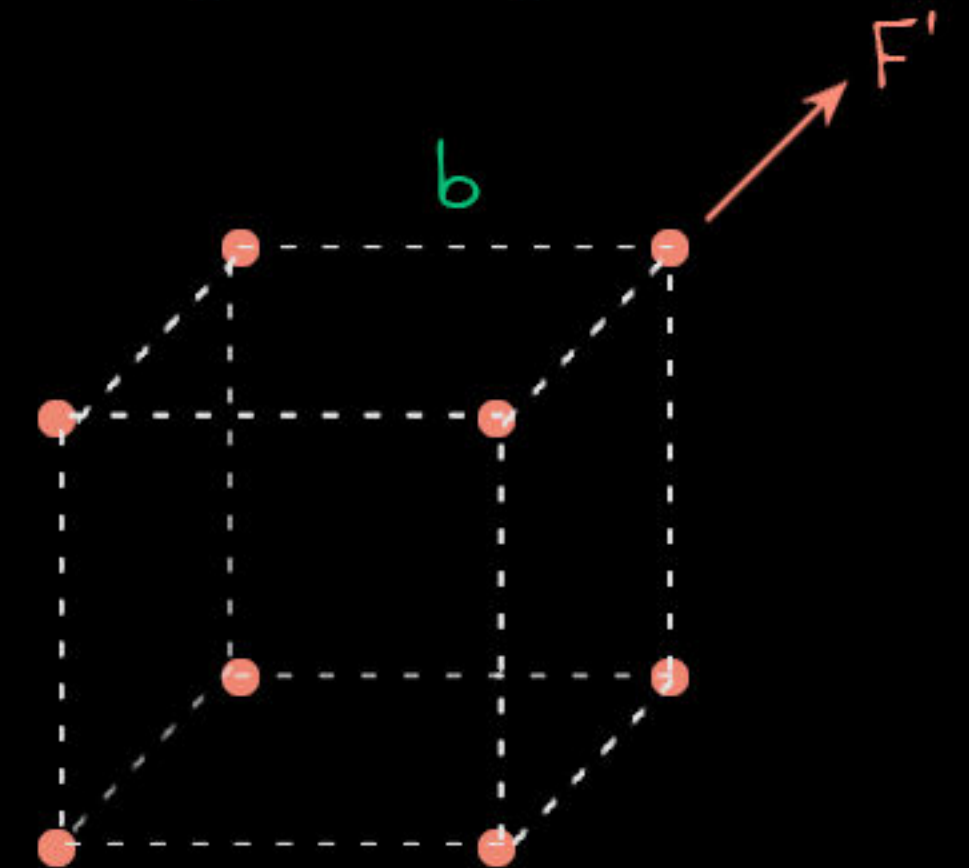
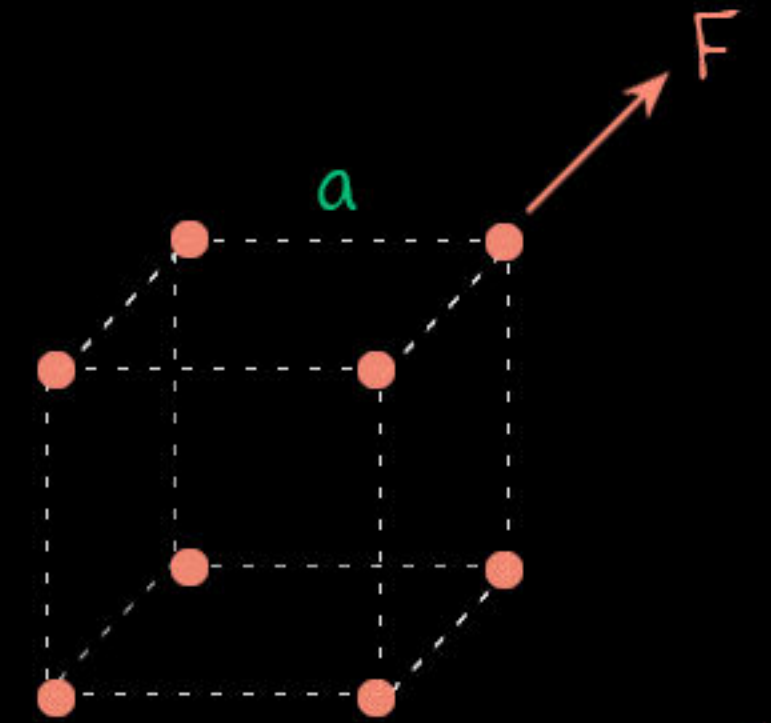
Scaling:

For all similar geometrical constructs: $F \propto \frac{q^2}{r^2}$, $E \propto \frac{q}{r^2}$, $V \propto \frac{q}{r}$

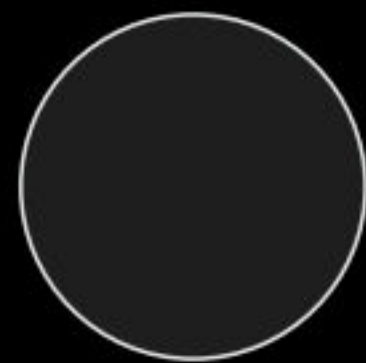
ex. cubes, cones, hemispheres etc.

Any length parameter
ex. side or diagonal of cube,
radius or height of cone,
radius of hemisphere etc.

Sol:



$$\frac{F}{F'} = \frac{q^2/a^2}{q^2/b^2} \Rightarrow F' = \frac{a^2 F}{b^2}$$



2. Two charged spheres are kept at a finite centre-to-centre spacing as shown in the figure. The force of electrostatic interaction between them is first **calculated assuming them point** like charges at their respective centres **and then force is measured experimentally**. If the calculated and the measured values are F_c and F_m respectively, which of the following **conclusion** can you certainly draw?

- (a) If $F_c > F_m$ for like charges and $F_c < F_m$ for unlike charges, both the spheres must be made of insulating materials.
- (b) If $F_c > F_m$ for like charges and $F_c < F_m$ for unlike charges, both the spheres must be made of conducting materials.
- (c) Irrespective of their materials, $F_c < F_m$ for like charges and $F_c > F_m$ for unlike charges.
- ✓ (d) Irrespective of their materials, $F_c > F_m$ for like charges and $F_c < F_m$ for unlike charges.

Concept: Lets keep the charge at center and let the charges shift or molecules polarise (depending on conducting or insulating material.)

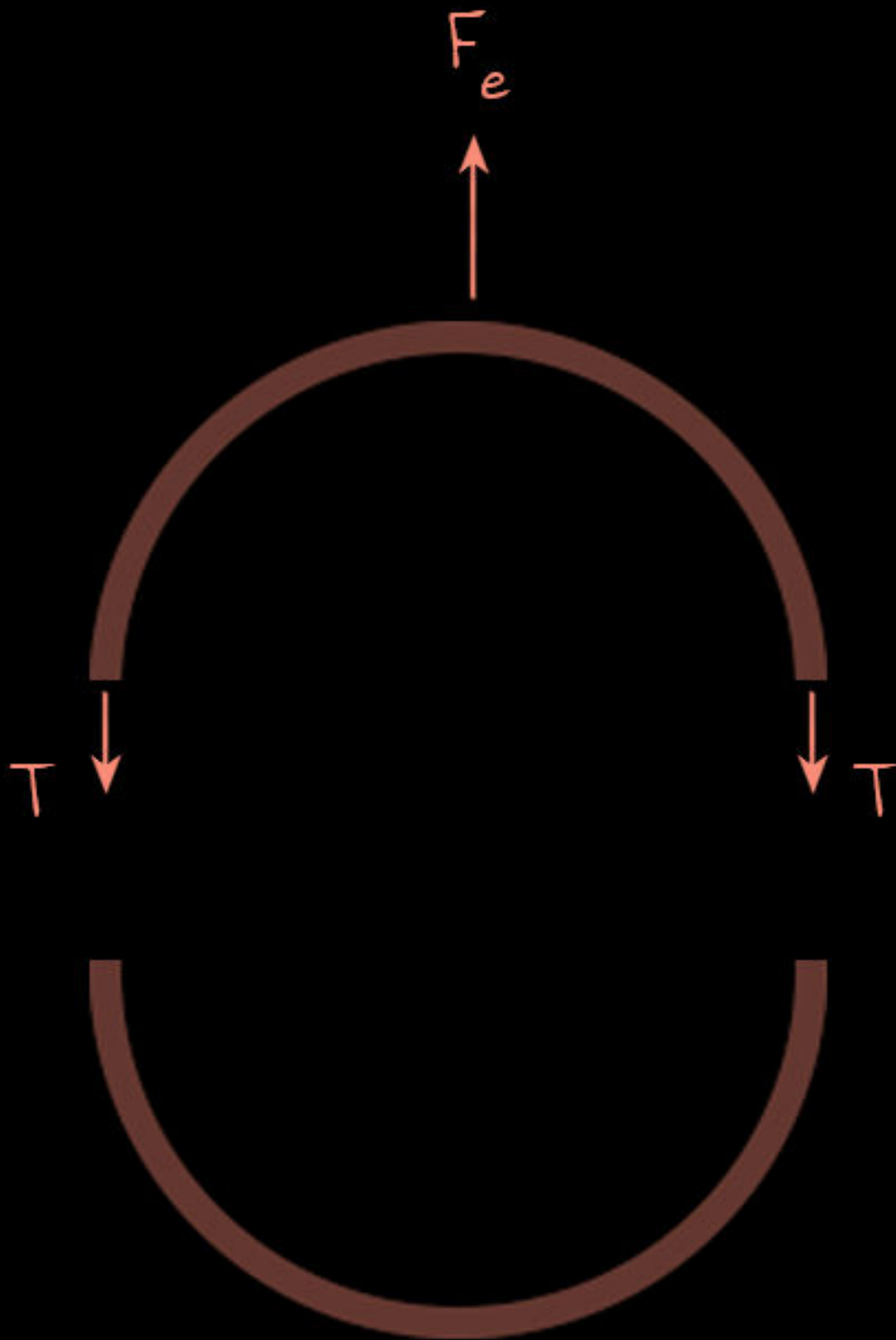
This shifting will lower the energy of system.

In two like charged bodies, lower energy implies less repel force.

And in unlike charged bodies, lower energy implies more attractive force.

3. A thin conducting ring is ruptured when it is given a charge q . Consider another thin conducting ring, radius of which is n times and tensile strength is k times of the former ring. How much maximum charge can this second ring be given without rupturing?

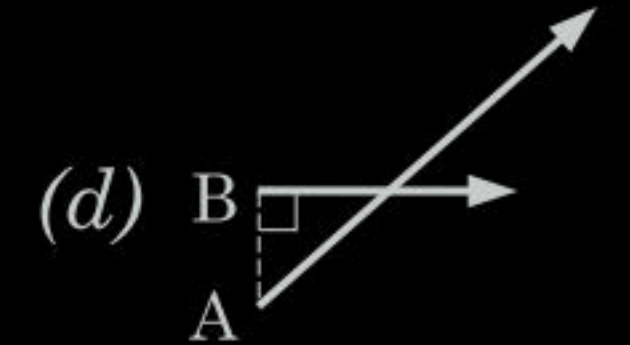
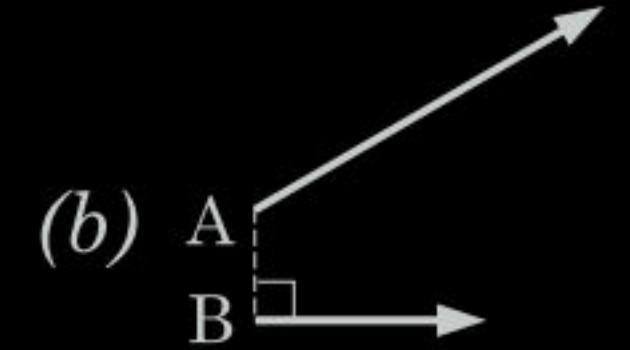
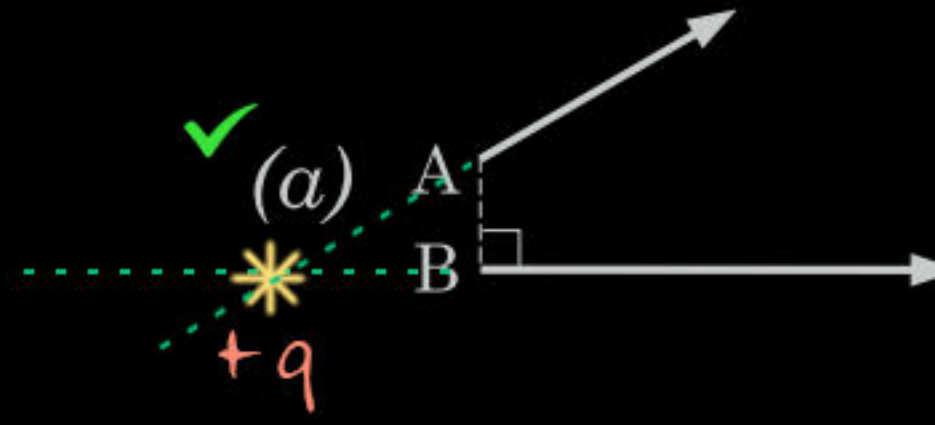
- (a) $< qnk$ (b) $< qn\sqrt{k}$
 (c) $< qn^2\sqrt{k}$ (d) Insufficient information

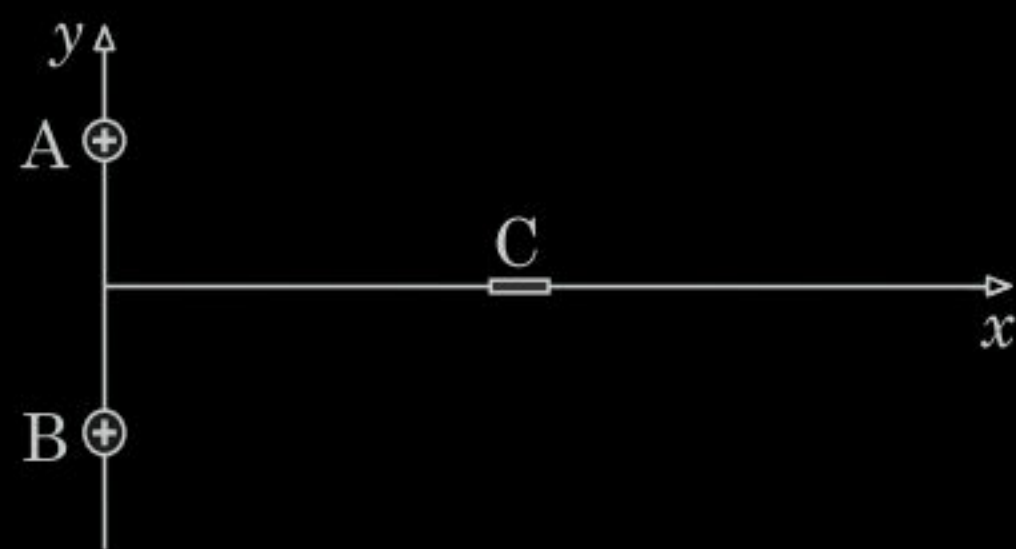


$$2T = F_e \propto \frac{q^2}{r^2} \begin{cases} T_0 = C \frac{q^2}{r^2} \\ kT_0 = C \frac{q'^2}{(nr)^2} \end{cases}$$

Dividing: $q' = qn\sqrt{k}$

4. Each of the following figures shows electric field vectors at two points A and B in an electric field. In which figure or figures can the illustrated field be created by a single point charge?





5. Two equal point-like positive charges A and B are fixed on the y-axis equidistant from the origin. What can you say for the net electrostatic force on a small electrically neutral rod placed on the x-axis as shown in the figure?

(a) No force at all.

(c) It points towards the origin.

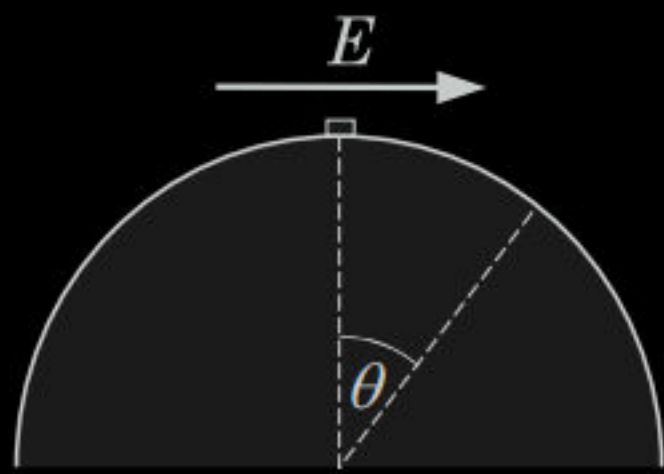
(b) It points away from the origin.

✓ (d) Information is insufficient.

Need not be uniform.



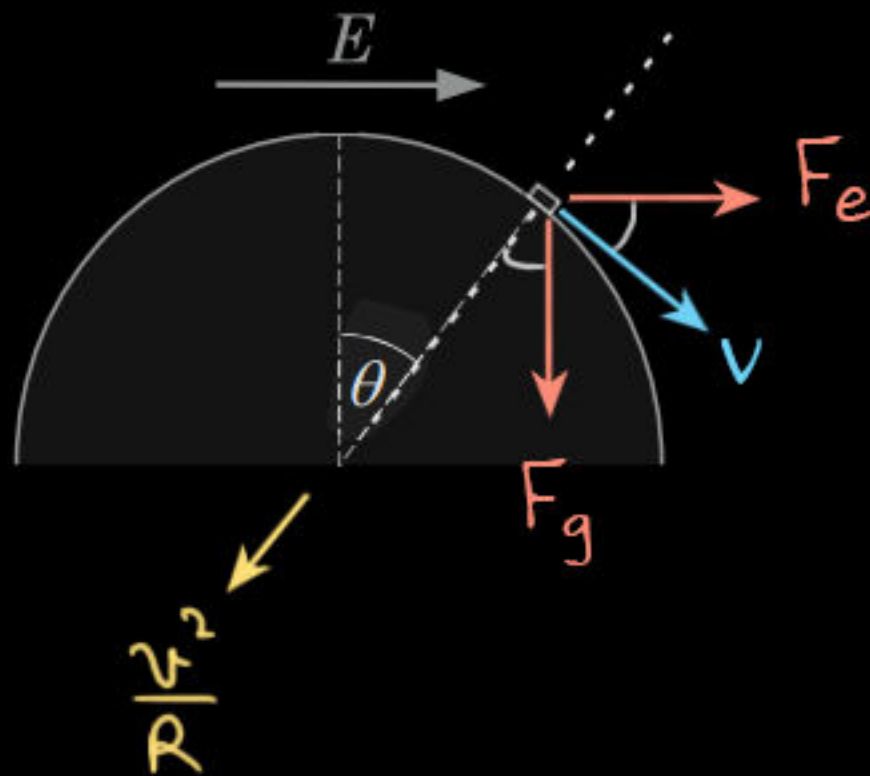
So it can be a small dipole orientation of which will determine the direction of force.



6. A positively charged small disc is released on the top of fixed hemispherical frictionless dome in presence of a uniform horizontal electric field. If the disc leaves the dome after an angular displacement $\theta = \sin^{-1}(3/5)$, find ratio of gravitational and electrostatic forces on the disc. Assume that the dome does not exhibit any electrical property.

(a) $4/3$
(c) $9/2$

(b) $3/4$
(d) 12

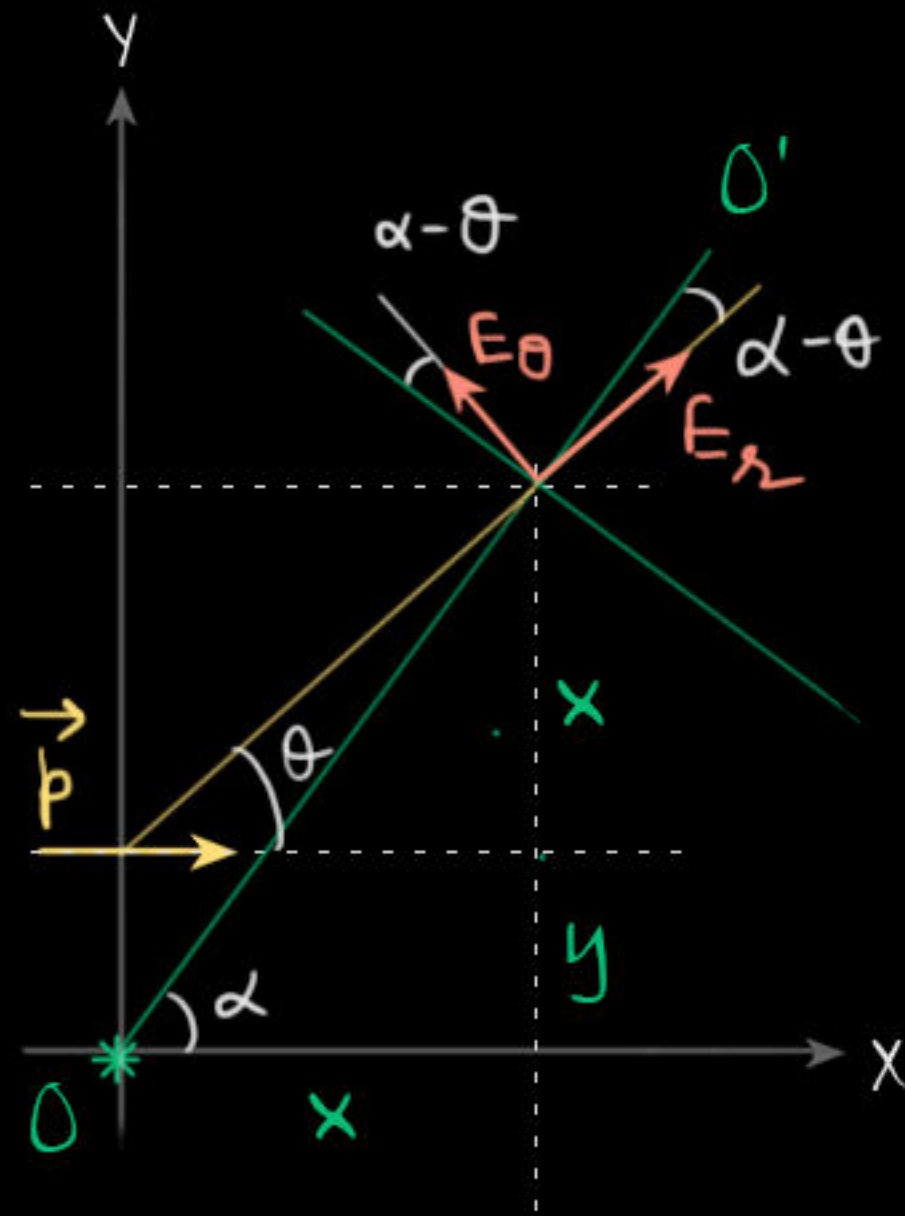


Force: $f_g \cos \theta - f_e \sin \theta = \frac{mv^2}{r}$ ①

Energy: $f_e r \sin \theta + f_g r (1 - \cos \theta) = \frac{1}{2} mv^2$ ②

Dividing: $\frac{f_g}{f_e} = \frac{3 \sin \theta}{3 \cos \theta - 2} = \frac{3(3/5)}{3(4/5) - 2} = \frac{9}{2} //$

Lets show the field due to dipole alone.



7. If a dipole of dipole moment $p\hat{i}$ is placed at point $(0, y)$ and a point charge at the origin of a coordinate system, net electric field at point $(x, x+y)$ vanishes. If x and y both are positive, the coordinate y is equal to

(a) x

(b) $2x$

(c) $2.5x$

(d) $3x$

$$\therefore E_2 \sin(\alpha - \theta) = E_\theta \cos(\alpha - \theta)$$

$$\therefore \tan \theta = 1$$

$$\Rightarrow \frac{2kp \cos \theta}{r^3} \sin(\alpha - \theta) = \frac{kp \sin \theta}{r^3} \cos(\alpha - \theta)$$

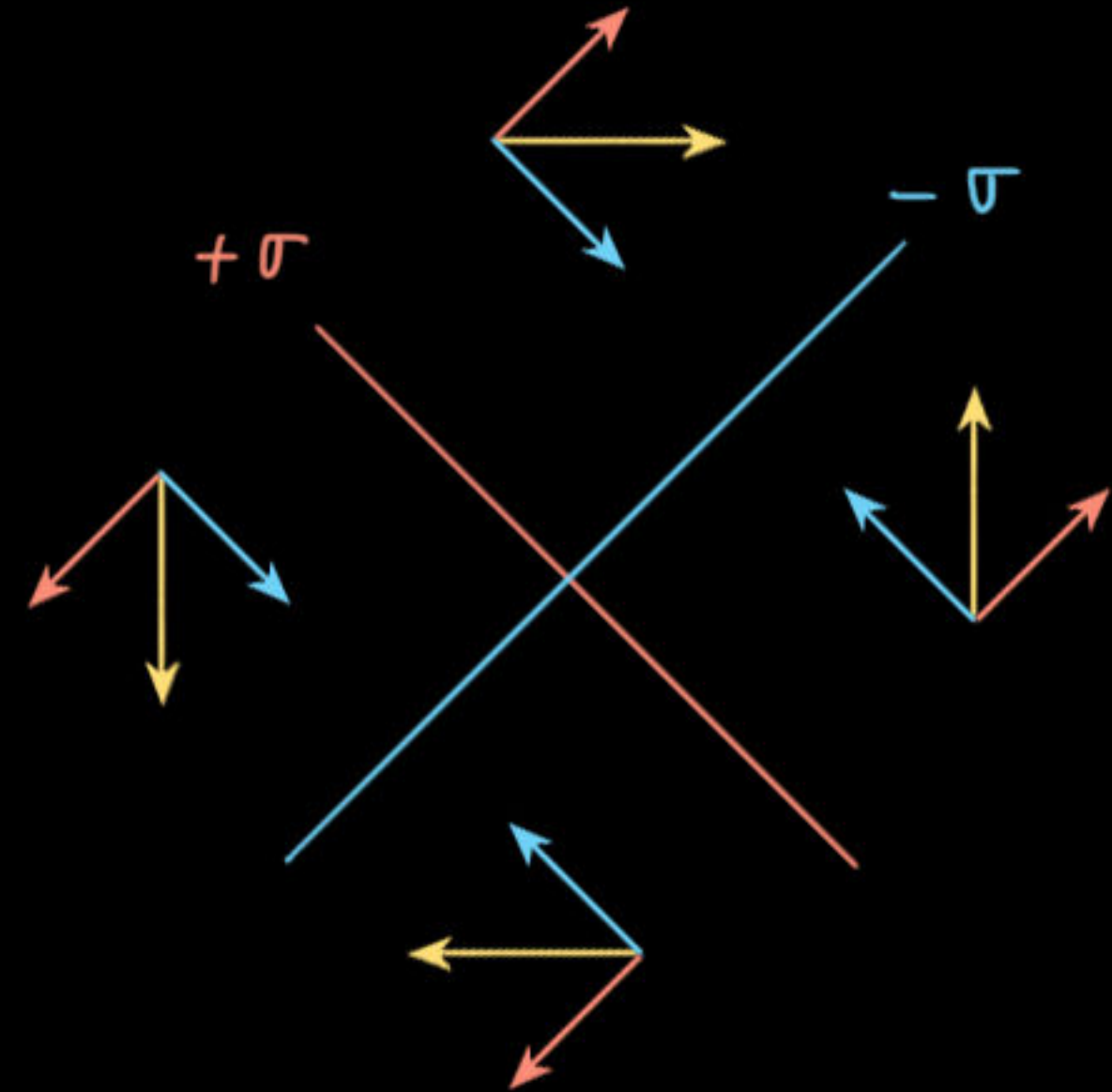
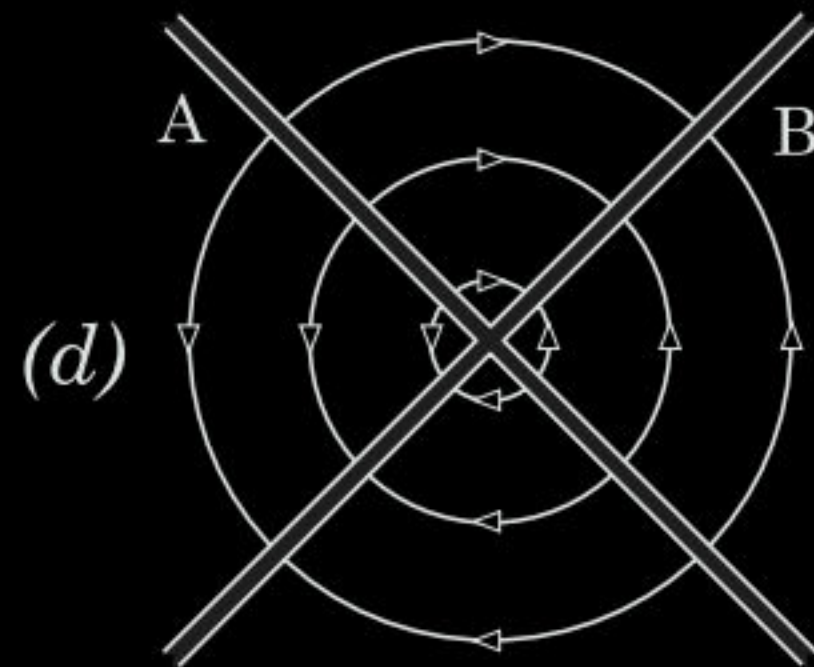
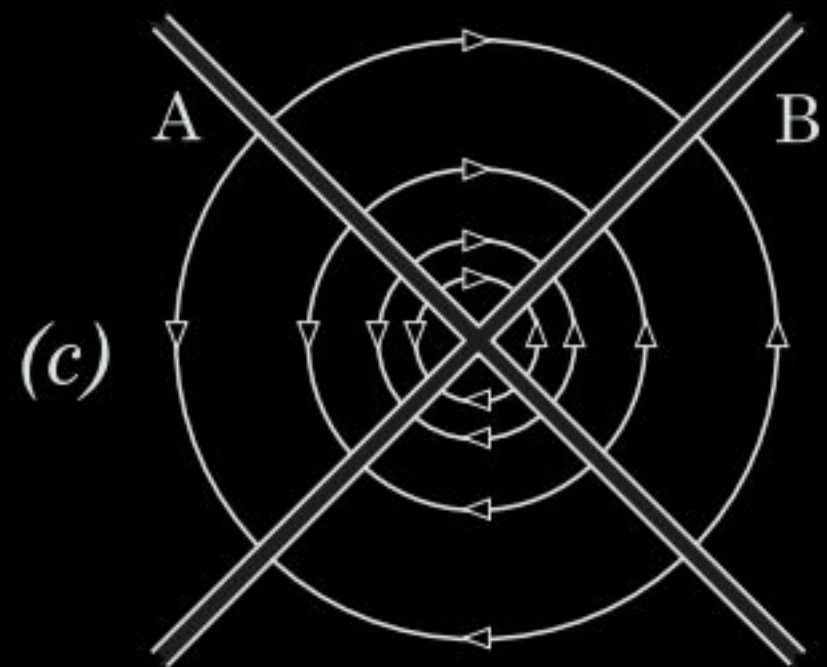
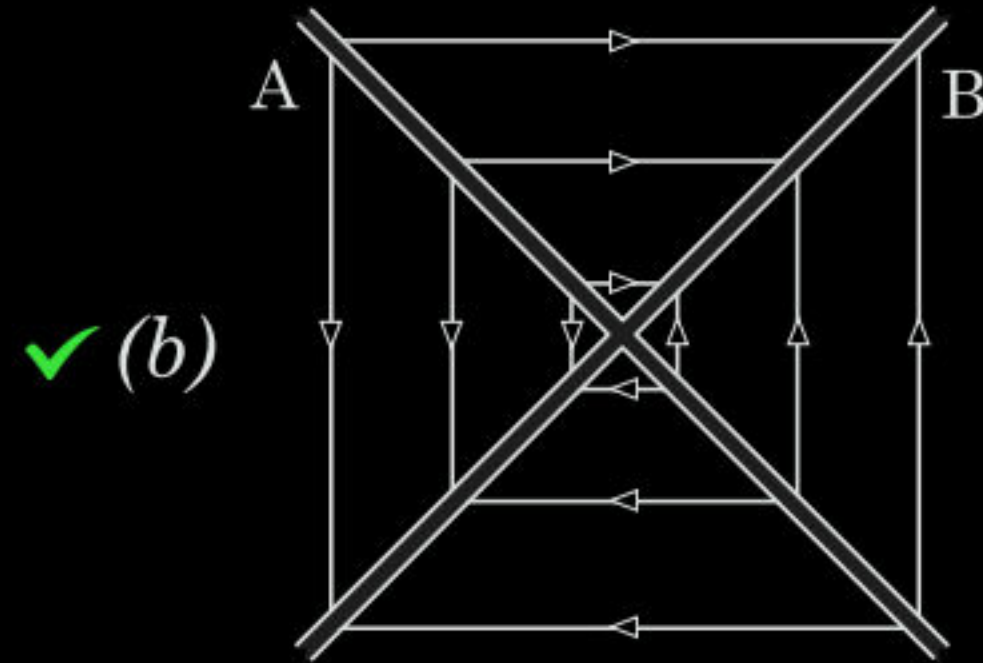
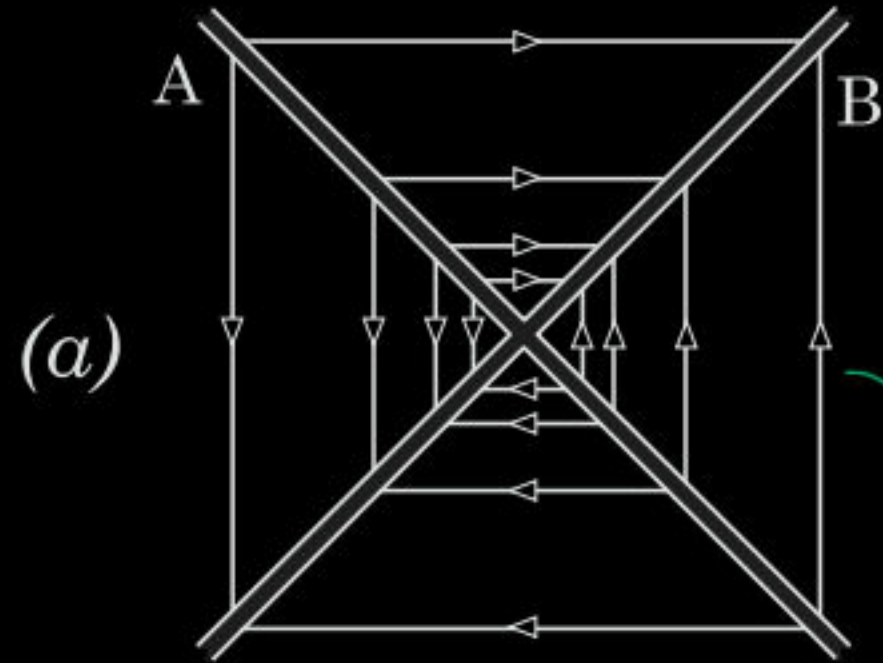
$$\Rightarrow 2 \frac{\tan \alpha - \tan \theta}{1 + \tan \alpha \tan \theta} = 1$$

$$\Rightarrow 2 \frac{\frac{x+y}{x} - 1}{1 + \frac{x+y}{x}} = 1 \Rightarrow y = 2x //$$

Concept: Field along OO' can be balanced by adjusting the charge at O .

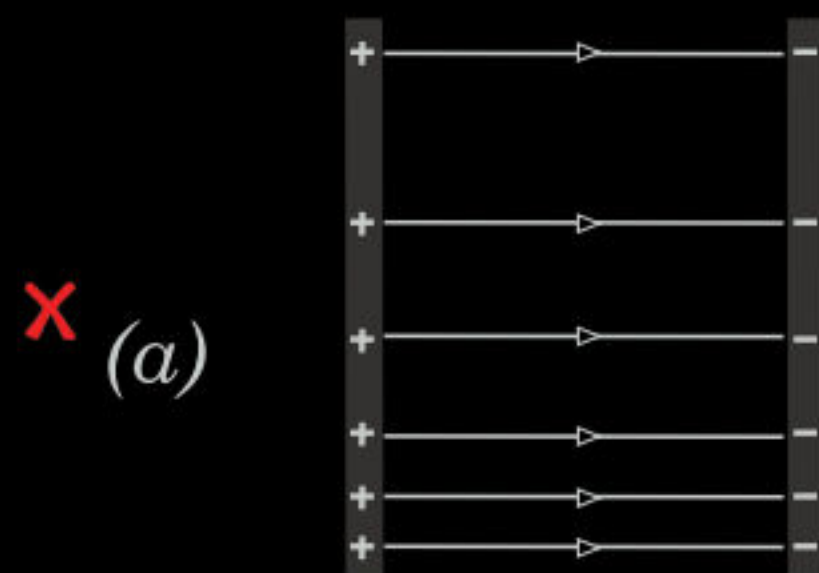
But field perpendicular to OO' are only due to dipole and must be balanced regardless of charge at O .

8. Two infinitely large planes A and B intersect each other at right angles and carry uniform surface charge densities $+\sigma$ and $-\sigma$. Which of the following figures best represents electric field lines?

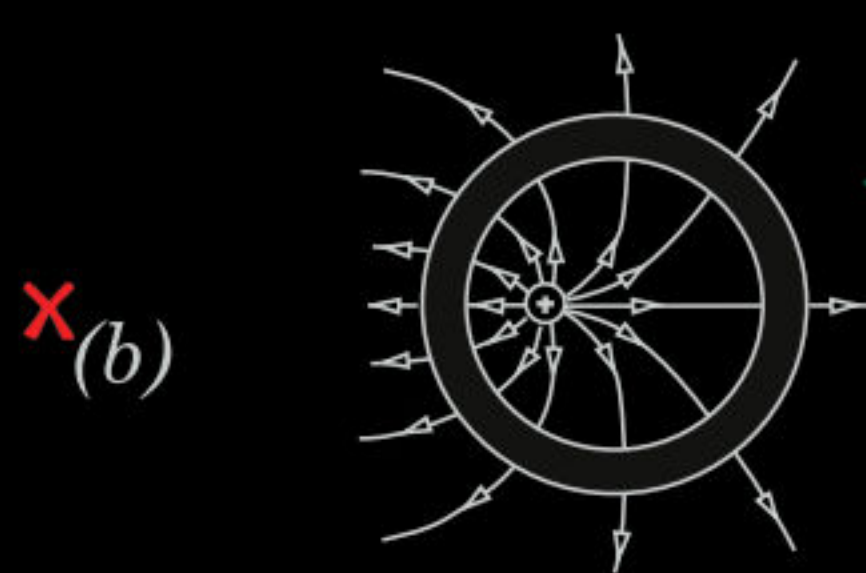


Parallel field lines have to be equally spaced (As Electric field is conservative). Here, they are not. So not possible.

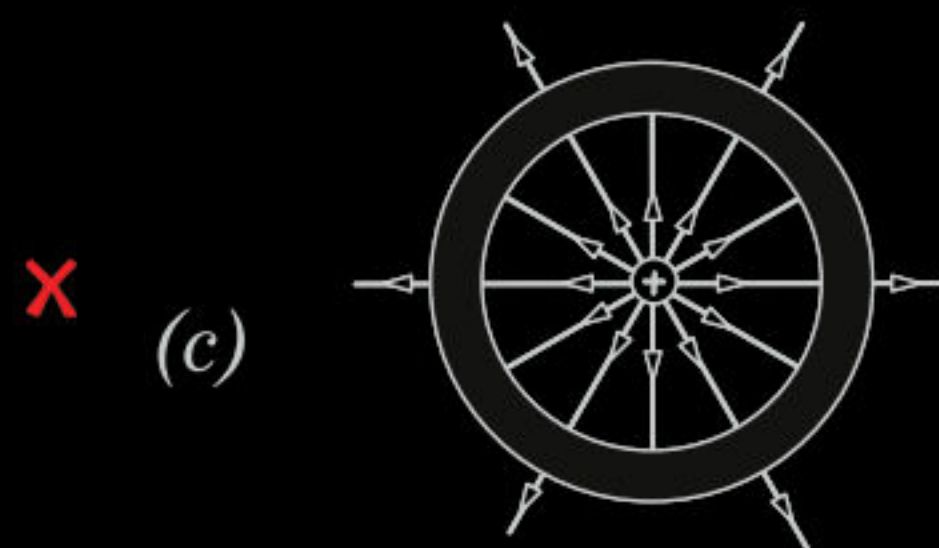
9. In the following figures, electric field lines of some electrostatic fields are shown. Which of them are incorrect representations?



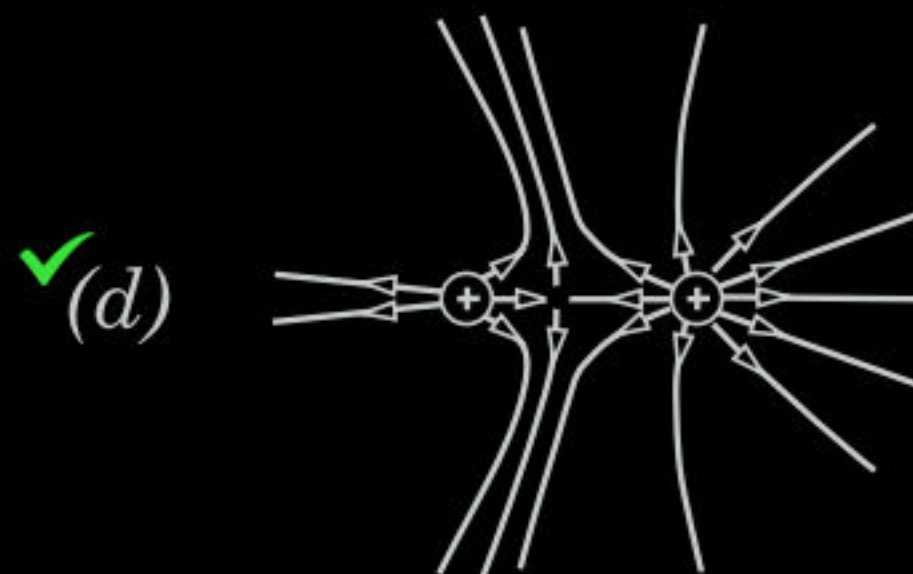
Two infinitely large charged layers



Point charge in a neutral conducting shell



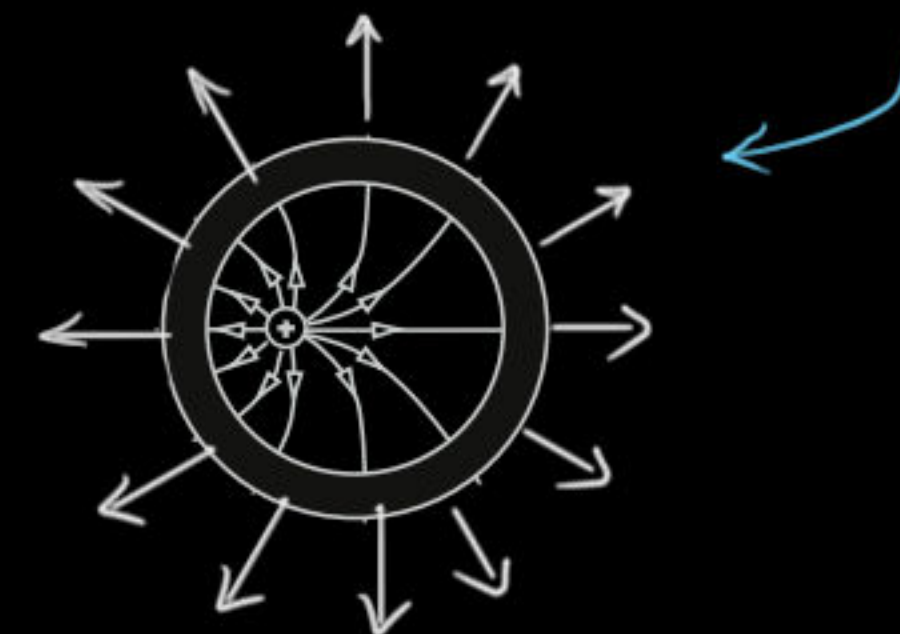
Point charge in a neutral conducting shell



Two unequal point charges

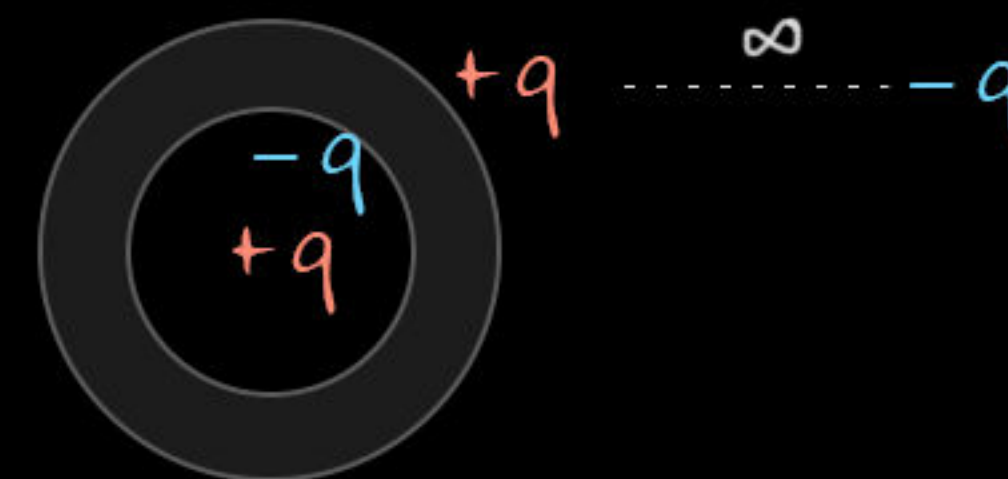
Parallel field lines have to be equally spaced (As Electric field is conservative). Here, they are not. So not possible.

Inside cavity field lines can be correct. But outside conductor it will be as shown:

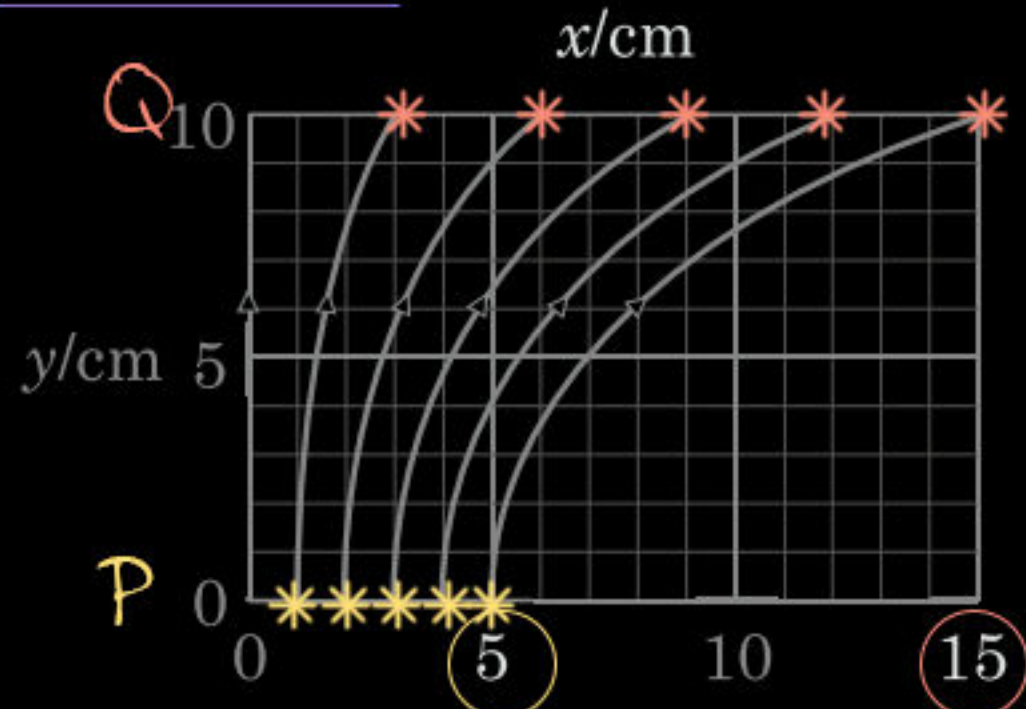


Number of lines inside and outside will be equal too (12)

∴ Between $+q$ and $-q$, number of lines have to be same.



Front view:



Top view:



10. Electric field lines of a portion of an electric field that is symmetrical with respect to rotation about y-axis are shown in the figure. A charge particle stays in equilibrium at point (0, 10). If modulus of charge of the particle is changed gradually till it shifts to another equilibrium position (0, 0), estimate fractional change in the charge modulus.

- (a) 1/3
(c) 1/9

- (b) 2/3
(d) 8/9

Concept:

$$\text{Field} \propto \frac{\text{Number of field lines}}{\text{Area}}$$

$$E_p = \frac{C \cdot 5}{\pi (5)^2}$$

$$E_q = \frac{C \cdot 5}{\pi (15)^2}$$

$$\Rightarrow E_p = 9 E_q \quad (1)$$

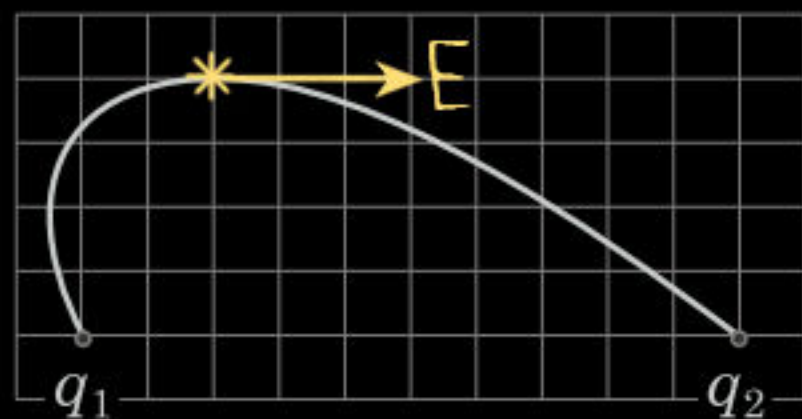
$$\text{Initially: } mg = q E_q$$

$$\text{Finally: } mg = q' E_p$$

$$\frac{q'}{q} = \frac{E_q}{E_p} \quad (2)$$

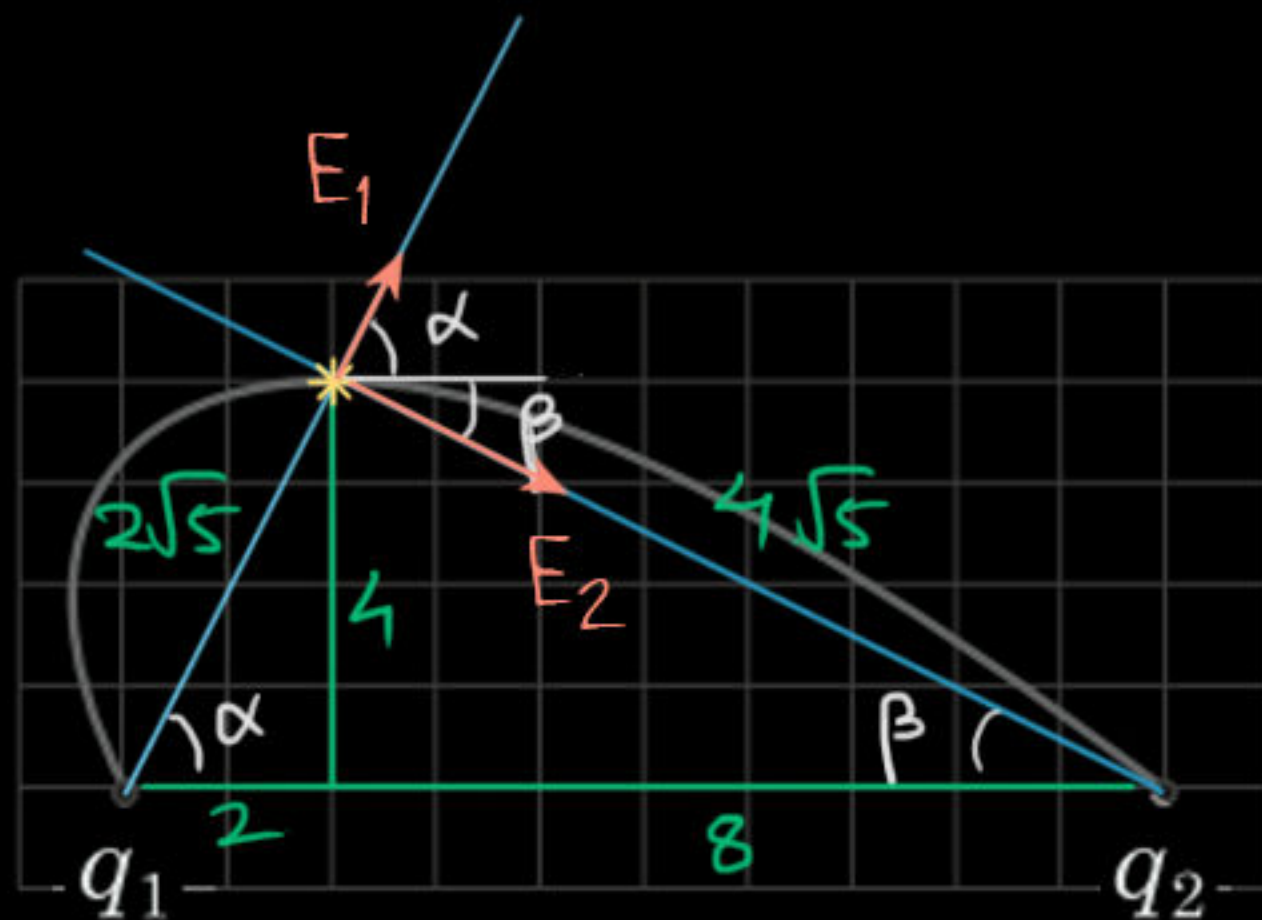
$$\therefore \text{Required fractional change in } q = \frac{q' - q}{q} = \frac{q'}{q} - 1 = \frac{E_q}{E_p} - 1 = \left| \frac{1}{9} - 1 \right| = \frac{8}{9}$$

-ve sign indicates, q is lessened



11. In the figure, a line of electric field created by two point charges q_1 and q_2 is shown. If it is known that $q_1 = 1 \mu\text{C}$, the charge q_2 is closest to
- (a) $-2 \mu\text{C}$ (b) $-4 \mu\text{C}$
 (c) $-6 \mu\text{C}$ (d) $-8 \mu\text{C}$

Charges must be of opposite sign. As field line starts at q_1 and terminates at q_2 .

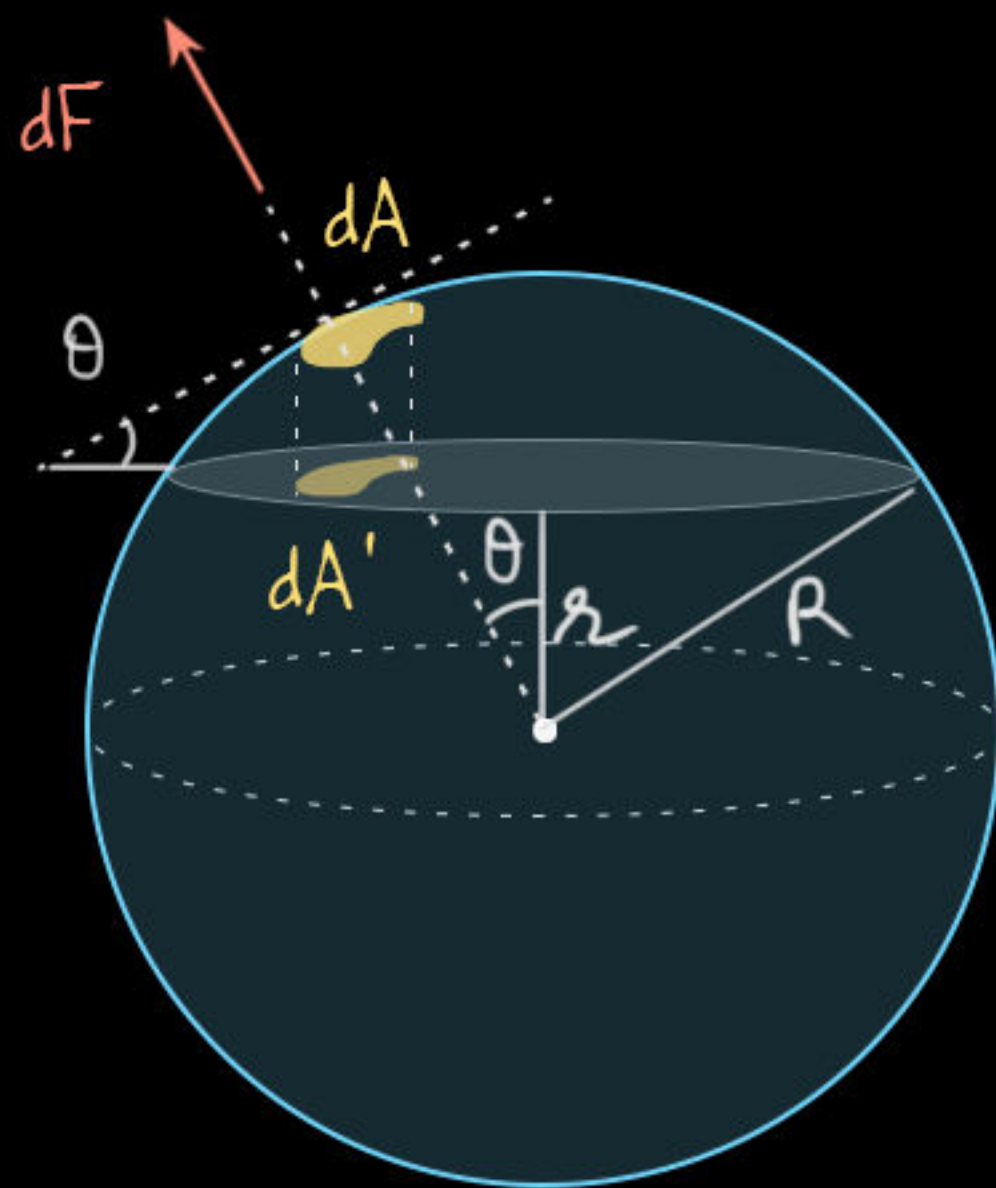


$$|E_1 \sin \alpha| = |E_2 \sin \beta|$$

$$\Rightarrow \frac{kq_1}{(2\sqrt{5})^2} \cdot \frac{4}{2\sqrt{5}} = \frac{kq_2}{(4\sqrt{5})^2} \cdot \frac{4}{4\sqrt{5}}$$

$$\Rightarrow q_2 = 8q_1$$

$$\therefore q_2 = -8 \mu\text{C}$$



12. A conducting shell of radius R has charge Q . Electrostatic force between two parts of the shell which are on either sides of a plane that is at a distance r ($r < R$) from the centre of the shell is

(a) $\frac{Q^2}{32\pi\epsilon_0 R^2} \left(1 + \frac{r}{R}\right)$

(b) $\frac{Q^2}{32\pi\epsilon_0 R^2} \left(1 - \frac{r}{R}\right)$

(c) $\frac{Q^2}{32\pi\epsilon_0 R^2} \left(1 + \frac{r^2}{R^2}\right)$

✓ (d) $\frac{Q^2}{32\pi\epsilon_0 R^2} \left(1 - \frac{r^2}{R^2}\right)$

On a small element dA on the shell which is at angle θ from Y axis:

$$\begin{aligned} dF &= dq E \\ &= (\sigma dA) \frac{\sigma}{2\epsilon_0} \\ &= \frac{\sigma^2}{2\epsilon_0} dA \end{aligned}$$

Net force will be in Y direction.

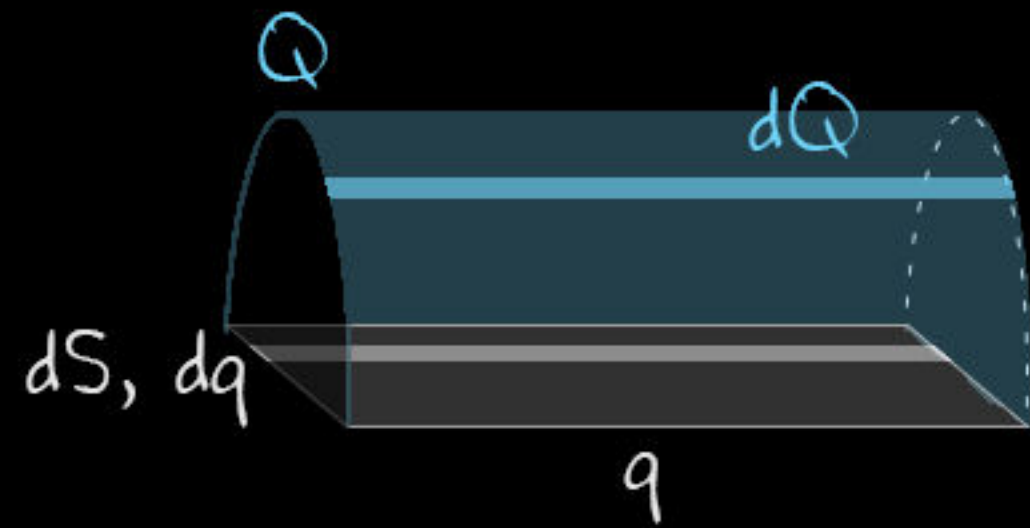
$$\therefore F_{\text{net}} = \int dF \cos\theta$$

$$= \frac{\sigma^2}{2\epsilon_0} \int dA \cos\theta$$

$$= \frac{\sigma^2}{2\epsilon_0} \int dA'$$

area of partition plate

$$= \frac{\sigma^2}{2\epsilon_0} \pi (R^2 - r^2) \xrightarrow{\sigma \rightarrow \frac{Q}{4\pi R^2}} \frac{Q^2}{32\pi R^2 \epsilon_0} \left(1 - \frac{r^2}{R^2}\right)$$



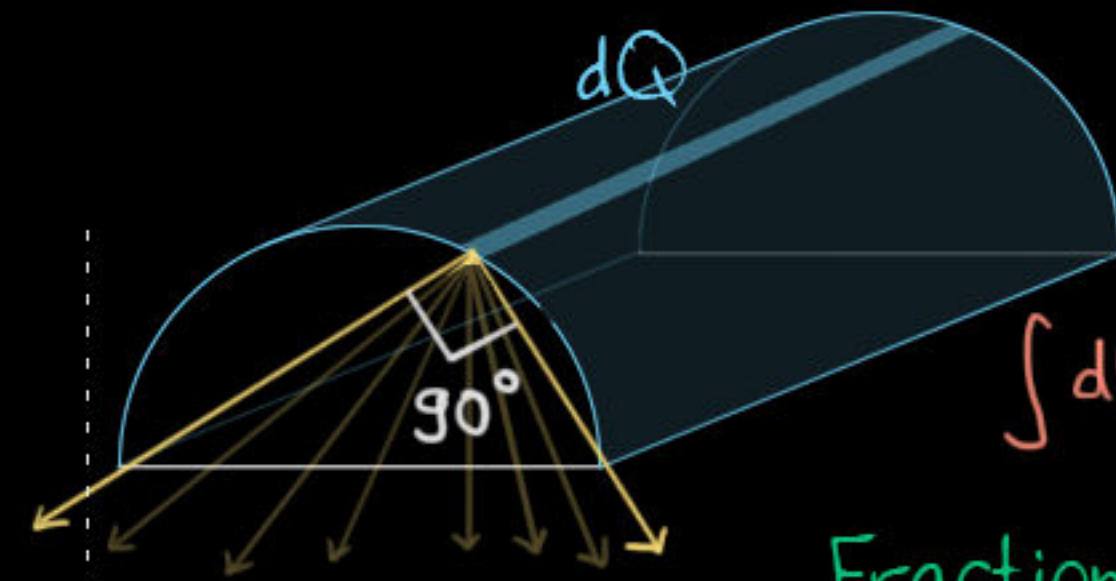
13. A half cylinder of radius R and length $L \gg R$ is formed by cutting a cylindrical pipe made of an insulating material along a plane containing its axis. The rectangular base of the half cylinder is closed by a dielectric plate of length L and width $2R$. A charge Q on the half cylinder and a charge q on the dielectric plate are uniformly sprinkled. Electrostatic force between the plate and the half cylinder is closest to (d) $\frac{qQ}{8\epsilon_0 RL}$

Net force on plate will be in Y direction:

$$\begin{aligned} \therefore dF_{\text{net}} &= dF \cos \theta \\ &= dq dE \cos \theta \\ &= (\sigma dS) dE \cos \theta \\ &= \sigma (dE dS \cos \theta) \\ &= \sigma dE \cdot dS \end{aligned}$$

$$F_{\text{net}} = \sigma \int \int dE \cdot dS \quad (1)$$

→ flux through dS element due to dQ
 → Net flux through entire plate due to dQ
 → Net flux through entire plate due to Q



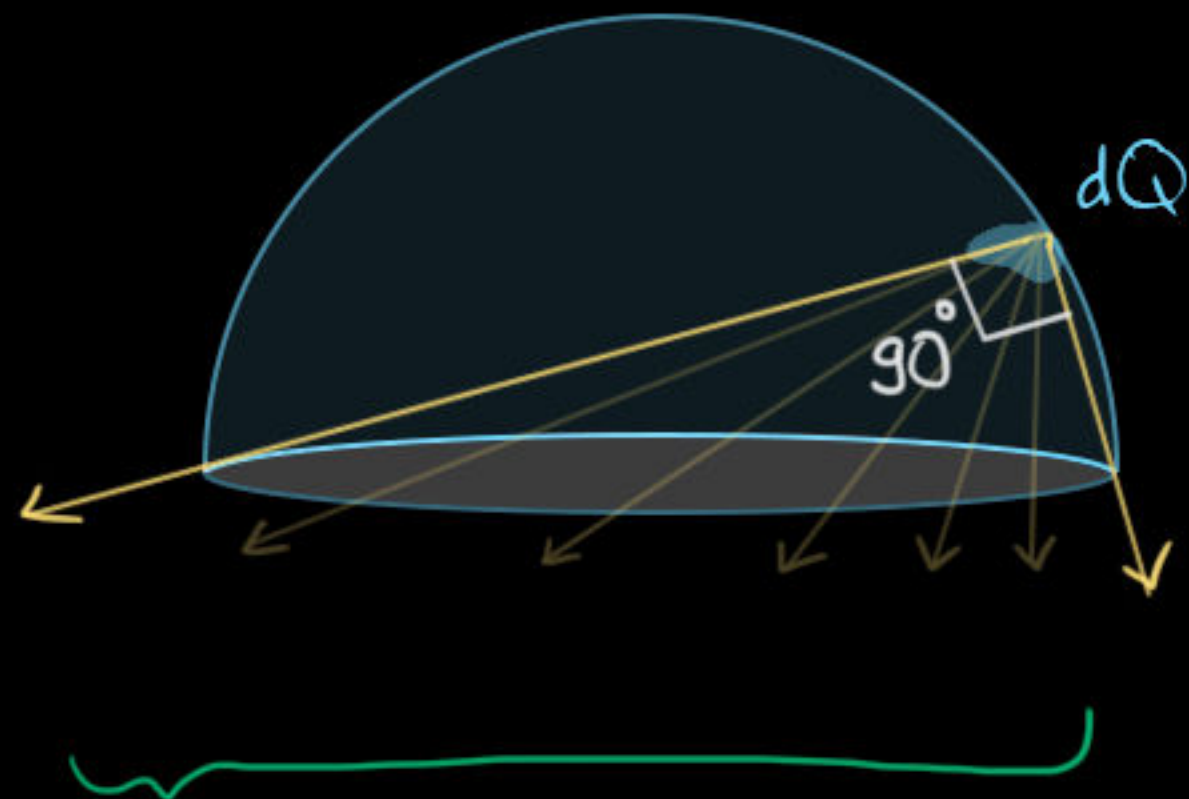
$$\int dE \cdot dS = \frac{1}{4} \frac{dQ}{\epsilon_0} \quad (2)$$

Fraction through plate.

$$\begin{aligned} \text{From 1, 2: } F_{\text{net}} &= \sigma \int \frac{1}{4} \frac{dQ}{\epsilon_0} \\ &= \frac{q}{2RL} \cdot \frac{1}{4} \frac{Q}{\epsilon_0} // \end{aligned}$$

Similarly, we can find force between a hemispherical shell and plate that covers it.

$$F_{\text{net}} = \sigma \int \int dE \cdot dS \quad (1)$$



$$\int dE \cdot dS = \underbrace{\frac{2\pi(1 - \cos 45^\circ)}{4\pi}}_{\text{Fraction through plate}} \frac{dQ}{\epsilon_0} = \frac{\sqrt{2}-1}{2\sqrt{2}} \frac{dQ}{\epsilon_0} \quad (2)$$

Flux is coming through
a cone of solid angle: $2\pi(1 - \cos 45^\circ)$

From 1,2:

$$F_{\text{net}} = \sigma \int \frac{\sqrt{2}-1}{2\sqrt{2}} \frac{dQ}{\epsilon_0}$$

$$= \frac{q}{\pi R^2} \frac{\sqrt{2}-1}{2\sqrt{2}} \frac{Q}{\epsilon_0}$$

Youtube / Arihant Senior

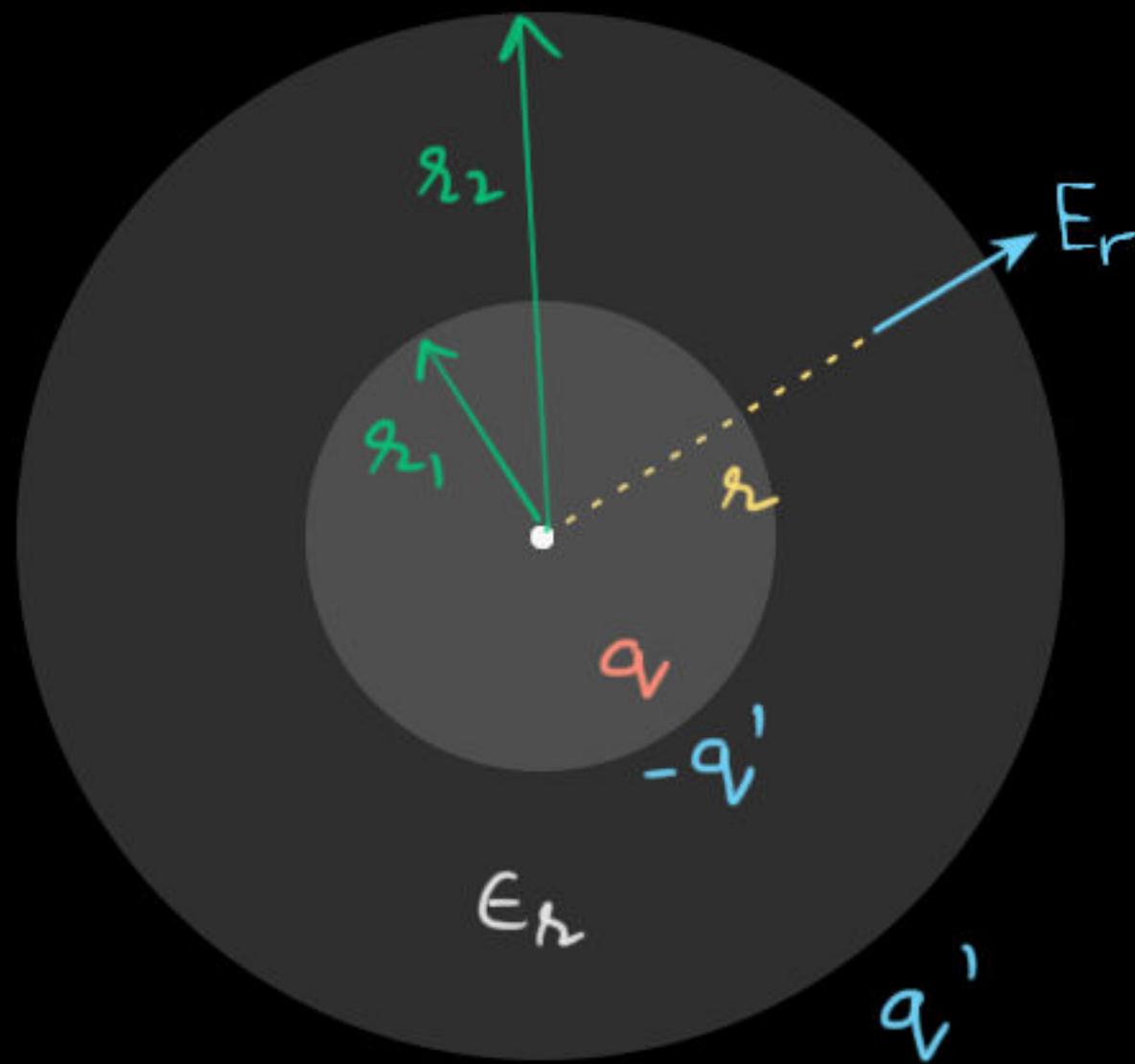
14. A conducting sphere of radius r_1 is surrounded by a dielectric layer of outer radius r_2 and dielectric constant ϵ_r . If the conducting sphere is given a charge q , determine surface density of polarization charges on the outer surface of the dielectric layer.

(a) $\frac{\epsilon_r q}{4\pi r_2^2}$

(b) $\frac{q}{4\pi \epsilon_r r_2^2}$

(c) $\frac{(\epsilon_r - 1)q}{4\pi r_2^2}$

(d) $\frac{(\epsilon_r - 1)q}{4\pi \epsilon_r r_2^2}$



$$E_r = \frac{1}{\epsilon_r} \frac{kq}{r^2} = \frac{k(q - q')}{r^2}$$

$$\Rightarrow q' = q \left(1 - \frac{1}{\epsilon_r}\right)$$

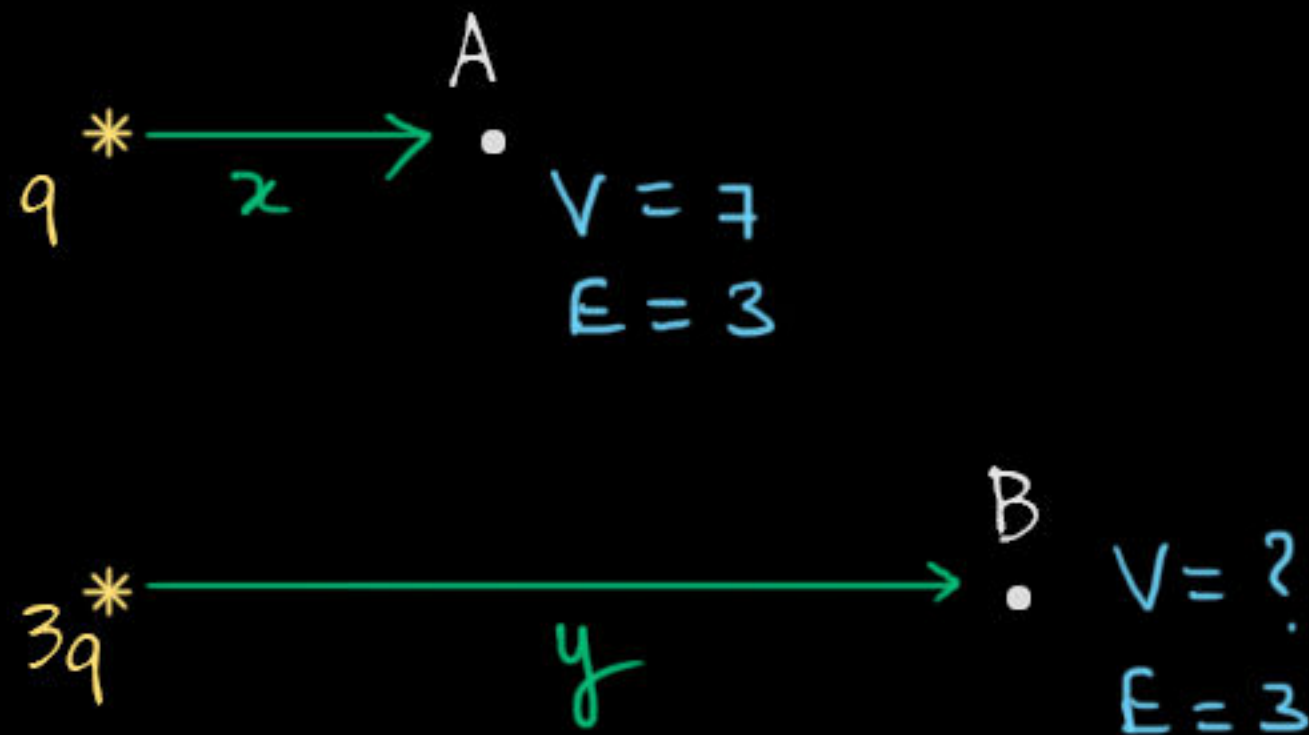
$$\therefore \text{Outer surface charge density} = \frac{q'}{4\pi r_2^2} = \frac{q(\epsilon_r - 1)}{4\pi \epsilon_r r_2^2} //$$

Independent of r_1 !!
Youtube / Arihant Senior

15. Due to a point charge, potential and electric field at a point A are 7 V and 3 V/m respectively and electric field at a point B is less than 3 V/m. Now magnitude of the charge is tripled. If electric field at B becomes 3 V/m, potential at B will become closest to

- (a) 7 V (b) 12 V
(c) 21 V (d) Insufficient information

Irrelevant I think!



$$\frac{kq}{x} = 7 \quad (1)$$

$$\frac{kq}{x^2} = 3 \quad (2)$$

$$\frac{k(3q)}{y^2} = 3 \quad (3)$$

From 1, 2, 3:

$$V = \frac{k(3q)}{y}$$

$$= 3 \left(\frac{kq}{y} \right)$$

$$= \frac{3 \left(\frac{49}{3} \right)}{\sqrt{\frac{49}{3}}} = 7\sqrt{3} \approx 12V //$$

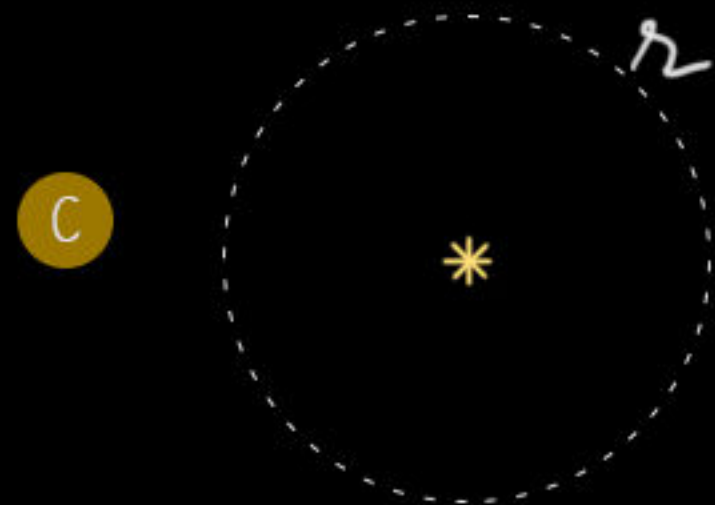
$$\vec{F} = \frac{Qq(1 - \sqrt{\alpha r})}{4\pi\epsilon_0 r^3} \vec{r}$$

Concept: If radial force vector is a function of 'r' (distance from origin), then it's a conservative force.

A $\vec{E} = \frac{\vec{F}}{q}$

B Yes: $\oint \vec{E} \cdot d\vec{l} = 0$

∴ Force is conservative



16. In another world, instead of the Coulomb's law, electric force $\vec{F}$ on a point like charge q due to another point like charge Q is found to obey the following law.

Here α is a positive constant and $\vec{r}$ is the position vector of charge q relative to the charge Q .

- ✓ (a) Electric field due to a point charge Q is $\vec{E} = \frac{Q(1 - \sqrt{\alpha r})}{4\pi\epsilon_0 r^3} \vec{r}$
- ✓ (b) Line integral of this electric field $\oint \vec{E} \cdot d\vec{l}$ over a closed path is also zero as in our world.
- (c) Gauss' law $\oint \vec{E} \cdot d\vec{s} = \frac{q_{\text{enclosed}}}{\epsilon_0}$ also holds true for this electric field.
- (d) All the above statements are true but this electric field is not conservative.

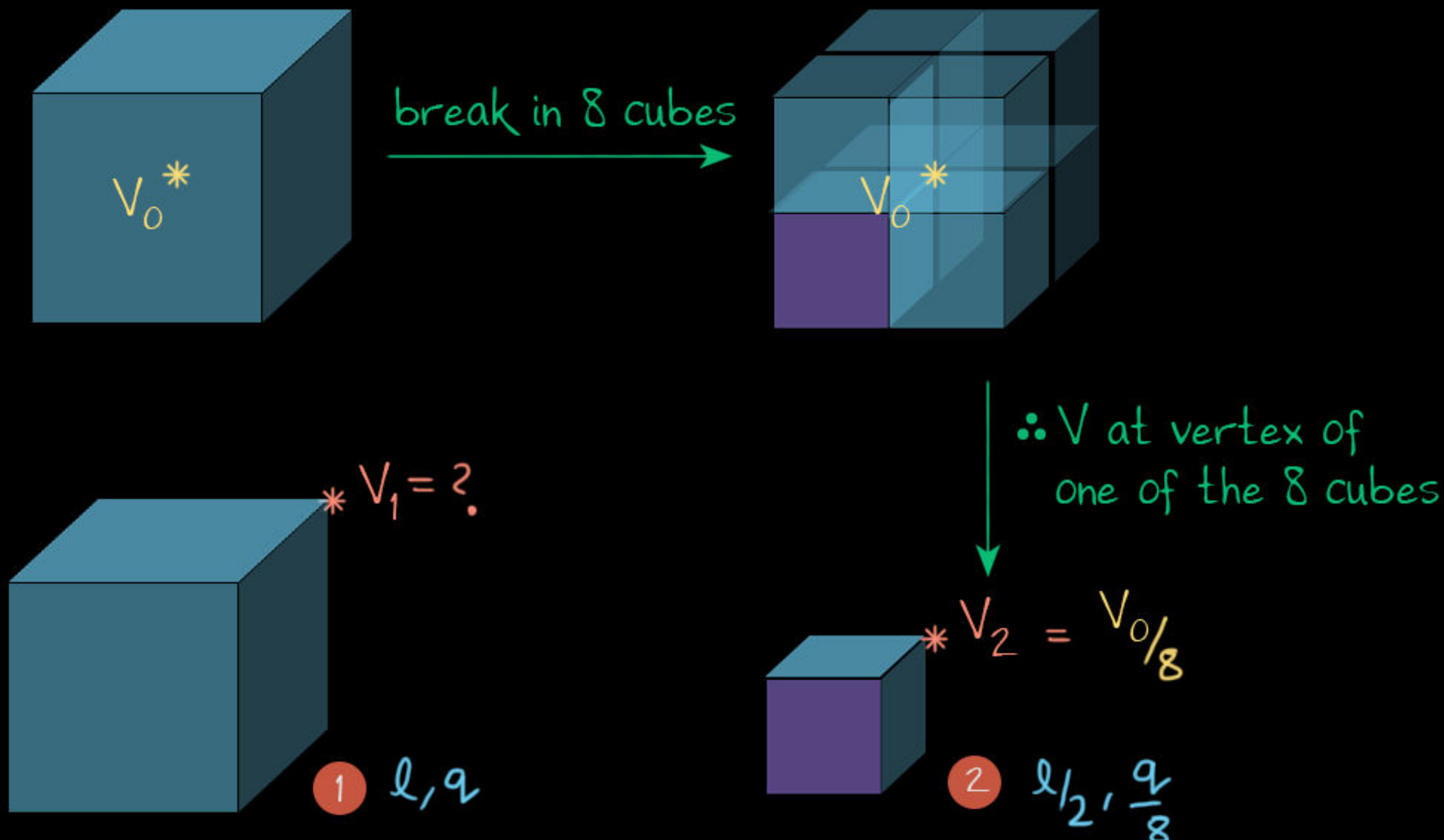
Lets take a spherical surface of radius r , and calculate flux through it.

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q(1 - \sqrt{\alpha r})}{4\pi\epsilon_0 r^2} \cdot (4\pi r^2) = \frac{Q}{\epsilon_0} (1 - \sqrt{\alpha r}) \neq \frac{Q}{\epsilon_0}$$

17. A cube made of an insulating material has uniform charge distribution throughout its volume. Assuming electric potential due to this charged cube at infinitely distant places to be zero, potential at the centre is found to be V_0 . What is electric potential at one of its corners?

- (a) V_0
(c) $V_0/4$

- (b) $V_0/2$
(d) $V_0/8$



For similar geometrical figures 1 and 2:

$$V \propto \frac{q}{l}$$

$$\Rightarrow \frac{Vl}{q} = \text{constant}$$

$$\Rightarrow \frac{V_1 l}{q} = \frac{V_2 (l/2)}{q/8}$$

$$\begin{aligned} \Rightarrow V_1 &= 4V_2 \\ &= 4\left(\frac{V_0}{8}\right) \\ &= V_0/2 \end{aligned}$$

Figure-I

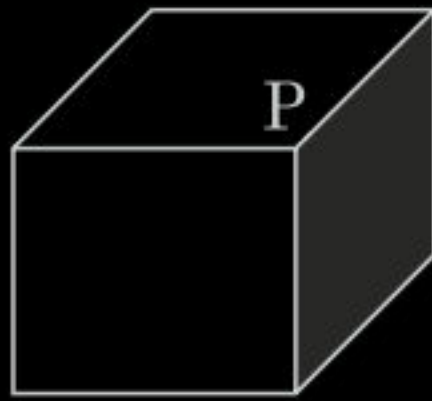
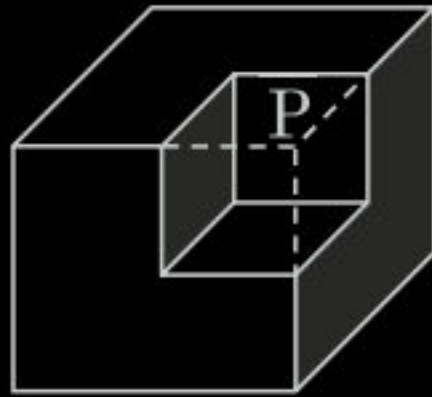
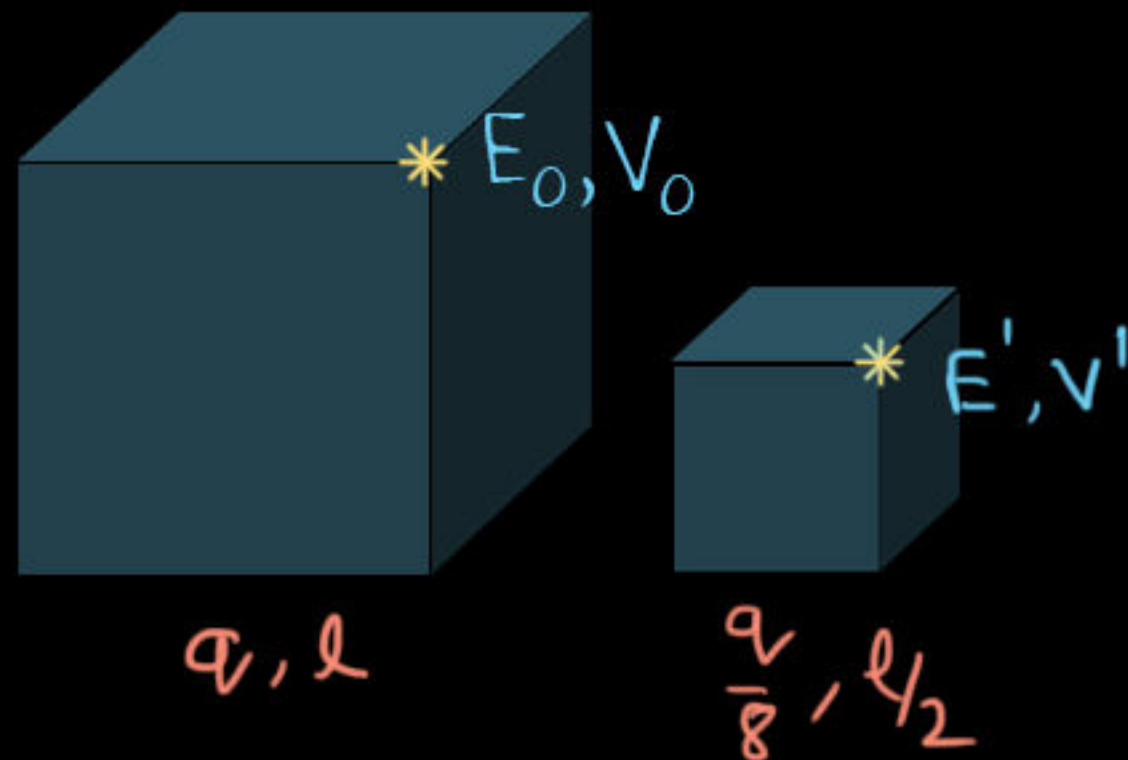


Figure-II



18. Consider a cube as shown in the figure-I; with **uniformly distributed charge in its entire volume**. Intensity of electrical field and potential at one of its **vertex P** are E_0 and V_0 respectively. A portion of half the size (half edge length) of the original cube is **cut and removed** as shown in the figure-II. Find modulus of **electric field and potential at the point P** in the new structure.

(a) $\frac{E_0}{2}$ and $\frac{3V_0}{4}$



$$E \propto \frac{q}{l^2} \Rightarrow \frac{El^2}{q} = \text{constant}$$

$$\Rightarrow \frac{E_0 l^2}{q} = \frac{E' (l/2)^2}{q/8}$$

$$\Rightarrow E' = E_0/2$$

$$V \propto \frac{q}{l} \Rightarrow \frac{Vl}{q} = \text{constant}$$

$$\Rightarrow \frac{V_0 l}{q} = \frac{V' (l/2)}{q/8}$$

$$\Rightarrow V' = V_0/4$$

∴ in figure-II:

$$E_P = E_0 - E' = E_0/2 //$$

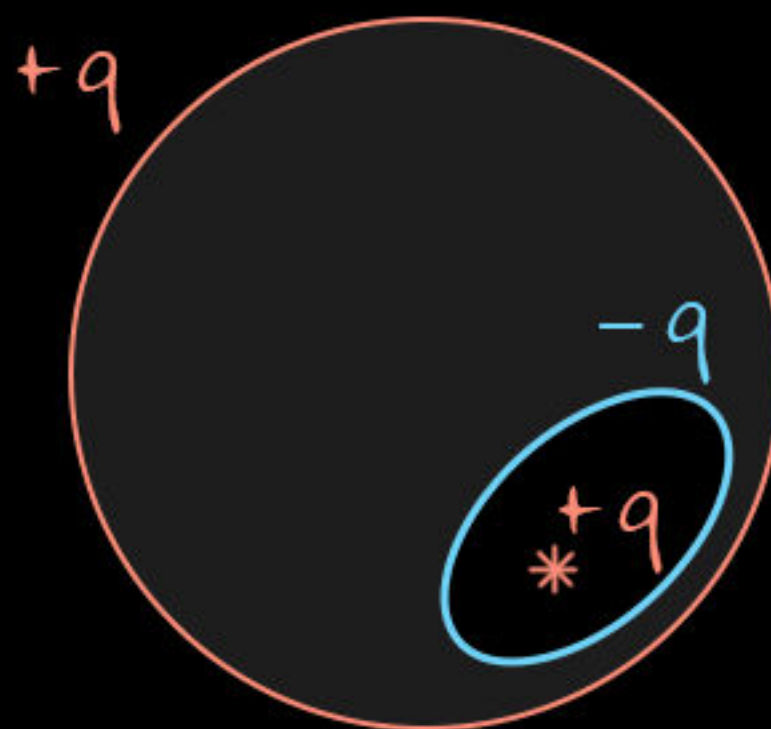
$$V_P = V_0 - V' = 3V_0/4 //$$

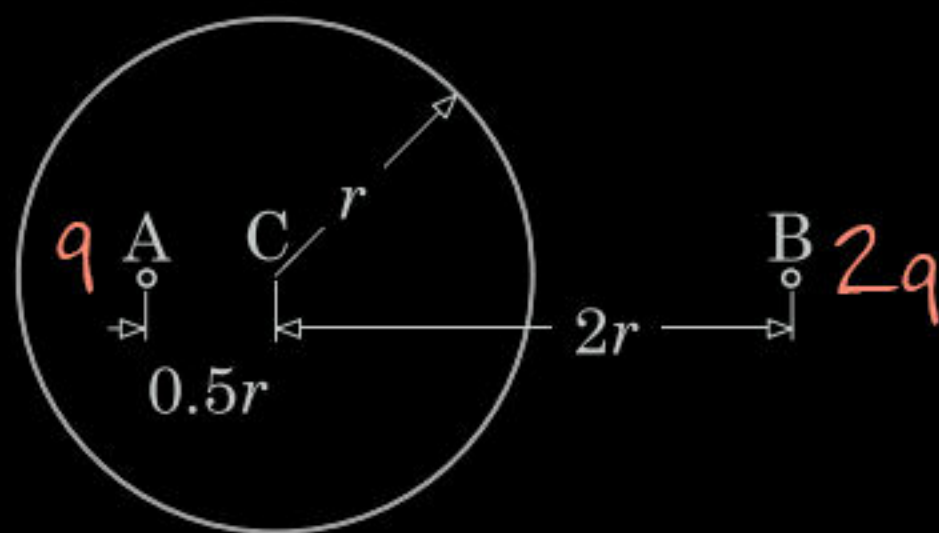
19. Electrostatic potential V has been measured everywhere outside a spherical body of radius R made of unknown material. It was found that the potential is spherically symmetric, i.e. depends only on the distance r from the centre of the sphere as given by the expression $V = A/r$, where A is a constant. No measurement of the potential inside the sphere has been done. What can you conclude for charge distribution of the body?

- (a) It must be uniform.
- (b) It may be nonuniform but must have spherical symmetry.
- ✓ (c) It may be uniform or nonuniform without any spherical symmetry.
- (d) To predict charge distribution precisely we need to know electric field outside the sphere.

A spherical conductor with a cavity containing a charge $+q$.

Example of non-uniform charge distribution, without any spherical symmetry.





20. Consider a thin conducting shell of radius r carrying total charge q . Two point charges q and $2q$ are placed on points A and B, which are at distances $0.5r$ and $2r$ from the centre C of the shell respectively. If the shell is earthed, how much charge will flow to the earth?

(a) $2q$

(b) $3q$

(c) $4q$

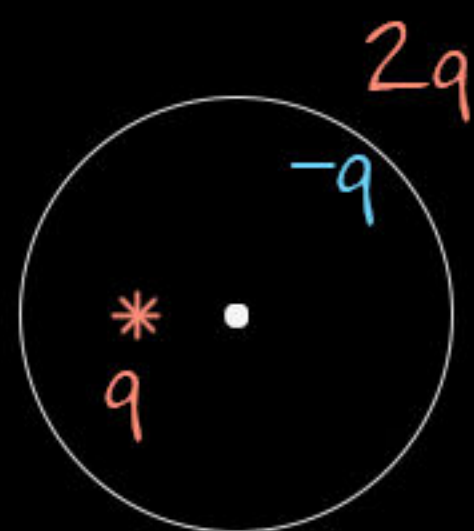
(d) More than $2q$ and less than $3q$

Concept: Inside shell is cavity.

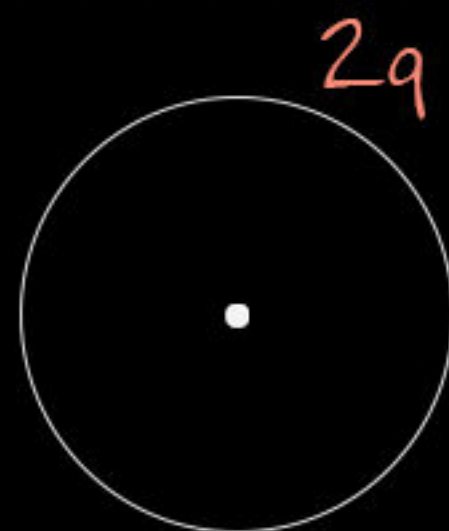
And charges inside cavity and those distributed on cavity's inner surface have no effect on system outside of it and vice versa.

So after equilibrium, we can ignore cavity and treat it as chargeless volume, as earthing is done outside of cavity.

Initially (after equilibrium):



$\equiv$

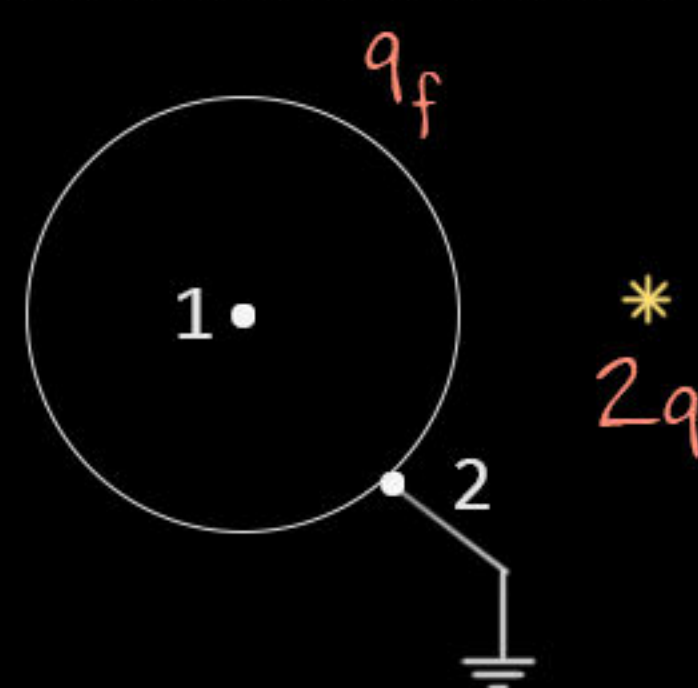


*

$2q$

→ This step is unaffected by q and $-q$.

On earthing:

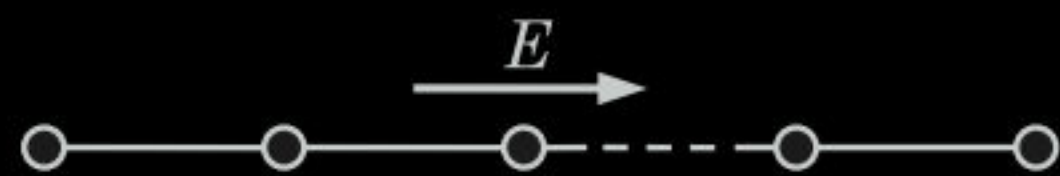


$$V_1 = V_2$$

$$\frac{kq_f}{R} + \frac{k(2q)}{2R} = 0$$

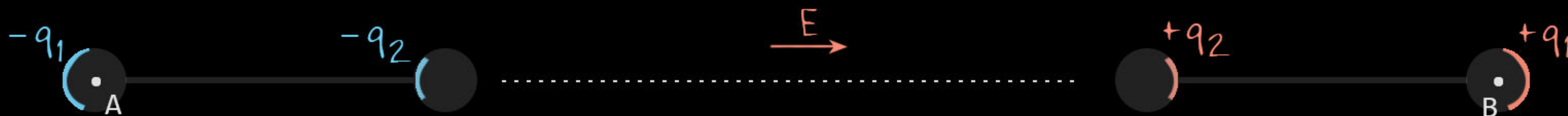
$$\Rightarrow q_f = -q$$

$$\therefore \text{charge flow} = 2q - q_f = 3q //$$



Concept 1: Charges will get distributed such that E inside each ball is zero.

Let them be as shown:



Concept 2: $V_A = V_B$

Concept 3: We can NOT comment on V_A or V_B independently as there is external field present.

21. A straight chain consisting of n identical metal balls is at rest in a region of free space as shown. In the chain, each ball is connected with adjacent balls by identical conducting wires. Length l of a connecting wire is much larger than the radius r of a ball. A uniform electric field E pointing along the chain is switched on in the region. Find magnitude of induced charges on one of the end ball.

(b) $2\pi\epsilon_0 r(n-1)lE$

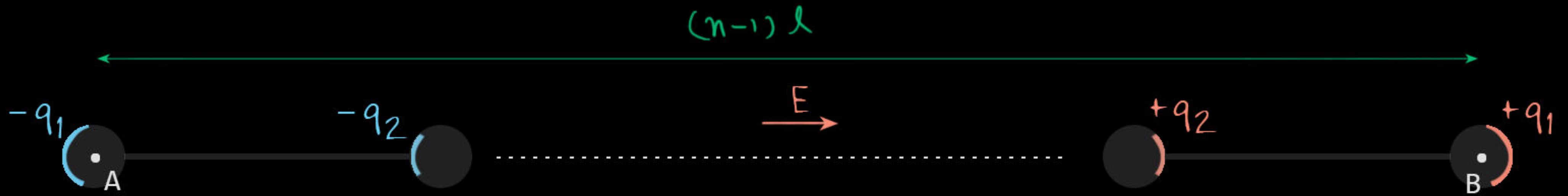
Approach: We can however find $V_A - V_B$ by thinking of resultant field distribution as superimposition of

- 1) External field E and
- 2) Field due to induced charges

i.e: $V_A - V_B = 0$

$$\Rightarrow (V_A - V_B)_{\text{due to external } E} + (V_A - V_B)_{\text{due to induced charges}} = 0$$

Sol:



$$\underbrace{(V_A - V_B)}_{\text{due to external } E} + \underbrace{(V_A - V_B)}_{\text{due to induced charges}} = 0$$

$$\Rightarrow E(n-1)l + \left[\frac{k(-q_1)}{r} + \cancel{\frac{k(-q_2)}{l}} + \dots \right] - \left[\frac{kq_1}{r} + \cancel{\frac{kq_2}{l}} + \dots \right] = 0$$

Note: The terms $\frac{k(-q_2)}{l}$ and $\frac{kq_2}{l}$ are crossed out with green arrows pointing to ≈ 0 .

$$\Rightarrow \frac{2kq_1}{r} = E(n-1)l$$

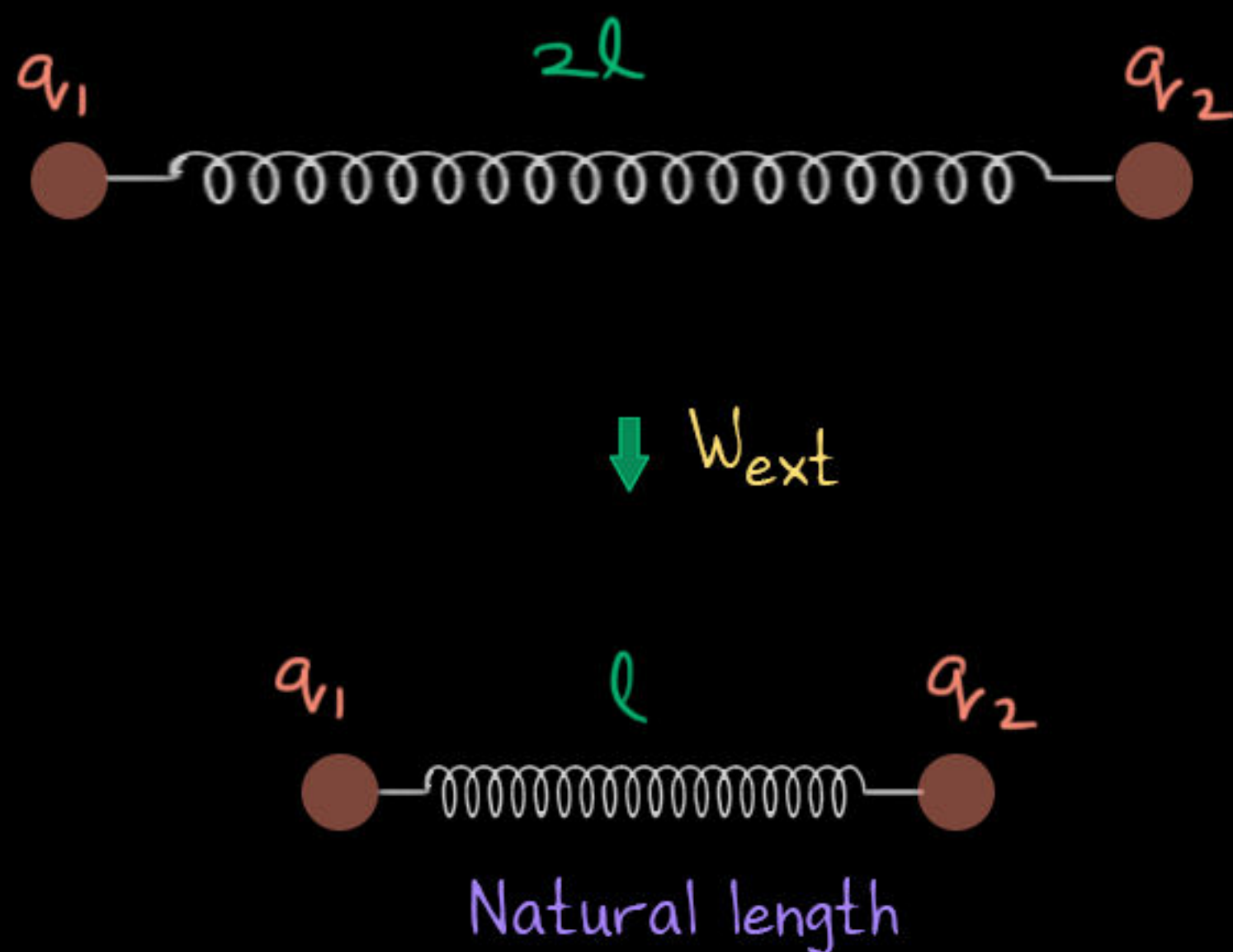
$$\Rightarrow q_1 = 2\pi\epsilon_0 E(n-1)lr //$$

Note: With similar approach we can find q_2, q_3 etc too.

Youtube / Arihant Senior

22. Small identical balls are attached at each end of a spring of force constant $k = 200 \text{ N/m}$ and relaxed length $l = 20 \text{ cm}$. Unknown amount of charges are gradually transferred to the balls in unequal amounts until the spring length becomes twice of its relaxed length. What amount of work must be done by an external agency in slowly compressing the spring back to its relaxed length?

Let spring constant be k' .



Force: $\frac{kq_1q_2}{(2l)^2} = k'l$

Energy: $W_{\text{ext}} = \Delta PE + \cancel{\Delta KE}$

$$= \left(0 - \frac{1}{2}k'l^2\right) + kq_1q_2\left(\frac{1}{l} - \frac{1}{2l}\right)$$

$$= -\frac{1}{2}k'l^2 + 4k'l^3\left(\frac{1}{l} - \frac{1}{2l}\right)$$

$$= k'l^2\left(-\frac{1}{2} + 4 - 2\right)$$

$$= (200)(0.2)^2 \cdot \frac{3}{2} = 12 \text{ J}$$

23. Electrostatic potential V at a point on circumference of a circular layer of uniform charge and radius r is given by equation $V = 4\sigma r$, here σ is surface charge density in the layer. Which of the following expression correctly represents electrostatic energy stored in the electric field of a similar charge layer of radius R ?

i.e disc

(a) $\frac{1}{3}\pi\sigma^2 R^3$

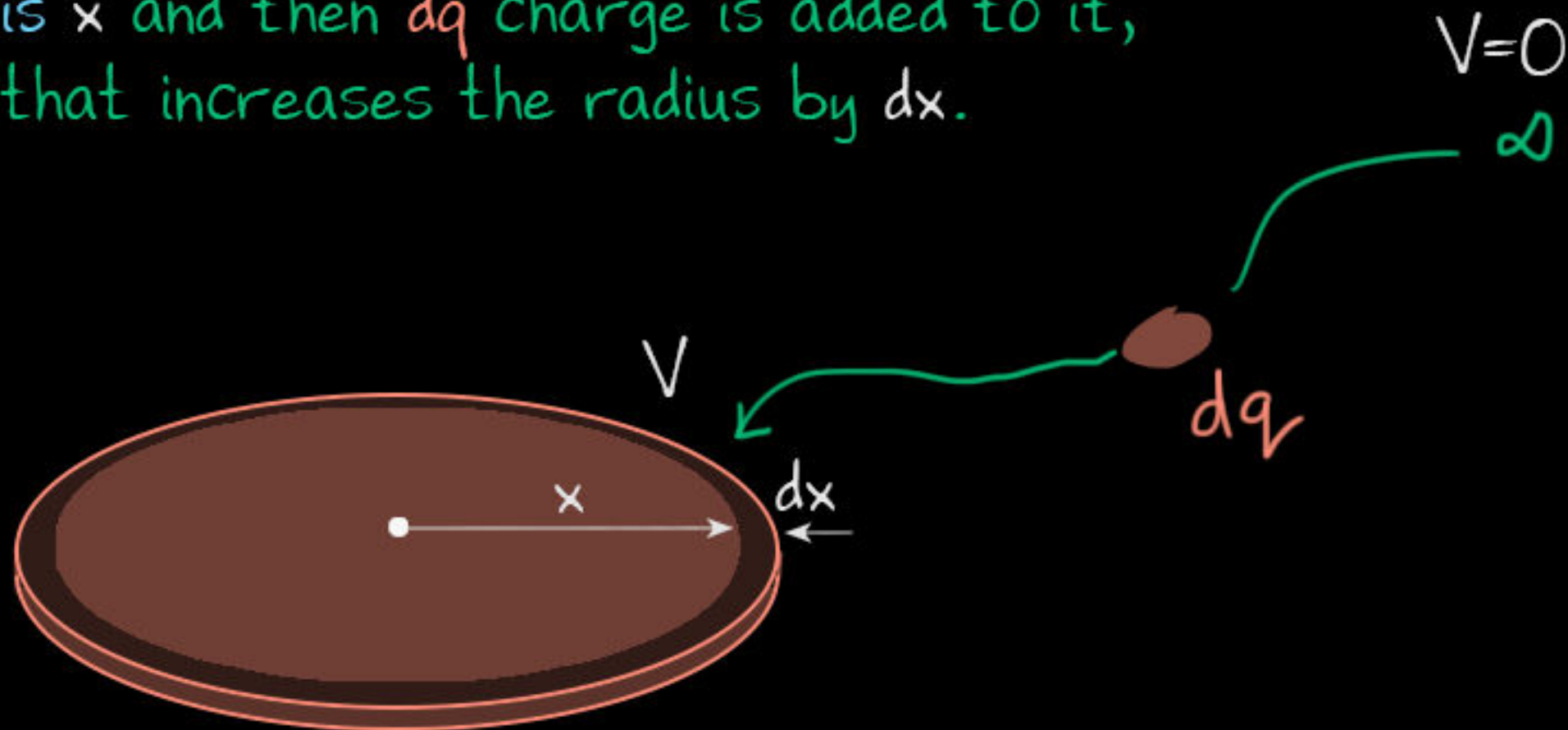
(b) $\frac{2}{3}\pi\sigma^2 R^3$

(c) $\frac{4}{3}\pi\sigma^2 R^3$

(d) $\frac{8}{3}\pi\sigma^2 R^3$

Energy stored is external work needed to build the given disc.

While building the disc let's say, its radius is x and then dq charge is added to it, that increases the radius by dx .



$$\begin{aligned}
 dW_{\text{ext}} &= dq V \\
 &= (\sigma 2\pi x dx) 4\sigma x \\
 &\Rightarrow W_{\text{ext}} = 8\pi\sigma^2 \int_0^R x^2 dx \\
 &= \frac{8\pi\sigma^2 R^3}{3} //
 \end{aligned}$$

⊕ ⊕ ⊕
⊖ ⊖ ⊖

We know:

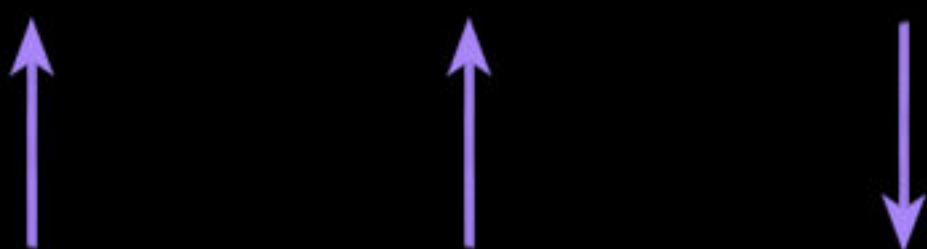
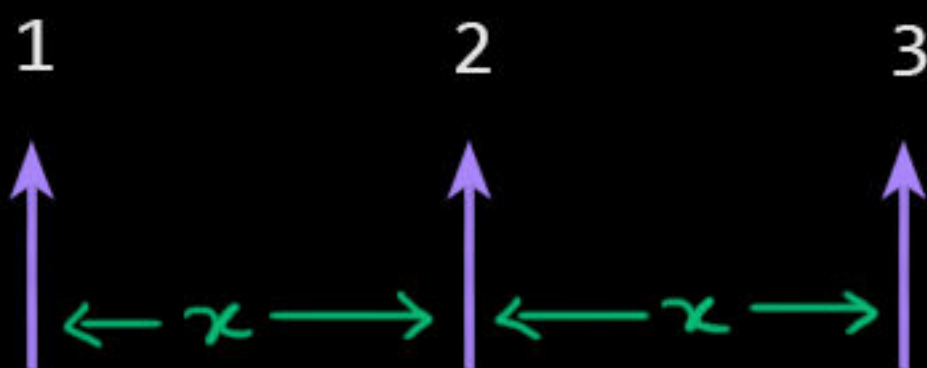


B $U_{\text{dipole}} = -\vec{p} \cdot \vec{E}$

24. Three identical electric dipoles are arranged parallel to each other at equal separations as shown in the figure. The separation between the charges of a dipole is negligible as compared to the separation between the dipoles. In the given configuration, total electrostatic interaction energy of these dipoles is U_0 . Now one of the end dipole is gradually reversed, how much work is done by the electric forces? = $-W_{\text{ext}}$

(a) $\frac{17U_0}{18}$ (b) $-\frac{17U_0}{18}$

✓ (c) $\frac{18U_0}{17}$ (d) $-\frac{18U_0}{17}$



For each pair

$$U_0 = -2 \times \left[-\frac{k p^2}{x^3} - \frac{k p^2}{x^3} - \frac{k p^2}{(2x)^3} \right] = \frac{17 k p^2}{4 x^3}$$

$$U_f = -2 \times \left[-\frac{k p^2}{x^3} + \frac{k p^2}{x^3} + \frac{k p^2}{(2x)^3} \right] = -\frac{k p^2}{4 x^3}$$

$$W_{\text{elec}} = U_0 - U_f$$

$$= U_0 + \frac{k p^2}{4 x^3}$$

$$= U_0 + \frac{1}{4} \cdot \frac{4 U_0}{17}$$

$$= \frac{18 U_0}{17}$$

Youtube / Arihant Senior

by Ankit Singhvi



We know,

Polarized charge

on dielectric: $Q_p = Q(1 - \frac{1}{k})$

Force on a

capacitor plate: $F = \frac{Q^2}{2A\epsilon_0}$

q_0, A $q_0, 2A$



1 2

$C_0 V = 2q_0$

A $V_1 = V_2$

$\Rightarrow \frac{q_0}{C_1} = \frac{q_0}{C_2}$

$\Rightarrow C_1 = C_2$

$\Rightarrow \frac{A(k\epsilon_0)}{d} = \frac{2A\epsilon_0}{d}$

$\Rightarrow k = 2 //$

B $Q_p = q_0(1 - \frac{1}{k}) = q_0/2 = \frac{C_0 V}{4} //$

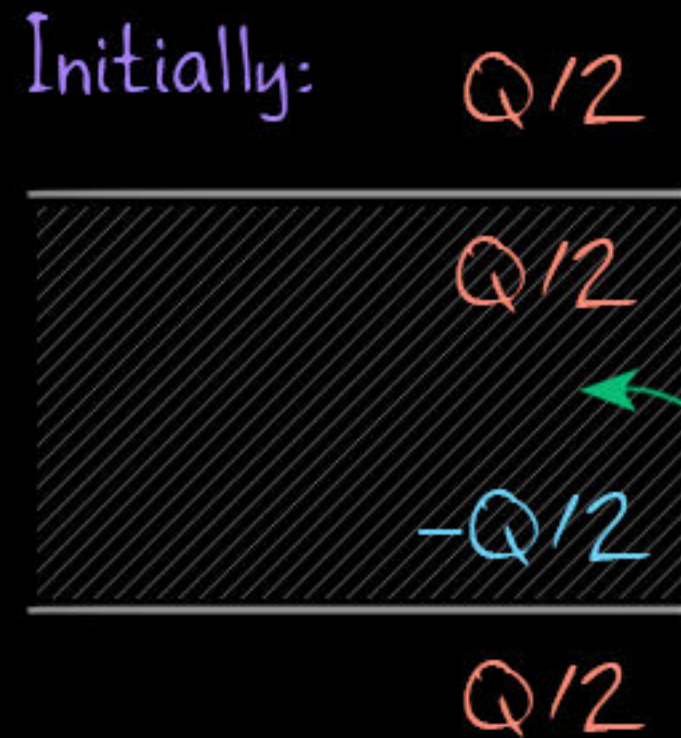
C $F_1 = \frac{q_0^2}{2A\epsilon_0}$, $f_2 = \frac{q_0^2}{2(2A)\epsilon_0} \therefore f_1 \neq f_2$

D $F_i = \frac{(2q_0)^2}{2(3A)\epsilon_0}$, $f_f = F_1 + f_2 = \frac{3q_0^2}{4A\epsilon_0} \Rightarrow \frac{F_f}{F_i} = \frac{9}{8} //$

25. A parallel plate capacitor of capacitance C_0 is charged to a voltage V and then the battery is disconnected. A dielectric covering one-third area of each plate is now inserted as shown in the figure. If charges on the capacitor plates get redistributed such that the portions covered with dielectric and not covered with the dielectric share equal amounts of charge, which of the following statements is/are true?

- ✓(a) Dielectric constant of the dielectric is 2.0.
- ✓(b) Charge appearing due to polarization on the surface of the dielectric is $0.25C_0V$.
- (c) Force of electrostatic interaction between portions of the plates covered with dielectric is equal to that between uncovered portions.
- ✓(d) Force of electrostatic interaction between the plates after insertion of the dielectric becomes $9/8$ times of its value before insertion of the dielectric.

26. A flat air capacitor C consists of two large plates that are close to each other. Initially, one of the plates was not charged, while the other had charge Q . If entire space between the plates is filled with a slab of finite electrical resistance, estimate total amount of energy lost in the slab.



(a) $\frac{Q^2}{2C}$

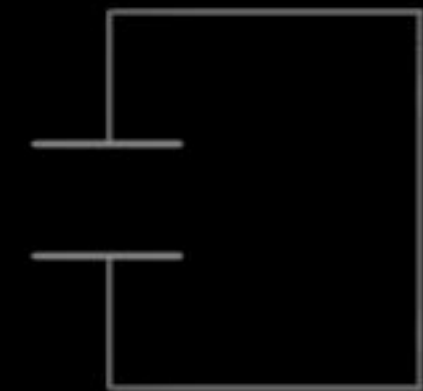
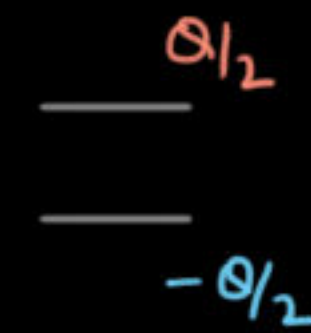
(c) $\frac{3Q^2}{8C}$

✓ (b) $\frac{Q^2}{8C}$

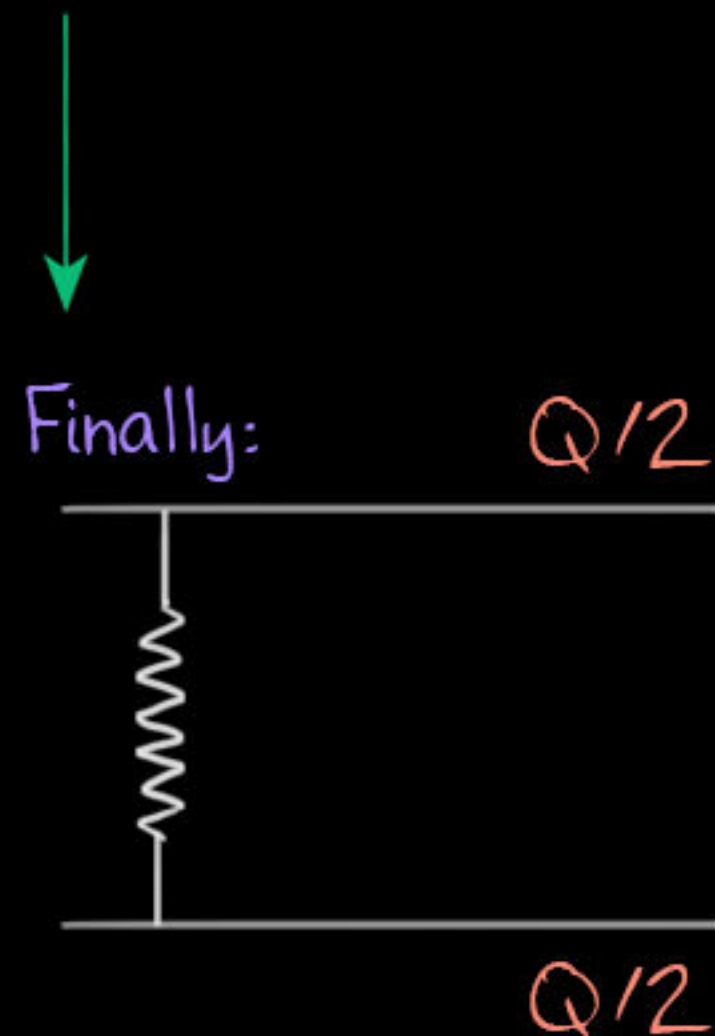
(d) $\frac{7Q^2}{8C}$

This field energy is lost later.

Which is same as energy lost by shorting a $Q/2$ charged capacitor, as charges on outside layers are unchanged.



Which is same as energy stored in the $Q/2$ charged capacitor.



$$\therefore E_{\text{lost}} = \frac{(Q/2)^2}{2C} = \frac{Q^2}{8C} //$$

Youtube / Arihant Senior

At $v_{\max}$: $\frac{dv}{dt} = 0$

$\Rightarrow f = 0$

27. Two particles each of mass 100 g and charge $10 \mu\text{C}$ are released on a horizontal plane at a distance 1.0 m from each other. Coefficient of friction between the particles and the plane is 0.1 and acceleration of free fall is 10 m/s^2 . Maximum speed acquired by the particles after they are released is closest to

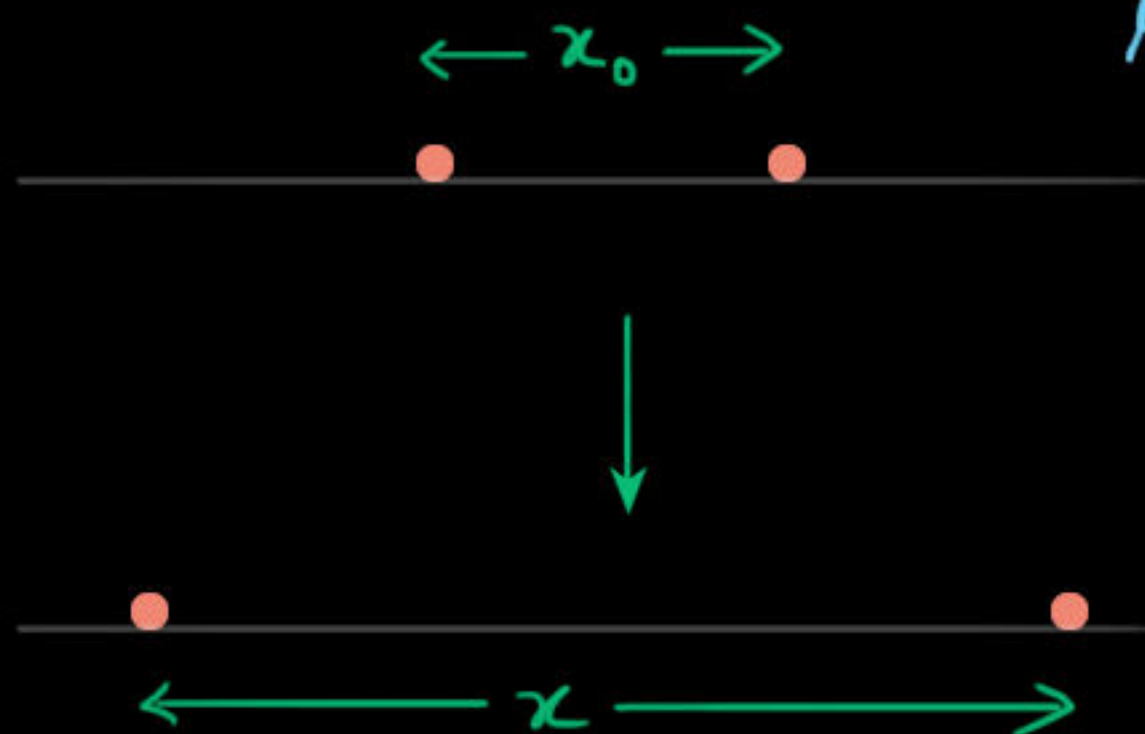
Given:

$m = 0.1$

$q = 10 \cdot 10^{-6}$

$x_0 = 1$

$\mu = 0.1$



✓(a) 2.0 m/s

(c) 3.0 m/s

(b) 2.8 m/s

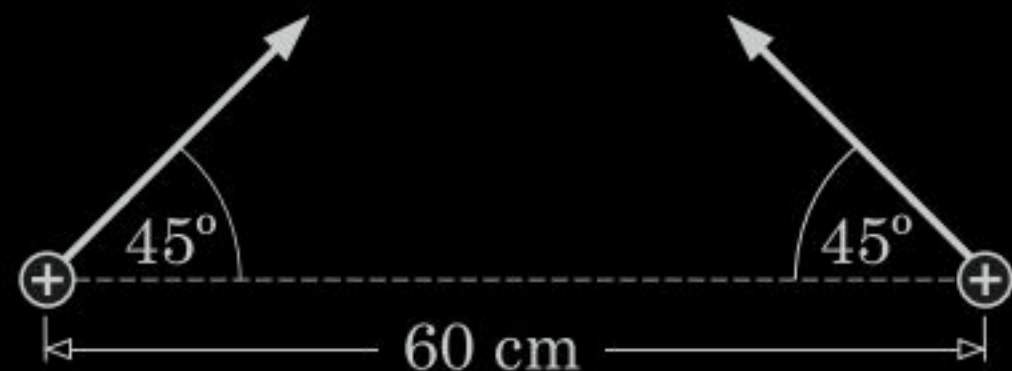
(d) 4.2 m/s

Force on one charge: $\frac{kq^2}{x^2} = \mu mg$ ①

Energy of system: $W_f = \Delta U + \Delta KE$

$\Rightarrow -\mu mg (x - x_0) = \left(\frac{kq^2}{x} - \frac{kq^2}{x_0} \right) + 2 \left(\frac{1}{2} mv^2 \right)$ ②

From 1, 2: $v = 2 \text{ m/s}$ //
Youtube / Arihant Senior



28. Two identical point charges are moving in free space, when they are 60 cm apart; their **velocity vectors are equal in modulus** and make angles of 45° from the line joining them as shown in the figure. **If at this instant, their total kinetic energy is equal to their potential energy**, what will be the **distance of closest approach** between them?

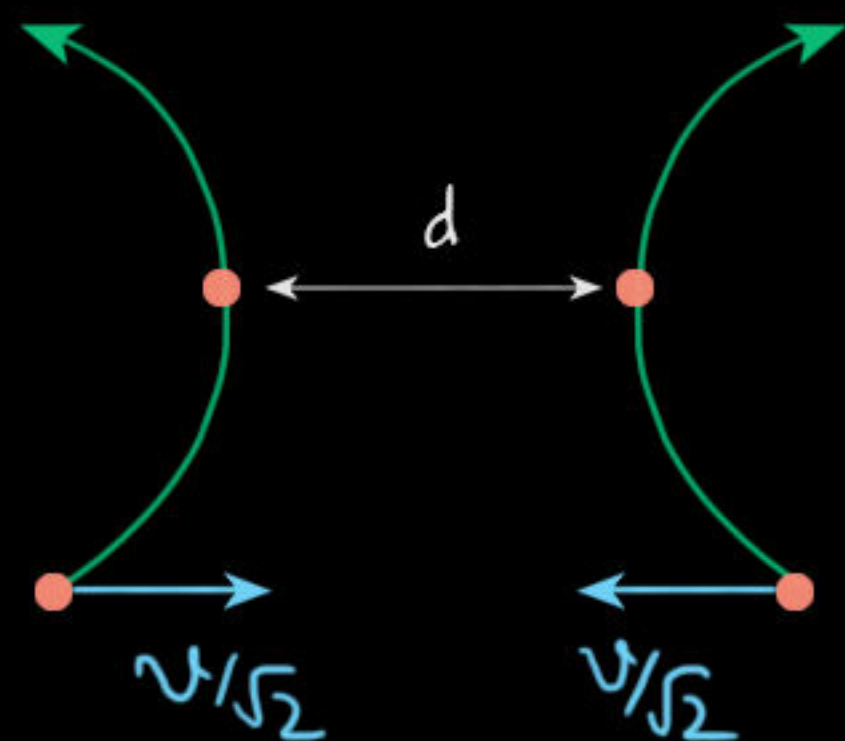
- (a) 20 cm (b) 30 cm
 ✓ (c) 40 cm (d) 45 cm

Given:

$$2 \left(\frac{1}{2} m v^2 \right) = \frac{k q^2}{(0.6)} \quad (1)$$

v_y remains unchanged as force is always in X direction.

So we can conserve Energy by writing KE due to v_x alone!



$$\Delta U + \Delta KE = 0$$

$$k q^2 \left(\frac{1}{d} - \frac{1}{0.6} \right) = 2 \times \frac{1}{2} m \left(\frac{v}{\sqrt{2}} \right)^2 \quad (2)$$

From 1,2:

$$0.6 \left(\frac{1}{d} - \frac{1}{0.6} \right) = \frac{1}{2}$$

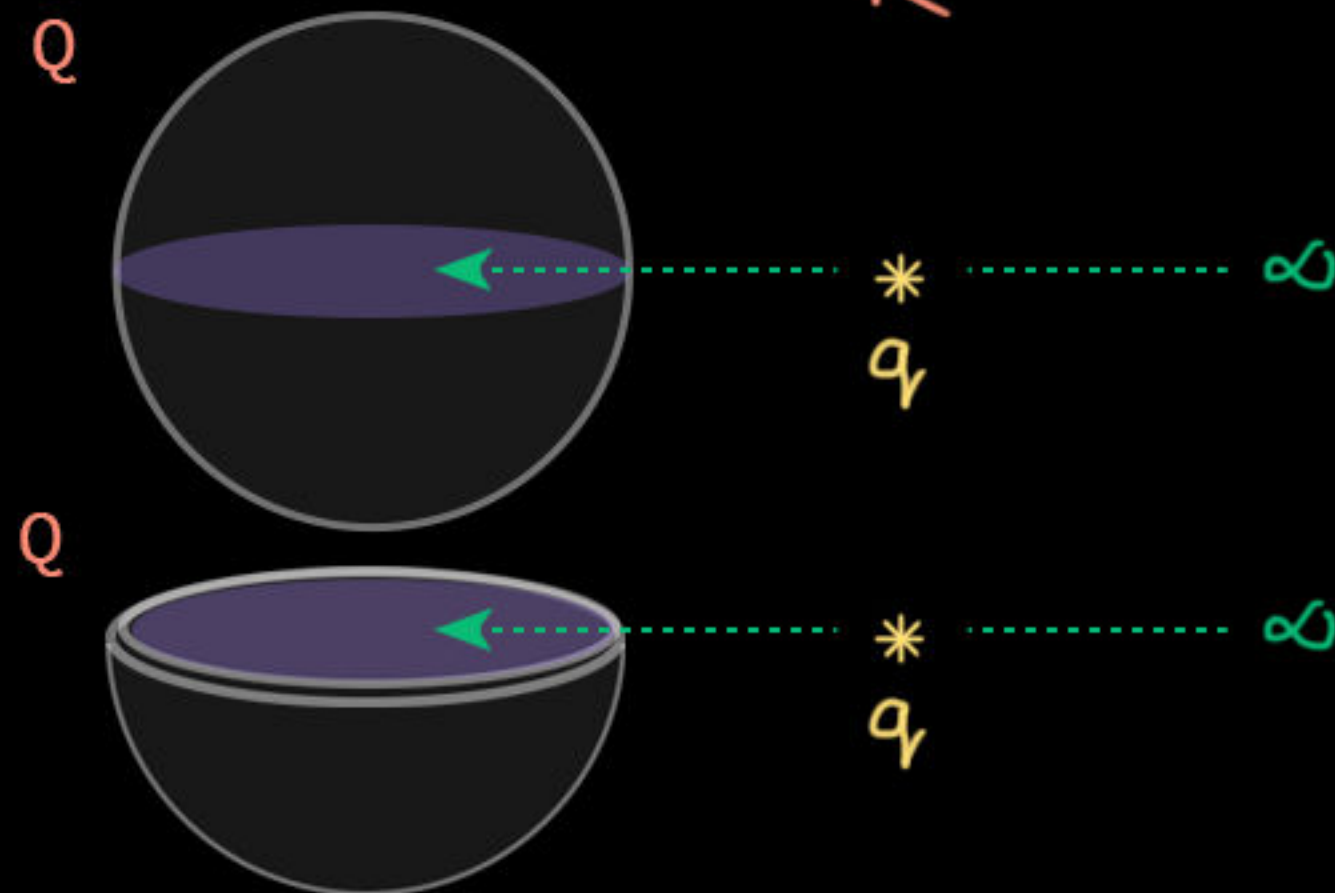
$$\Rightarrow d = 0.4 //$$

Lets break a spherical shell in two hemispherical cups and fit them one inside other.

Clearly, for a test charge to move from infinity to circular plane, W/D will be same.

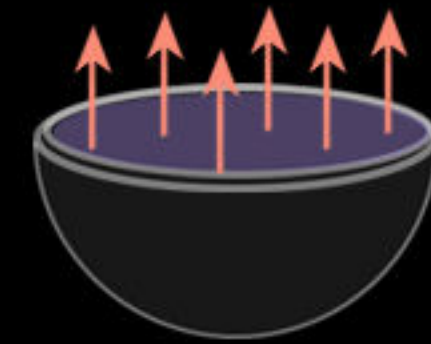
Also inside shell, potential is same.

Hence potential at every point of disc in both cases is same. And it is: $V = \frac{kQ}{R}$ ①



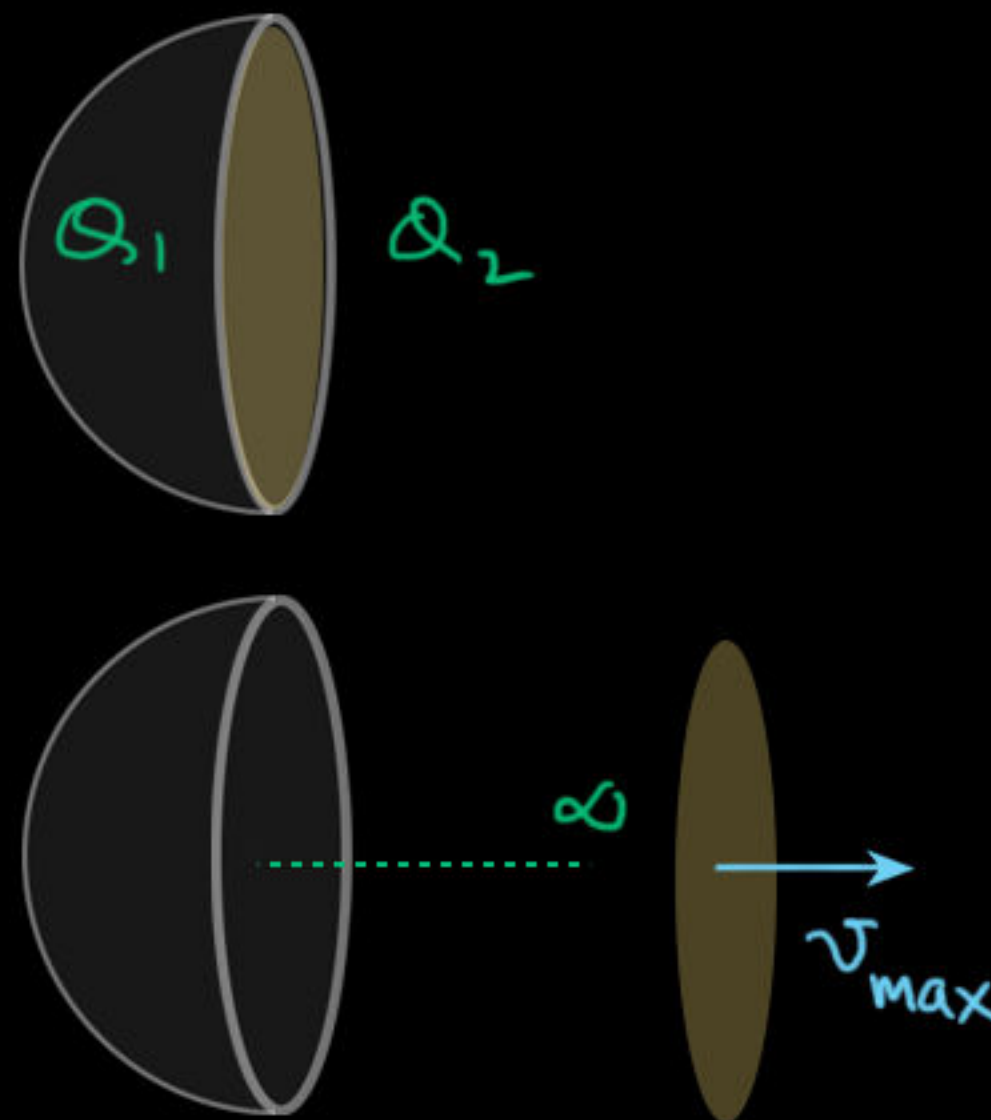
29. A thin disk of radius R is held closing the opening of a thin hemispherical shell of the same radius. Both the bodies are made of insulating materials and have uniform charges of surface charge density σ each. The plate is released keeping the shell fixed. How much maximum kinetic energy will the plate acquire after it is released?

From 1, its also clear that in the plane of hemispherical disc, field lines are all parallel to each other. (E is not same though)



$$\checkmark (c) \frac{\pi R^3 \sigma^2}{2\epsilon_0}$$

Sol:

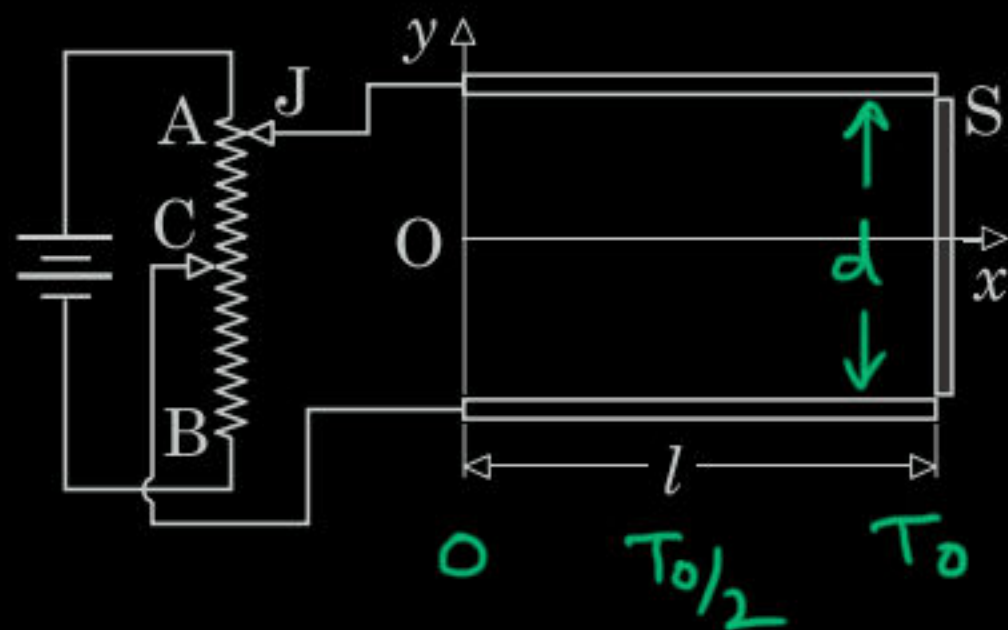


by Ankit Singhvi

$$\begin{aligned} \text{Energy: } \Delta PE + \Delta KE &= 0 \\ \Rightarrow [0 - Q_2 V] + KE_{\max} &= 0 \\ &\quad \quad \quad \frac{kQ_1}{R} \text{ (from 1)} \end{aligned}$$

$$\begin{aligned} \Rightarrow KE_{\max} &= \frac{kQ_1 Q_2}{R} \\ &= \frac{1}{2} \frac{\sigma^2 \pi R^2 \cdot \sigma \pi R^2}{4\pi\epsilon_0 R} = \frac{\pi \sigma^2 R^3}{2\epsilon_0} // \end{aligned}$$

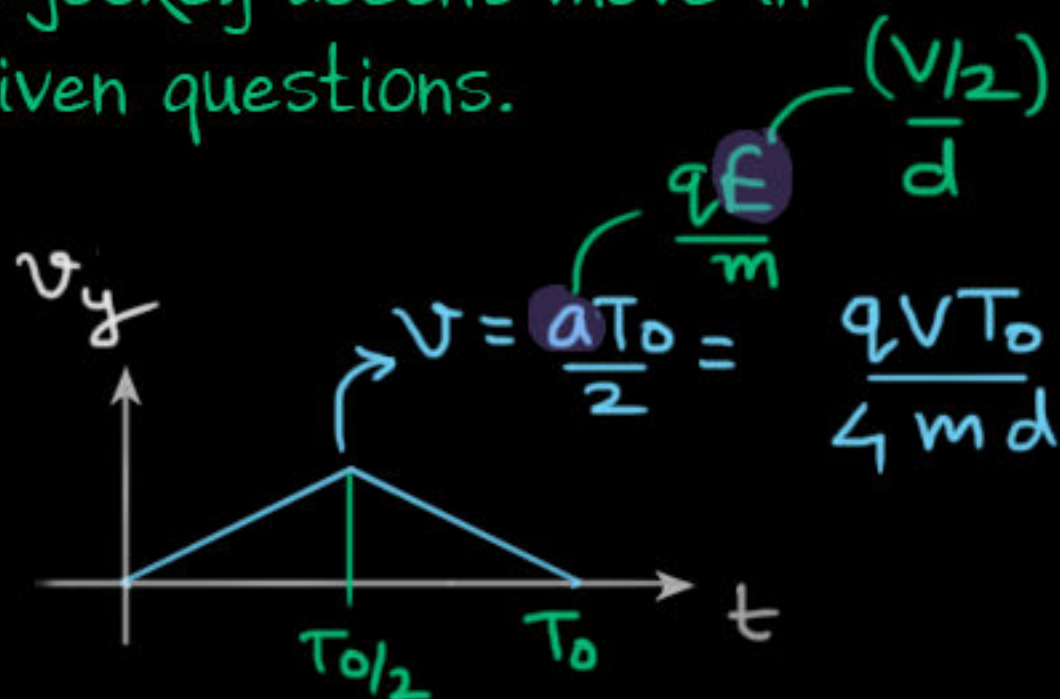
Youtube / Arihant Senior



Some points:

- 1 $v_x = \frac{l}{T_0} = \text{constant}$
- 2 In T_0 time, it reaches the end screen. So in $T_0/2$, it reaches half length $l/2$.

- 3 C jockey doesn't move in given questions.



Questions 30 to 32 are based on the following write-up.

Electrons (mass m and charge e) can be projected with a certain velocity from a point O between two parallel plate like electrodes separated by a distance d as shown in the figure. The bottom plate is connected to mid-point C of a rheostat, while the upper plate is connected to the rheostat through a sliding jockey J . The end terminals A and B of the rheostat are connected across an ideal battery of electromotive force V . When the jockey is held at C , electrons move along the x -axis that is parallel to the plates and hit a phosphorescent screen S in time T_0 .

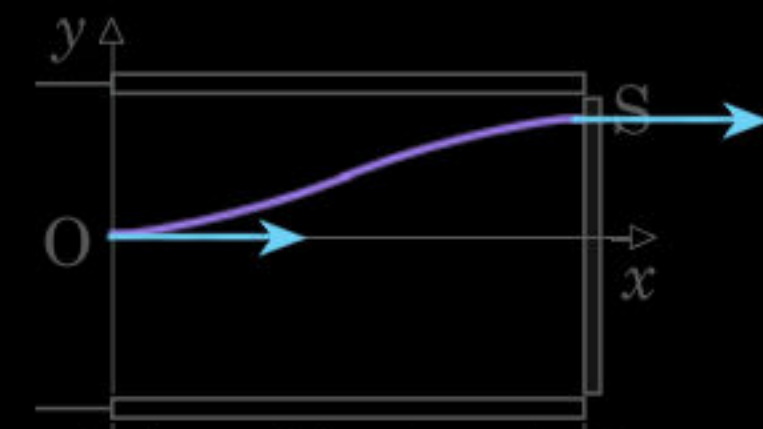
Distance d between the electrodes is large enough so that none of the electrons strikes the electrodes. Ignore magnetic effects and electromagnetic radiations.

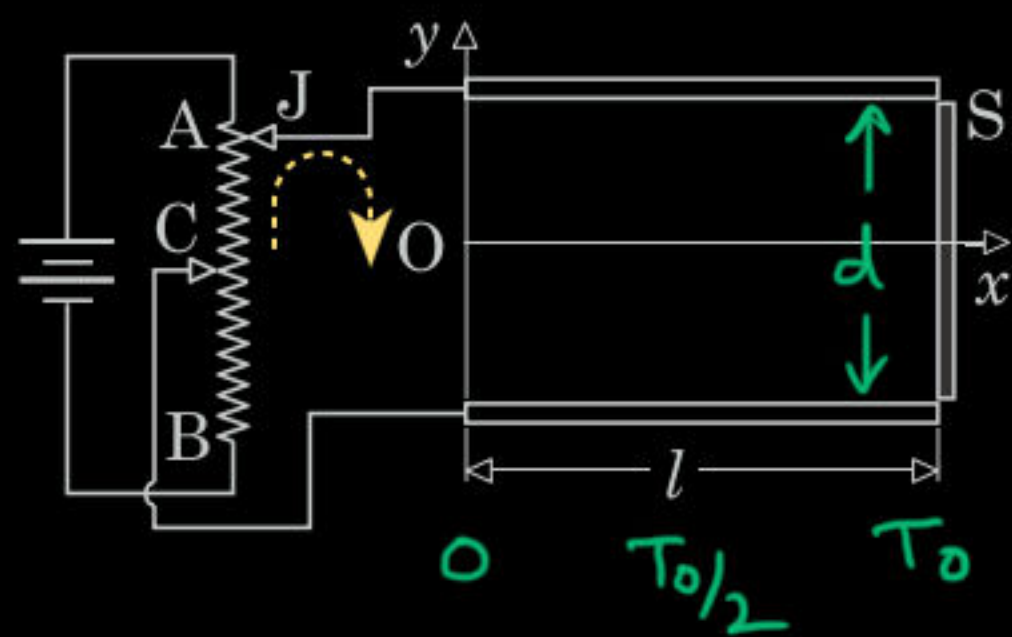
30. In a trial, the jockey J is held at A and an electron is projected. A time interval $0.5T_0$ after the electron is projected, the jockey is suddenly made to jump to the end B . Where on the screen does the electron make a spot?

✓(b) $\frac{eVT_0^2}{8md}$ above the x -axis.

Displacement $y =$

Area under triangle $= \frac{1}{2} T_0 \left(\frac{qVT_0}{4md} \right) = \frac{qVT_0^2}{8md}$





31. In second trial, the jockey J is made to slide from C to A and then back to C with the same speed in a total time interval T_0 . Where will the electron hit the screen?

(a) On the x-axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i}$.

(b) On the x-axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i} - \frac{eVT_0}{4md} \hat{j}$.

✓(c) Above the x-axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i} + \frac{eVT_0}{4md} \hat{j}$.

Jockey J moves with constant speed



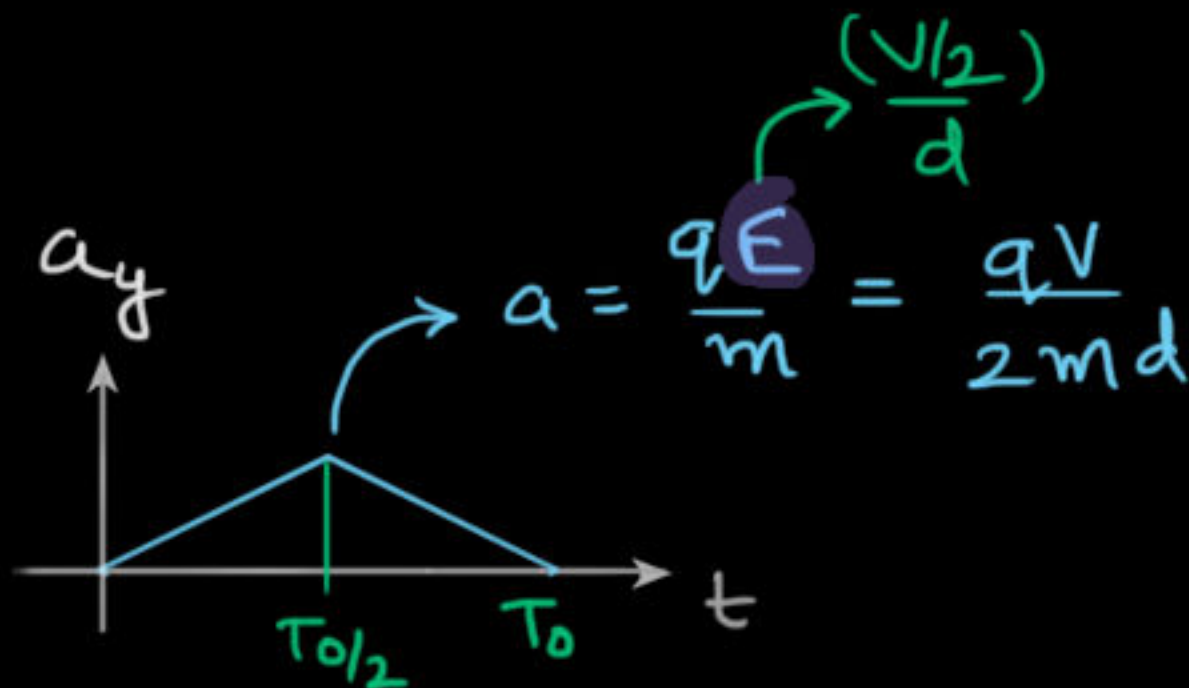
Field between plates vary linearly.



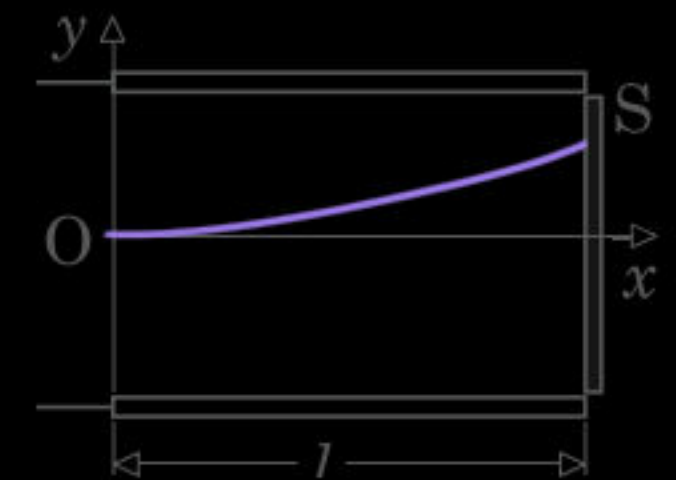
Acceleration varies linearly.

$$v_y = \text{Area under triangle} = \frac{1}{2} T_0 \left(\frac{qV}{2md} \right) = \frac{qVT_0}{4md} \hat{j}$$

$$\text{Unchanged } v_x = \frac{l}{T_0} \hat{i}$$

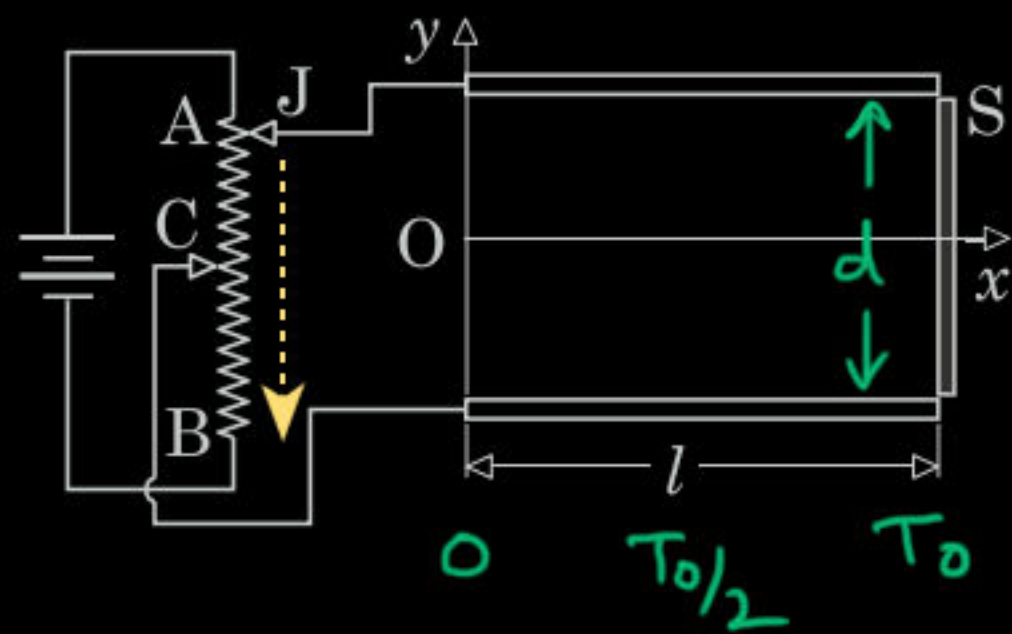


Also, we can see a_y is always positive, so it will strike above x-axis



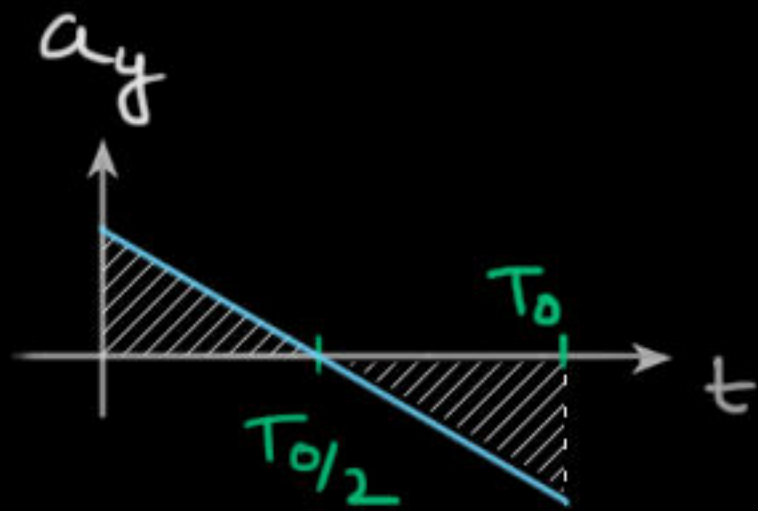
Youtube / Arihant Senior

by Ankit Singhvi



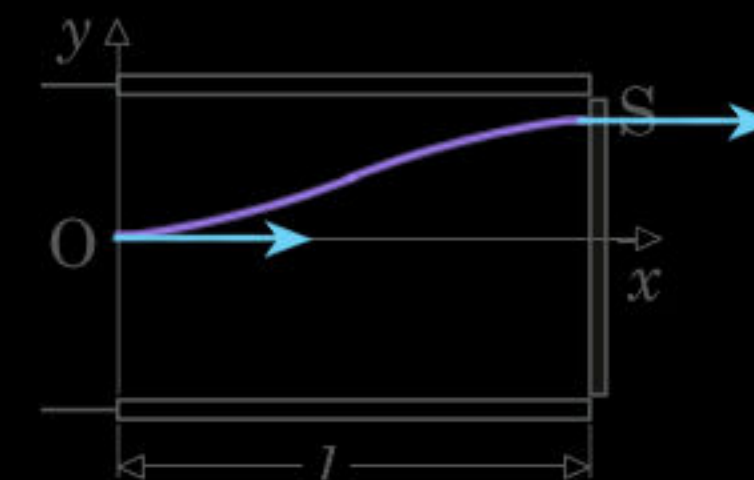
32. In third trial, the jockey is made to slide from A to B with a constant speed in the time interval T_0 . Where will the electron hit the screen?

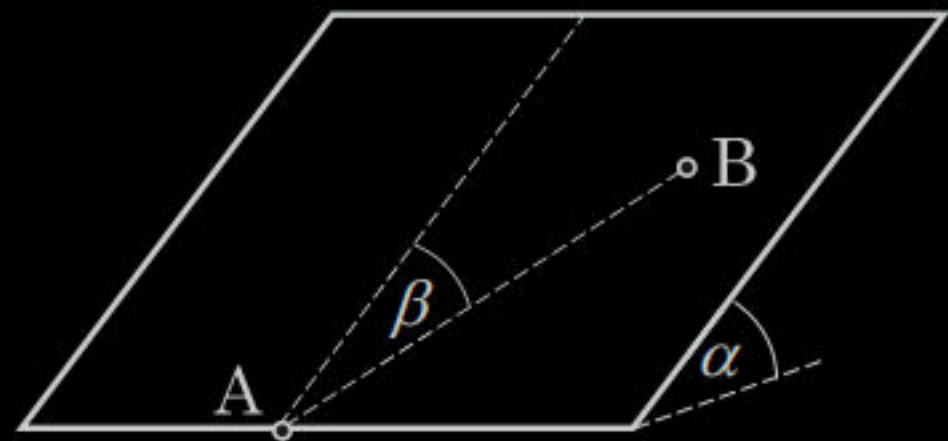
- (a) On the x -axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i}$.
- ✓ (b) Above the x -axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i}$.
- (c) Below the x -axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i}$.
- (d) On the x -axis with velocity $\vec{v} = \frac{l}{T_0} \hat{i} + \frac{eVT_0}{2md} \hat{j}$.



$v_y = \text{Area under curve} = 0$ (At the very end, otherwise its positive throughout)

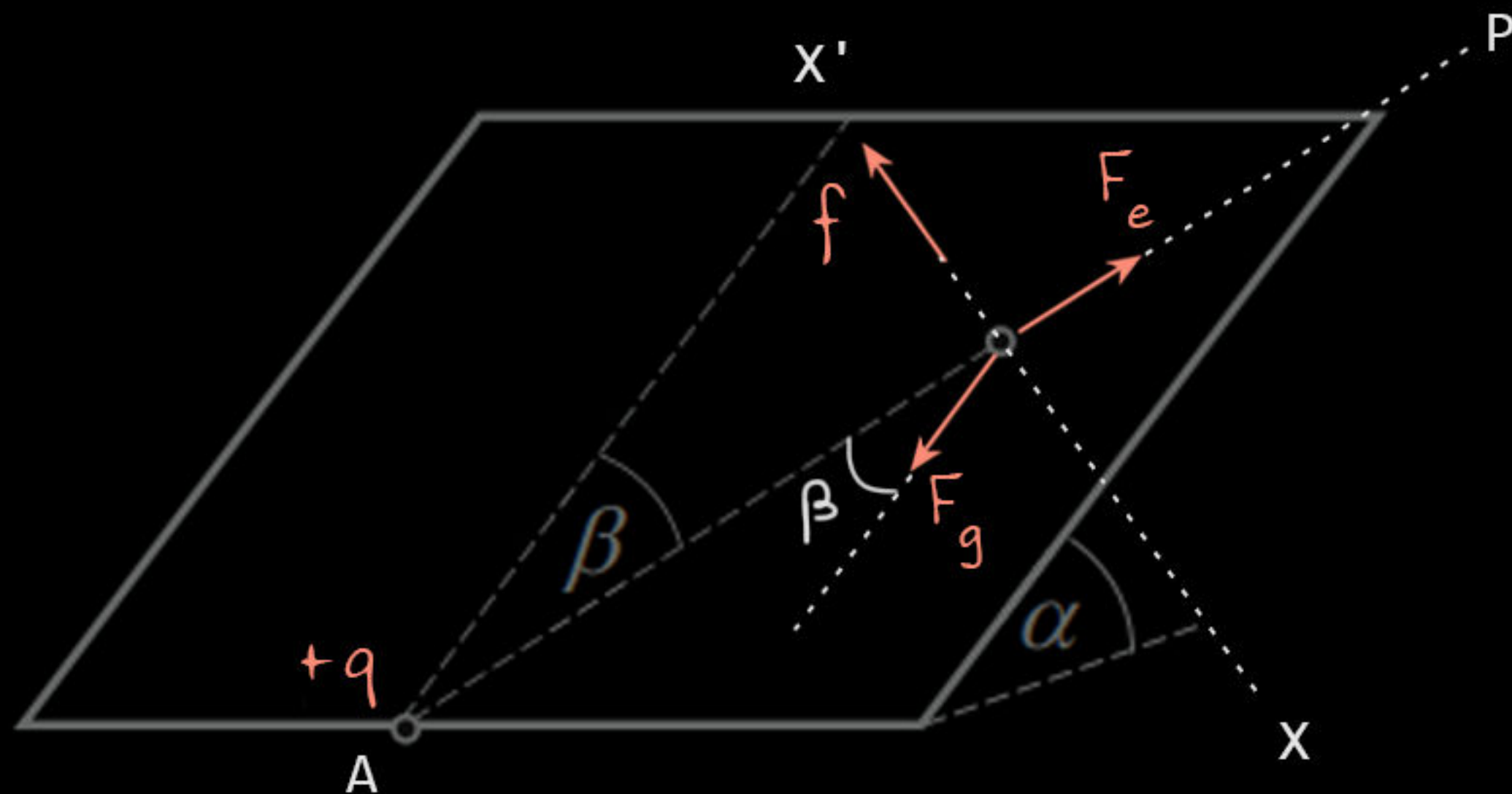
Unchanged $v_x = l/T_0 \hat{i}$





1. A charge particle A is fixed at the base of a uniform slope of inclination α . Another charge particle B is placed on the slope at an angular position β from the line of greatest slope through the position of the first particle. Coefficient of friction between the particle B and the slope is μ ($\mu < \tan \alpha$). For the particle at B to stay in equilibrium, what could be the maximum value of the angle β ?

Writing forces along the plane:



$$F_g = mg \sin \alpha$$

q can be suitably adjusted to balance forces along AP.

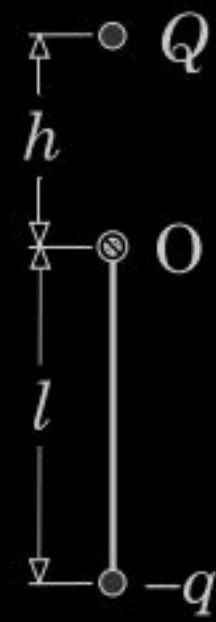
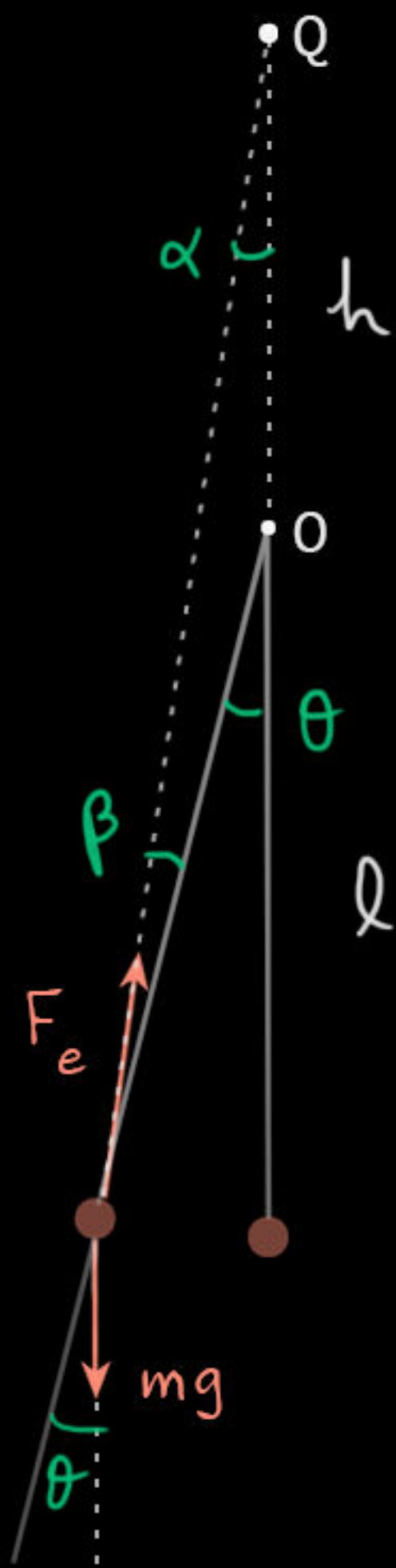
Along XX' however, only counter to F_g 's component is friction's component.

- ∴ At maximum β all the friction will be used to counter F_g 's component along XX'

$$\text{i.e.: } F_g \sin \beta = f < \mu mg \cos \alpha$$

$$\Rightarrow (\cancel{mg} \sin \alpha) \sin \beta < \mu \cancel{mg} \cos \alpha$$

$$\Rightarrow \sin \beta < \mu \cot \alpha //$$



2. One end of an insulating **rigid rod** of negligible mass and length l is pivoted to a fixed point O. A small ball of mass m having a **negative charge of modulus q** is attached to the lower end of the rod. Another small ball carrying a positive charge Q is fixed at a height h above the point O as shown in the figure. **What should the range of values of mass m of the lower ball be so that the rod remains in a state of stable equilibrium?** Acceleration due to gravity is g .

On disturbing a bit, torque must be restoring for rod to be in equilibrium.

$$\therefore mg\theta > F_e \beta \Rightarrow m > \frac{F_e}{g} \frac{\beta}{\theta} \quad (1)$$

$\frac{kqQ}{(l+h)^2}$

$$\alpha + \beta = \theta \Rightarrow \frac{\alpha}{\theta} + \frac{\beta}{\theta} = 1 \quad (2)$$

By geometry {

$$(l+h)\alpha = l\theta \Rightarrow (l+h) \frac{\alpha}{\theta} = l \quad (3)$$

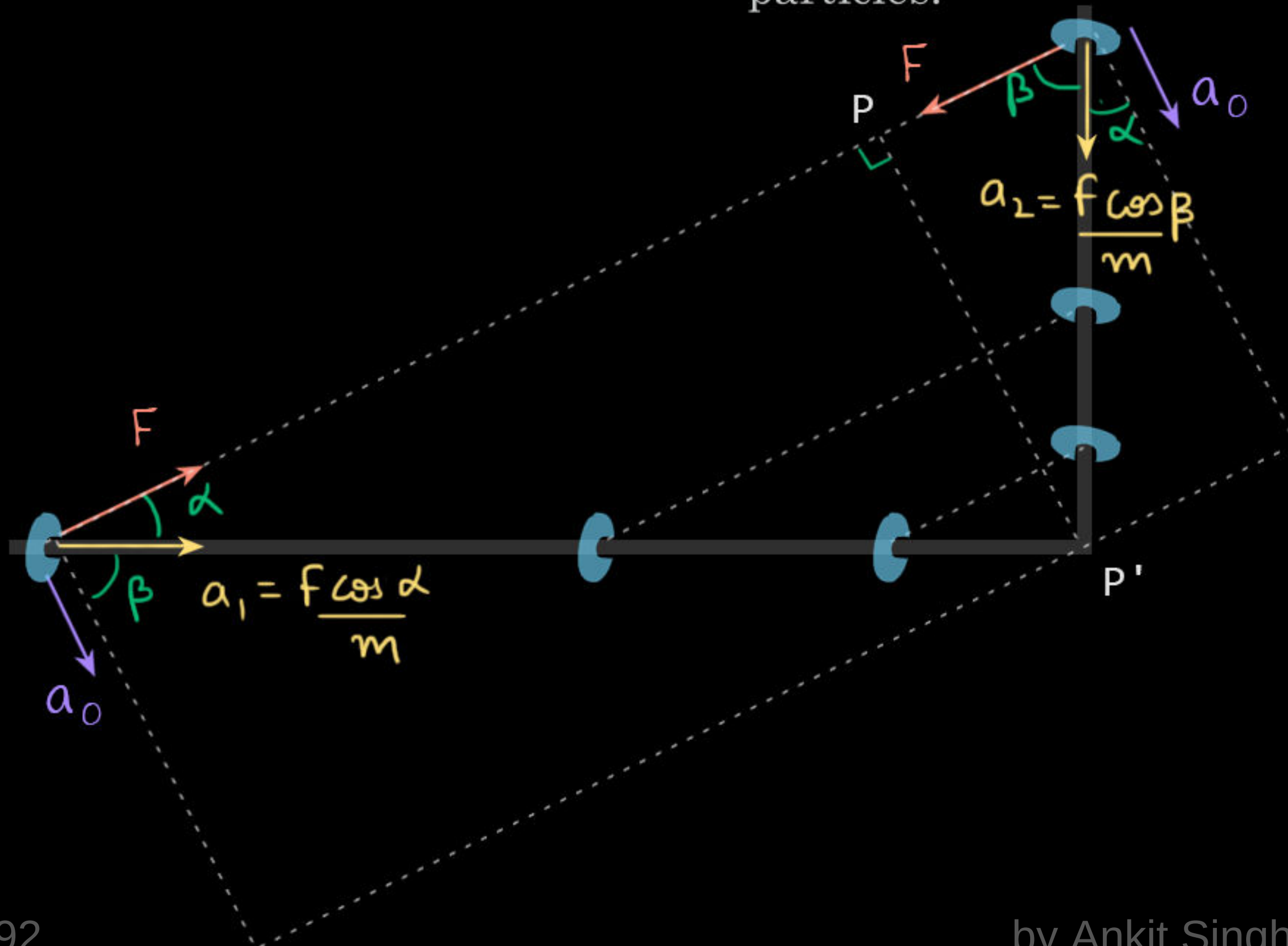
From 1,2,3:

$$m > \frac{kqQ h}{g(l+h)^3}$$

Youtube / Arihant Senior



3. Two beads of equal mass m and unlike charges of modulus q_1 and q_2 can slide on a fixed frictionless non-conducting rod, bent at right angle. Initially, the beads are held at rest at distances d and $2d$ from the corner as shown and then released simultaneously. When one of the beads reaches the corner, where will the other bead be? Treat the beads as particles.



Actual accelerations are a_1 and a_2 .
Components of these along PP' are:

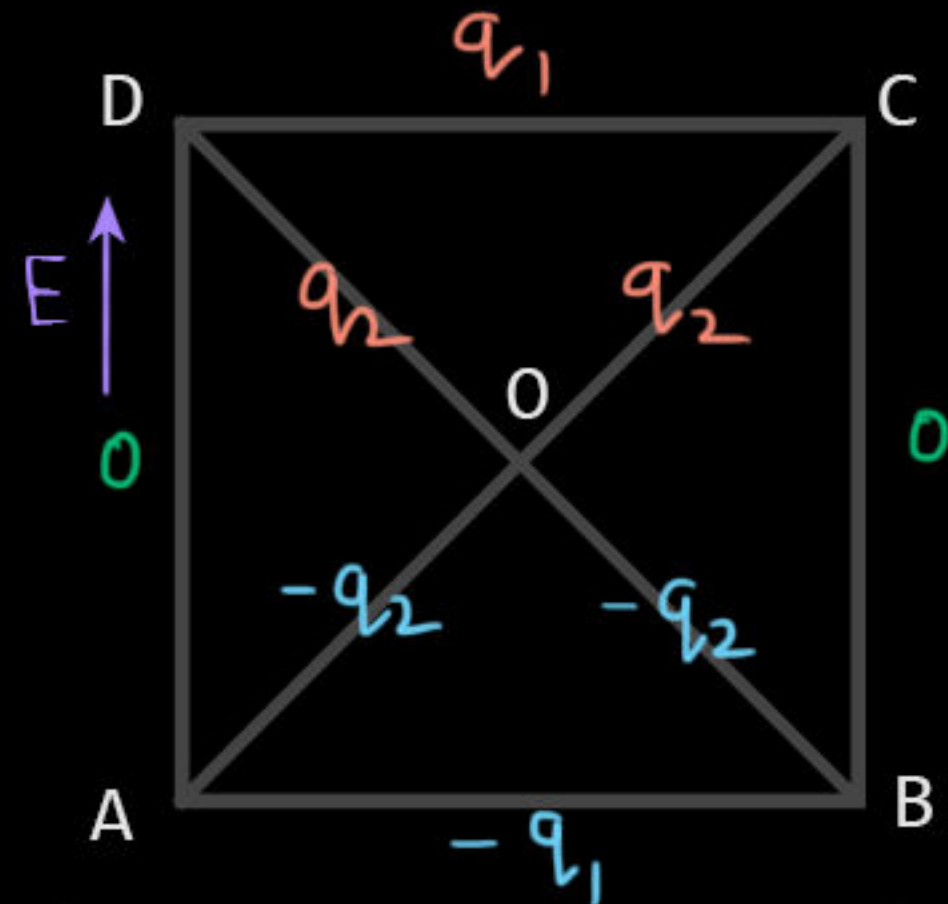
$$a_1 \cos \beta = a_2 \cos \alpha = \frac{F \cos \alpha \cos \beta}{m} = a_0$$

So both start from rest and their accelerations along PP' is always equal. (even though non-uniform).

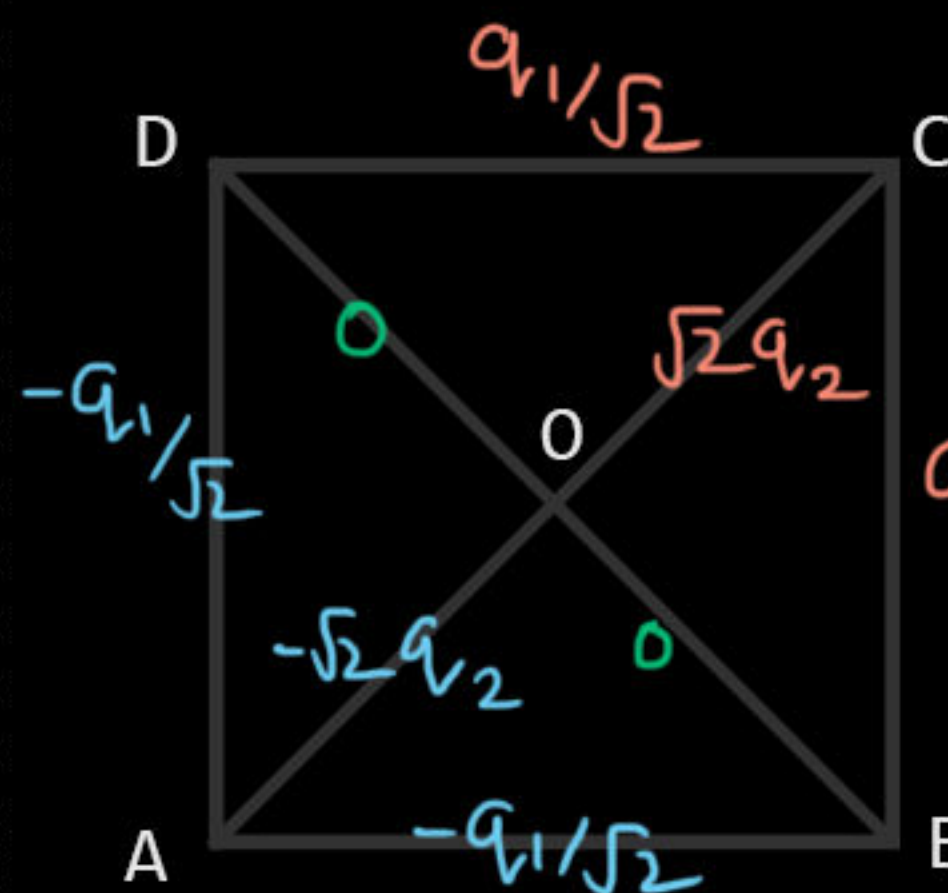
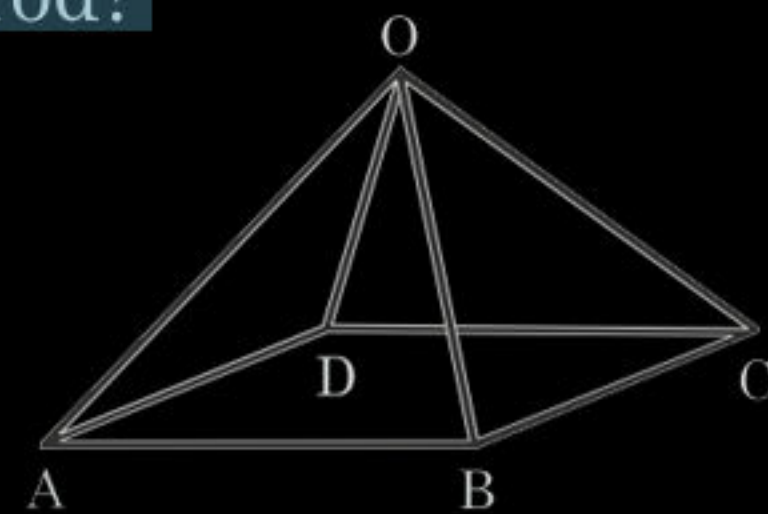
- They will travel equal distance in equal amount of time along PP' , and eventually meet at P' at the same time.

4. A rigid frame in the shape of a right pyramid is made of conducting rods. The base ABCD is a square and the apex O is vertically above the centre of the base. The frame is electrically neutral. When it is placed in a uniform electric field of intensity E pointing from the corner A towards the corner D, total charges induced on the rods DC and OC are known to be q_1 and q_2 respectively. Now the frame is rotated to make the electric field pointing from the corner A towards the corner C. What are the charges induced on each rod?

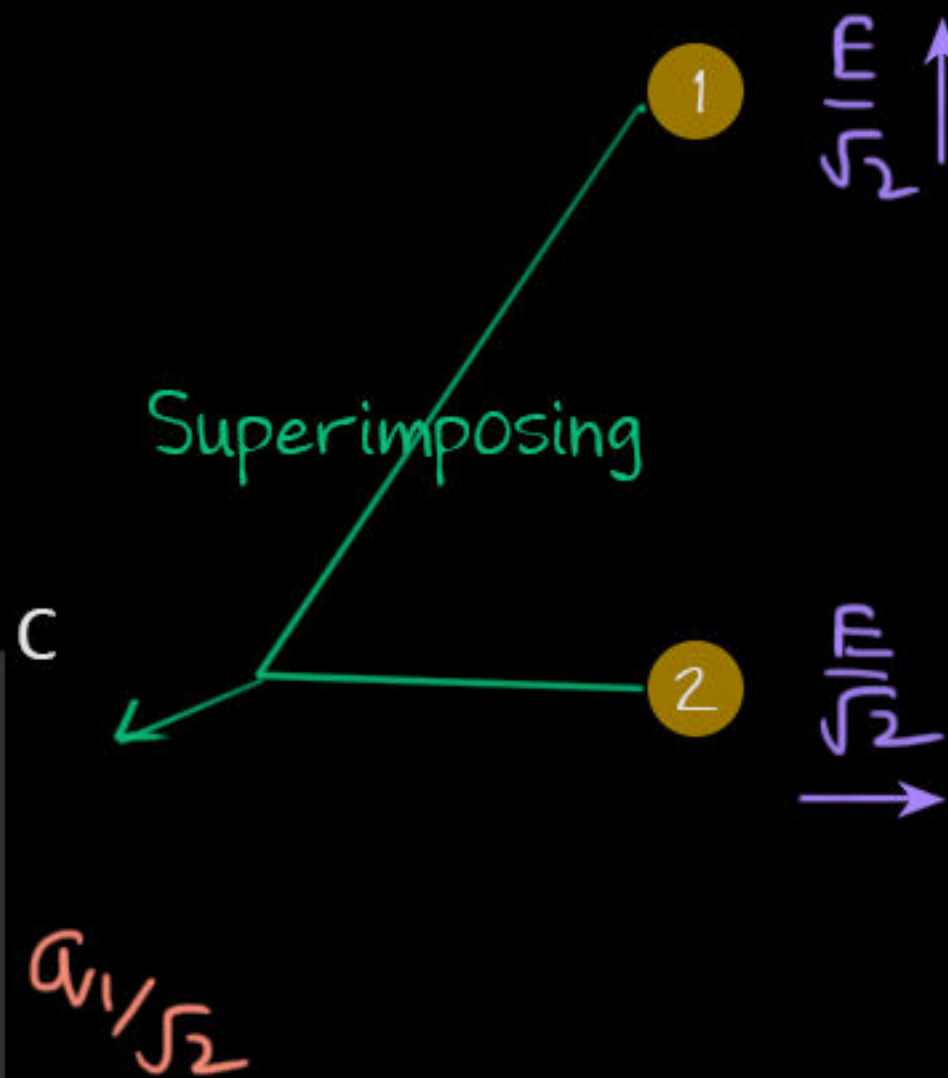
Concept: Induced charges in a conductor are proportional to external electric field.



By symmetry, AD and BC will have net zero charge.

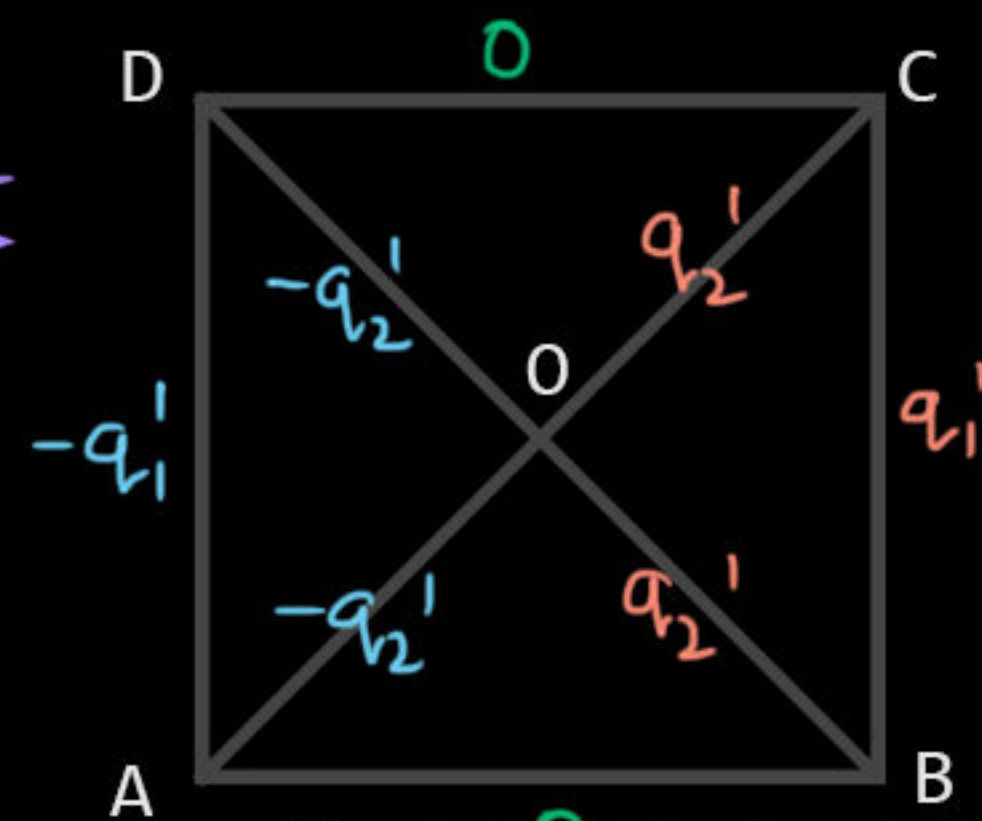
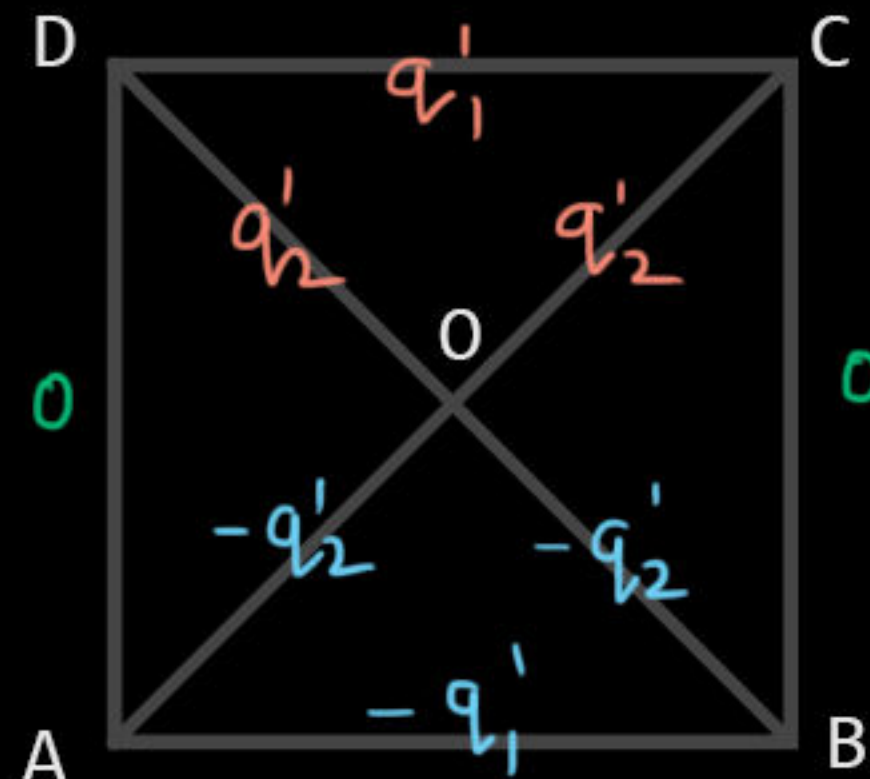


Superimposing



$$q'_1 = q_1/\sqrt{2}$$

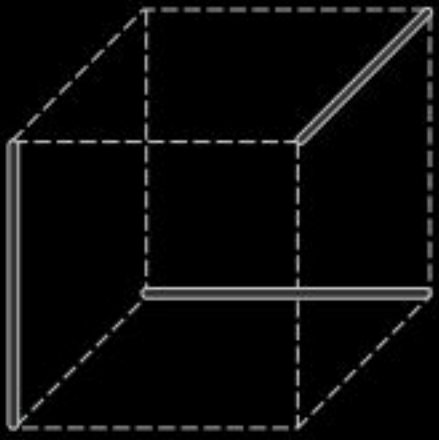
$$q'_2 = q_2/\sqrt{2}$$



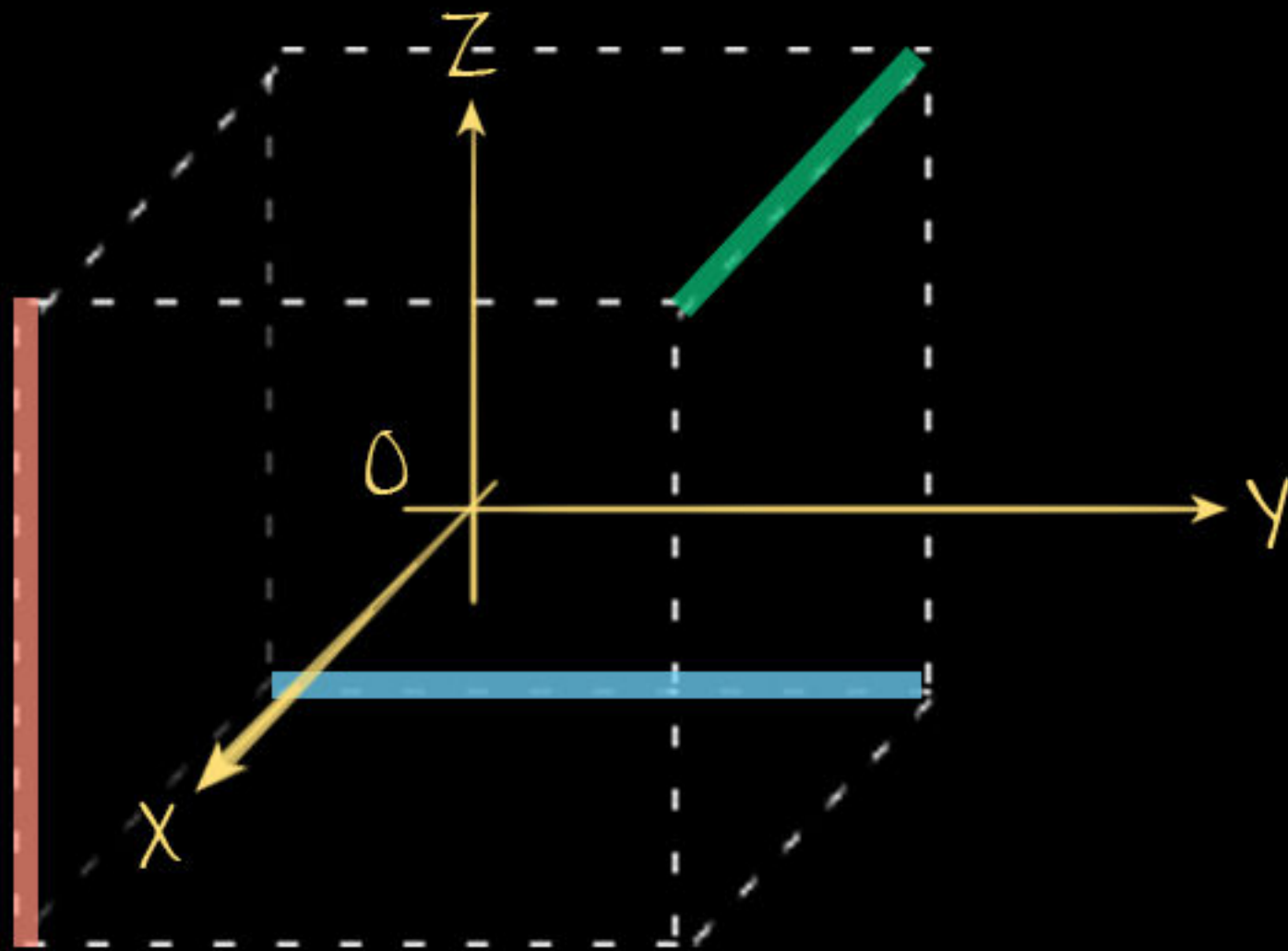
Answer

by Ankit Singhvi

Youtube / Arihant Senior

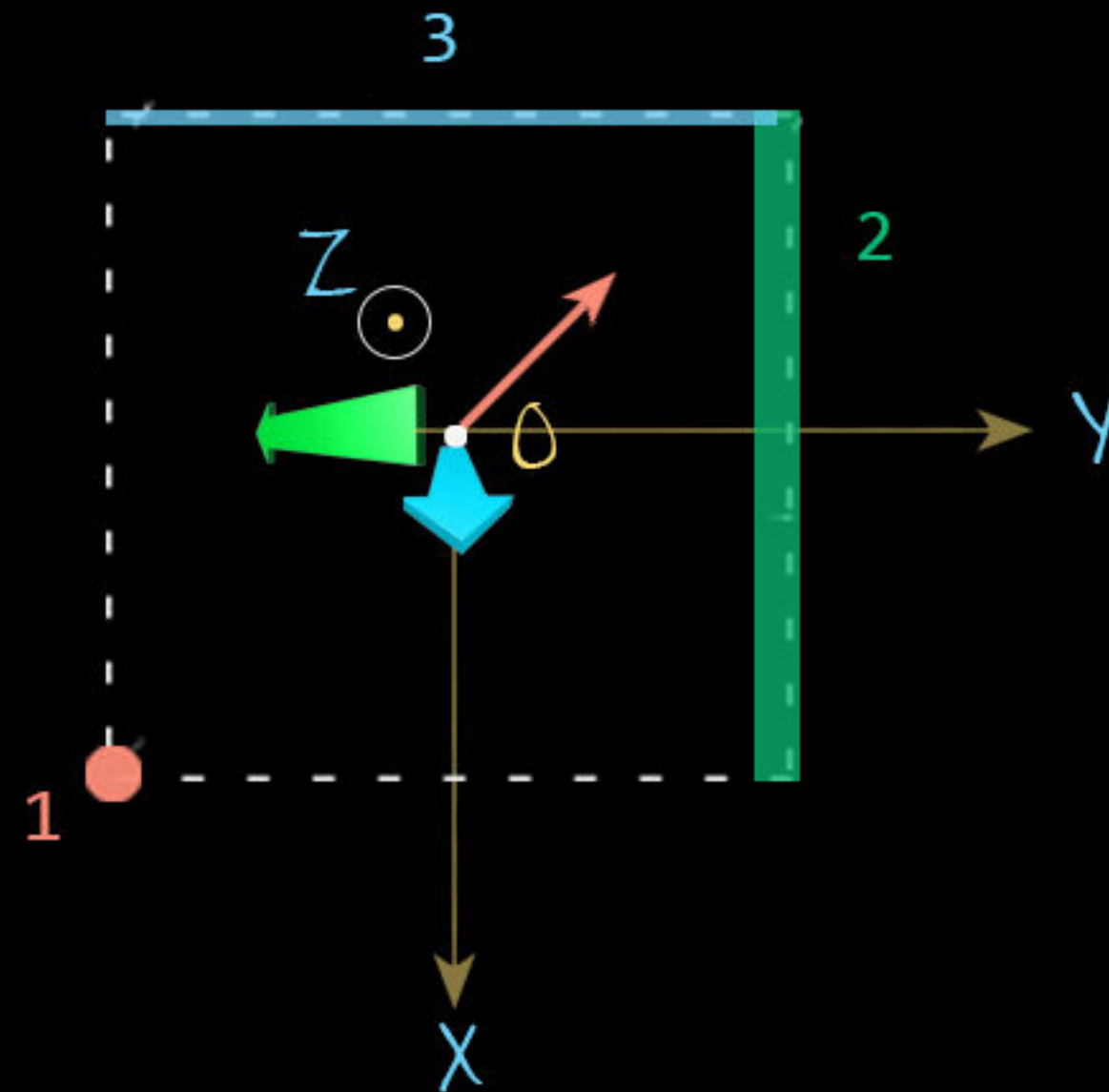


6. Three identical thin uniformly charged filaments are fixed along three sides of a cube as shown in the figure. Length of each filament is l and line charge density on each of them is λ . Determine electric field at the centre of the cube. The cube is a geometrical construct and not made of any matter.



Let the center of cube be origin.

Top view:



$$E_1 = k(-\hat{i} + \hat{j})$$

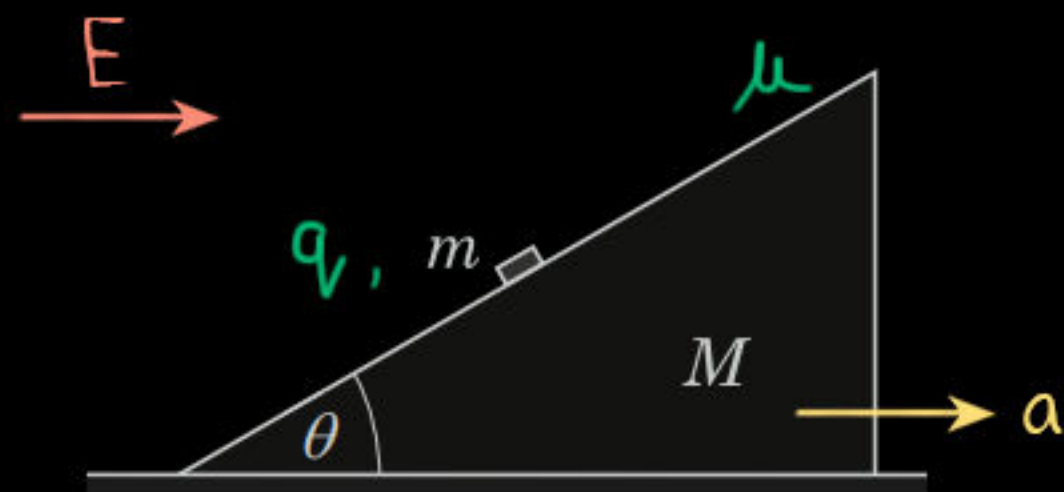
$$E_2 = k(-\hat{j} - \hat{k})$$

$$E_3 = k(\hat{i} + \hat{k})$$

$$\therefore E_1 + E_2 + E_3 = 0$$

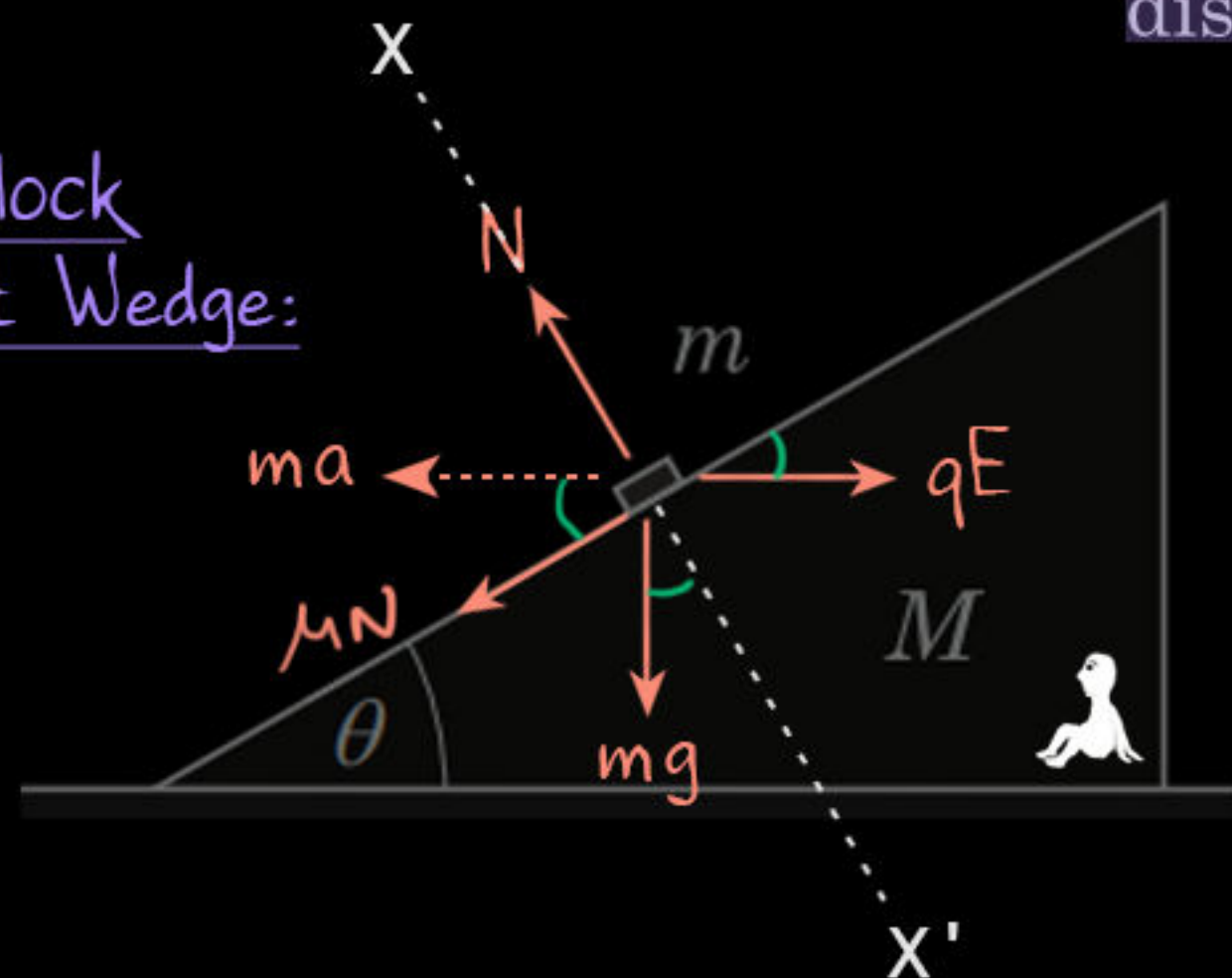
Youtube / Arihant Senior

by Ankit Singhvi



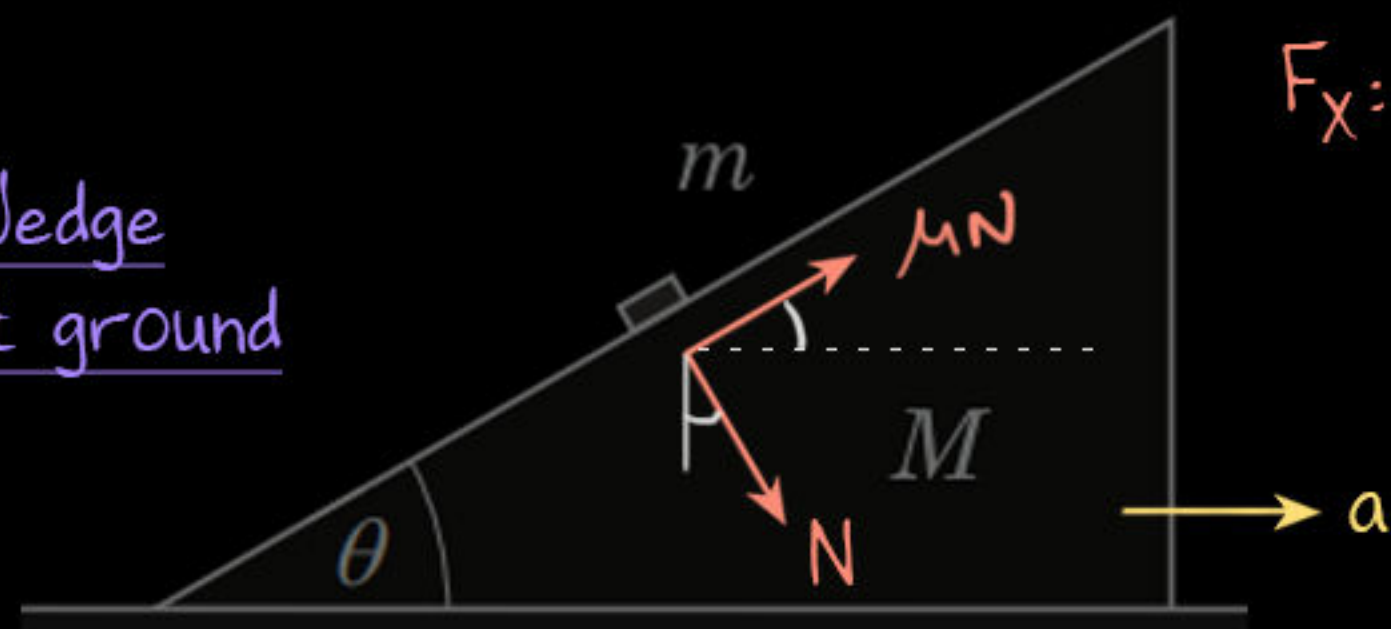
7. In presence of a uniform horizontal electric field E , a small non-conducting disc of mass m and charge q is released on a non-conducting triangular prism of mass M placed on a frictionless horizontal floor as shown in the figure. Slant face of the prism makes an angle θ with the horizontal and coefficient of friction between it and the disc is μ . If the disc accelerates up the slant face, find acceleration of the prism.

on block
w.r.t Wedge:



$$F_{xx'}: N = mg \cos \theta + (qE - ma) \sin \theta \quad (1)$$

on Wedge
w.r.t ground



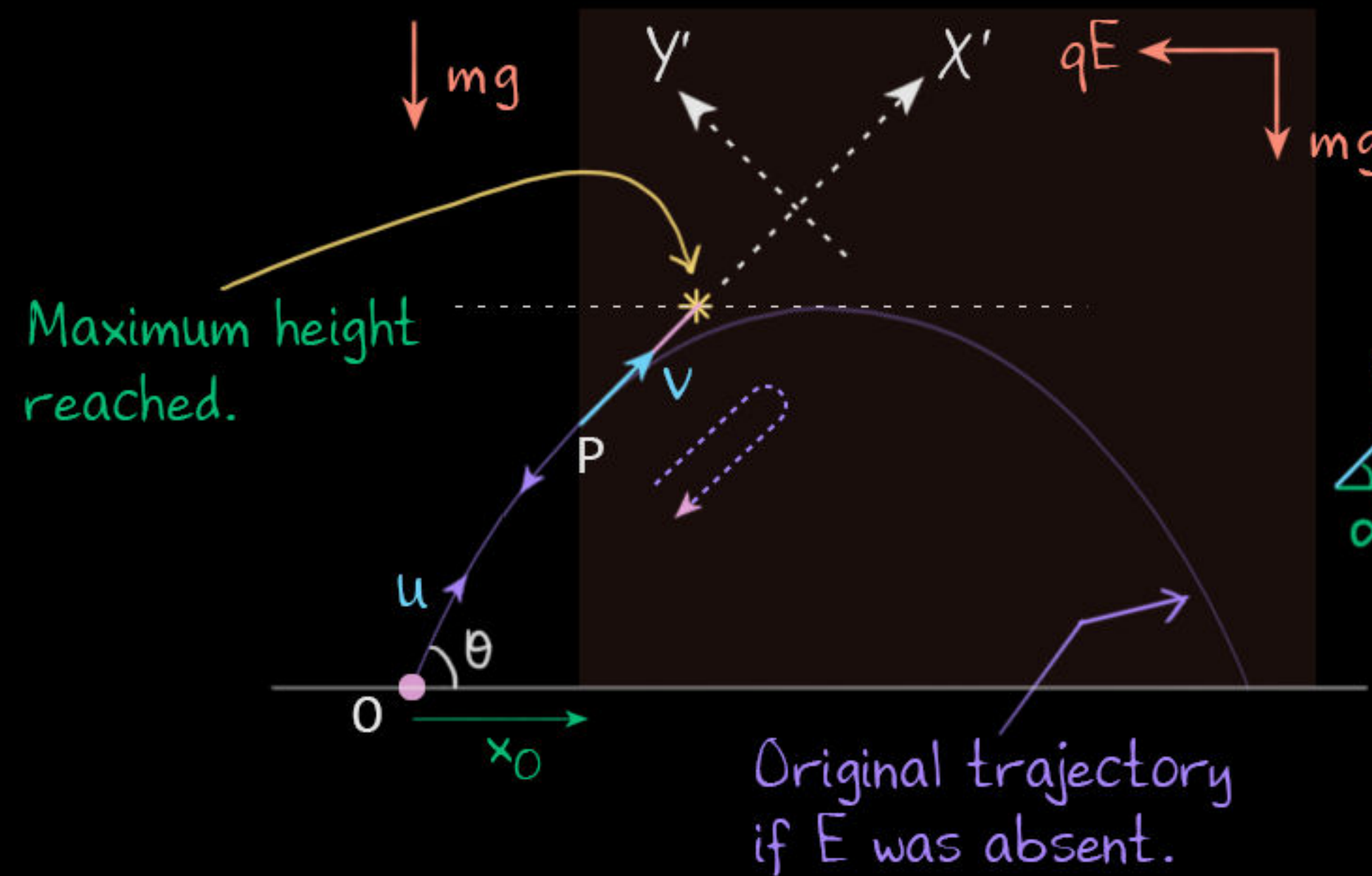
$$F_x: Ma = \mu N \cos \theta + N \sin \theta \quad (2)$$

From 1,2:

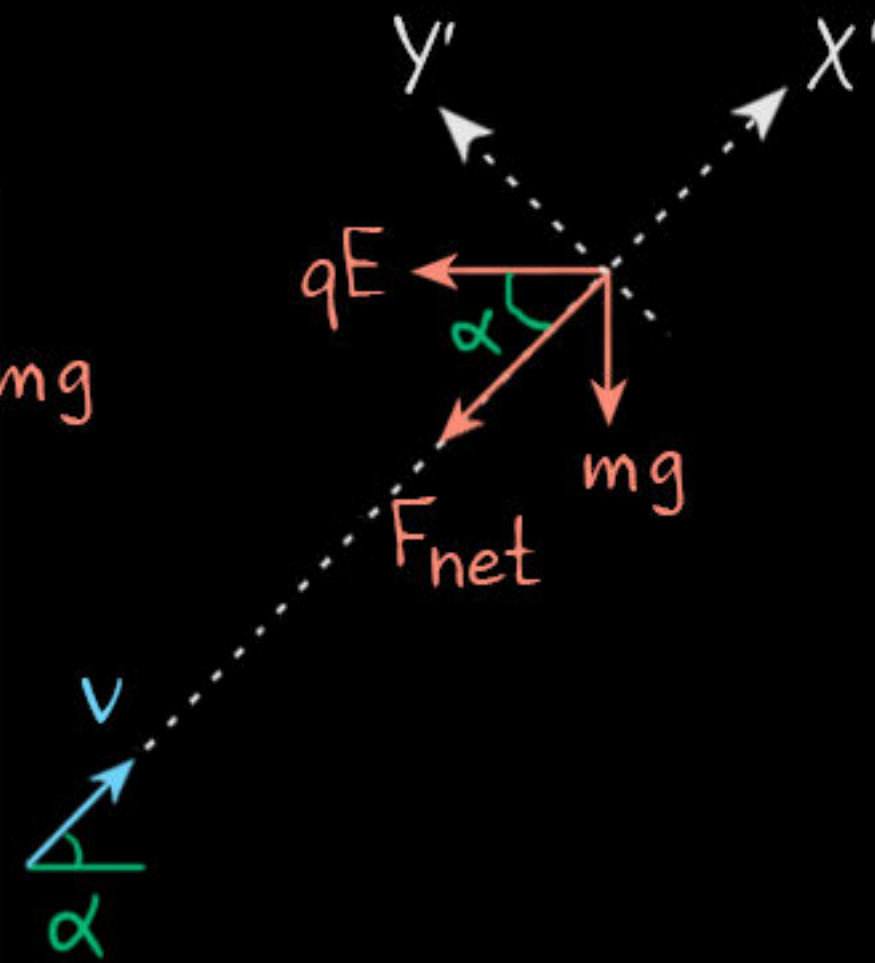
$$a = \frac{(mg \cos \theta + qE \sin \theta) (\mu \cos \theta + \sin \theta)}{M + m \sin \theta (\mu \cos \theta + \sin \theta)}$$

Particle's a_y' must be zero inside field, so it doesn't deviate. So, F_{net} must be along $-X'$ direction.

It will go in straight line, reach maximum height and retrace the trajectory back to point O.



8. A particle of mass m and charge q projected with a speed u at an angle θ above the horizontal, after travelling a horizontal distance x_0 , enters a region where in addition to gravity a uniform static horizontal electric field also exists. Boundary of this region is vertical. If after some time, the particle returns to the point of projection, find magnitude of the electric field and time of flight of the particle. Acceleration of free fall is g and influence of the air is to be neglected.



$$T = \frac{2u \sin \theta}{g}$$

$$\tan \alpha = \frac{v_y}{v_x} = \frac{mg}{qE}$$

$(u \sin \theta - gt_0) \frac{x_0}{u \cos \theta}$

$$\Rightarrow \frac{u \sin \theta - g \left(\frac{x_0}{u \cos \theta} \right)}{u \cos \theta} = \frac{mg}{qE}$$

$$\Rightarrow E = \frac{mg u^2 \cos^2 \theta}{q (u^2 \sin \theta \cos \theta - g x_0)}$$

Let the time it takes for particle to enter the region at distance x_0 is t_0 .

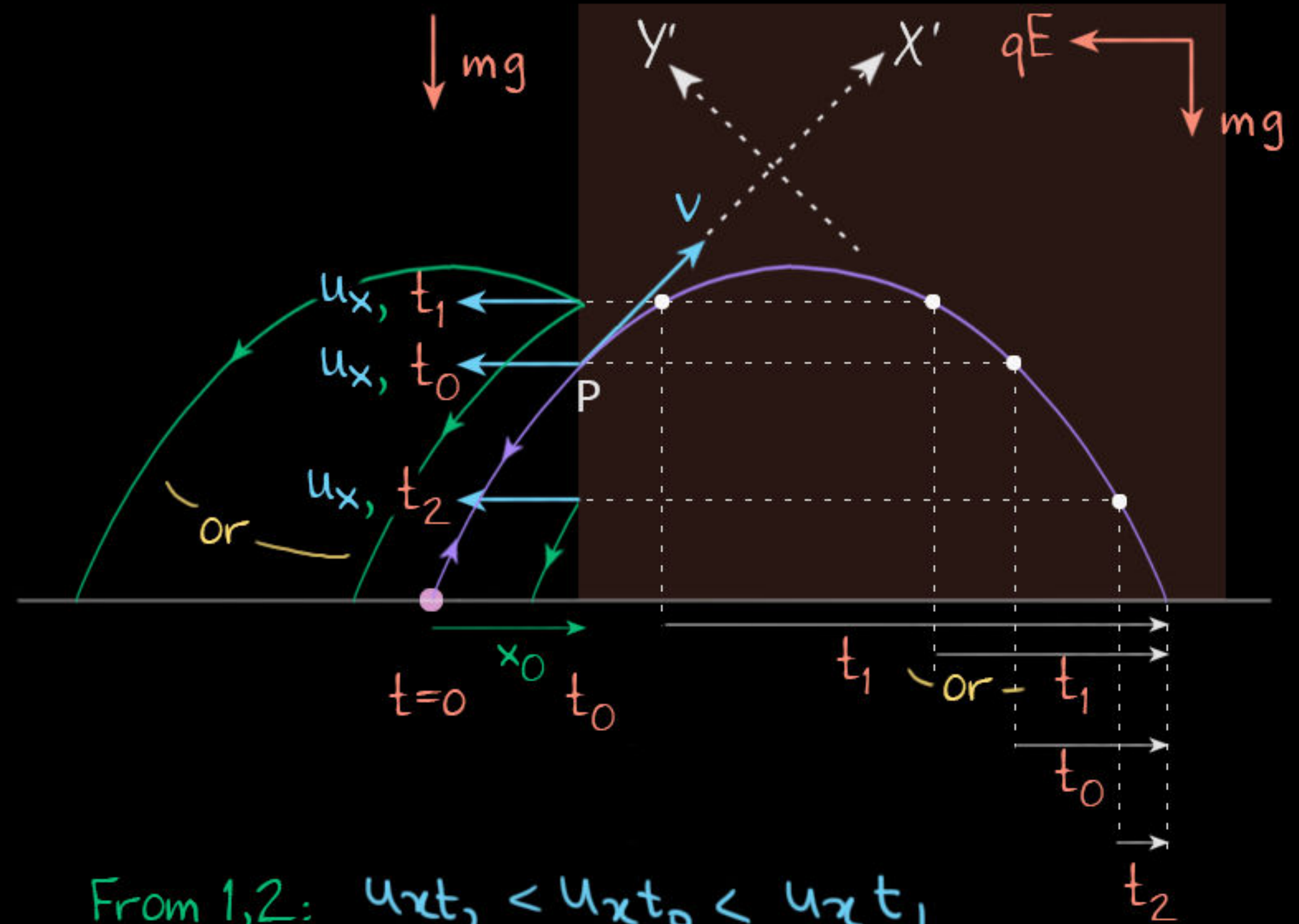
As qE is conservative, when particle reappears its u_x will be unchanged.

mg is conservative too, so when it reappears, its v_y will be same as that of original trajectory's v_y at same height.

Now, flight time after reappearance depends on v_y , so that too will be of original trajectory's remaining flight time at same height as shown.

∴ Clearly: $t_2 < t_0 < t_1$ ①

Also, after reappearance: $x = u_x t$ ②



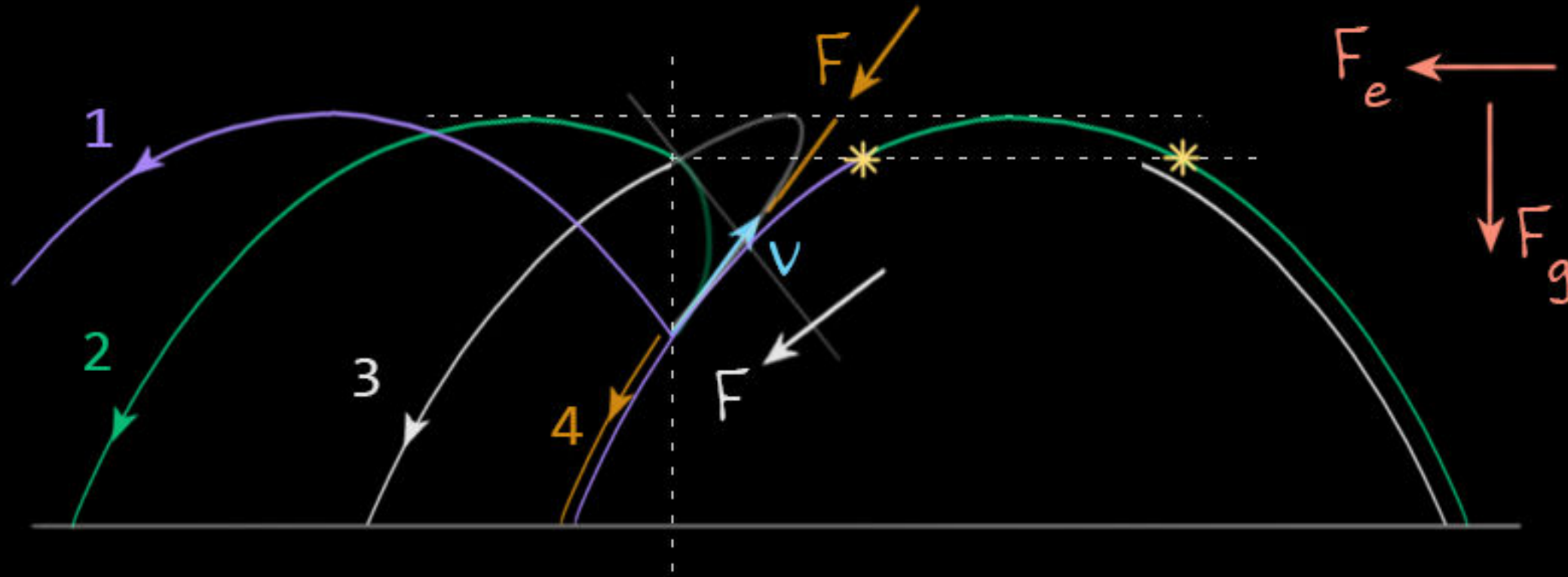
From 1,2: $u_x t_2 < u_x t_0 < u_x t_1$

$x_2 < x_0 < x_1$

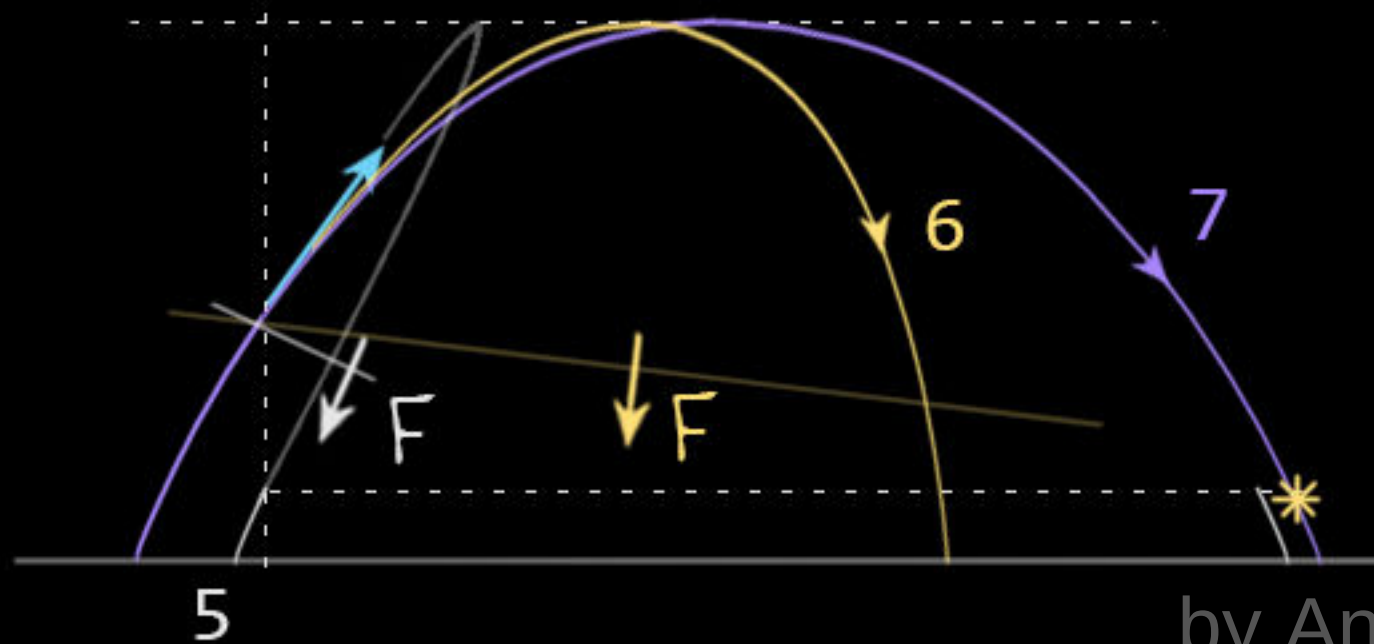
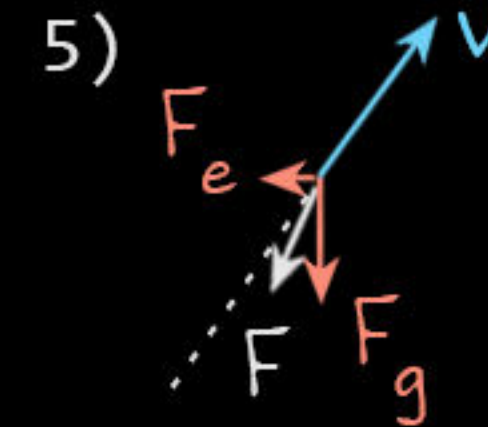
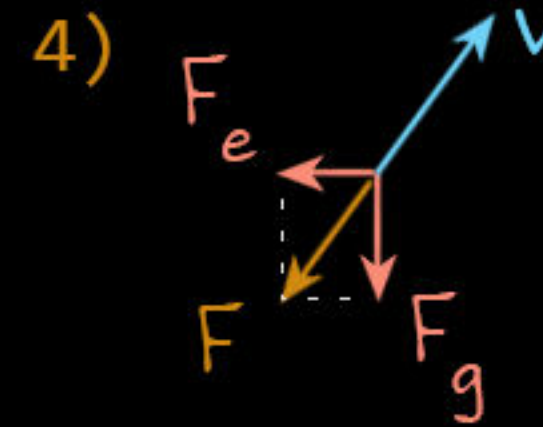
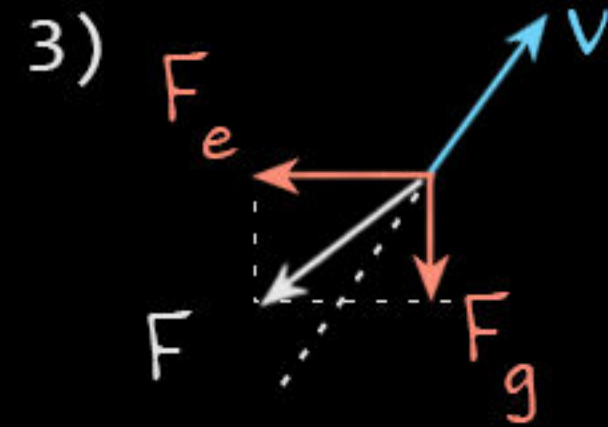
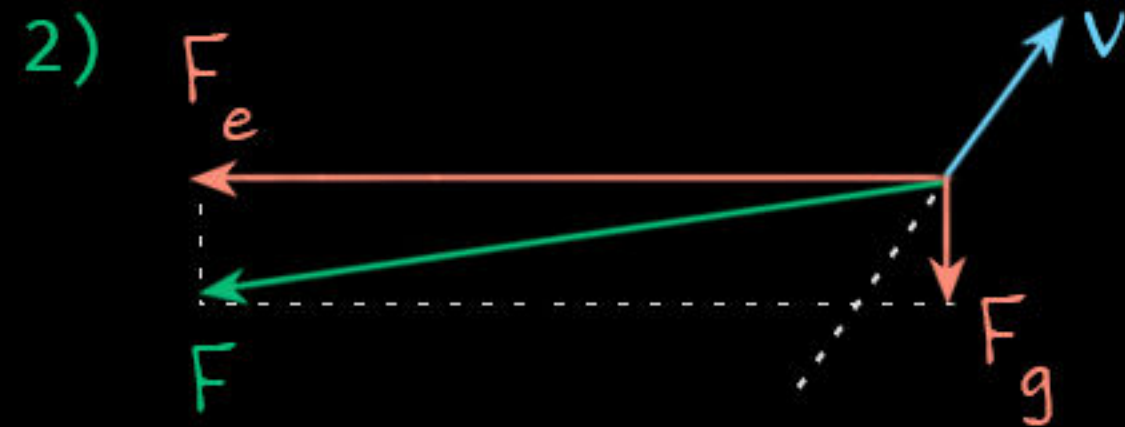
⇒ Particle must reappear at same point P to return to point of projection.

⇒ Particle's a_y must be zero inside field, so it doesn't deviate.

Lets see all 7 cases as F_e decreases from infinity to zero.

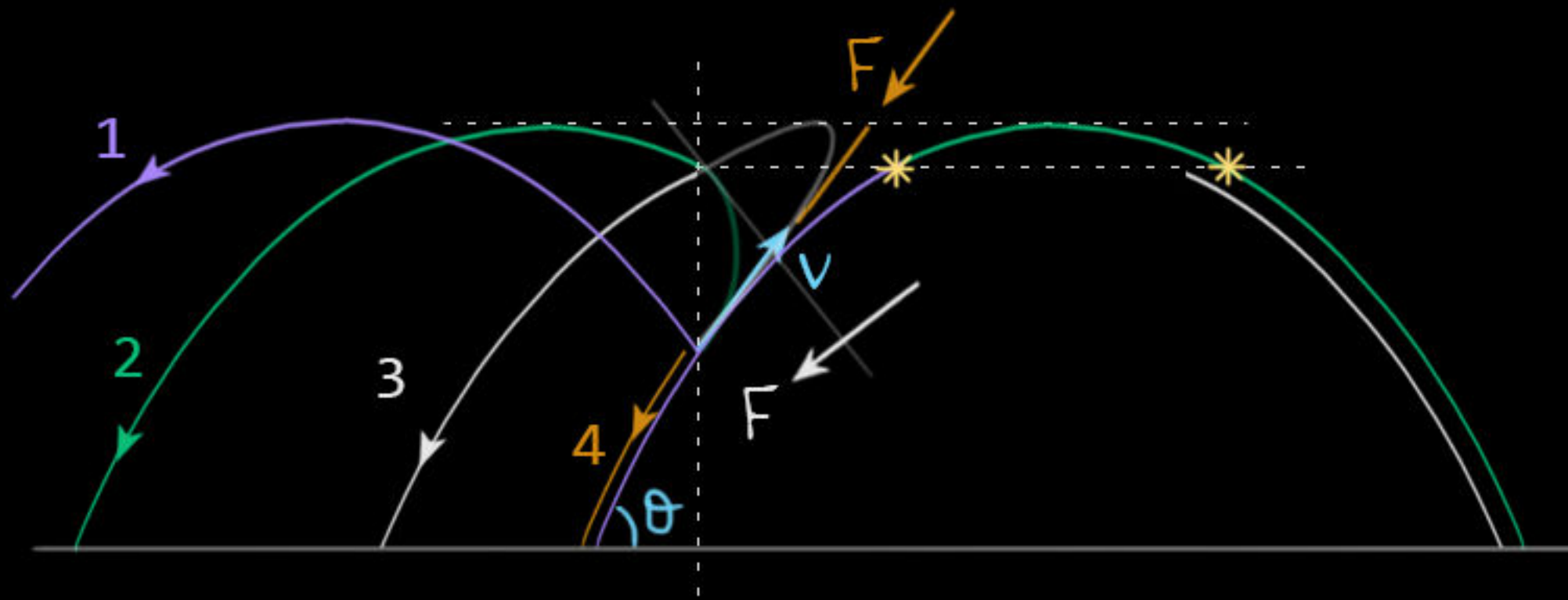


1) F_e is infinitely high
(it will bounce off)

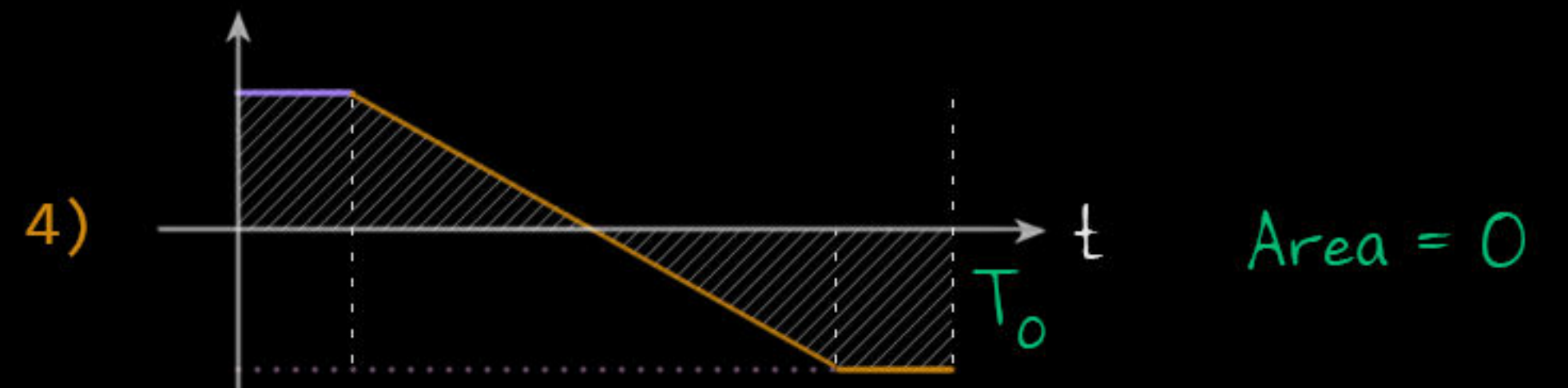
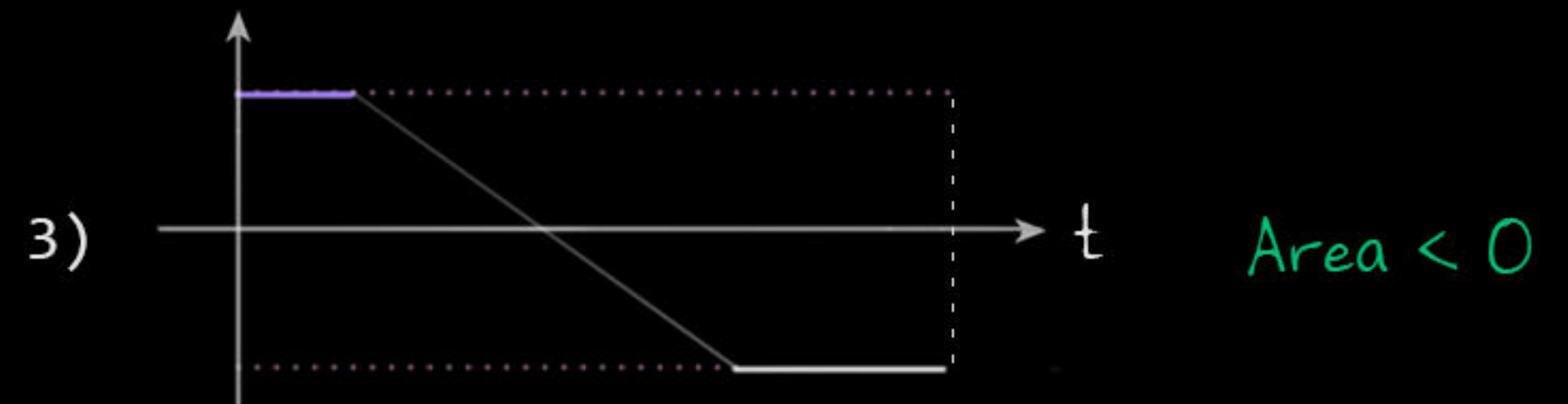
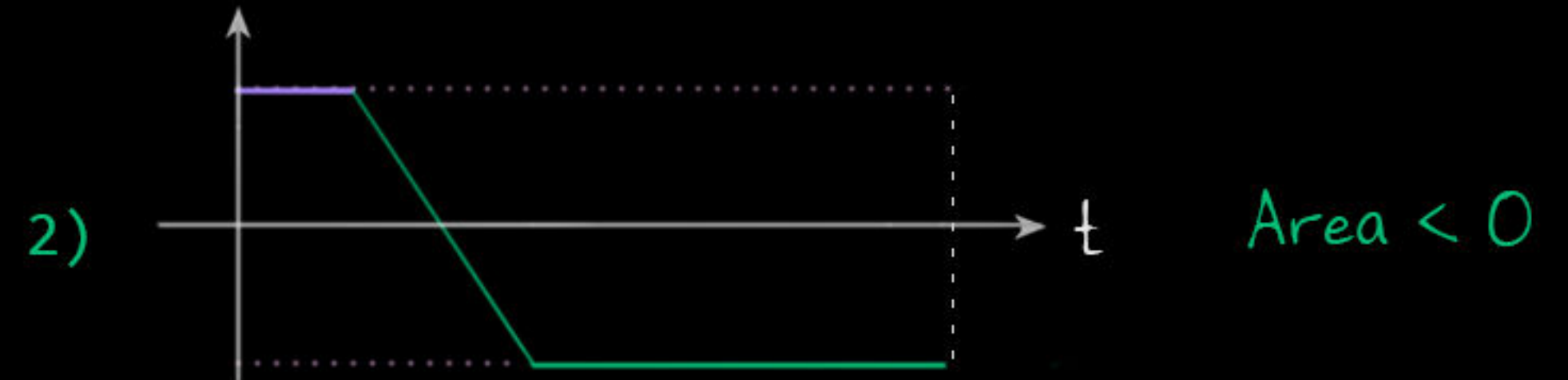


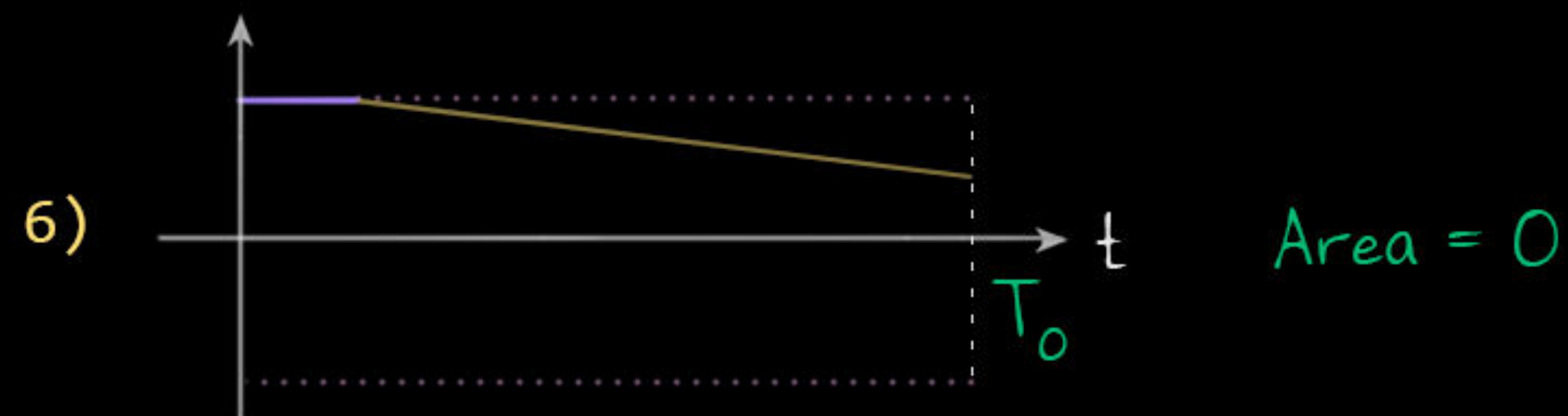
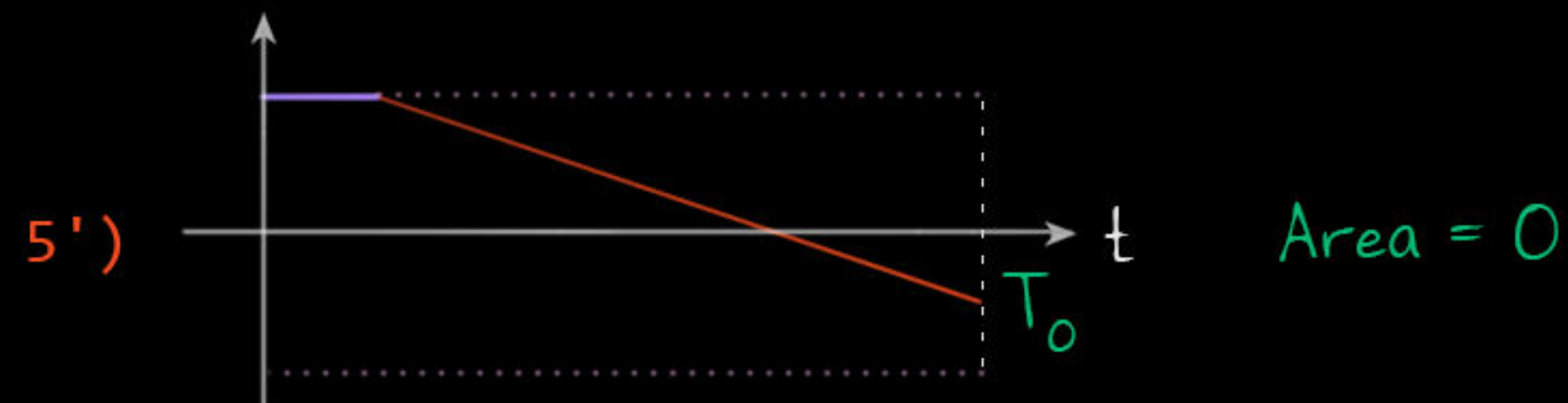
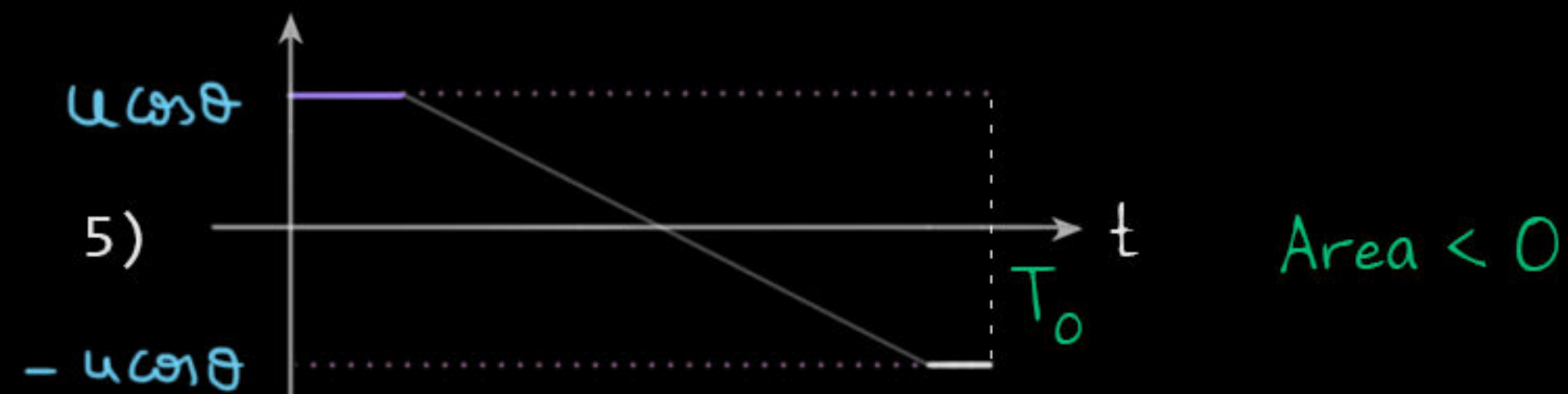
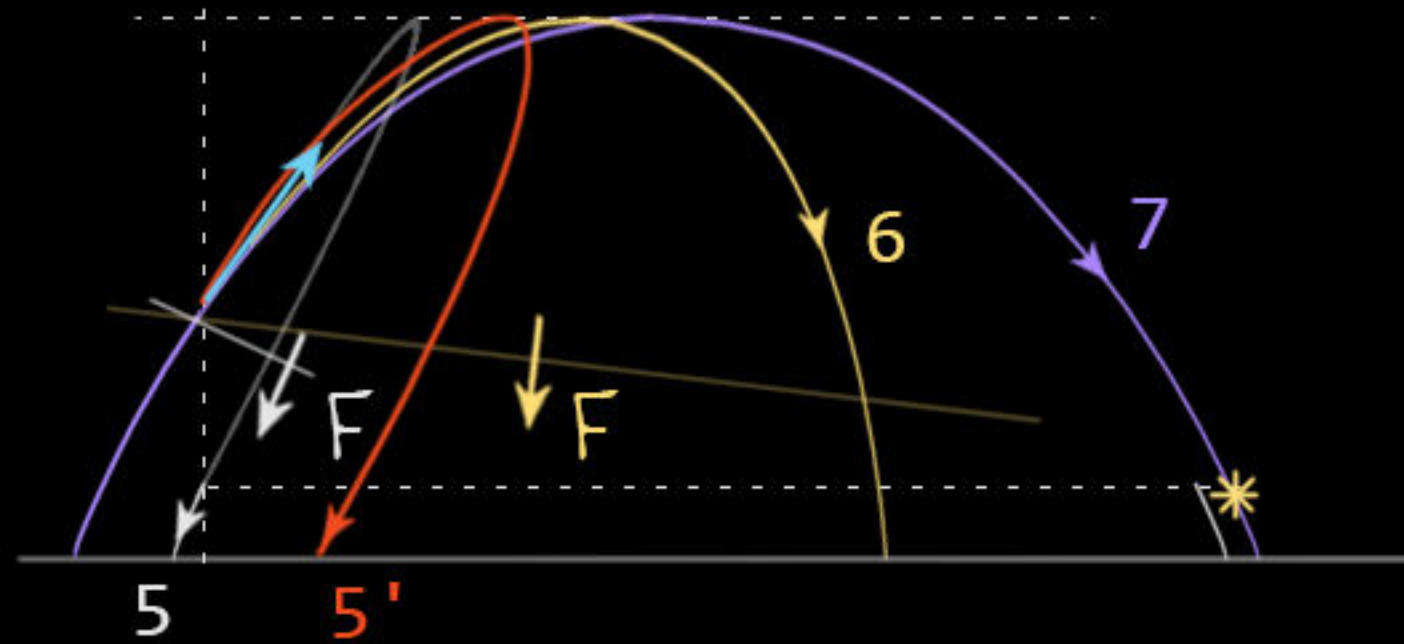
6) F_e is really small.
(it wont come back)

7) $F_e = 0$
(default original path)



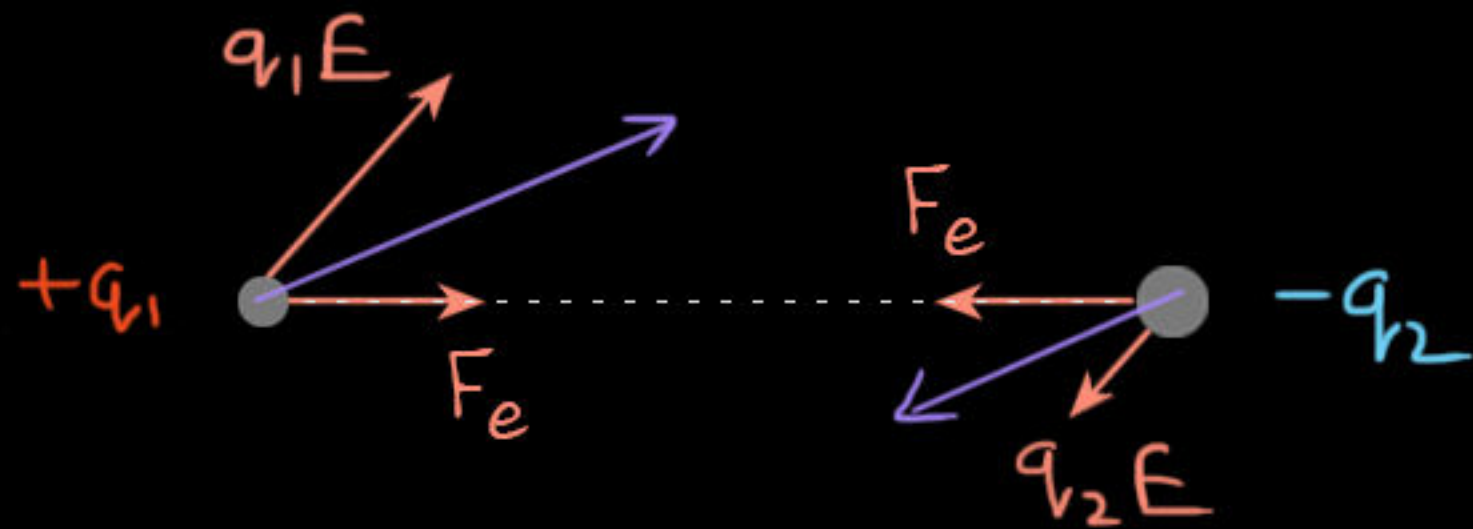
Time of flight, $T_0 = \frac{2u \sin \theta}{g}$





9. Two particles of charges and masses $(+q_1, m_1)$ and $(-q_2, m_2)$ are released at different locations in a uniform electric field E in free space. If their separation remains unchanged, find the separation between them.

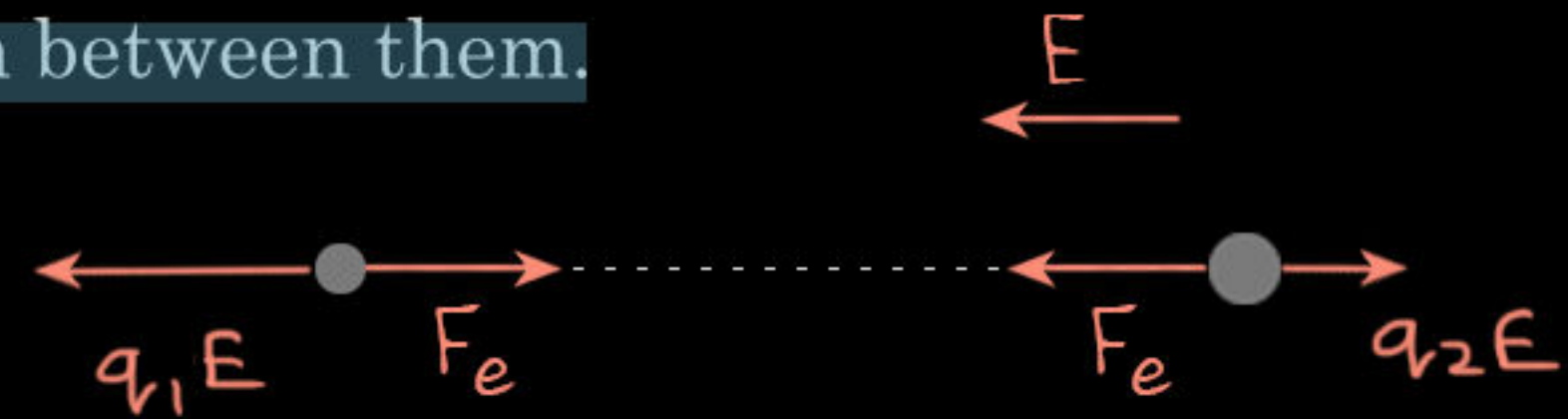
Let E be in general direction as shown: $E \nearrow$



Clearly, accelerations of both particles won't be in same direction (regardless of magnitudes of qE 's).

But for gap to be same, their accelerations must be equal.

Hence E has to be parallel to line joining the two. Also, it must be towards left.

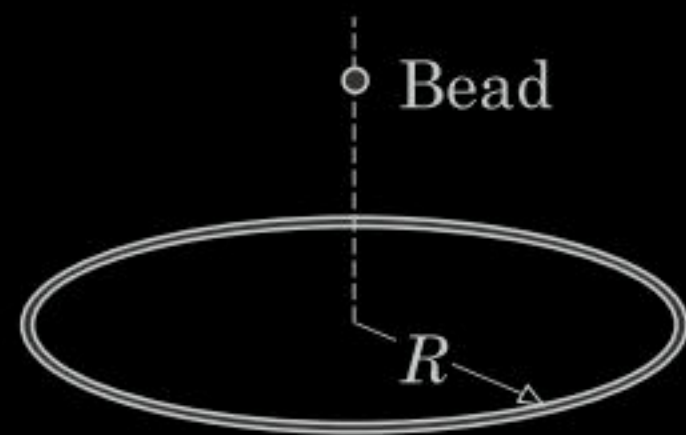


$$\frac{q_1 E - F_e}{m_1} = \frac{F_e - q_2 E}{m_2}$$

$$\Rightarrow F_e = \frac{k q_1 q_2}{r^2} = \frac{(m_1 q_2 + m_2 q_1) E}{(m_1 + m_2)}$$

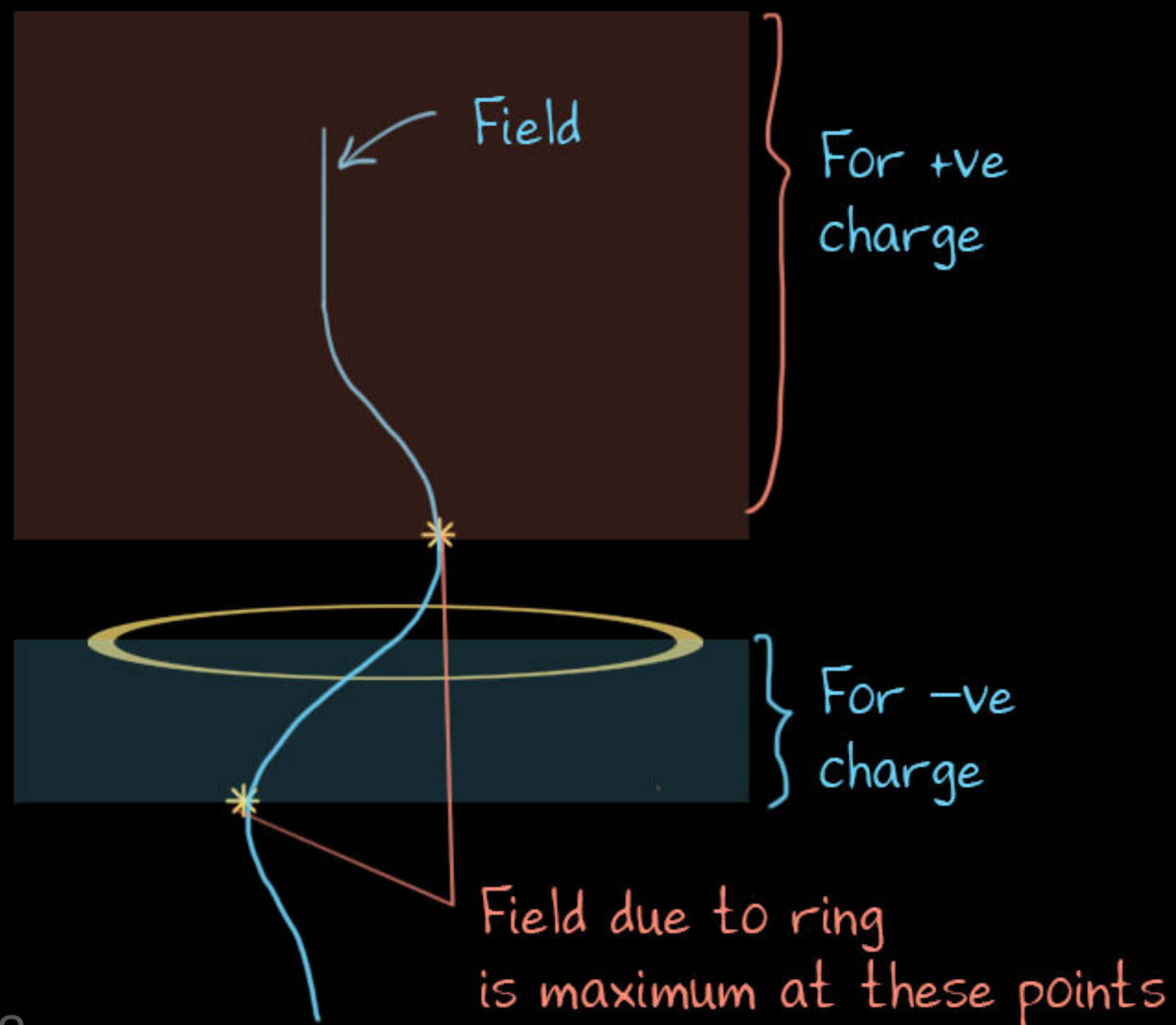
$$\Rightarrow r = \sqrt{\frac{k q_1 q_2 (m_1 + m_2)}{E (m_1 q_2 + m_2 q_1)}}$$

Note: We didn't bother with signs, as we dealt with arrows.



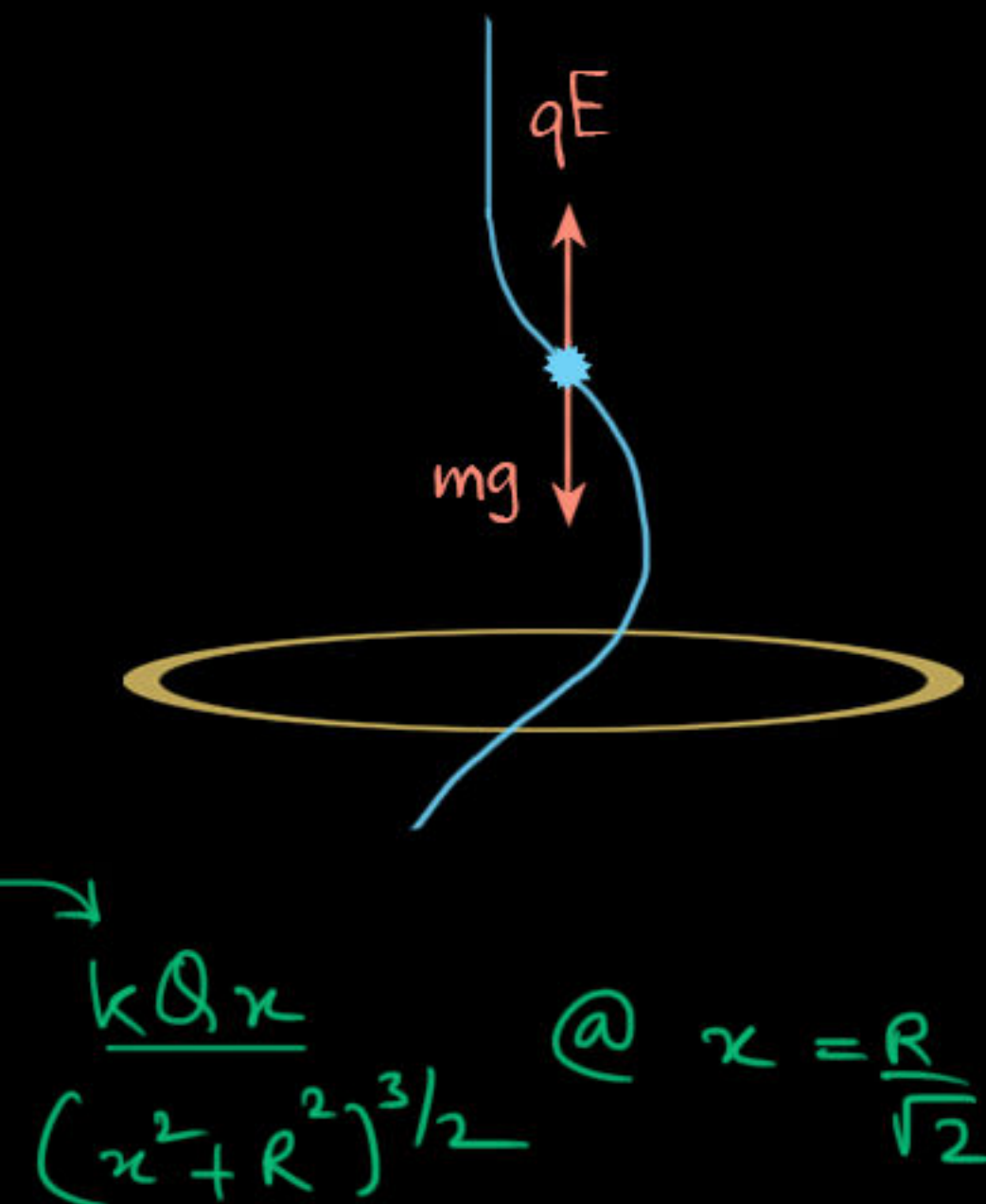
10. A ring of radius R having a uniformly distributed charge Q is fixed in a horizontal plane. A bead of mass m and an unknown charge stays in equilibrium at such point on the vertical axis of the ring that if it is displaced slightly up or down, it undergoes oscillatory motion. Find charge on the bead. Intensity of gravitational field is g .

Possible equilibrium positions



$$mg < qE_{\max}$$

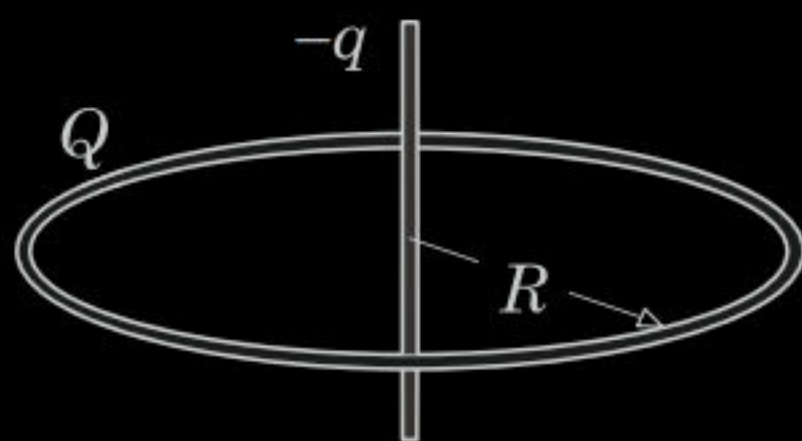
$$\Rightarrow q > \frac{mg}{E_{\max}}$$



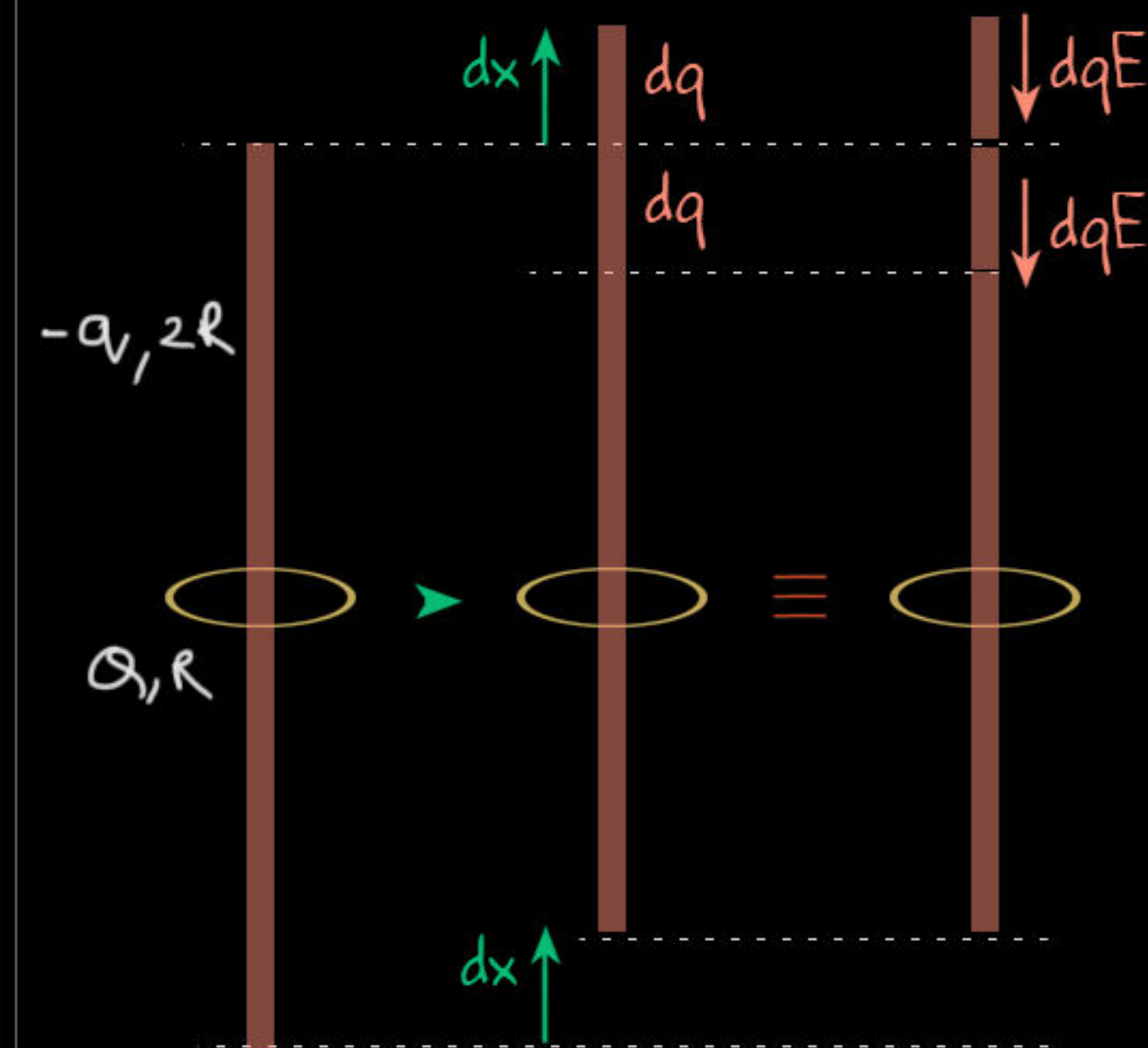
$$\Rightarrow |q| > \frac{3\sqrt{3} mg R^2}{2kq}$$

by Ankit Singhvi

Youtube / Arihant Senior



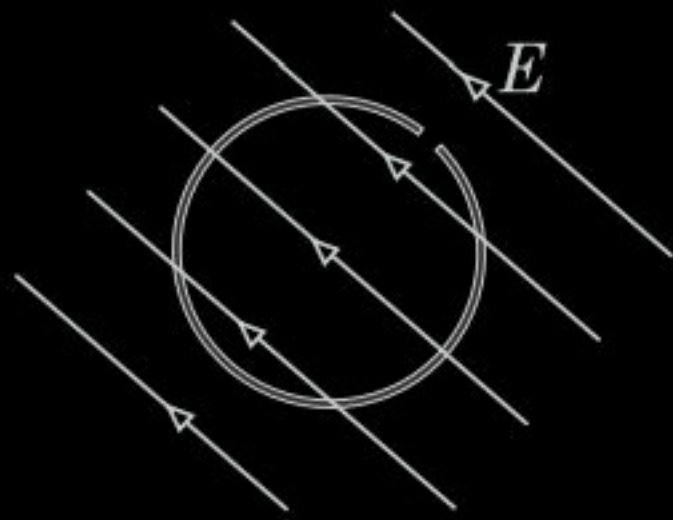
11. A thin rod of mass m carrying uniform negative charge $-q$ is placed symmetrically along the axis of a thin ring of radius R carrying uniformly distributed charge Q . The ring is held fixed in free space and length of the rod is $2R$. Find period of the small amplitude oscillations of the rod along the axis of the ring.



$$\begin{aligned} \text{Restoring force} &= 2 dqE \\ &= 2 \left(\frac{dx}{2R} \right) q \frac{kQR}{(R^2 + R^2)^{3/2}} \\ &= \frac{kqQ}{(2R^2)^{3/2}} dx \end{aligned} \quad \therefore \text{SHM}$$

$$\text{Time period} = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{\left(\frac{kqQ}{(2R^2)^{3/2}} \right)}}$$

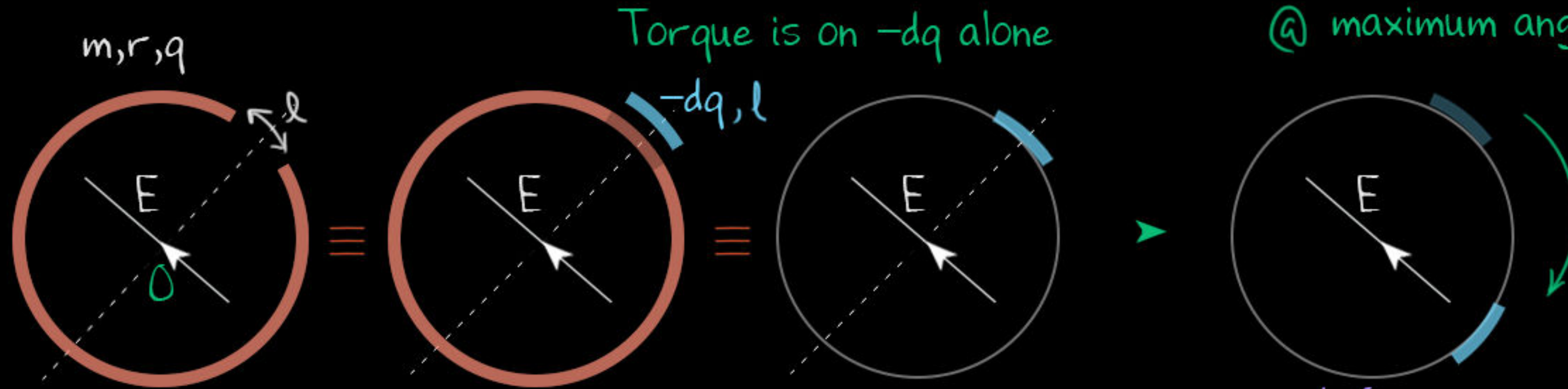
$$= 2\pi \sqrt{\frac{2\sqrt{2} m R^3}{kqQ}}$$



12. A thin rigid insulating ring of radius r and mass m has a very small gap of length l ($l \ll r$) and carries a uniformly distributed charge q . The ring is at rest in free space and a uniform electric field E in the plane of the ring and parallel to the gap is switched on as shown in the figure. Find the maximum angular speed of the ring in subsequent motion.

Correction in problem: gap is only in charge, ring is physically complete otherwise.

w.r.t O



@ maximum angular velocity

Rotational kinetic energy in ring is due to torque on shown element alone.

∴ w.r.t O

$$W_E = \Delta KE \Rightarrow dq E R = \frac{1}{2} I \omega^2$$

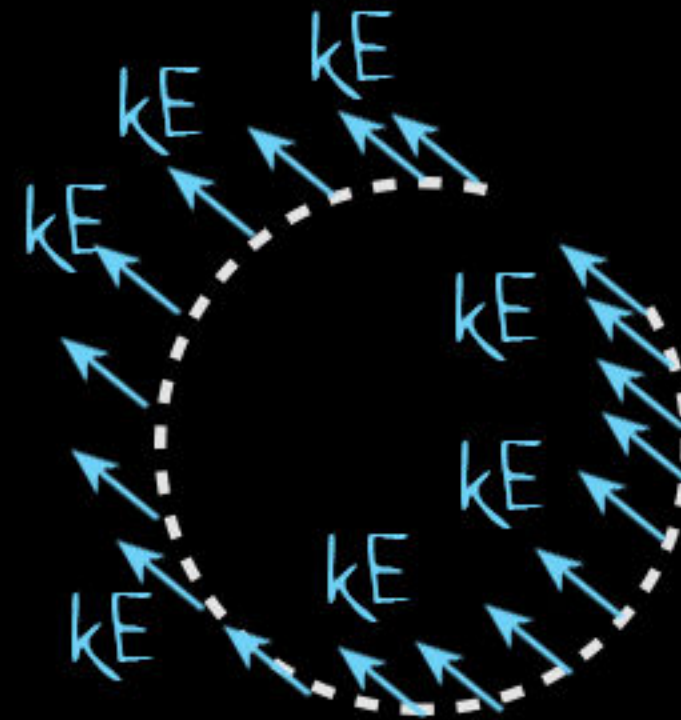
$$\Rightarrow \left(\frac{l}{2\pi R} \cdot q \right) E R = \frac{1}{2} m R^2 \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{q E l}{\pi m R^2}}$$

Note: If there is a physical gap as well, then ring will not rotate at all.

Just imagine the ring being composed of system of particles. Each will then have the same accn = $\frac{dq E}{dm} = \frac{\lambda_1 dx}{\lambda_2 dx} E = k E$

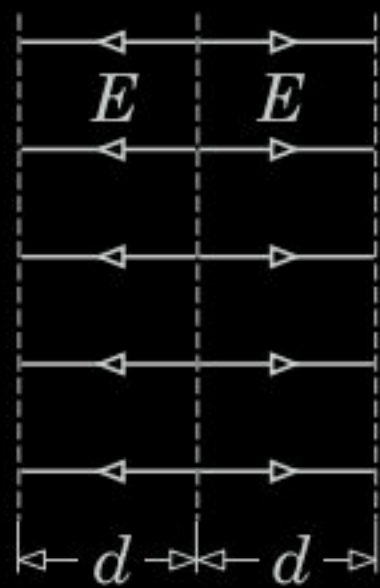
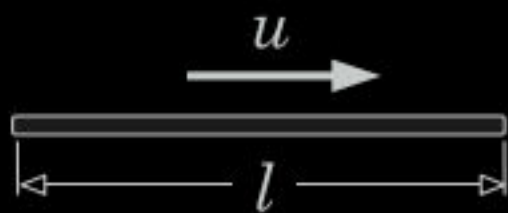
Or imagine a similar uncharged ring with gap, free falling. It won't rotate either.



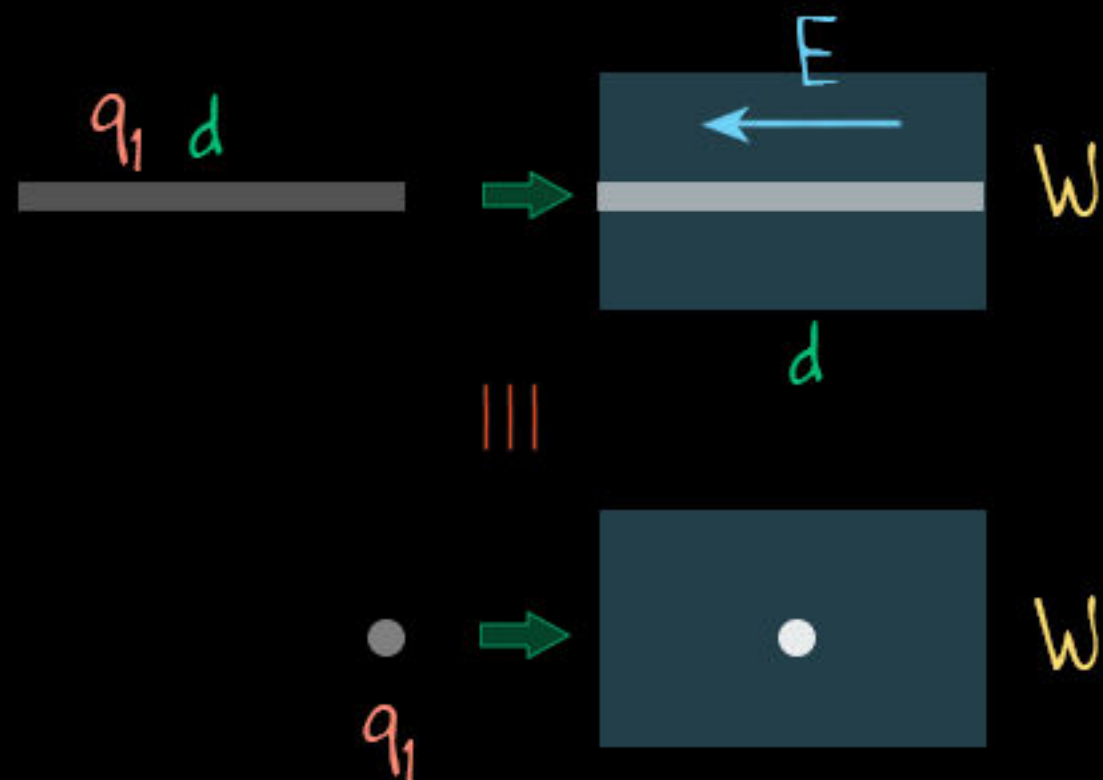
by Ankit Singhvi

Youtube / Arihant Senior

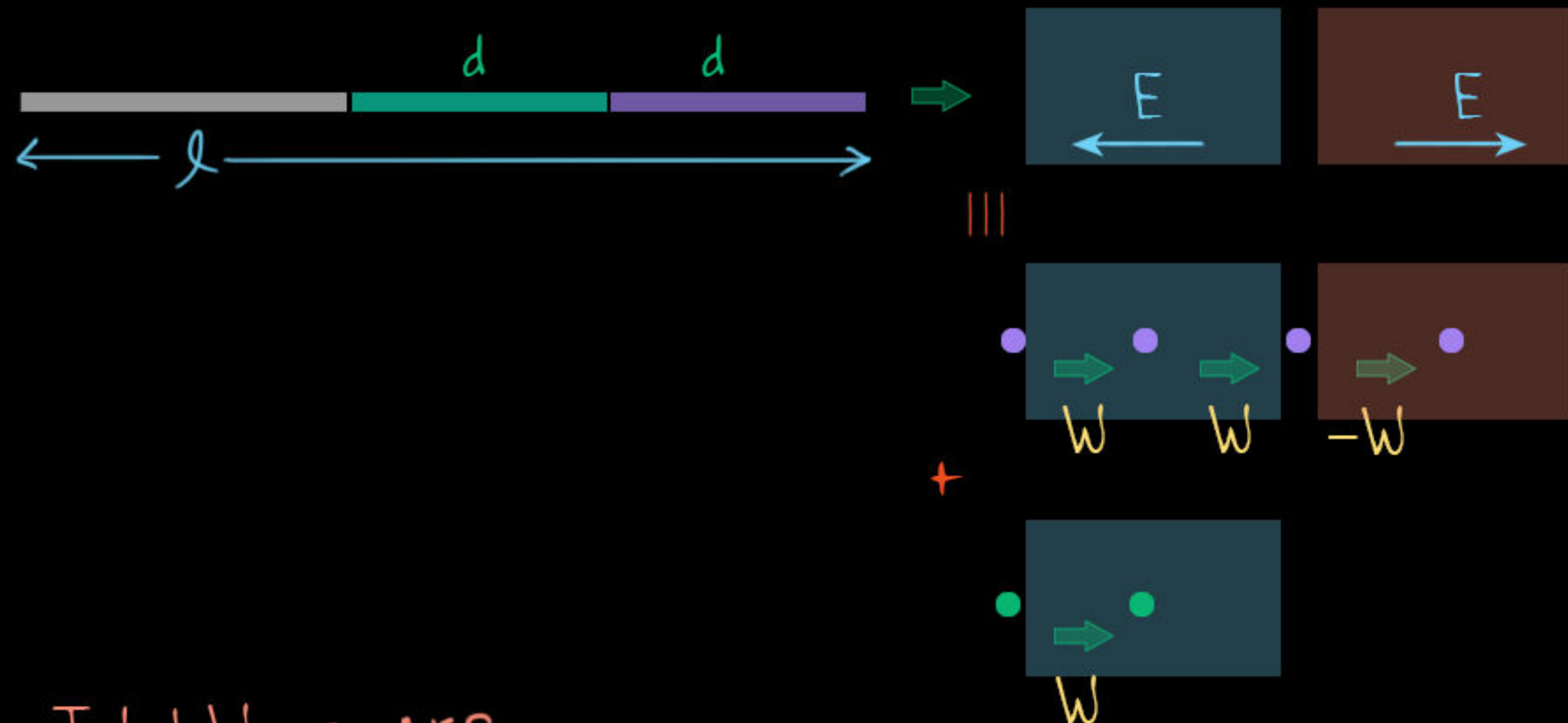
m, l, q



Concept: As field is constant, W_E in shown two cases is same and is $W = -q_1 \frac{Ed}{2}$



13. (a) What should the minimum velocity u of the rod be so that it will completely pass through both the sections?



$$\text{Total } W_E = \Delta KE$$

$$\therefore \text{Total } W_E = 3W - W = 2W = 2 \left(-q_1 \frac{Ed}{2} \right) = \left(0 - \frac{1}{2} m u^2 \right)$$

$$\Rightarrow 2 \left(\frac{d}{2} q_1 \frac{Ed}{2} \right) = \frac{1}{2} m u^2$$

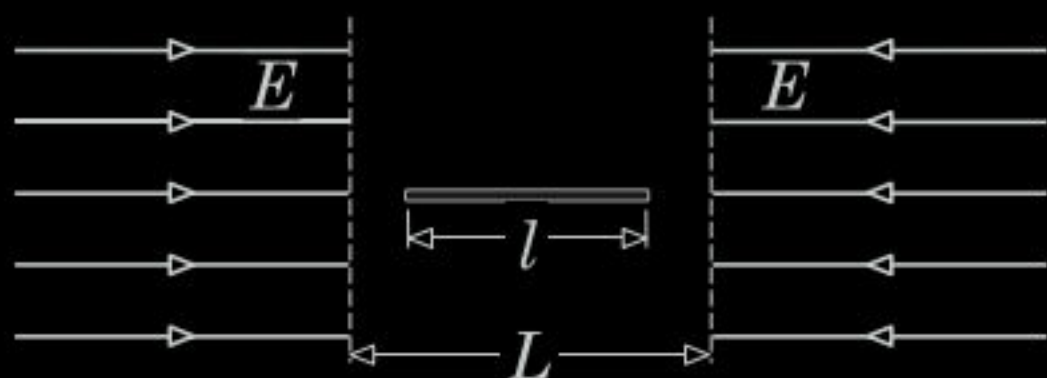
$$\Rightarrow u = \sqrt{\frac{2q_1 E d^2}{m}}$$

by Ankit Singhvi

Youtube / Arihant Senior

(b) If the rod enters the first section with a velocity, which is double of that obtained in the previous part, with what speed will it emerge out of the second section?

It will come out of the field. Therefore, total W_F will be zero.
So, its exit velocity will be same as enter velocity, ie $2u$.



When inside the field:



$$\text{Restoring force} = \left(\frac{x}{l} q\right) E$$

$$\therefore \text{SHM with } T_1 = 2\pi \sqrt{\frac{m}{qE/l}}$$

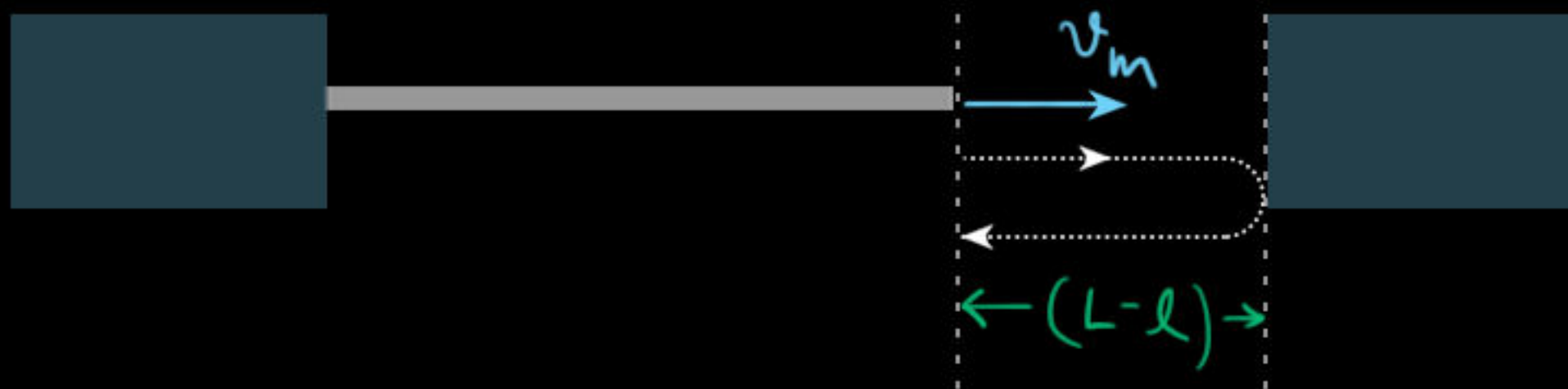
$$= 2\pi \sqrt{\frac{ml}{qE}}$$

14. A charge q is uniformly distributed on a thin rod of mass m and length l that stays at rest in free space. Two regions of uniform electric field of intensity E are created on both sides of the rod as shown in the figure. In the region of width L where the rod is initially placed, there is no electric field. Now the rod is projected along its length so that it cannot enter completely into any of the regions of electric field. Find period of oscillations of the rod as a function of the maximum distance x_m entered by a leading end of the rod into a region of electric field.

When outside the field:

($\because$ SHM)

$$\text{Velocity with which it comes out, } v_m = \omega x_m = \sqrt{\frac{qE/l}{m}} x_m$$



$$\therefore T_2 = \frac{2(L-l)}{v_m} = \frac{2(L-l)}{x_m} \sqrt{\frac{ml}{qE}}$$

Total time period =

$T_1 + T_2$
Youtube / Arihant Senior

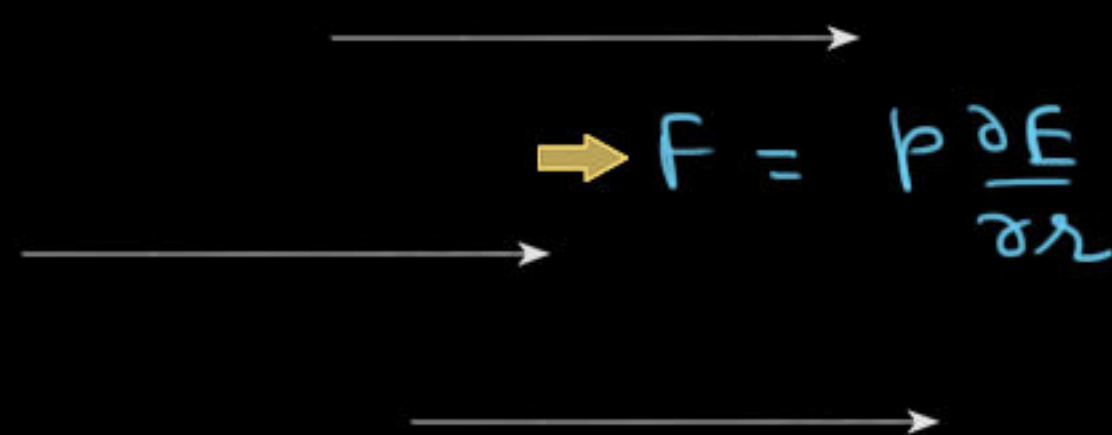
by Ankit Singhvi

We know,



$$E = \frac{2kp}{r^3}$$

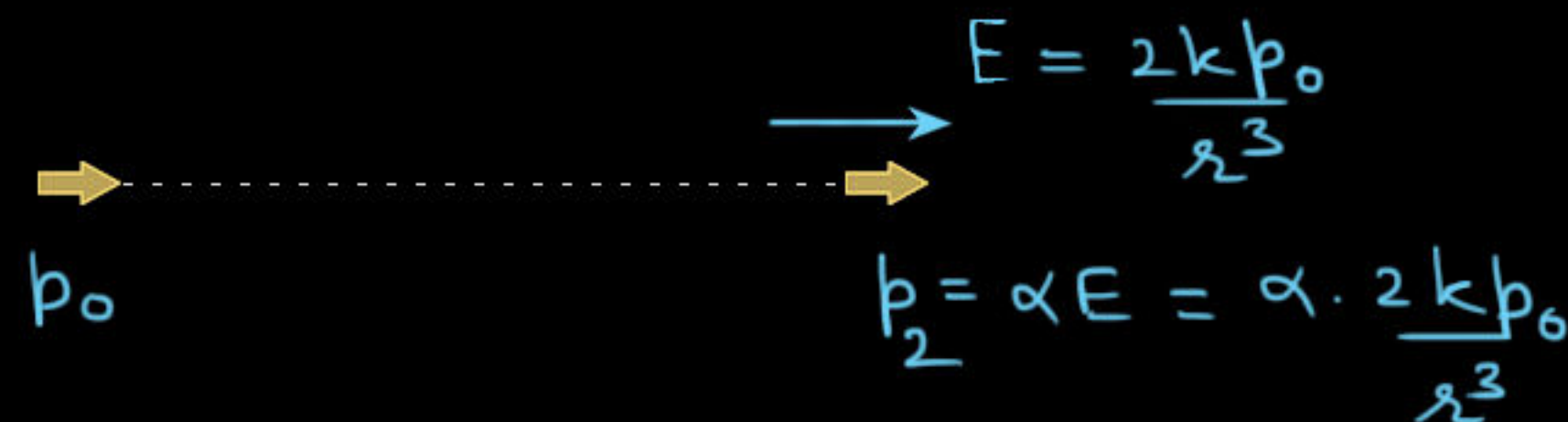
Also, for a dipole kept in a varying field E (parallel to dipole), force is given by:



$$F = p \frac{\partial E}{\partial r}$$

15. In an external electric field a molecule is polarized. The dipole moment thus induced is given by equation $\vec{p} = \alpha \vec{E}$, where α known as polarizability is a positive constant and $\vec{E}$ is the external electric field. Consider a molecule of polarizability α located at a distance r from a dipole of dipole moment p_0 on its axis. Find force of electrostatic interaction between the molecule and the dipole.

Sol:



$$E = \frac{2kp_0}{r^3}$$

$$p_2 = \alpha E = \alpha \cdot \frac{2kp_0}{r^3}$$

$$F_2 = p_2 \frac{\partial E}{\partial r} = p_2 \frac{\partial}{\partial r} \left(\frac{2kp_0}{r^3} \right)$$

$$= p_2 \left[\frac{6kp_0}{r^4} \right]$$

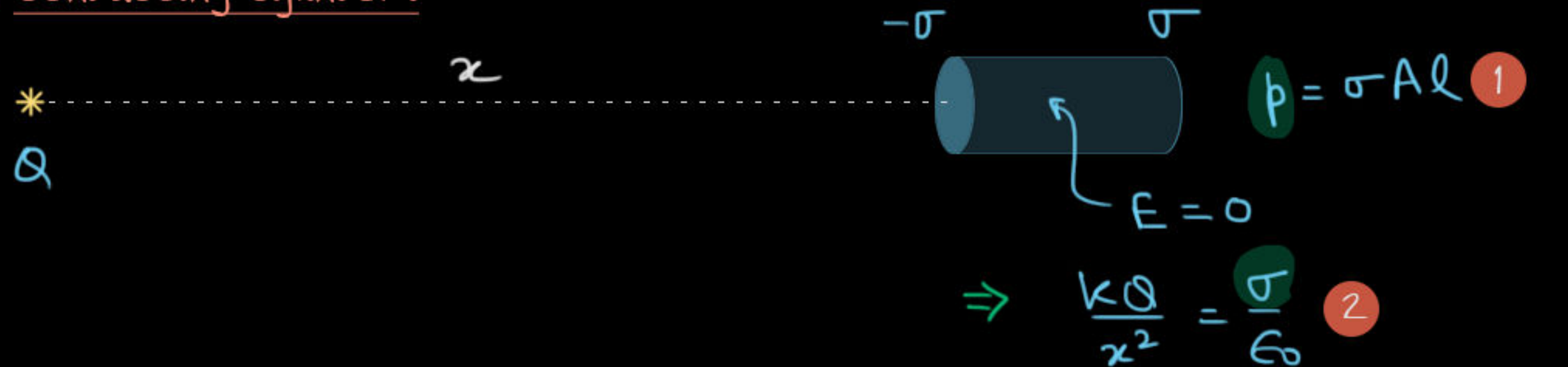
$$= \alpha \cdot \frac{2kp_0}{r^3} \cdot \frac{6kp_0}{r^4} = \frac{3}{4} \frac{\alpha p_0^2}{\pi^2 \epsilon_0^2 r^7} //$$



Charges will get induced. So cylinder will act as dipole.

16. A particle of charge Q is placed on the axis of a neutral cylinder of volume V at a distance x from one end of the cylinder as shown in the figure. Here the distance x is much larger than the linear dimensions of the cylinder. Using suitable approximations, find force F of electrostatic interaction between the charge particle and the cylinder considering the cases - the cylinder is made of a conducting material and the cylinder is made of an insulating material of relative permittivity ϵ_r .

A Conducting cylinder:



From 1,2: $p = \frac{kQ\epsilon_0}{x^2} A l$

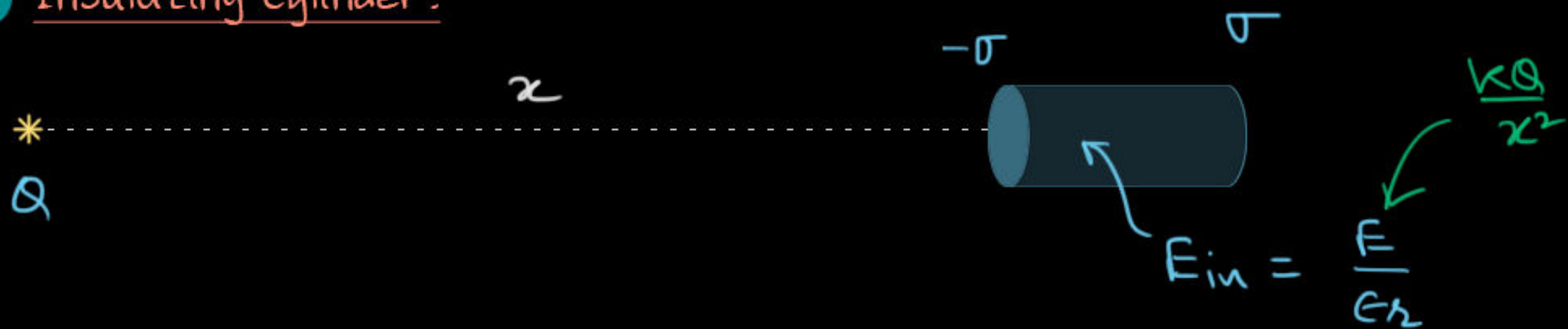
$\therefore$ Force on $Q = Q \cdot E = Q \frac{2kp}{x^3}$

$= Q \cdot \frac{2k}{x^3} \left[\frac{kQ\epsilon_0}{x^2} A l \right]$

$= \frac{Q^2 V}{8\pi^2 \epsilon_0 x^5}$

Youtube / Arihant Senior

B Insulating cylinder:



$$F = \frac{2kpQ}{x^3} \quad (1)$$

$$p = \sigma Al \quad (2)$$

$$\Rightarrow E - \frac{\sigma}{\epsilon_0} = \frac{E}{\epsilon_r}$$

$$\Rightarrow E \left(1 - \frac{1}{\epsilon_r}\right) = \frac{\sigma}{\epsilon_0} \quad (3)$$

From 1,2,3:

$$F = \frac{Q^2 v}{8\pi^2 \epsilon_0 x^5} \left(1 - \frac{1}{\epsilon_r}\right) //$$

17. Find electric field everywhere on a plane containing the borders of two parallel half-planes extending in opposite directions. Distance between the half-planes is d and they have uniform surface charge density σ each. Angle between the plane and the half-planes is θ .

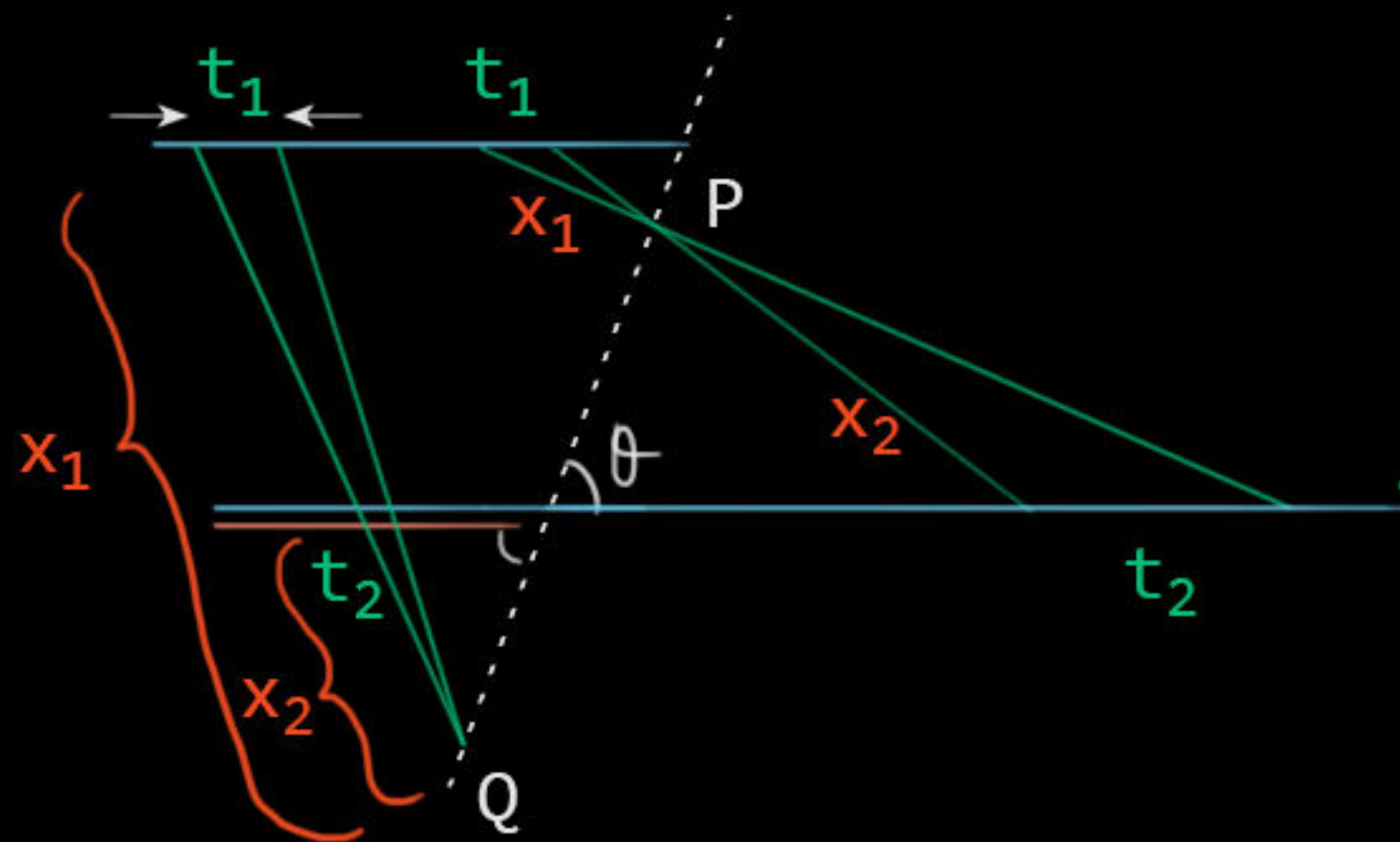
$$E_P = E_{P_1} + E_{P_2}$$

$$E_{P_1} = \frac{2k\lambda_1}{x_1} = 2kC \frac{t_1}{x_1}$$

$$E_{P_2} = \frac{2k\lambda_2}{x_2} = 2kC \frac{t_2}{x_2}$$

$$\therefore E_P = 0_{//}$$

Taking elemental strips on both planes as shown below.



$$E_Q = E_{\text{full plane}} + E_{t_1} - E_{t_2}$$

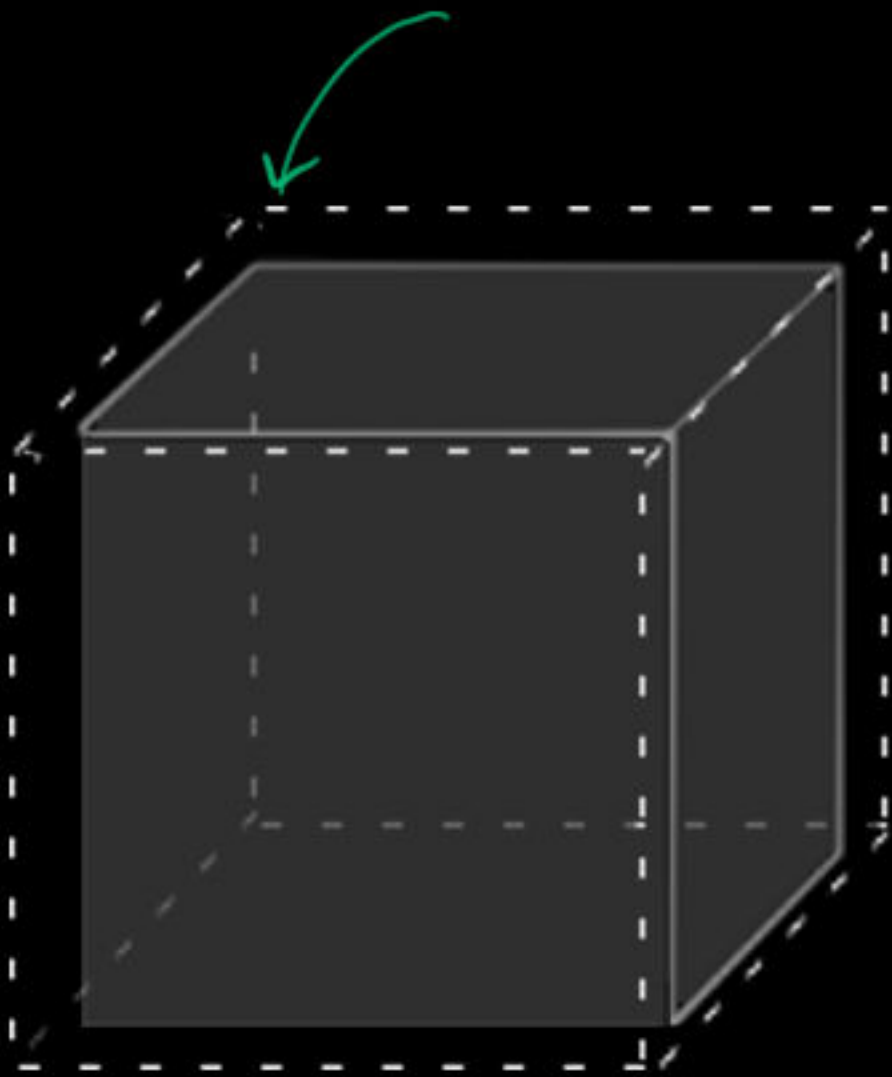
$$= \frac{\sigma}{2\epsilon_0} + \frac{2k\lambda_1}{x_1} - \frac{2k\lambda_2}{x_2}$$

$$= \frac{\sigma}{2\epsilon_0}$$

by Ankit Singhvi

Youtube / Arihant Senior

Cubical envelope
surrounding the cube.

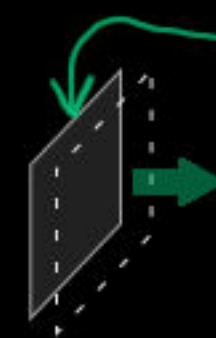


18. An insulating hollow cube of side l carries uniform charge on all of its surfaces. The surface charge density is σ . Determine the force acting on a face due all the other faces.

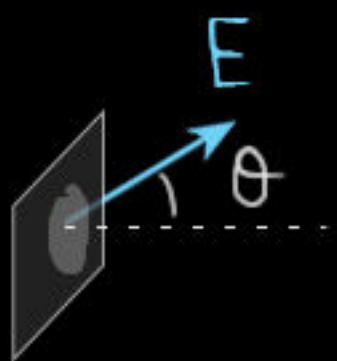
Total flux through envelope: $\phi_{\text{total}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{6(\sigma A)}{\epsilon_0}$

∴ Flux through one surface of envelope: $\phi_1 = \frac{\phi_{\text{total}}}{6} = \frac{\sigma A}{\epsilon_0}$

Also, flux through one surface of envelope due to square surface of cube next to it:



$$\phi_2 = \frac{1}{2} \left(\frac{q_{\text{in}}}{\epsilon_0} \right) = \frac{\sigma A}{2\epsilon_0}$$

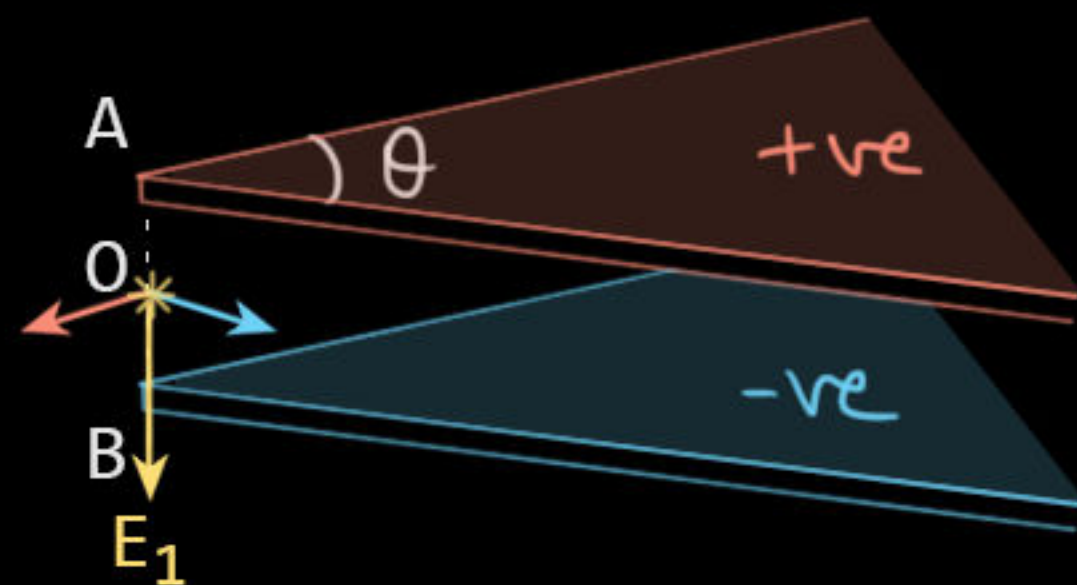
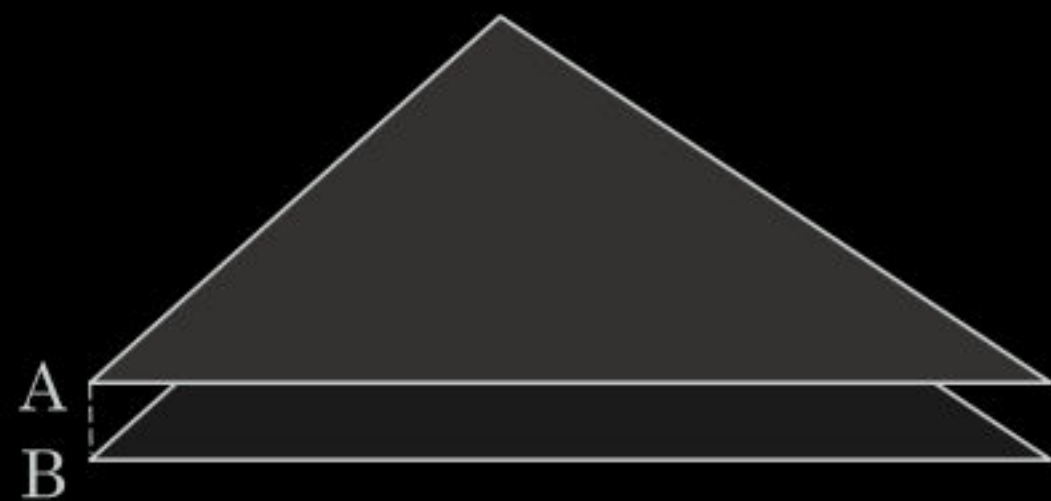


Force on plate = $df = \sigma dS E \cos \theta$

$\Rightarrow F = \sigma \oint \vec{E} \cdot d\vec{S}$

= $\sigma \times$ flux through  due to other five plates

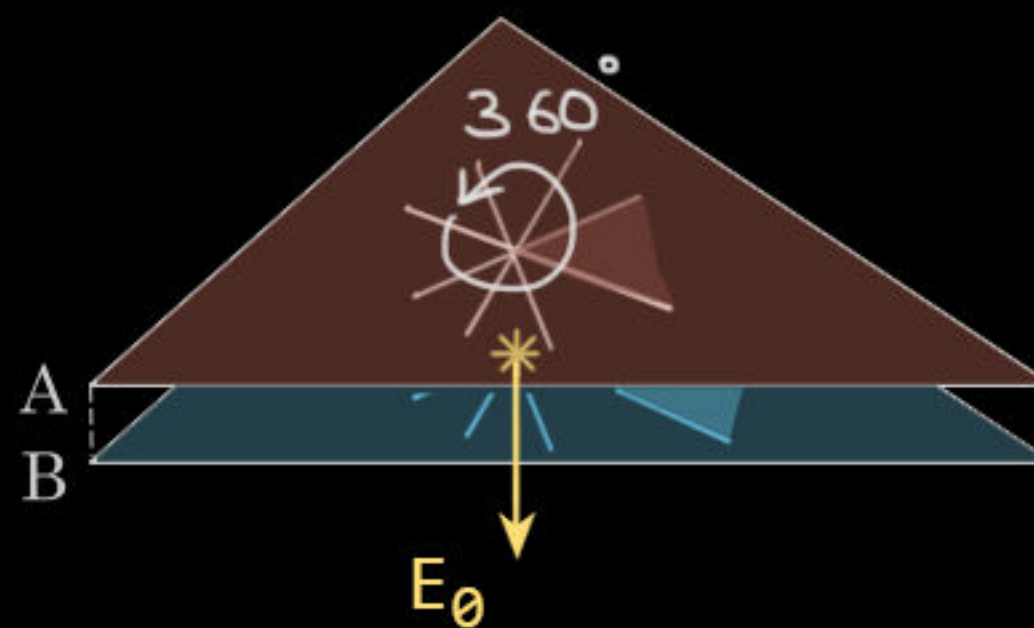
= $\sigma \times (\phi_1 - \phi_2) = \frac{\sigma^2 A}{2\epsilon_0}$



O is midpoint, so Electric field from both wedges will be symmetrical to AB.

∴ Net field will be perpendicular to surface.

19. Two parallel equilateral triangular plates are placed overlapping each other very close to each other. The plates carry **uniformly distributed unlike charges of equal moduli**. Deep inside the region between the plates, electric field is uniform but near the edges, it becomes nonuniform. If modulus of electric field deep inside the plates is E_0 , find modulus of **electric field at the midpoint of line AB**. What happens if the plates have the shape of a regular pentagon?



E_0 too is perpendicular to surface, and can be thought of as resultant of lots of smaller wedge pairs.

$$\therefore E_1 = \frac{\theta}{360} \cdot E_0$$

A For equilateral triangular plate: $E_1 = \frac{60}{360} \times E_0 = \frac{E_0}{6}$

B For regular pentagonal plate: $E_1 = \frac{108}{360} \times E_0 = \frac{3E_0}{10}$



20. An ideal gas is trapped in a thin glass tube of cross section area A between two metallic pistons that can slide without friction. Initially the pistons stay in equilibrium a distance d apart that is small compared to their diameter. If the pistons are given unlike charges each of modulus q , what will the distance between them become? Electrical permittivity of the gas is ϵ and atmospheric pressure is p_0 .

Concept: Force on a plate due to charge on other plate is: $\frac{q^2}{2A\epsilon_0}$ It is NOT $\frac{q^2}{2A\epsilon}$

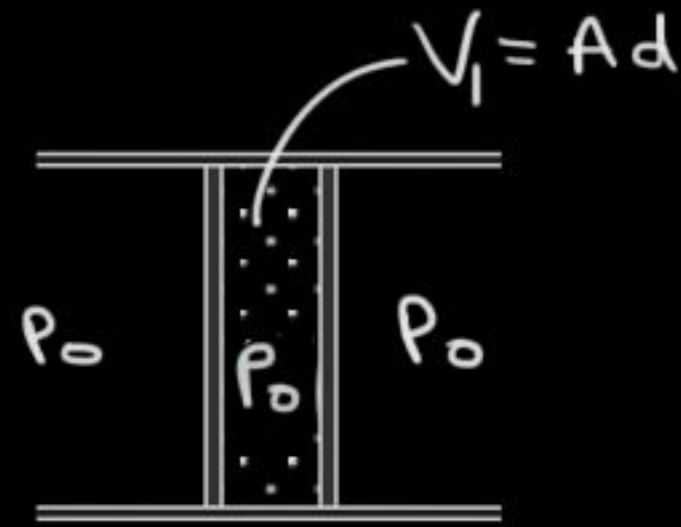
Reason:

Force is $\frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$ for point charges only.

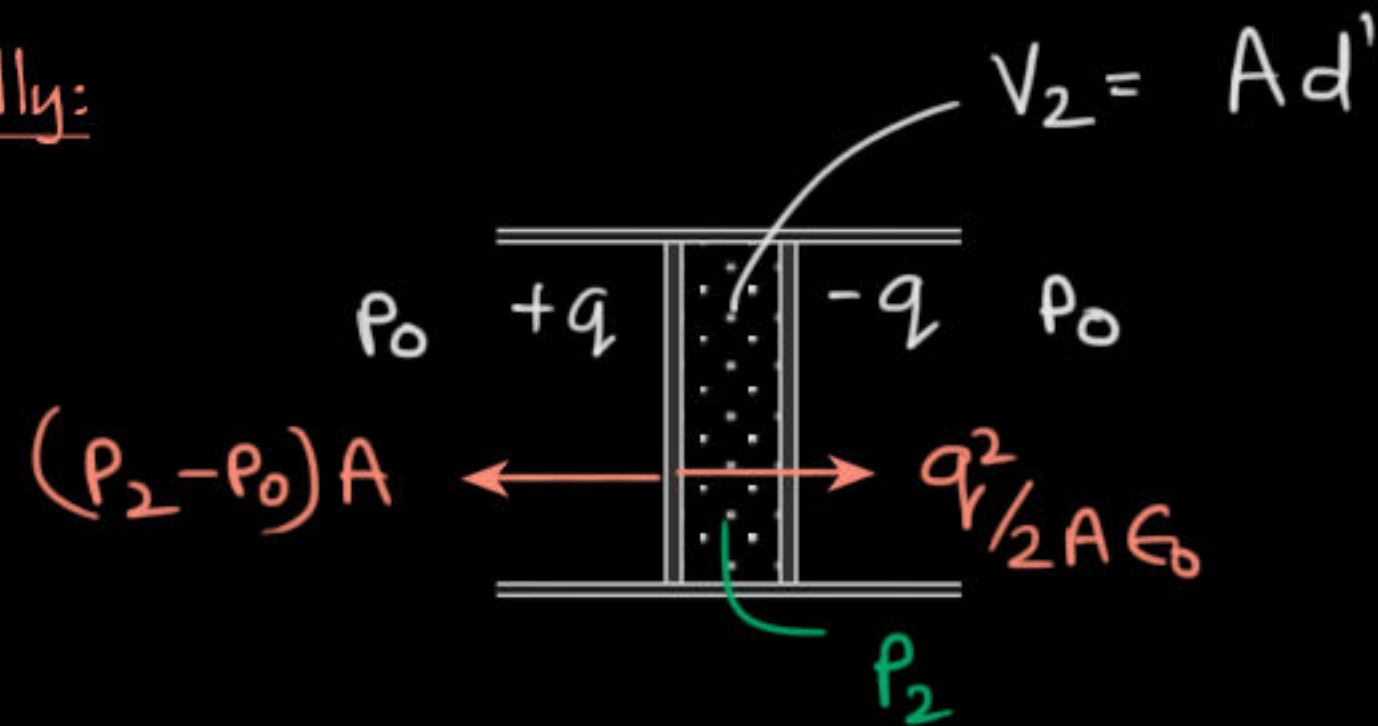
Here we have large plates (as d is small compared to diameter), so polarized gas molecules will not cause a net force on plate, as that effect will get canceled out from polarized charges on other plate.



Initially:



Finally:

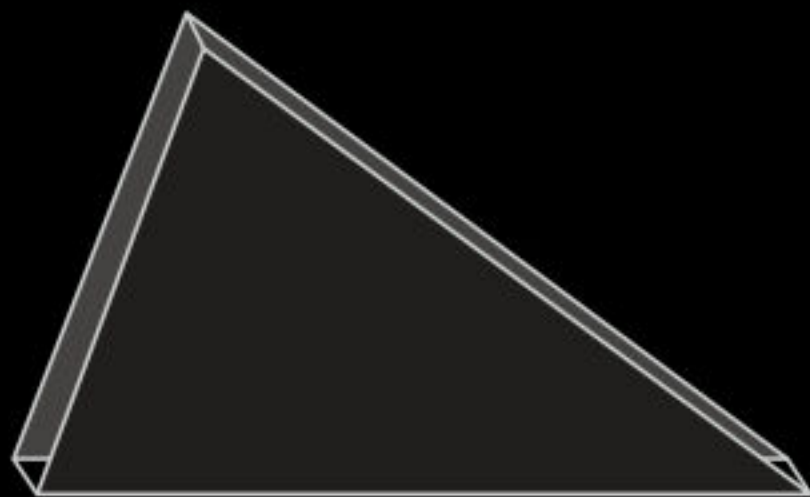


$$(P_2 - P_0)A = \frac{q^2}{2A\epsilon_0} \quad (1)$$

Also, $P_0 V_1 = P_2 V_2$

$$\Rightarrow P_0 Ad = P_2 Ad' \quad (2)$$

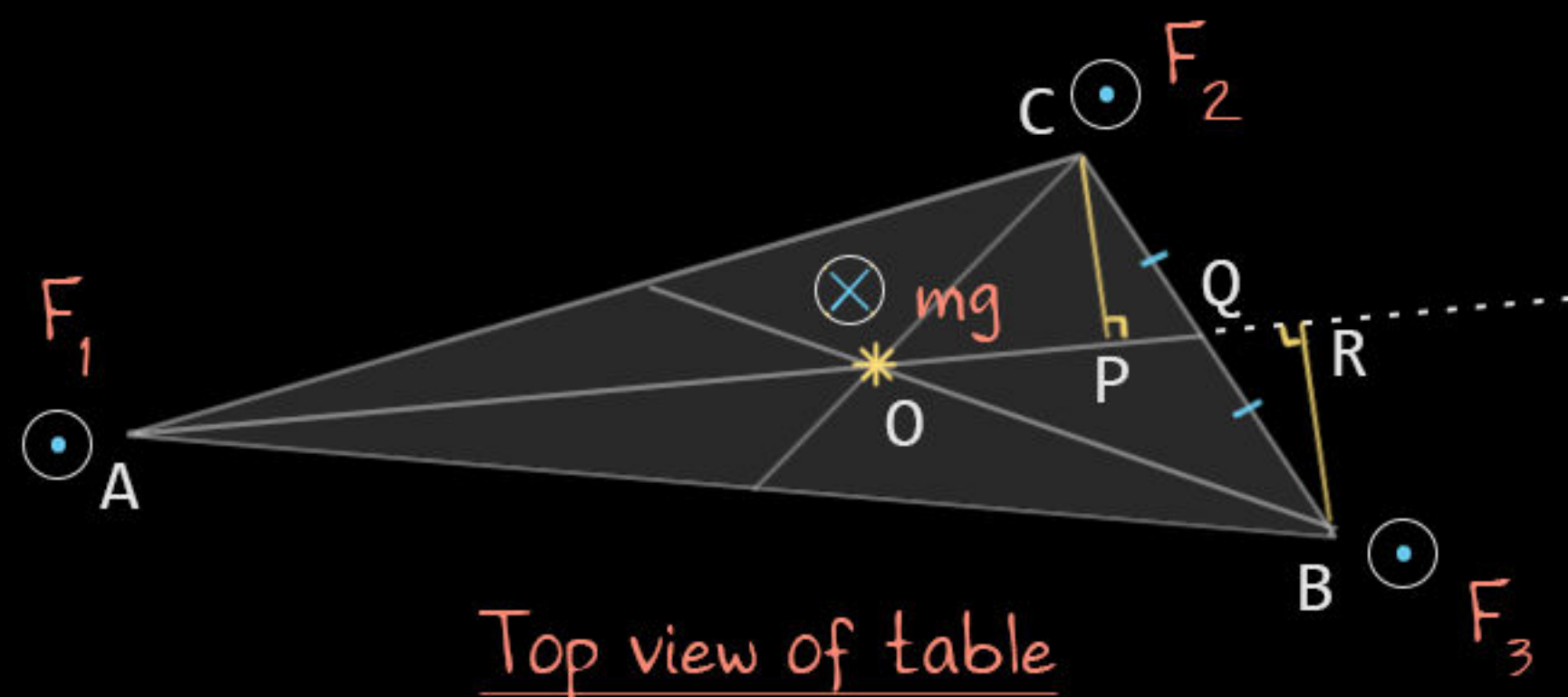
From 1,2: $d' = \frac{P_0 d}{\frac{q^2}{2A^2\epsilon_0} + P_0}$ //



21. Two identical nonconducting triangular plates each of area A carrying uniform positive charges $+3q$ and $-2q$ are kept parallel to each other by inserting three identical insulating rods between their corners as shown in the figure. Distance between the plates is much smaller than their linear dimensions. Assuming the assembly to be isolated from other bodies, find compressive forces developed in the rods.

Concept: We have a triangle shaped table with its legs on vertex.
Then the legs will share equal load of table top. i.e. $mg/3$.

Proof: There are four forces on table top:



mg acts on centroid O and table is at rest

$$\therefore \tau_{AOQ} = 0$$

$$\Rightarrow F_2 \cdot CP = F_3 \cdot BR \quad \{ \because \triangle CPQ \cong \triangle BRQ \}$$

$$\Rightarrow F_2 = F_3$$

$$\text{Similarly: } F_1 = F_2 \text{ \& } F_3 = F_1 \Rightarrow F_1 = F_2 = F_3$$

As the dimensions of plates are large and gap between them small, E due to each plate can be considered uniform.



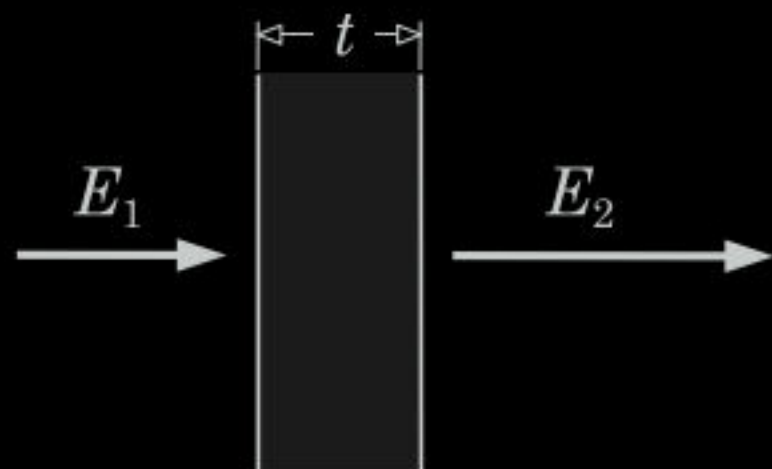
So force on each element of plate is proportional to its surface area. Just like gravitational force on a flat plate.



Force on each joint is therefore $1/3$ of total coulomb force.

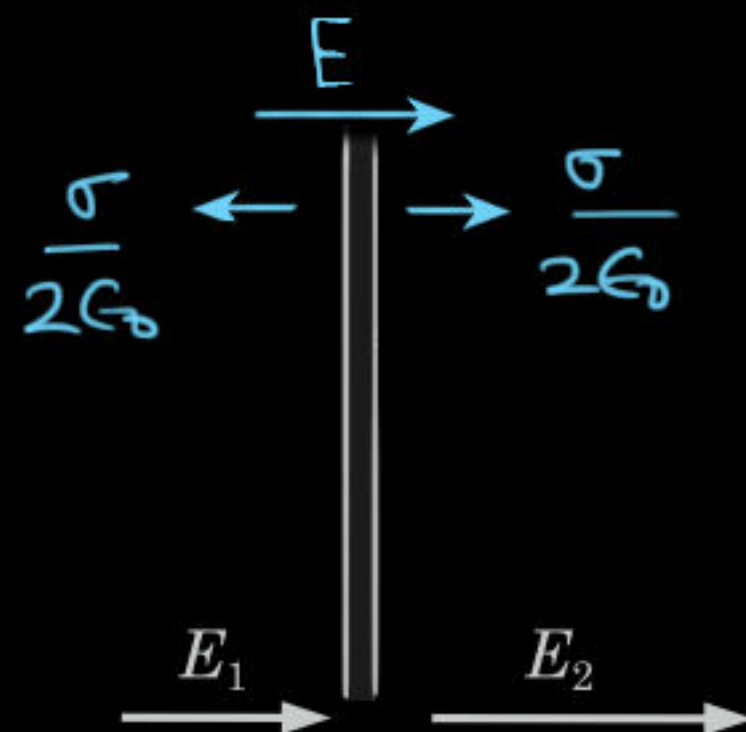
$$\text{Electrostatic force between plates} = \frac{q_1 q_2}{2A\epsilon_0} = \frac{(\cancel{2}q)(3q)}{\cancel{2}A\epsilon_0} = \frac{3q^2}{A\epsilon_0}$$

$$\therefore \text{Stress in a single joint} = \frac{1}{\cancel{3}} \left(\frac{\cancel{3}q^2}{A\epsilon_0} \right) = \frac{q^2}{A\epsilon_0} //$$



22. An infinitely large layer of charge of uniform thickness t is placed normal to an existing uniform electric field. Presence of this charge layer so alters the electric field that it remains uniform on both the sides and assumes values E_1 and E_2 as shown in the figure. Charge distribution in the layer is not uniform and depends only on distance from its faces. Find expression for the force per unit area experienced by the charge layer.

Lets compress the sheet into a thin film.
Let its surface charge density be σ
And let external field be E .



$$E + \frac{\sigma}{2\epsilon_0} = E_2$$

$$E - \frac{\sigma}{2\epsilon_0} = E_1$$

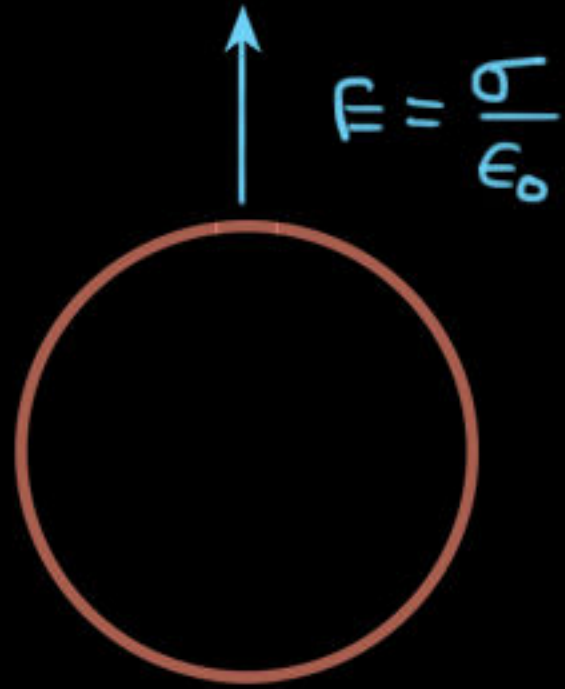
$$\Rightarrow E = \frac{E_2 + E_1}{2} \quad \& \quad \frac{\sigma}{\epsilon_0} = E_2 - E_1$$

1

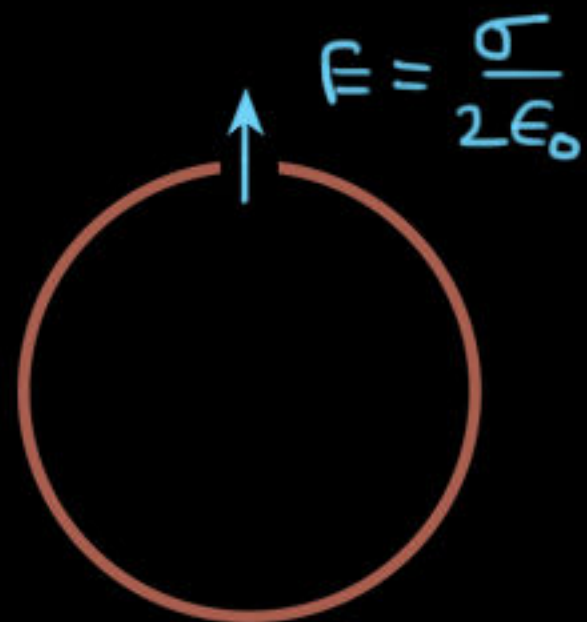
2

$$\therefore \text{Force per unit area on the sheet} = \sigma E \xrightarrow{1,2} = \frac{\epsilon_0}{2} (E_2^2 - E_1^2)$$

We know that field due to a uniformly charged shell just outside its surface is.

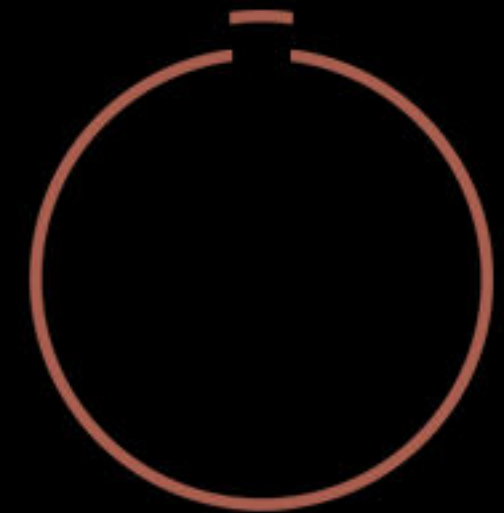


And if we cut a small portion out, field is halved i.e.



23. A small circular metal disc of mass m and radius r is glued on a heavy porcelain ball of radius R with a special type of glue that loses its strength very quickly. Before the glue loses its strength, a total charge Q is sprinkled uniformly on the surface of the assembly. The metal disc jumps off the ball, immediately after the glue loses its strength. Find acceleration of the metal disc at the time of separation. Neglect effects of gravity and assume area of the disc negligibly small as compared to the surface area of the ball.

Disc that jumps up can be considered as cut portion of shell.

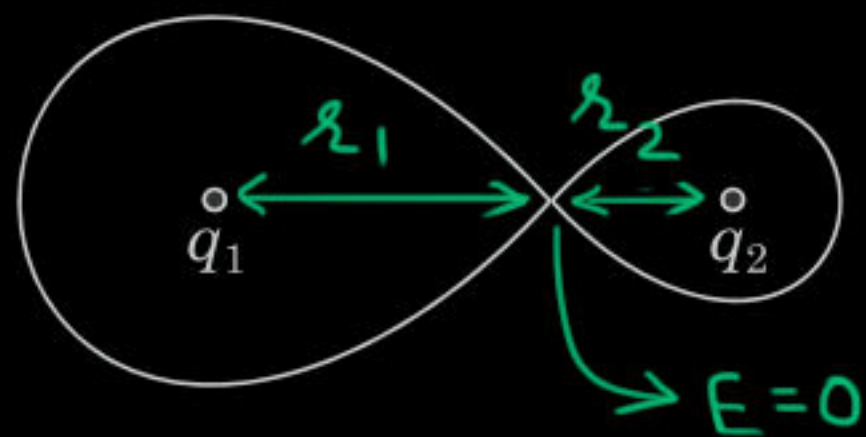


$$\text{Force on that disc} = qE = ma$$

$$\Rightarrow (\sigma \pi r^2) \frac{\sigma}{2\epsilon_0} = ma$$

$\swarrow \quad \quad \quad \nearrow \quad Q/4\pi R^2$

$$\Rightarrow a = \frac{Q^2 r^2}{32 \pi \epsilon_0 m R^4}$$



24. Two point like charges $q_1 = 4.0 \text{ nC}$ and $q_2 = 1.0 \text{ nC}$ occupy fixed positions in free space. If at every point on a curve surrounding the charges as shown in the figure, net electrostatic potential V created by these charges is 900 V , what should the separation between the charges be?

Concept: At a point (or line or plane) where two equipotential surfaces cut, $E=0$.

As field lines are perpendicular to equipotential surfaces and they can not have two directions.

$$\therefore \frac{kq_1}{r_1^2} = \frac{kq_2}{r_2^2} \quad (1)$$

Also, $\frac{kq_1}{r_1} + \frac{kq_2}{r_2} = 900 \quad (2)$

From 1,2:

$$r_1 = 6 \text{ cm}$$

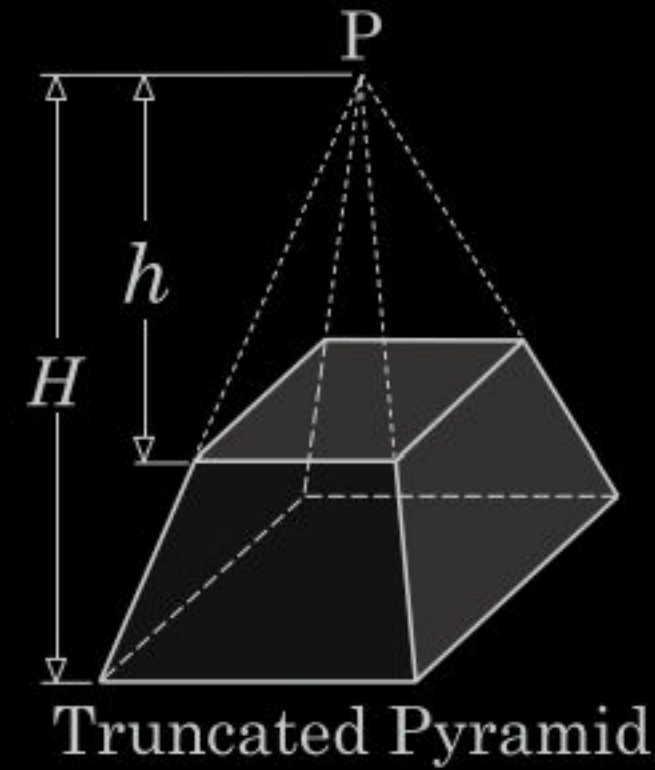
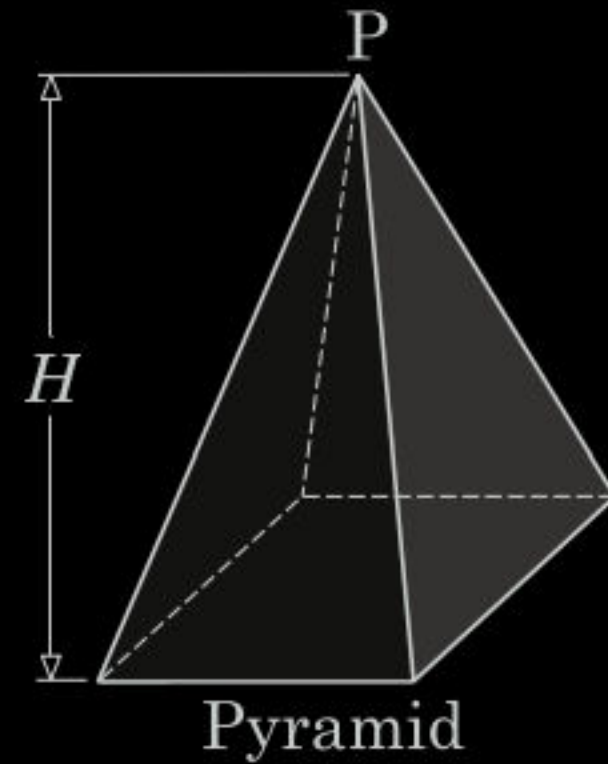
$$r_2 = 3 \text{ cm}$$

$$\Rightarrow r_1 + r_2 = 9 \text{ cm}$$

Youtube / Arihant Senior

25. A right pyramid of square base and height H has uniform charge distributed everywhere within its volume. Modulus of electric field and potential at the apex P of the pyramid are E_0 and V_0 . A symmetrical portion of height h from the apex has been removed. Find modulus of electric field and potential at the apex P of this truncated pyramid.

Prerequisite: MCQ 1



$$E \propto \frac{Q}{h^2} \rightarrow Q \propto h^3$$

$$\Rightarrow E \propto h$$

$$\Rightarrow E_0 = C_1 H$$

$$V \propto \frac{Q}{h} \rightarrow Q \propto h^3$$

$$\Rightarrow V \propto h^2$$

$$\Rightarrow V_0 = C_2 H^2$$

$$E_2 = E_0 - C_1 h$$

$$= C_1 H - C_1 h$$

$$= C_1 (H - h)$$

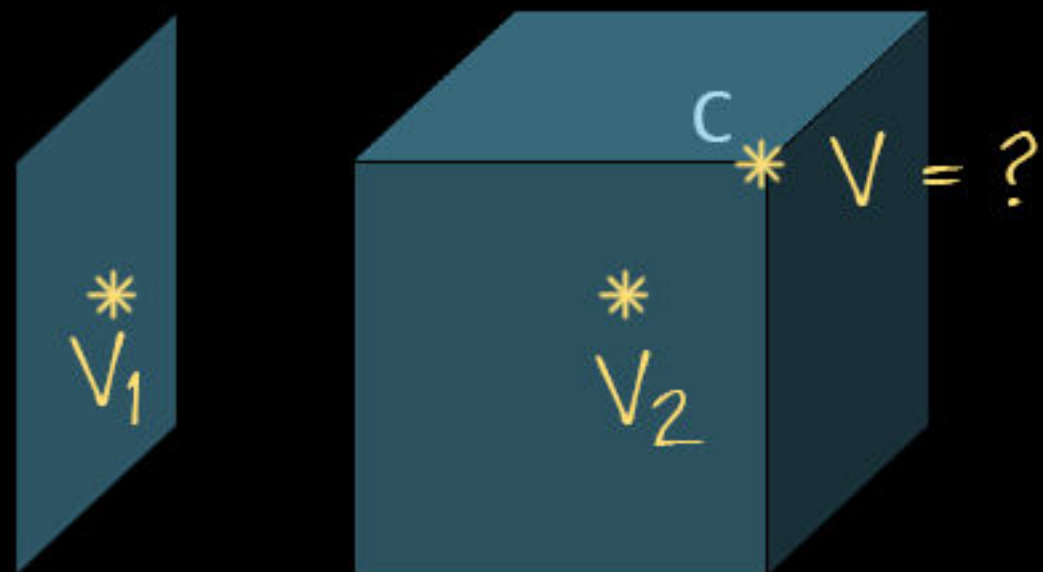
$$= \frac{E_0}{H} (H - h)$$

$$V_2 = V_0 - C_2 h^2$$

$$= C_2 H^2 - C_2 h^2$$

$$= C_2 (H^2 - h^2)$$

$$= \frac{V_0}{H^2} (H^2 - h^2)$$



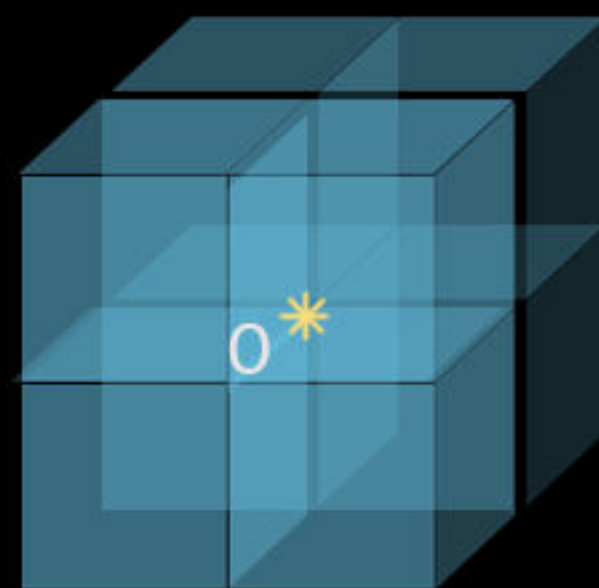
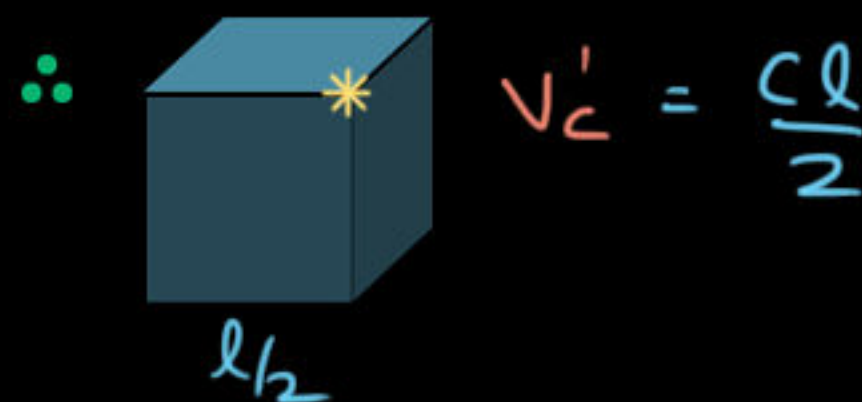
26. If a charge q is uniformly spread on a thin dielectric square plate, the electric potential at its centre is found to be V_1 . If six such charged plates are joined to make a hollow cube, the potential at the centre of the cube is found to be V_2 . Find potential at one of the vertex of this cube. In all the cases, potential at infinitely distant points has been assumed zero.

Prerequisite: MCQ 1

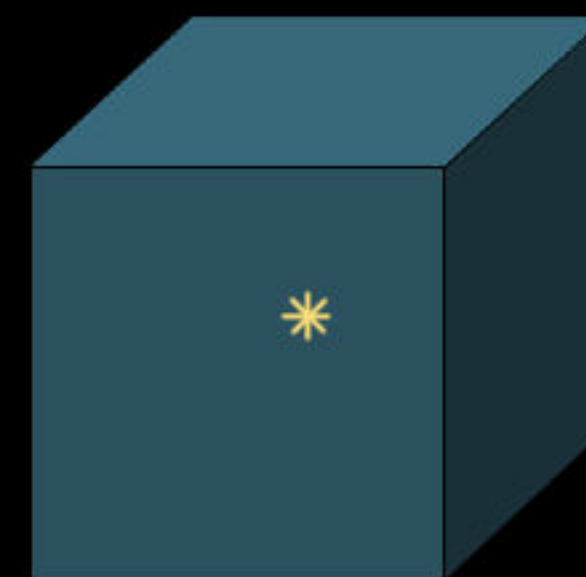
@ corner: $V_c \propto \frac{Q}{l}$ $Q \propto l^2$

$$\Rightarrow V_c \propto l$$

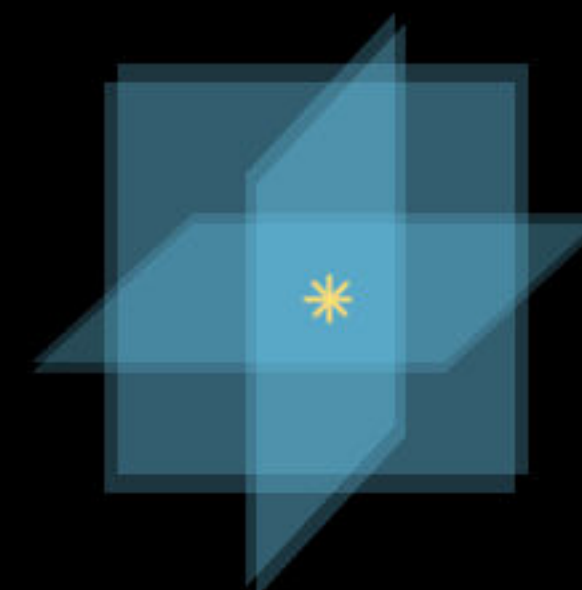
$$\Rightarrow V_c = Cl$$



8 small cubical shells



1 large cubical shell



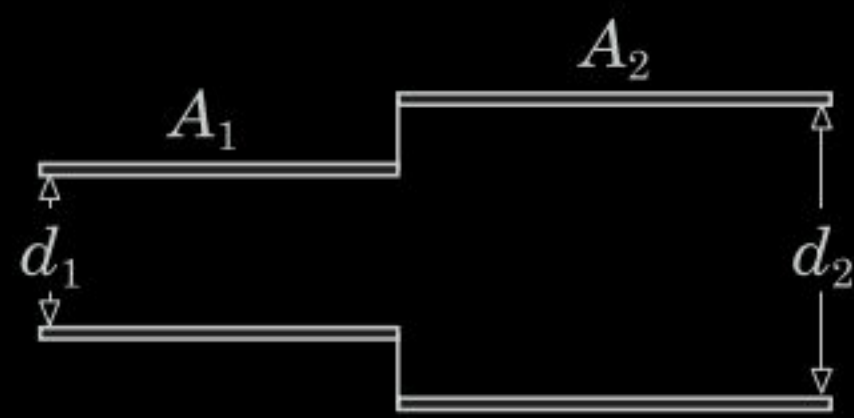
6 square plates

$$\therefore V_0 = 8V'_c = V_2 + 6V_1$$

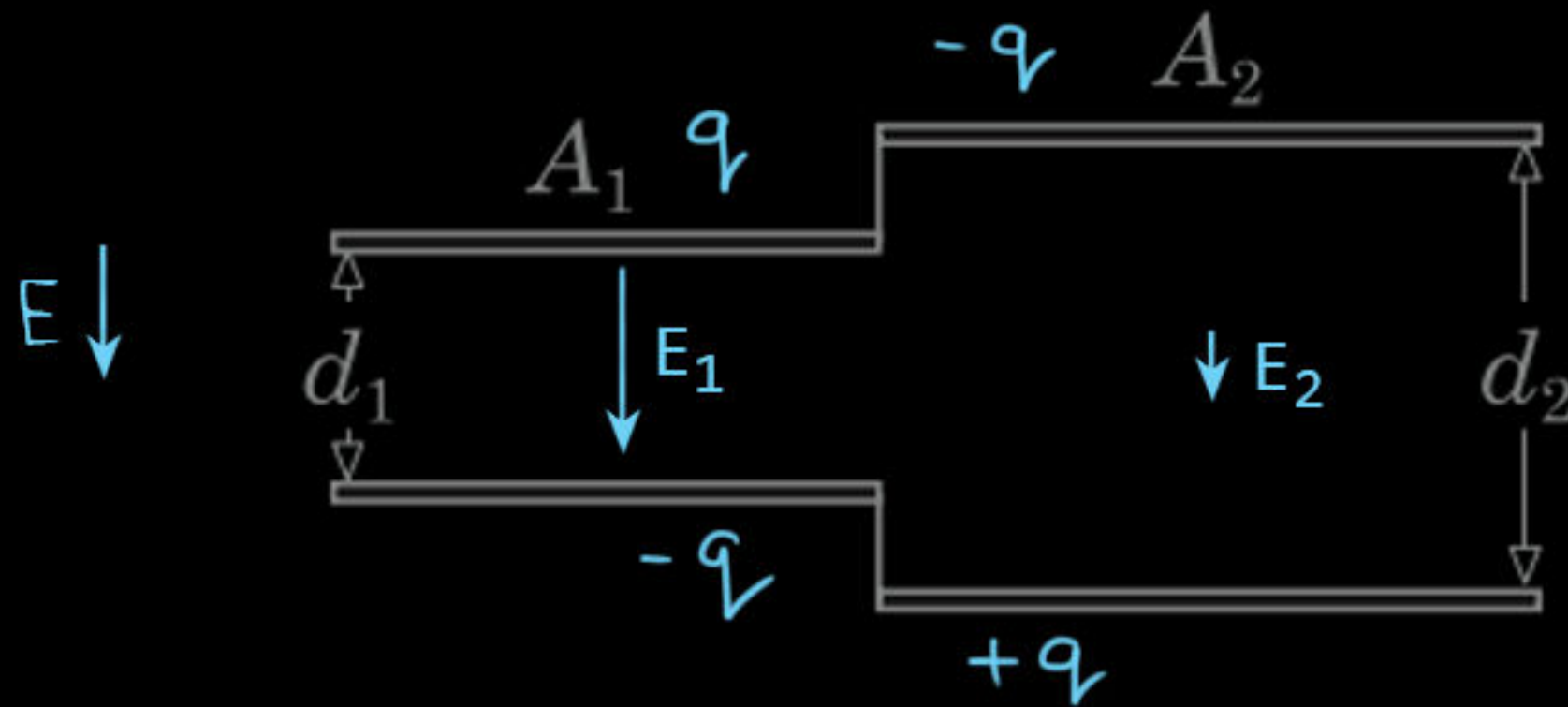
$$\Rightarrow V_c = \frac{V_2 + 6V_1}{4}$$

by Ankit Singhvi

Youtube / Arihant Senior



27. A parallel plate capacitor has two sections. In these sections, areas of plates are A_1 and A_2 and distances between the plates are d_1 and d_2 respectively. The distances between the plates are much smaller than their linear dimensions. A uniform electric field E that is perpendicular to the plates is switched on. Neglecting edge effects, deduce expressions for the intensities E_1 and E_2 of electric fields established in both the sections of the capacitor.



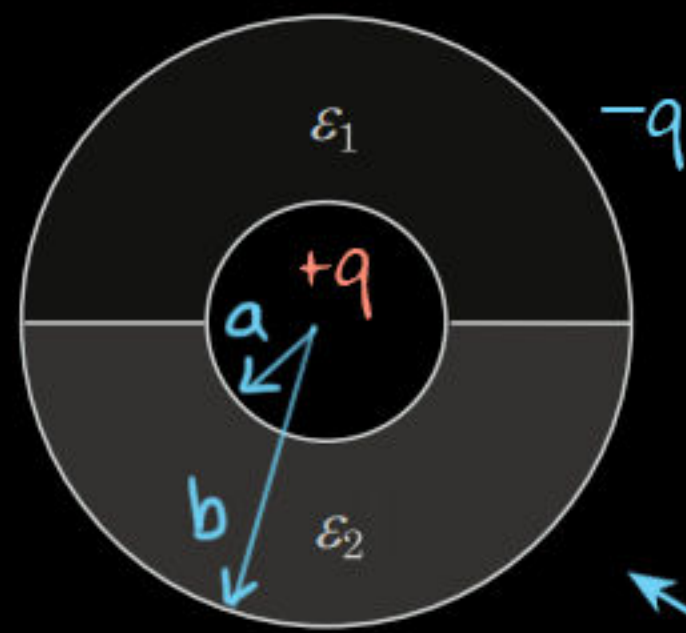
$$E_1 d_1 = E_2 d_2 \quad (1)$$

$$E_1 = E + \frac{(q/A_1)}{\epsilon_0} \quad (2)$$

$$E_2 = E - \frac{(q/A_2)}{\epsilon_0} \quad (3)$$

$$\text{From 1,2,3: } E_1 = E d_2 \left[\frac{A_1 + A_2}{A_1 d_2 + A_2 d_1} \right]$$

$$E_2 = E d_1 \left[\frac{A_1 + A_2}{A_1 d_2 + A_2 d_1} \right]$$



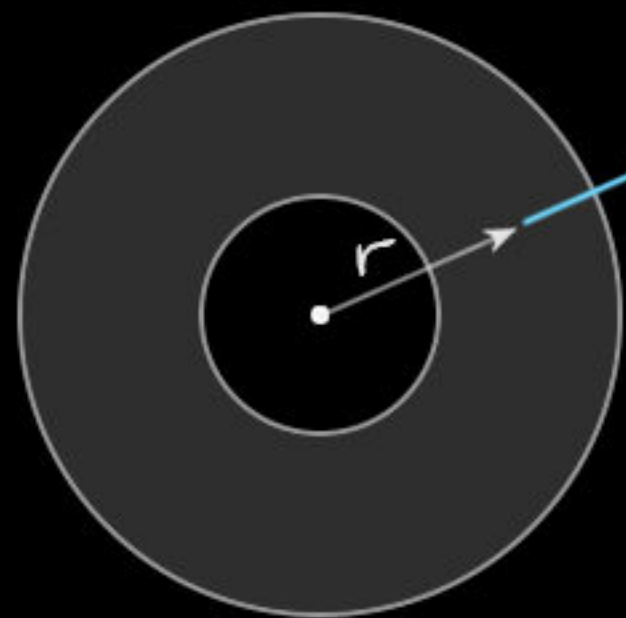
28. Radii of the inner and outer concentric conducting spheres of a spherical capacitor are a and b . One half of the space between the spheres is filled with a linear isotropic dielectric of permittivity ϵ_1 and the other half with another linear isotropic dielectric of permittivity ϵ_2 as shown in the figure. The inner and the outer spheres are given charge $+q$ and $-q$ respectively. Find expressions for the electric fields in both the dielectrics and potential difference between the spheres.

We know, capacitance of a hemispherical capacitor is:

$$C = \frac{1}{2} \left[\frac{4\pi\epsilon ba}{b-a} \right]$$

$C_{eq} = C_1 + C_2$ (As both hemispherical capacitors are in parallel)

$$= \frac{1}{2} \left[\frac{4\pi\epsilon_1 ba}{b-a} \right] + \frac{1}{2} \left[\frac{4\pi\epsilon_2 ba}{b-a} \right] = \frac{2\pi ba}{b-a} (\epsilon_1 + \epsilon_2)$$

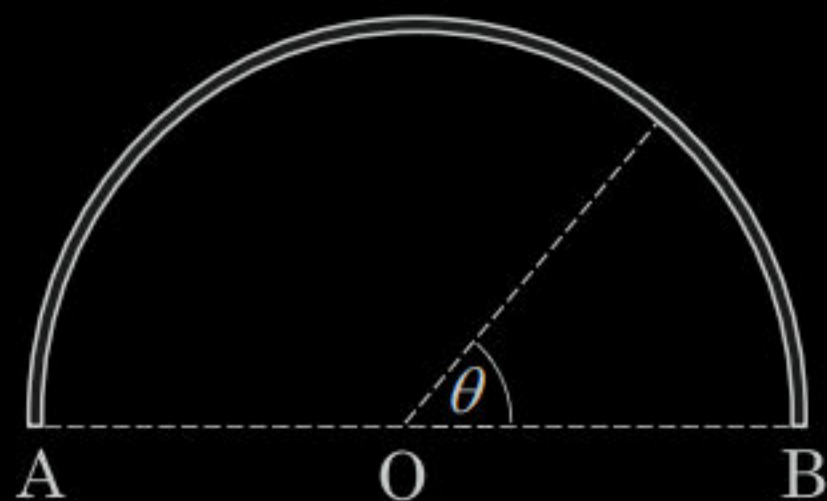


$$E = \frac{dV_r}{dr} = \frac{d}{dr} \left[\frac{q(r-a)}{2\pi(\epsilon_1 + \epsilon_2)ra} \right]$$

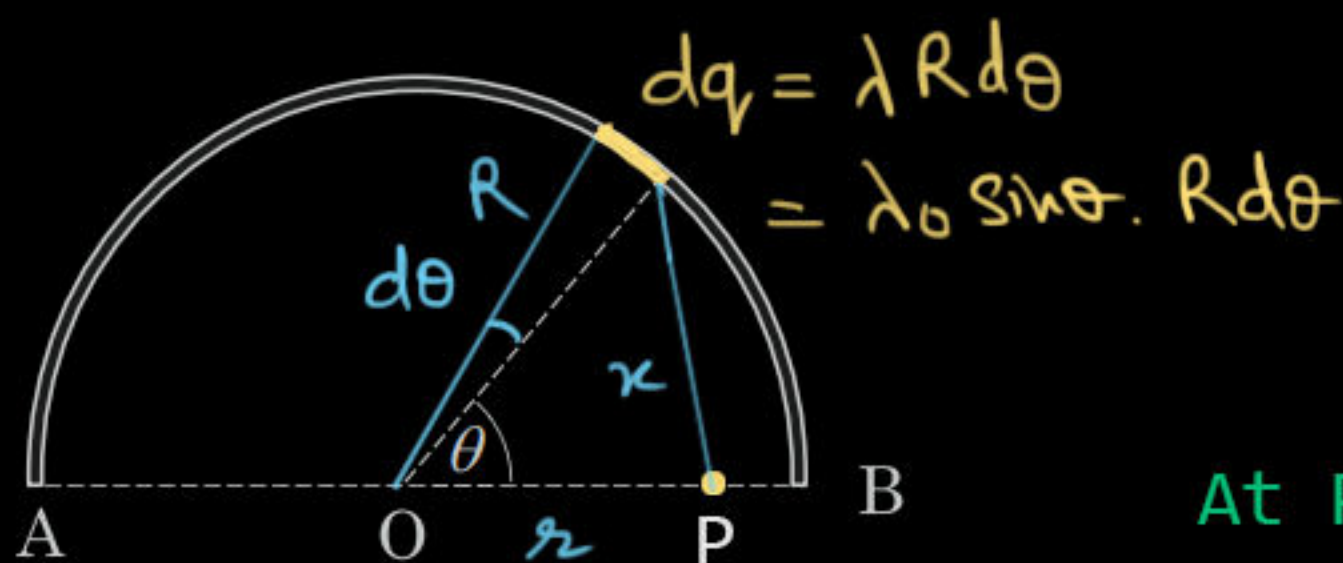
$$= \frac{q}{2\pi(\epsilon_1 + \epsilon_2)} \frac{1}{r^2} //$$

$$\therefore V = \frac{q}{C_{eq}} = \frac{q(b-a)}{2\pi(\epsilon_1 + \epsilon_2)ba} //$$

Replacing b with r



29. Charge distribution on a rod bent as a semicircular arc of radius R follows a function $\lambda = \lambda_0 \sin \theta$. Here λ is line charge density at a point on the rod, λ_0 is a positive constant and θ is angular position of the point as shown in the figure. Find the ratio of electrostatic potentials at the centre O and at a general point on the diameter AB at distance r ($r \leq R$) from the centre O .



$$\text{At } O: V_0 = \frac{kQ}{R} = \frac{k \int_0^\pi \lambda_0 \sin \theta R d\theta}{R} = 2k\lambda_0 //$$

$$\text{At } P: dV_P = \frac{k dq}{x} = \frac{k (\lambda_0 \sin \theta R d\theta)}{\sqrt{R^2 + r^2 - 2Rr \cos \theta}}$$

$$\begin{aligned} \text{Integrating by substituting } R^2 + r^2 - 2Rr \cos \theta &\rightarrow x^2 \\ \Rightarrow 2Rr \sin \theta d\theta &\rightarrow 2x dx \end{aligned}$$

$$\therefore V_P = kR\lambda_0 \int_{(R-r)}^{(R+r)} \frac{2x dx}{2xR} = \frac{k\lambda_0}{r} [(R+r) - (R-r)] = 2k\lambda_0 //$$

Fraction of charge removed everytime:

$$= \frac{\cancel{\pi r^2}}{\cancel{4\pi} R^2} = \frac{r^2}{4R^2}$$

∴ If initial

Q

After 1st

$$Q \left(1 - \frac{r^2}{4R^2}\right)$$

2nd

$$Q \left(1 - \frac{r^2}{4R^2}\right)^2$$

⋮

n^{th}

$$Q \left(1 - \frac{r^2}{4R^2}\right)^n$$

$$\approx Q \left(1 - \frac{n r^2}{4R^2}\right) \quad (1)$$

30. A conducting sphere of radius $R = 1.0$ m is charged to a potential $V_i = 1000$ V. A thin metal disc of radius $r = 1.0$ cm mounted on an insulating handle is touched with the sphere making contact with one of its flat faces and then separated. After separation the disc is earthed and the process is repeated until the potential of the sphere becomes $V_f = 999$ V. Approximately how many times has this process been repeated?

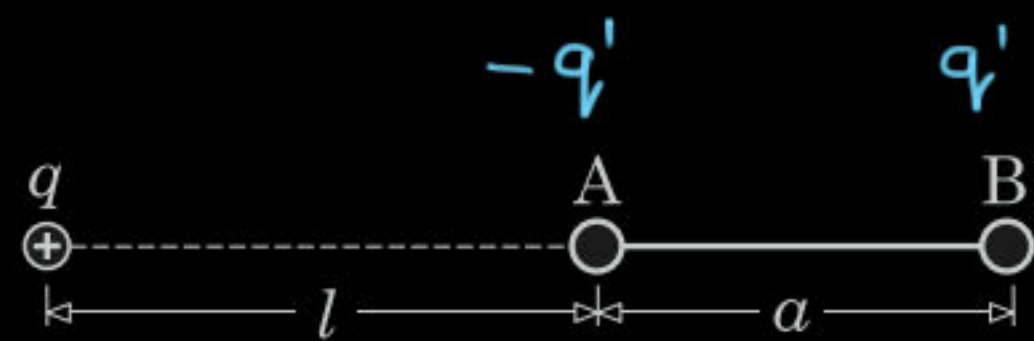
$$V_i = \frac{kQ}{R}$$

$$V_f = \frac{kQ}{R} \left(1 - \frac{1}{1000}\right) \quad (2)$$

From 1,2:

$$\frac{n r^2}{4R^2} = \frac{1}{1000}$$

$$\Rightarrow n = \frac{4}{1000} \frac{R^2}{r^2} = 40 //$$



31. Two small metal spheres A and B each of radius r supported on insulating stands, located at a distance a ($a \gg r$) from each other are connected by a thin conducting wire. A point charge q is placed on the line joining centres of the spheres at distance l ($l \gg r$) from the ball A. What are the moduli of charges induced on the spheres?

$$\Rightarrow \frac{kq}{l} - \frac{kq'}{r} + \frac{kq'}{a} = \frac{kq}{(l+a)} + \frac{kq'}{r} - \frac{kq'}{a} \quad \begin{matrix} V_A = V_B \\ a \gg r \end{matrix}$$

(Note: In the original image, green arrows point from the $\frac{kq'}{a}$ terms to ≈ 0 .)

$$\Rightarrow \frac{2kq'}{r} = \frac{kq}{l} - \frac{kq}{l+a}$$

$$\Rightarrow q' = \frac{qa}{2l(l+a)}$$

32. Consider a **dumbbell** like structure consisting of **two conducting balls** connected by a **conducting rod**. Radius r of the balls is much smaller than the length l of the connecting rod. The dumbbell is placed midway between two unlike point charges Q and $-Q$ that are a distance $3l$ apart. Rod of the dumbbell is collinear with the line joining the charges. By what amount will the net force on a charge change after placing the dumbbell?



$$V_A = V_B$$

$$\Rightarrow \frac{kQ}{l} - \frac{kq}{r} - \frac{kQ}{2l} + \frac{kq}{l} = \frac{kQ}{2l} + \frac{kq}{r} - \frac{kQ}{l} - \frac{kq}{l}$$

$l \gg r \Rightarrow \approx 0$

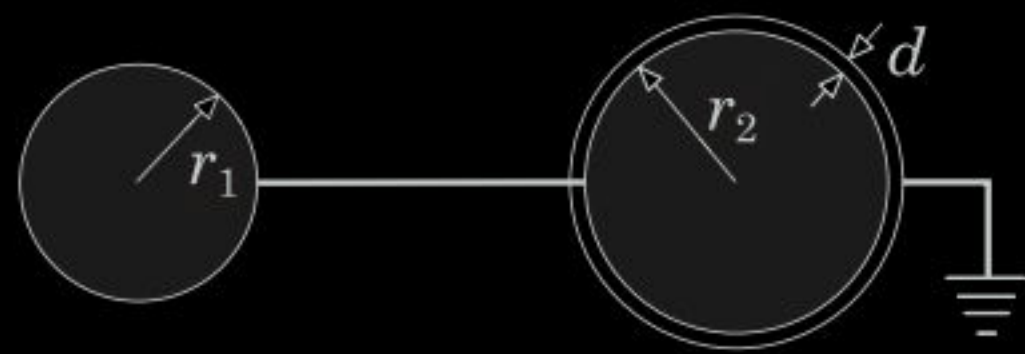
$$\Rightarrow q = \frac{rQ}{2l} \quad (1)$$

$$\Delta f_Q = -\frac{kQq}{l^2} + \frac{kQq}{(2l)^2}$$

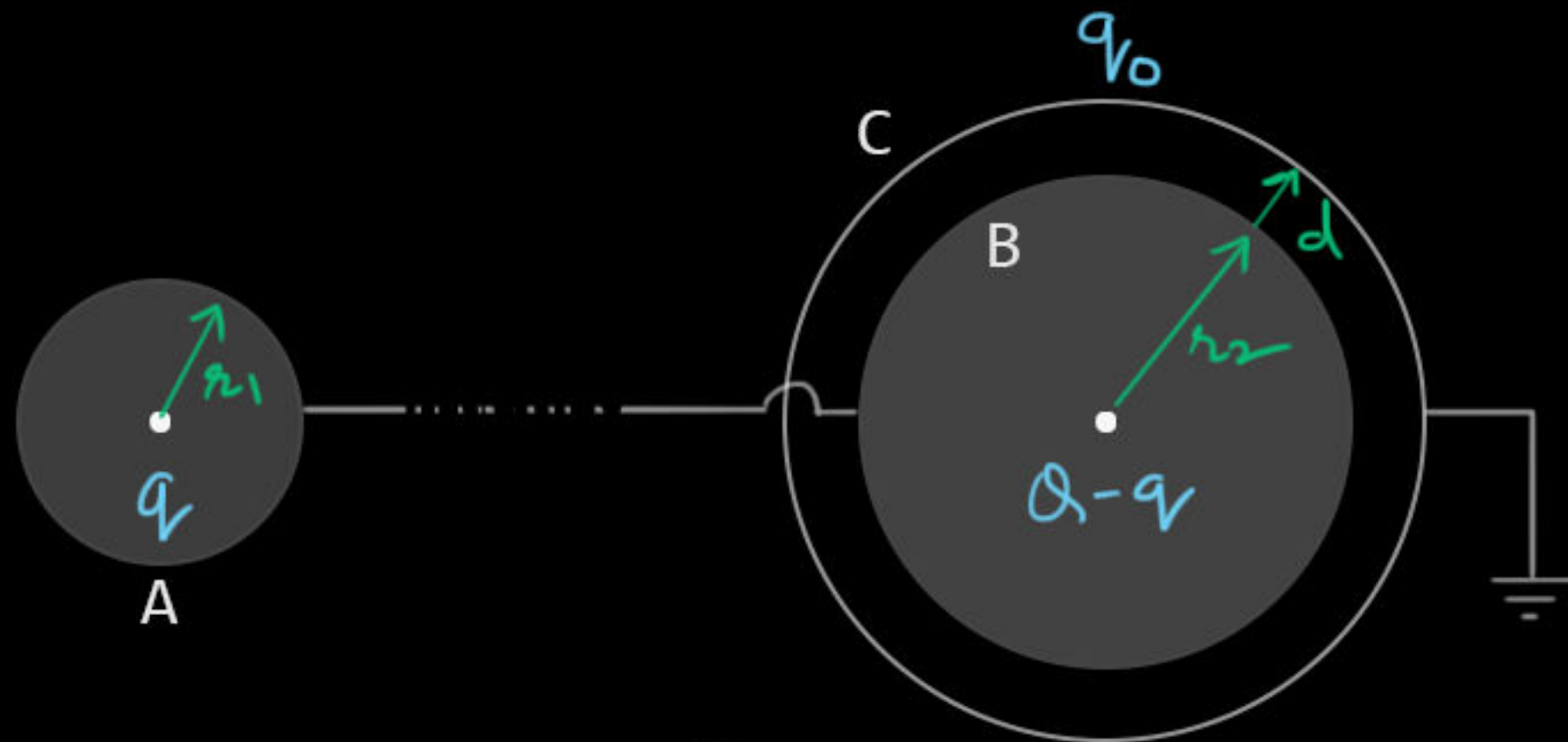
$$= \frac{3kqQ}{4l^2}$$

From 1

$$= \frac{3}{8} \frac{kQ^2 r}{l^3} //$$



33. Two metal spheres of radius r_1 and r_2 are connected by a thin conducting wire. The second sphere is surrounded by a grounded concentric conducting shell with uniform separation d between their facing surfaces. If the second sphere is given a charge Q , find charges acquired by all the three spheres. Assume that $d \ll r_2$, and the distance between the spheres is much larger than their radii.



$$V_C = 0$$

$$\Rightarrow \frac{kq_0}{r_2+d} + \frac{k(Q-q)}{r_2+d} = 0$$

$$\Rightarrow q_0 + Q - q = 0 \quad (1)$$

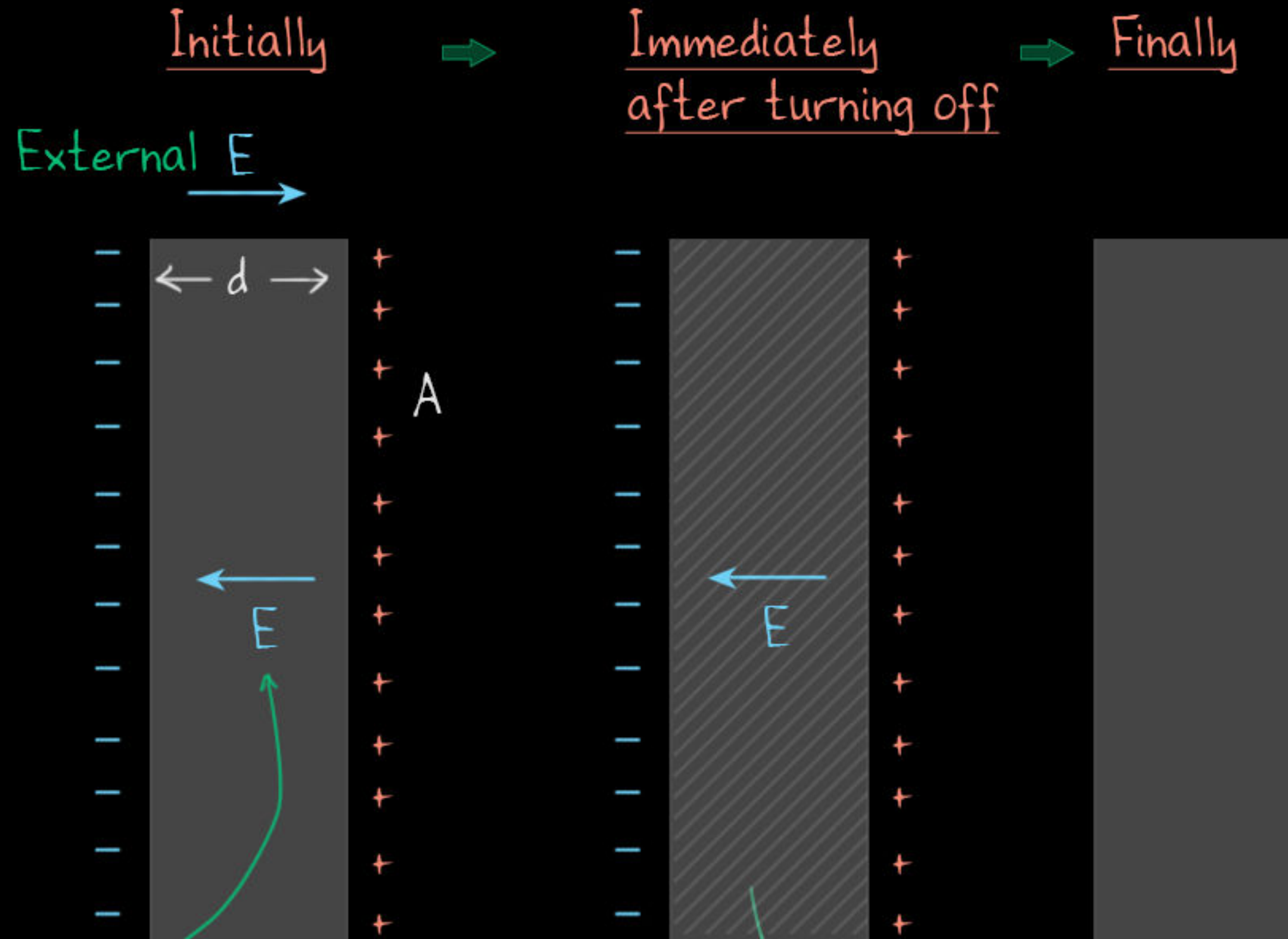
$$V_A = V_B$$

$$\Rightarrow \frac{kq}{r_1} = \frac{kq_0}{r_2+d} + \frac{k(Q-q)}{r_2} \quad (2)$$

$$\text{From 1,2: } q = \frac{r_1 d Q}{r_1 d + r_2^2} //$$

$$Q - q = \frac{r_2^2 Q}{r_1 d + r_2^2} //$$

34. A large conducting plate of surface area A and thickness d is placed perpendicular to a uniform electric field E . How much heat will be dissipated in the plate after the electric field is switched off?



We know,
electrostatic energy density

$$\frac{U}{V} = \frac{\epsilon_0 E^2}{2}$$

$$\therefore \text{Heat} = U = \frac{\epsilon_0 E^2}{2} \cdot Ad //$$

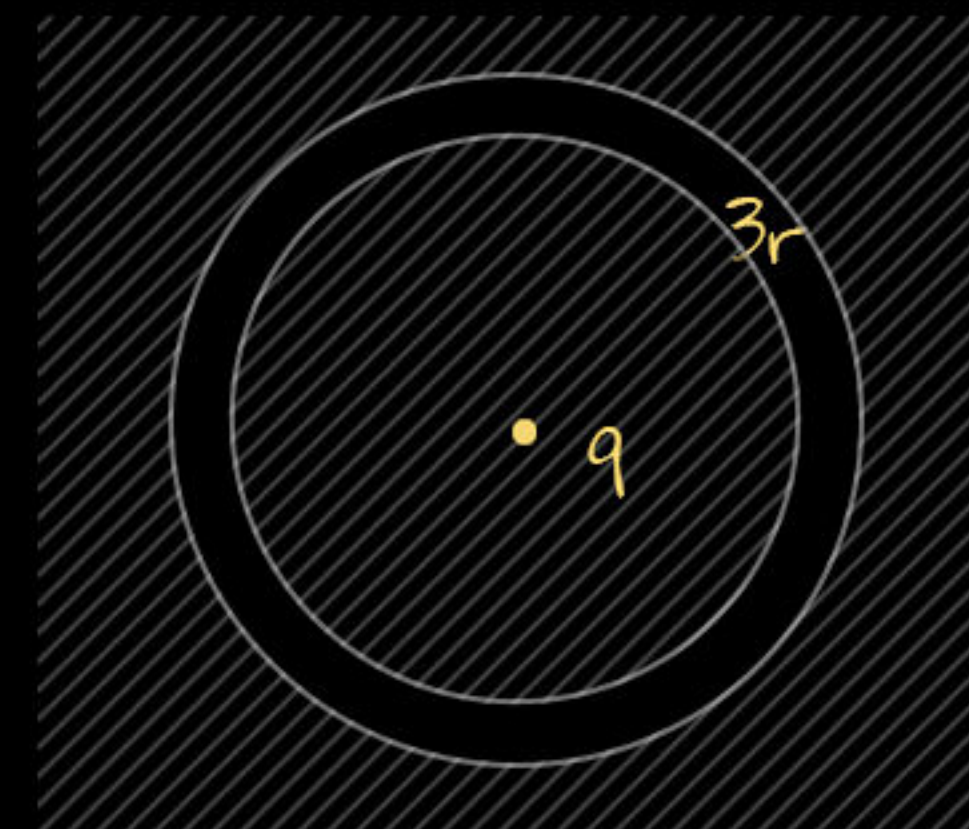
Due to induced charges

Electrostatic energy stored in this region will be lost as heat.



Electrostatic field energy is stored here

35. Two conducting spheres of external radii r and $3r$ have wall thickness $0.05r$ each. Point charges q and $2q$ are placed at the centres of these spheres respectively. The spheres are kept at great separation so that they do not affect charge distribution on the surfaces of each other and their interaction energy is negligible. Deduce expression for the minimum amount of work done by an external agency to exchange the position of the charges. The holes made to enable exchange of charges are so small that their size can be neglected.



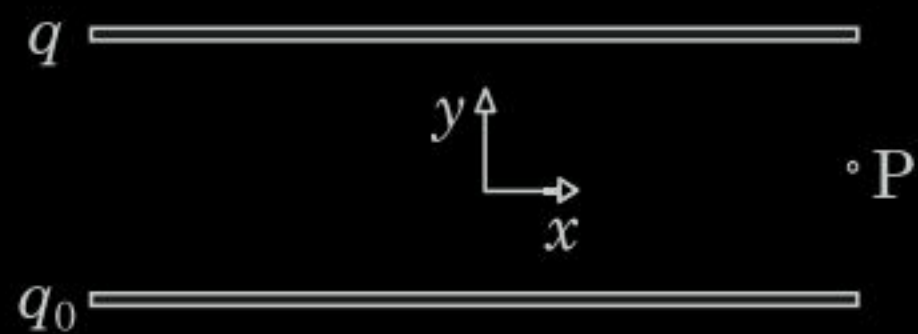
$$U_q = \left(\frac{kq}{r^2} \right)^2 \frac{\epsilon_0}{2} 4\pi r^2 dr \quad (1)$$

$$U_{2q} = \left(\frac{k(2q)}{(3r)^2} \right)^2 \frac{\epsilon_0}{2} 4\pi (3r)^2 dr \quad (2)$$

$$U_{2q} = \left[\frac{k(2q)}{r^2} \right]^2 \frac{\epsilon_0}{2} 4\pi r^2 dr \quad (3)$$

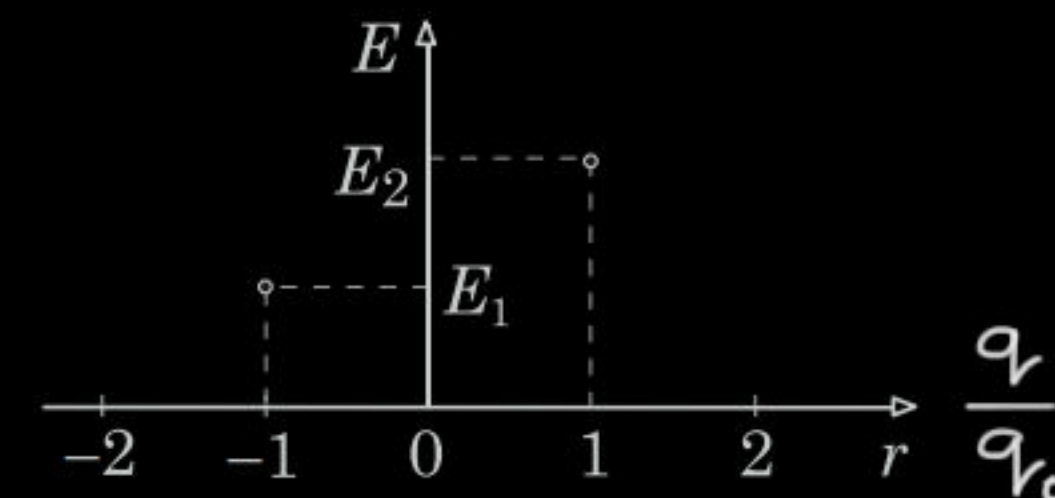
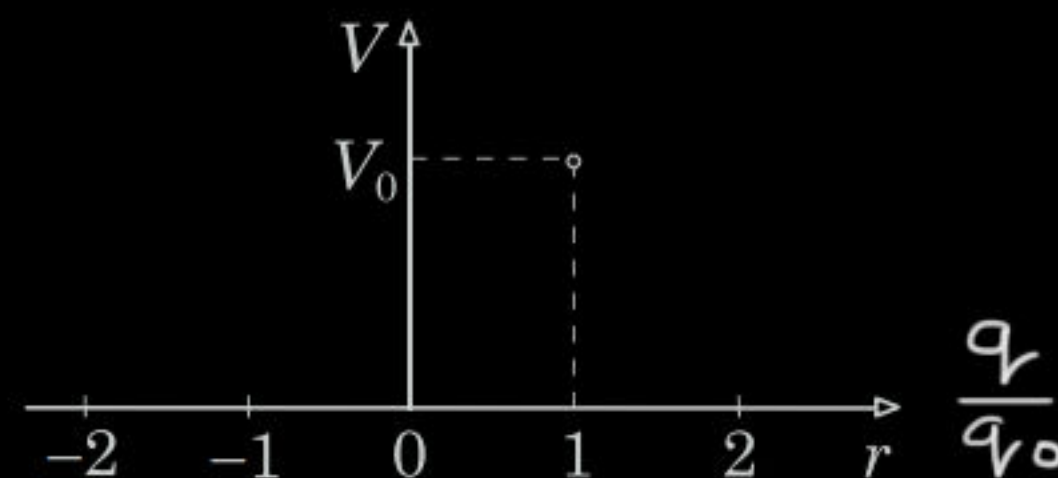
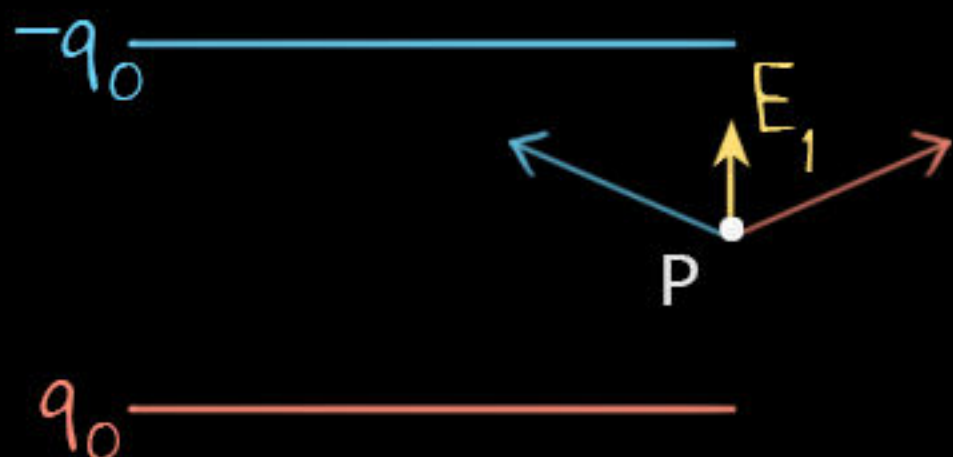
$$U_q = \left[\frac{kq}{(3r)^2} \right]^2 \frac{\epsilon_0}{2} 4\pi (3r)^2 dr \quad (4)$$

$$W = U_3 + U_4 - [U_1 + U_2] = - \frac{k^2 q^2}{r^2} \cdot \frac{\epsilon_0}{2} \cdot 4\pi \cdot 0.05r \left[4 + \frac{1}{9} - 1 - \frac{4}{9} \right] = - \frac{1}{15} \frac{kq^2}{r}$$

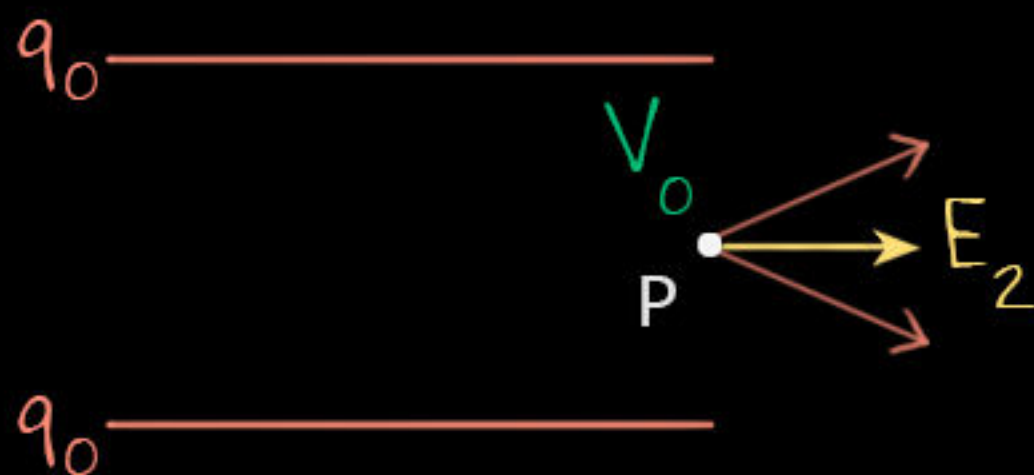


Lets represent graphical data in figures first.

Case 1 $q/q_0 = -1$



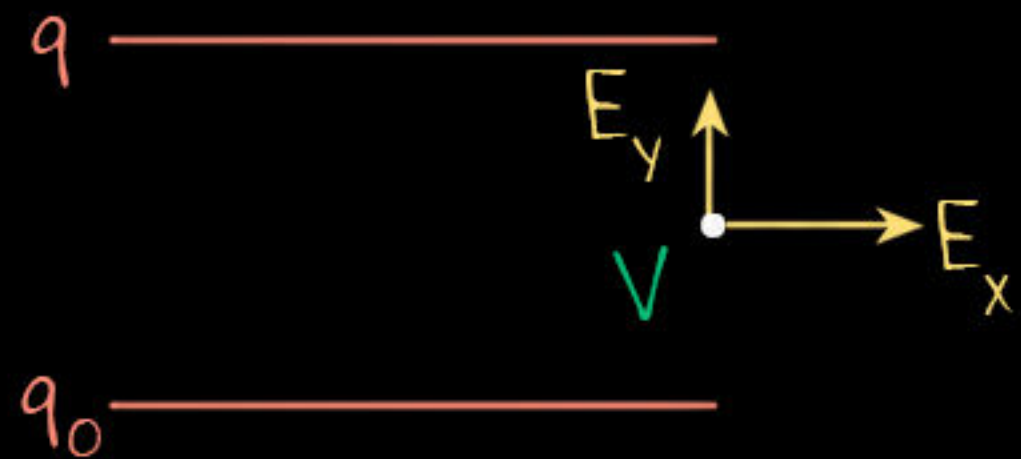
Case 2 $q/q_0 = 1$



Deduce analytical dependence of the potential V and the electric field vector $\vec{E}$ at point P on the ratio r of the charges.

36. A point P is located equidistant from both the plates and near one edge of a parallel plate capacitor as shown in the figure. In an experiment, maintaining charge q_0 on the bottom plate constant, potential V and modulus of electric field E at the point P are measured for different values of the charge q on the upper plate. In the following graphs the potential for one value of the charge q and modulus of electric field for two values of the charge q are shown as functions of the ratio of the charges $r = q/q_0$.

General case



$$E_x \propto q_0 + q$$

$$\& E_2 \propto 2q_0 \text{ (from 2)}$$

$$\Rightarrow \frac{E_x}{E_2} = \frac{q_0 + q}{2q_0}$$

$$\Rightarrow E_x = \frac{E_2 (q_0 + q)}{2q_0}$$

Case 1



$$E_y \propto q_0 - q$$

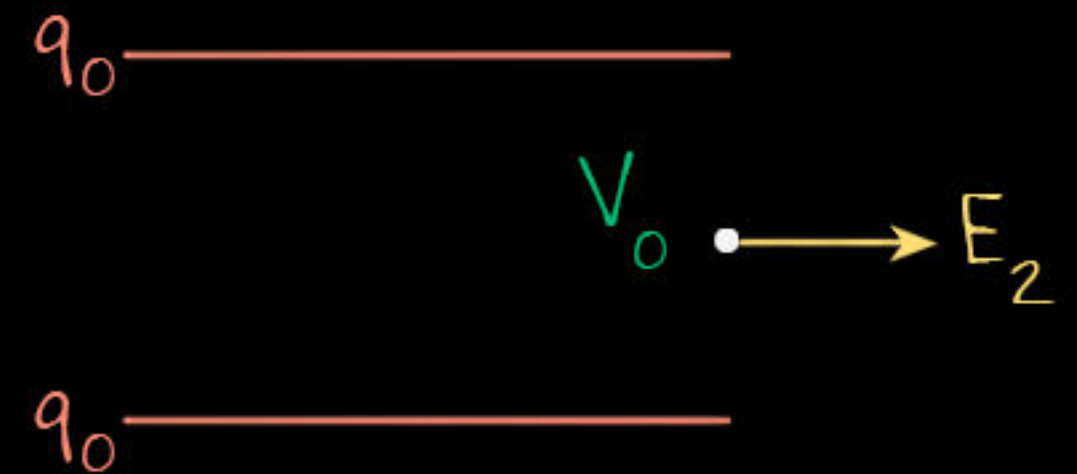
$$\& E_1 \propto 2q_0 \text{ (from 1)}$$

$$\Rightarrow \frac{E_y}{E_1} = \frac{q_0 - q}{2q_0}$$

$$\Rightarrow E_y = \frac{E_1 (q_0 - q)}{2q_0}$$

$$\therefore E = \frac{E_2}{2} (1 + \lambda) \hat{i} + \frac{E_1}{2} (1 - \lambda) \hat{j} //$$

Case 2



$$V \propto q_0 + q$$

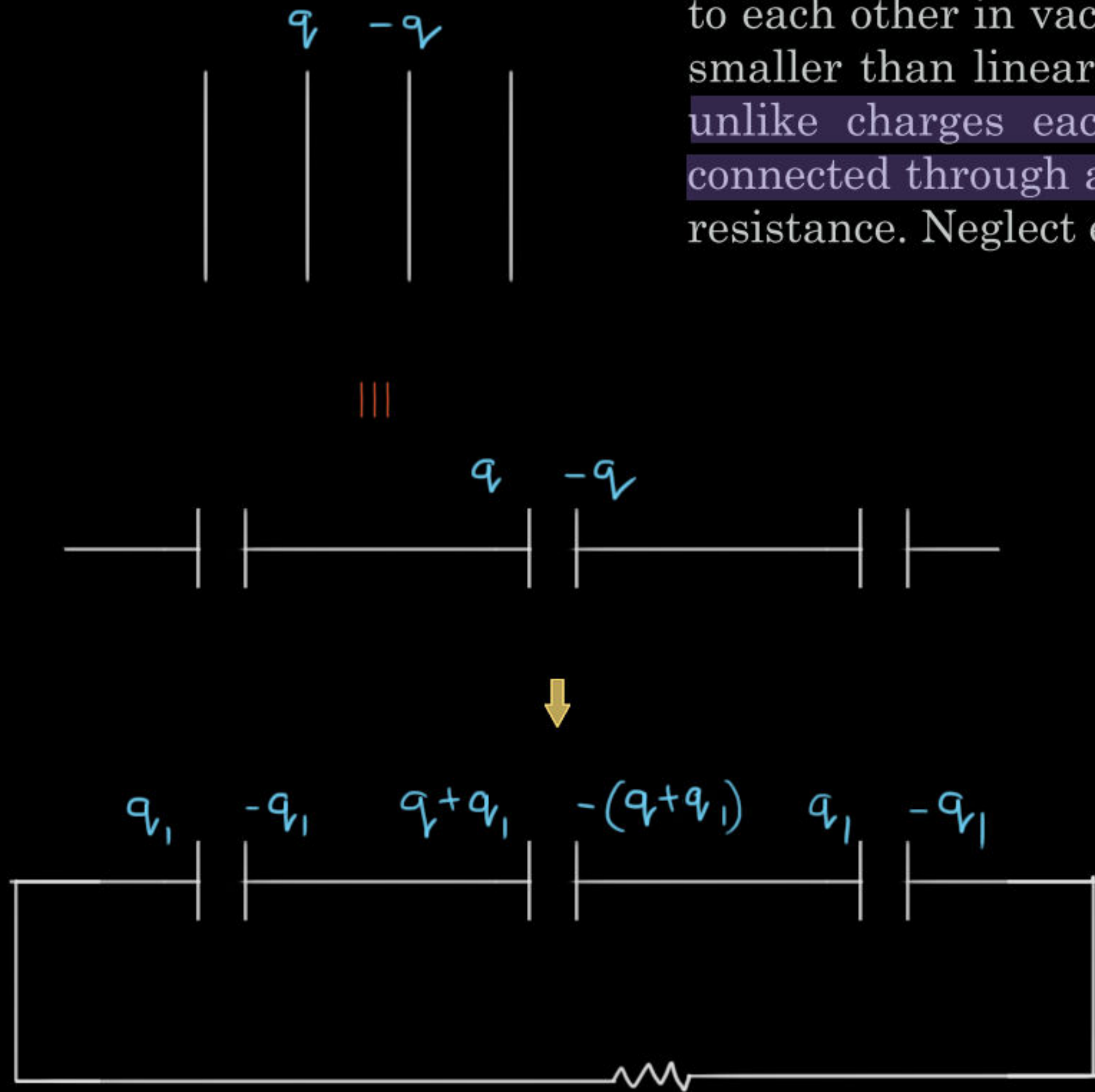
$$\& V_0 \propto 2q_0 \text{ (from 2)}$$

$$\Rightarrow \frac{V}{V_0} = \frac{q_0 + q}{2q_0}$$

$$\Rightarrow V = \frac{V_0 (q_0 + q)}{2q_0}$$

$$= \frac{V_0}{2} (1 + \lambda) //$$

37. Four uncharged identical metal plates, each of area A are placed parallel to each other in vacuum. Distances d between adjacent plates is much smaller than linear dimensions of a plate. The inner plates are given unlike charges each of modulus q and then the outer plates are connected through a resistance. Determine total heat dissipated in the resistance. Neglect effects of electromagnetic radiation.



$$\text{KVL: } \frac{q_1}{C} + \frac{q+q_1}{C} + \frac{q_1}{C} = 0$$

$$\Rightarrow q_1 = -q/3$$

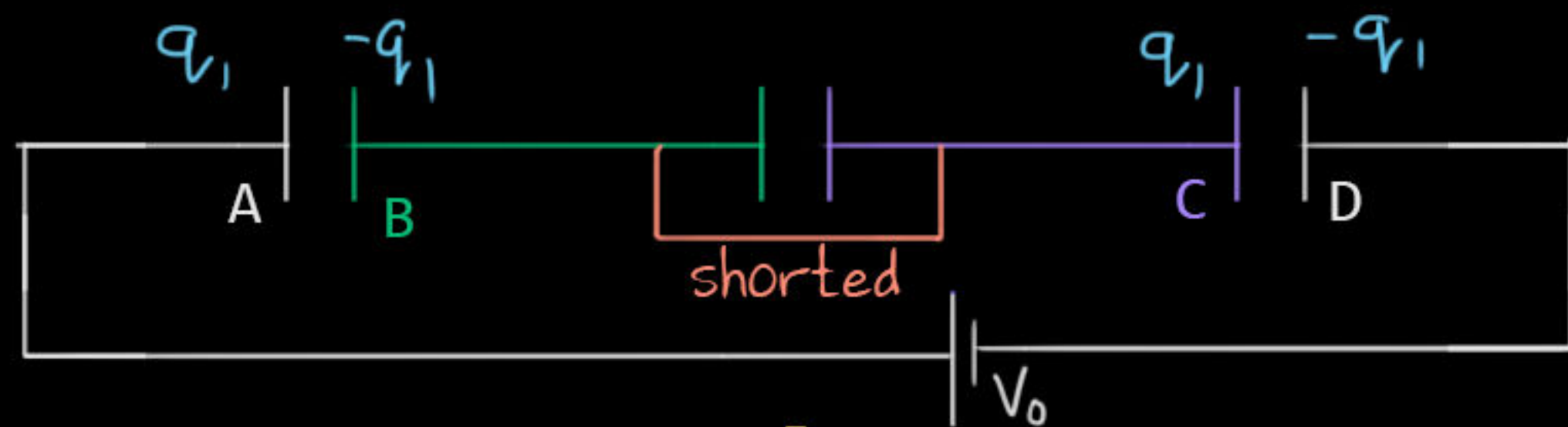
$$\Delta H = -\Delta U$$

$$= \frac{q^2}{2C} - \left[\frac{q_1^2}{2C} + \frac{(q+q_1)^2}{2C} + \frac{q_1^2}{2C} \right]$$

$$= \frac{q^2}{6C} = \frac{q^2 d}{6A\epsilon_0}$$



38. Four identical metallic plates are arranged very close and parallel to each other as shown in the figure. Plates A and D are connected across a voltage source, which maintains potential of plate A higher than that of D by an amount V_0 . Plates B and C are connected by a short conducting wire for a while and then the wire is removed. Now the voltage source is removed and then plates A and D are connected by another short conducting wire. Find potential differences between the pairs A-B, A-C and B-D.



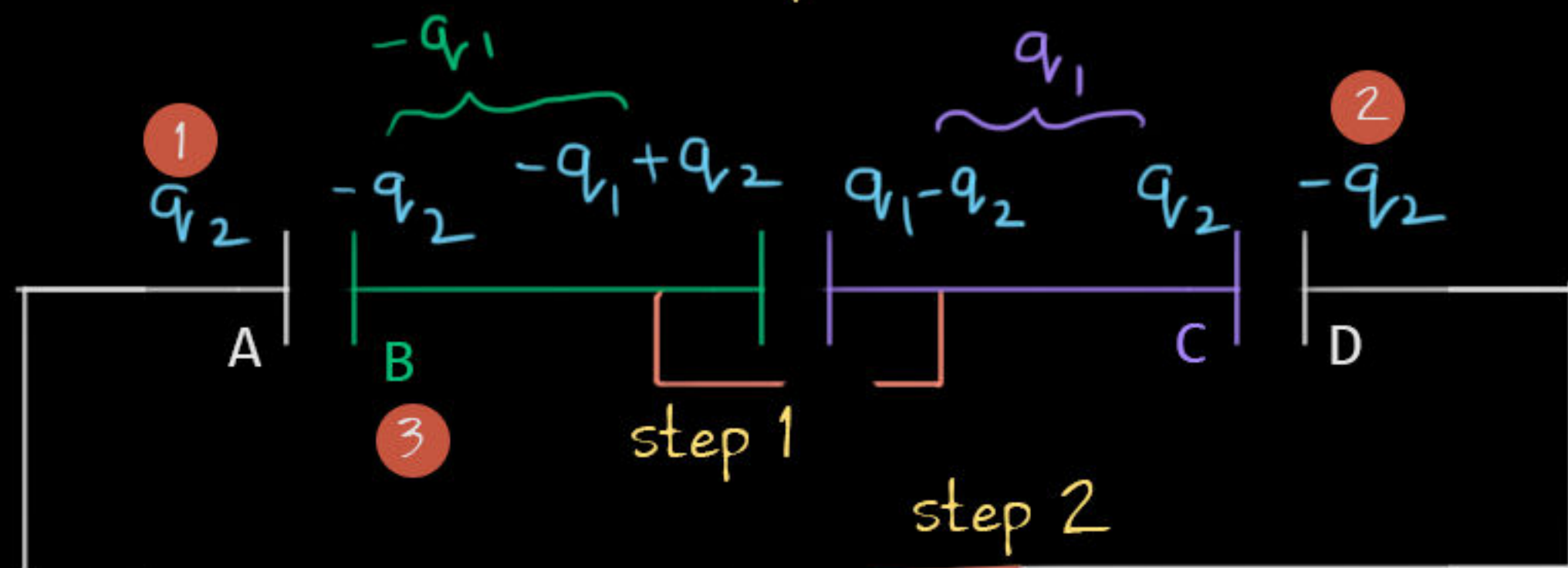
$$C_{eq} = C/2$$

$$\Rightarrow q_1 = \frac{CV_0}{2}$$

$$1 \quad V_{AB} = \frac{q_2}{C} = V_0/6 //$$

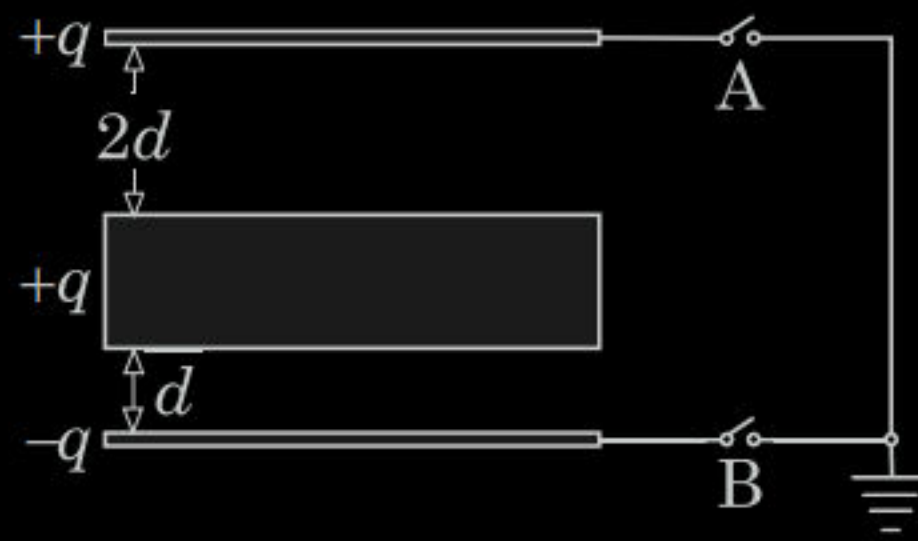
$$2 \quad V_{AC} = -\frac{q_2}{C} = -V_0/6 //$$

$$3 \quad V_{BD} = -\frac{q_2}{C} = -V_0/6 //$$



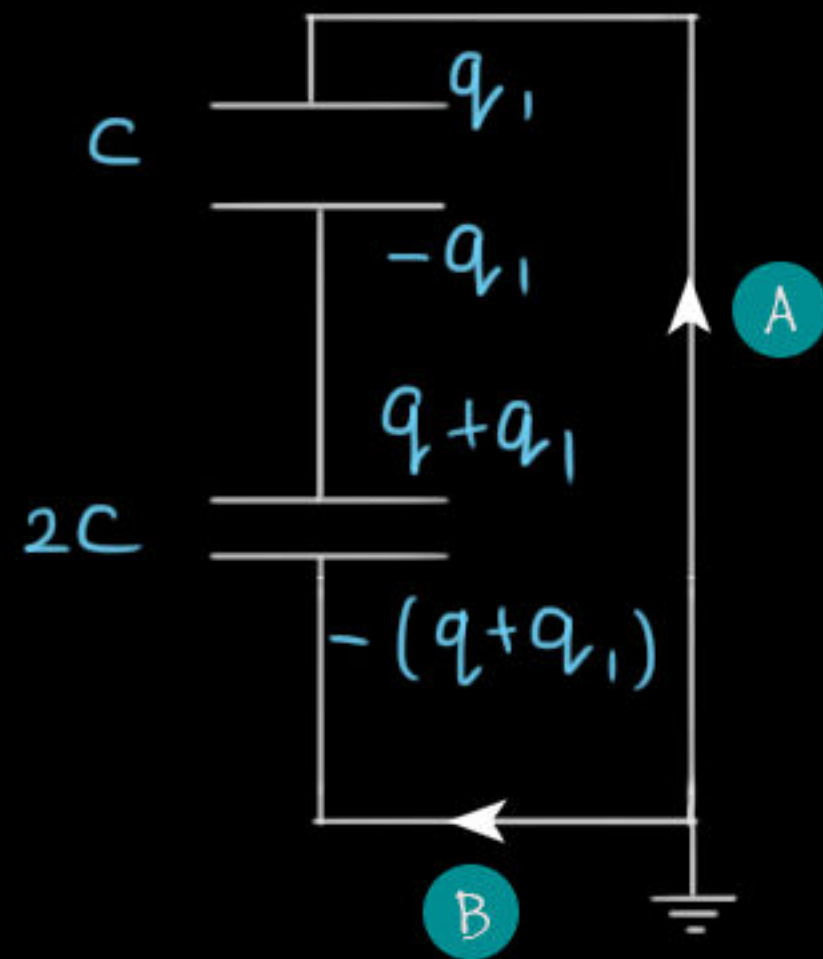
$$\frac{q_2}{C} + \frac{(-q_1 + q_2)}{C} + \frac{q_2}{C} = 0$$

$$\Rightarrow q_2 = q_1/3 = \frac{CV_0}{6}$$



|||

39. Between two thin parallel metal plates having charges $+q$ and $-q$ is inserted a thick metal slab having charge $+q$ so that distance between the upper plate and top face of the slab is $2d$ and the distance between lower plate and bottom face of the slab is d . Adjacent surfaces of the plates and that of the slab are equal in area and parallel to each other. Now the upper and the lower plates are earthed by closing the switches A and B simultaneously. Find amounts of the charges that pass through the switches.

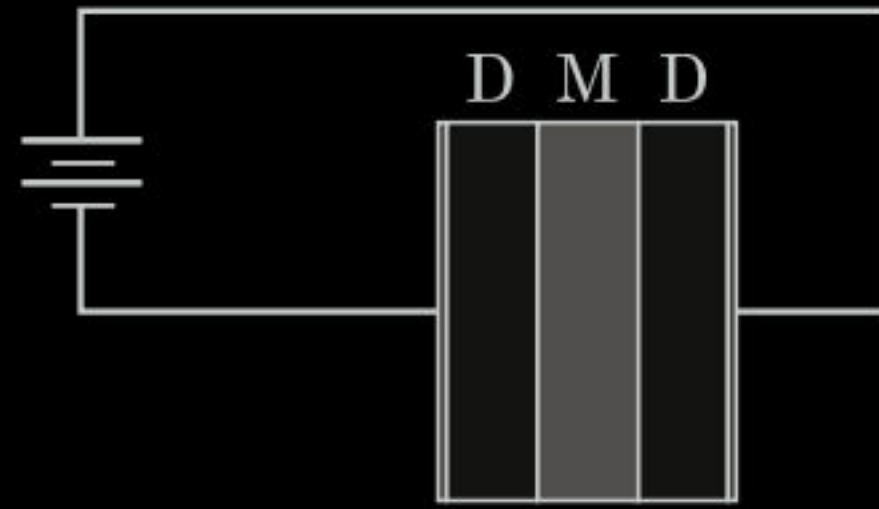


$$\text{KVL: } 0 + \frac{q_1 + q_1}{2C} + \frac{q_1}{C} = 0$$

$$\Rightarrow q_1 = -q/3$$

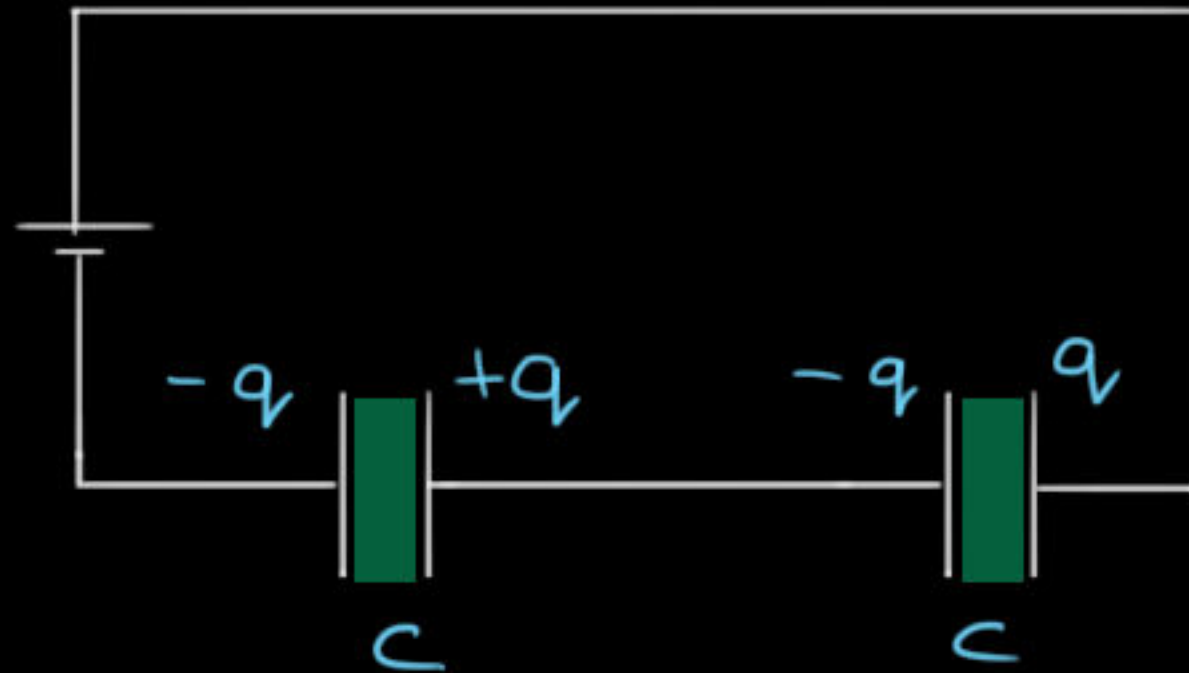
$$\therefore \Delta q_A = q_1 - q = -\frac{4q}{3} \quad \text{Charge goes from plate to switch}$$

$$\Delta q_B = -(q + q_1) - (-q) = q/3 \quad \text{Charge goes from switch to plate}$$



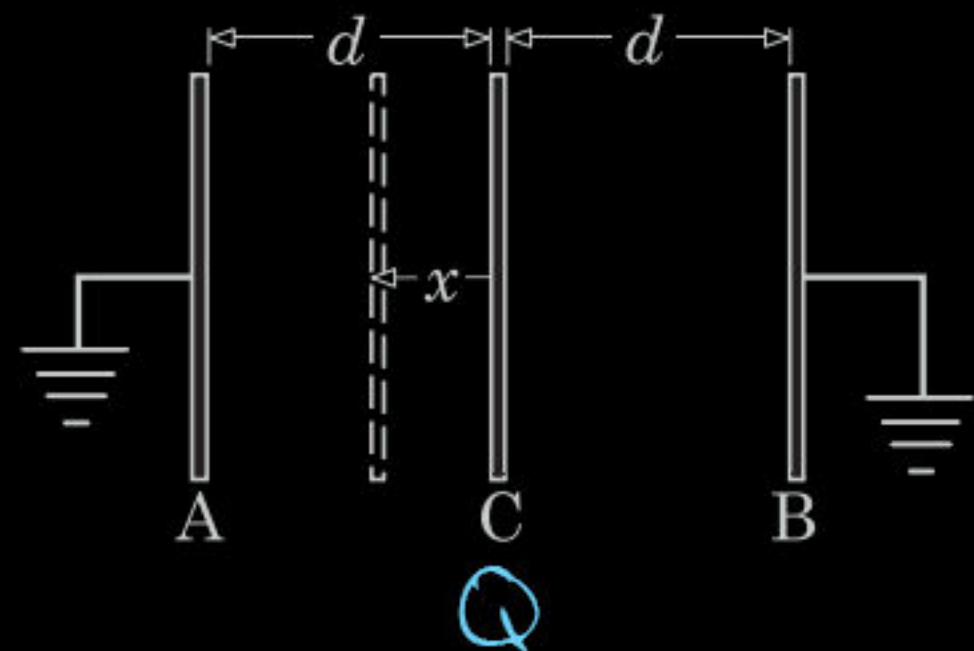
|||

40. Plates of a parallel plate capacitor have area A and distance between them is d . A potential difference V is maintained across the capacitor by a battery. Two identical dielectric slabs D and a metal slab M each of thickness $d/3$ are inserted in the capacitor as shown in the figure. Relative permittivity of each of the dielectric slab is ϵ_r . Find expression for modulus of the force of electrostatic interaction on either of the plates.

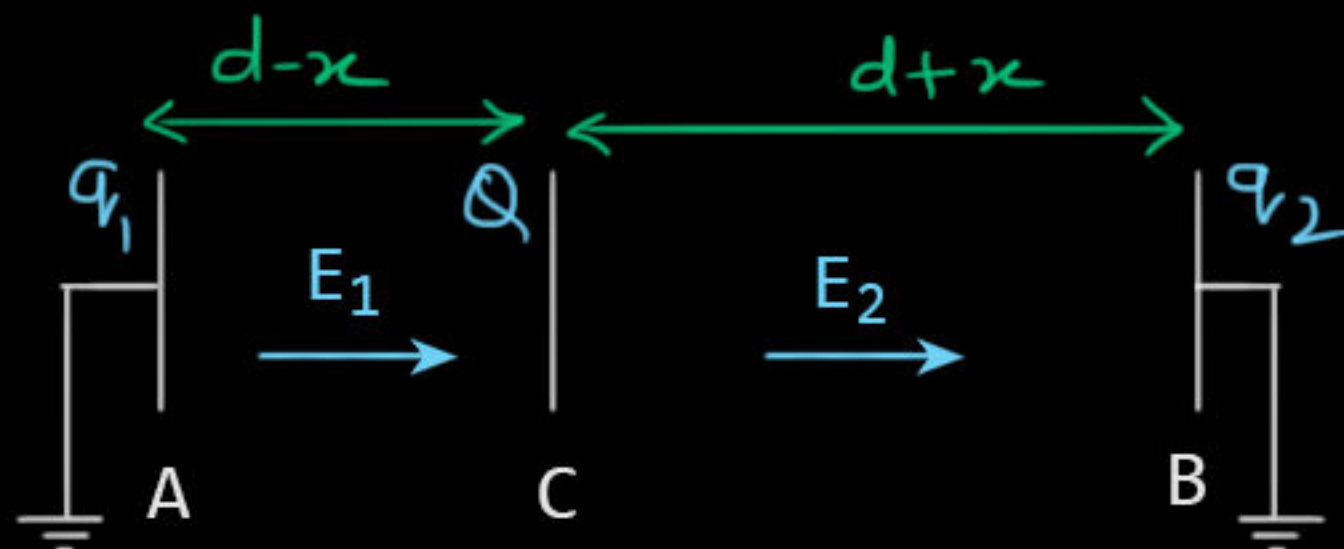


$$q = CeqV = \frac{1}{2} CV = \frac{1}{2} \left(\frac{A\epsilon_0\epsilon_r}{d/3} \right) V = \frac{3A\epsilon_0\epsilon_r V}{2d}$$

$$f = \frac{q^2}{2A\epsilon_0} = \frac{9A\epsilon_0\epsilon_r^2 V^2}{8d^2} //$$



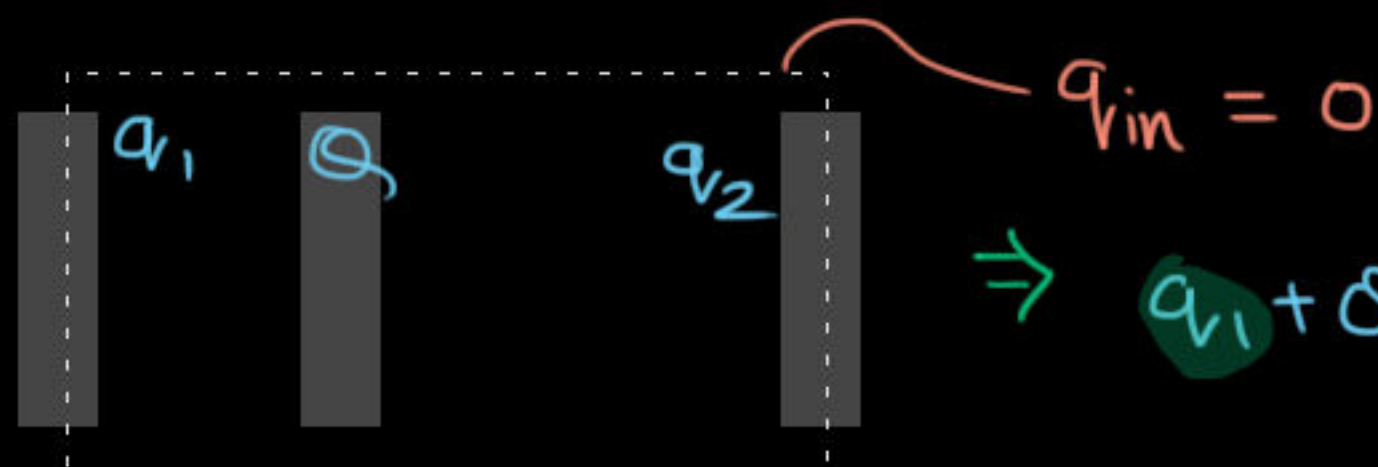
41. Two uncharged thin metal plates A and B each of area A are fixed parallel to each other and earthed. Another identical plate C made of a dielectric material is placed midway between the plates A and B. The plate C has a net positive charge Q uniformly distributed in its volume. Determine net electrostatic force on the plate C, if it is shifted a distance x towards the plate A. Neglect the end effects.



$$V_A = V_B$$

$$\Rightarrow E_1(d-x) + E_2(d+x) = 0$$

$$\Rightarrow (q_1 - Q - q_2)(d-x) + (q_1 + Q - q_2)(d+x) = 0 \quad (1)$$



$$q_{in} = 0$$

$$\Rightarrow q_1 + Q + q_2 = 0 \quad (2)$$

From 1,2: if needed:

$$q_1 = -\frac{Q}{2d}(x+d)$$

$$q_2 = \frac{Q}{2d}(x-d)$$

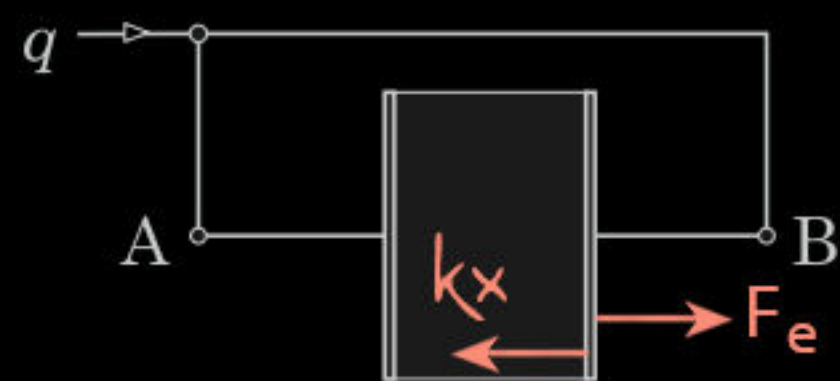
$$F_C = Q \left(\frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} \right) = \frac{Q}{2A\epsilon_0} (q_1 - q_2) \xrightarrow{\text{From 1}} -\frac{Q^2 x}{2A\epsilon_0 d} //$$

$q_1 - q_2 = -Qx/d$

by Ankit Singhvi

Youtube / Arihant Senior

A

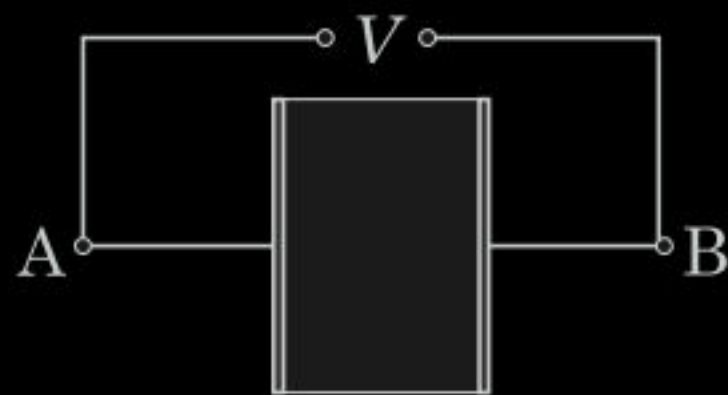


$$F_e = kx$$

$$\Rightarrow \frac{(q/2)^2}{2A\epsilon_0} = kx$$

$$\Rightarrow q = \sqrt{8A\epsilon_0 kx}$$

C



$$F_e = kx$$

$$CV = \frac{A\epsilon_0 \epsilon_r V}{(d-x)}$$

$$\Rightarrow \frac{q^2}{2A\epsilon_0} = kx$$

$$\Rightarrow V = \frac{d-x}{\epsilon_r} \sqrt{\frac{2kx}{A\epsilon_0}}$$

42. A device has been suggested to measure charge and electrostatic voltage. It consists of a cylindrical rubber pad with identical circular metal plates coaxially glued on both of its circular faces. Radius of the rubber pad and that of the metal plates are equal. One of the plates is held fixed and the other one can move while remaining parallel to the first one. In this way, a directly measured quantity from this device is the displacement x of the movable plate.

The area of a metal plates is S , the length of the rubber pad in undeformed state is d and its stiffness is k . Electrical permittivity of the free space is ϵ_0 and dielectric constant of the rubber is ϵ_r .

Neglect the fringing of the electric field at the edges of the plates, i.e. the electric field between the plates can be considered homogeneous.

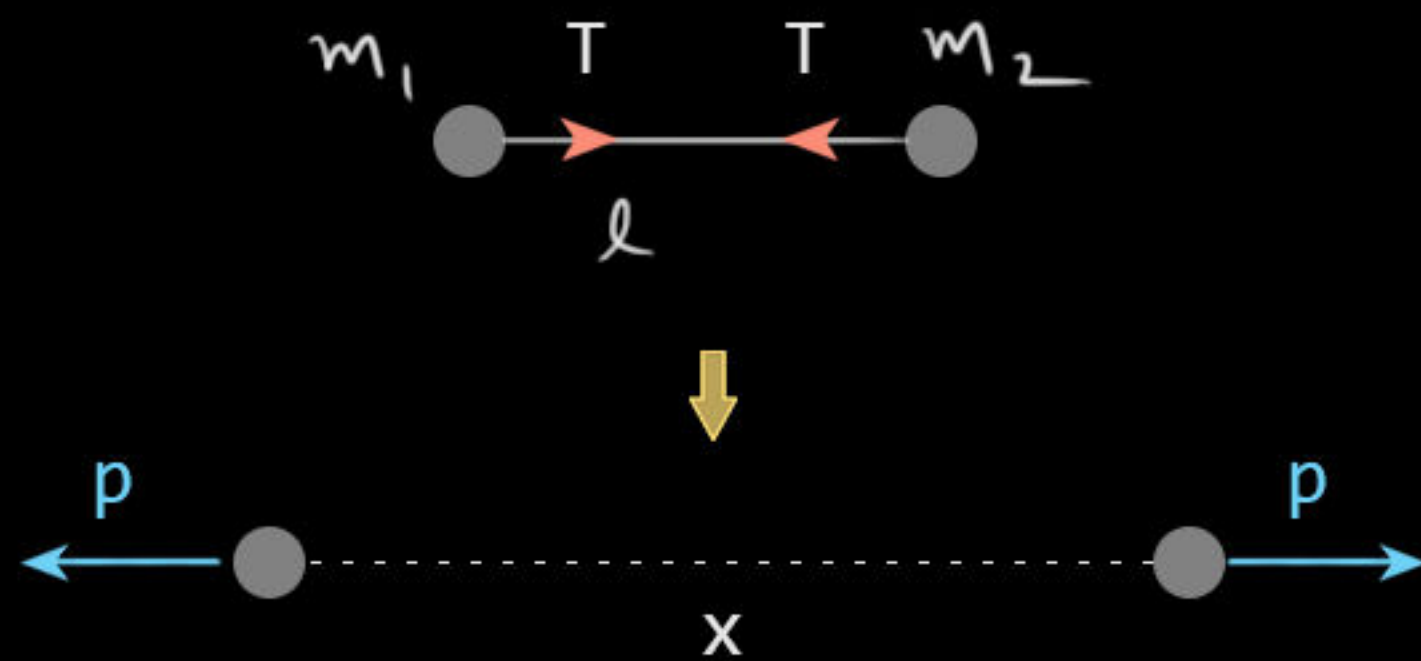
(a) To use the device as a charge meter, the charge q to be measured is fed to the plates through terminals A and B. Find relation between the plate displacement x and the charge q .

✗ (b) In which part of its range, is the device as a charge meter more accurate?

(c) To use the device as an electrostatic voltmeter, the electrostatic voltage V to be measured is applied between the terminals A and B. Find relation between the plate displacement x and the voltage V .

✗ (d) In which part of its range, is the device as an electrostatic voltmeter more accurate?

43. Two small balls of masses m_1 and m_2 having like charges are connected with each other by an insulating light thread of length l . Initially, the balls are at rest and tensile force in the thread is T . After the thread is cut, the balls move apart. Determine distance between the balls, when momentum of the first ball becomes p . Ignore effects of all forces other than the electrostatic forces between the balls.



Force:

$$\frac{kq_1q_2}{l^2} = T \quad (1)$$

Energy: $-\Delta U = \Delta KE$

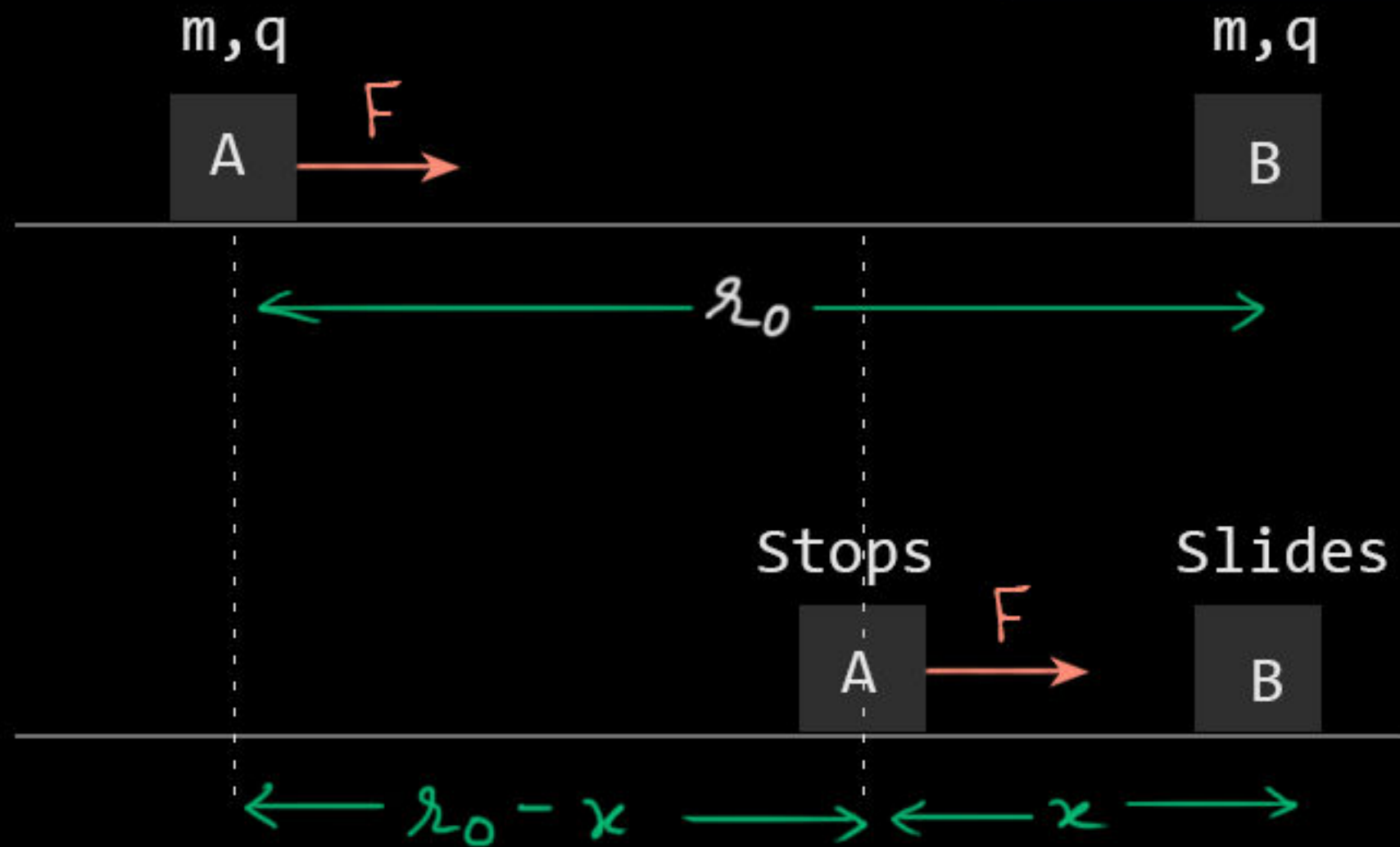
$$\Rightarrow \frac{kq_1q_2}{l} - \frac{kq_1q_2}{x} = \frac{p^2}{2m_1} + \frac{p^2}{2m_2} \quad (2)$$

From 1,2:

$$x = \frac{l^2}{l - \frac{p^2}{2T} \left(\frac{m_1 + m_2}{m_1 m_2} \right)} //$$

To use minimum force, A must stop as B just starts to slide.

44. Two particles each of mass $m = 50 \text{ g}$ and charge $q = 10 \mu\text{C}$ are at rest on a horizontal plane a distance $r_0 = 10 \text{ m}$ apart. Coefficient of friction between the particles and the plane is $\mu = 0.2$. What is the minimum constant horizontal force that must be applied to one of the particles so that the other one starts sliding?



Force: $\frac{kq^2}{x^2} = \mu mg$ (1)

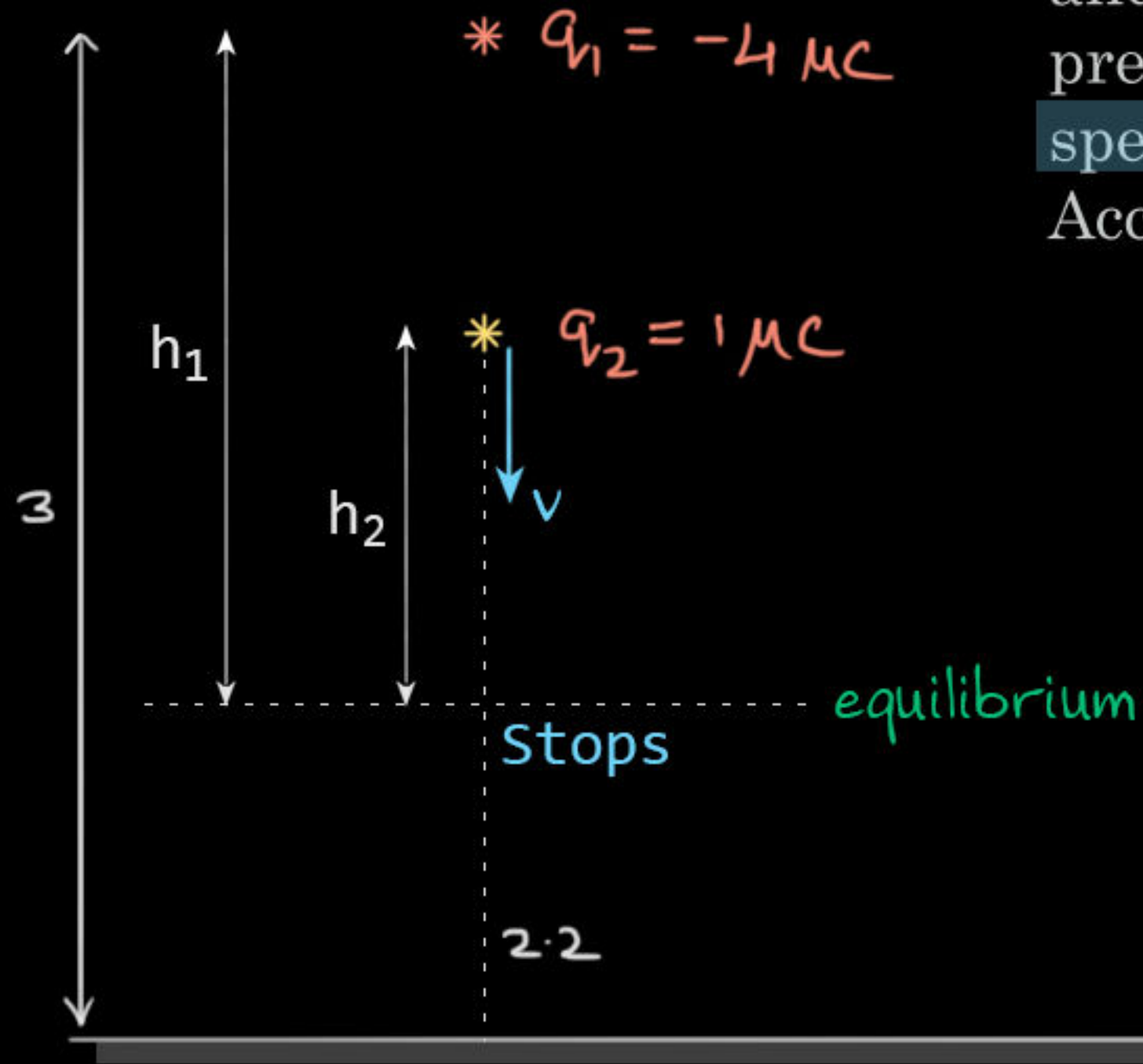
Energy: $W_{\text{ext}} = \Delta U + \Delta KE$

$$\Rightarrow F(r_0 - x) - \mu mg(r_0 - x) = \left[\frac{kq^2}{x} - \frac{kq^2}{r_0} \right]$$

$$\Rightarrow F - \mu mg = \frac{kq^2}{xr_0}$$
 (2)

From 1,2: $F = \frac{kq}{r_0} \sqrt{\frac{\mu mg}{k}} + \mu mg //$

45. A particle of charge $-4.0 \mu\text{C}$ is fixed at a height 3.0 m above the ground and another particle of mass 0.9 g and charge $1.0 \mu\text{C}$ is held under the previous particle at a height 2.2 m above the ground. What minimum speed must be given to the second particle so that it reaches the ground? Acceleration of free fall is 10 m/s^2 .



It doesn't matter where the ground is, if it's below the equilibrium point.

In all such cases, charge must be given a minimum velocity such that it just reaches the equilibrium with zero velocity.

Taking magnitudes of q_1 and q_2 in equations.

$$\text{Force: } mg = \frac{kq_1q_2}{h_1^2} \Rightarrow h_1 = 2 \quad (1) < 3$$

$\therefore$ equilibrium point is above ground

$$\text{Energy: } E_i = E_f$$

$$\Rightarrow \frac{1}{2}mv^2 + \left(-\frac{kq_1q_2}{h_1 - h_2} \right) + mgh_2 = -\frac{kq_1q_2}{h_1} \quad (2)$$

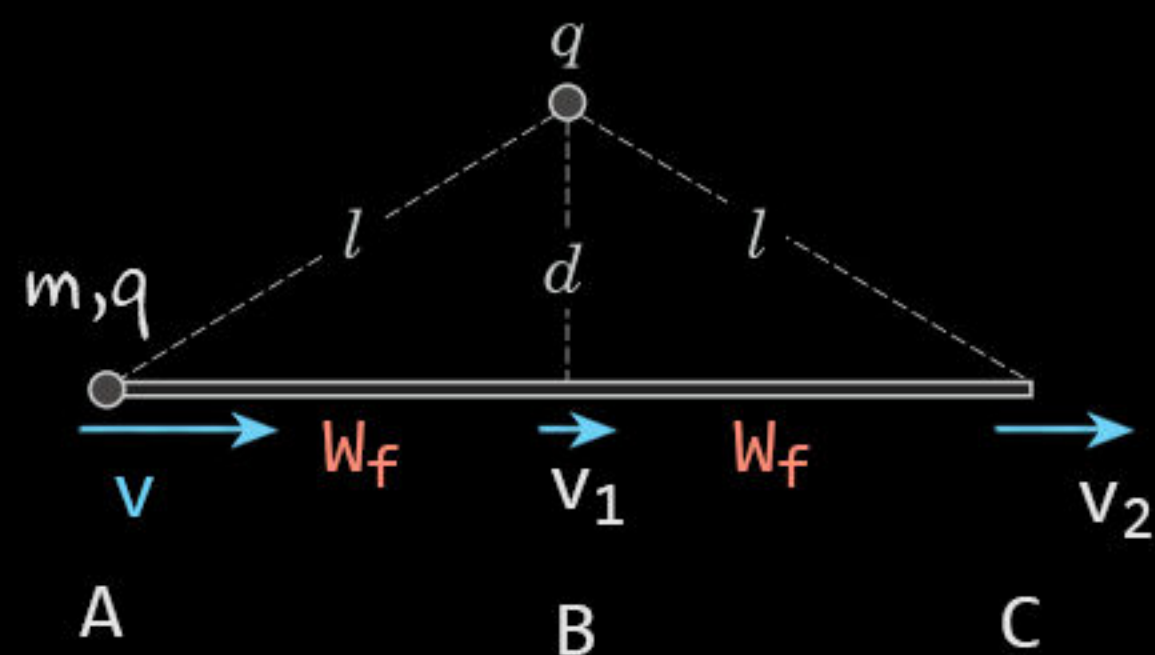
$$q_1 = 4, q_2 = 1$$

$$h_1 = 2, h_1 - h_2 = 3 - 2.2 = 0.8, h_2 = h_1 - 0.8 = 1.2$$

$$\text{From 1,2: } v = \sqrt{\frac{2}{m} \frac{kq_1q_2 h_2}{(h_1 - h_2)h_1} - 2gh_2} = \sqrt{60 - 24}$$

$$= 6$$

Youtube / Arihant Senior



46. A point like positive charge q is fixed at a distance d on the perpendicular bisector of thin electrically neutral nonconducting rod. Ends of the rod are at distance l from the point charge. A bead of mass m having the same charge q is threaded on the rod. The bead is projected from one end of the rod towards the other end. Velocity of the bead at the centre of the rod is observed to be v_1 and at the other end v_2 . If coefficient of friction between the bead and the rod is uniform everywhere, find velocity of projection of the bead.

Work done by friction in path AB is same as in path BC.

As $W_f = \int N_x dx$ is same

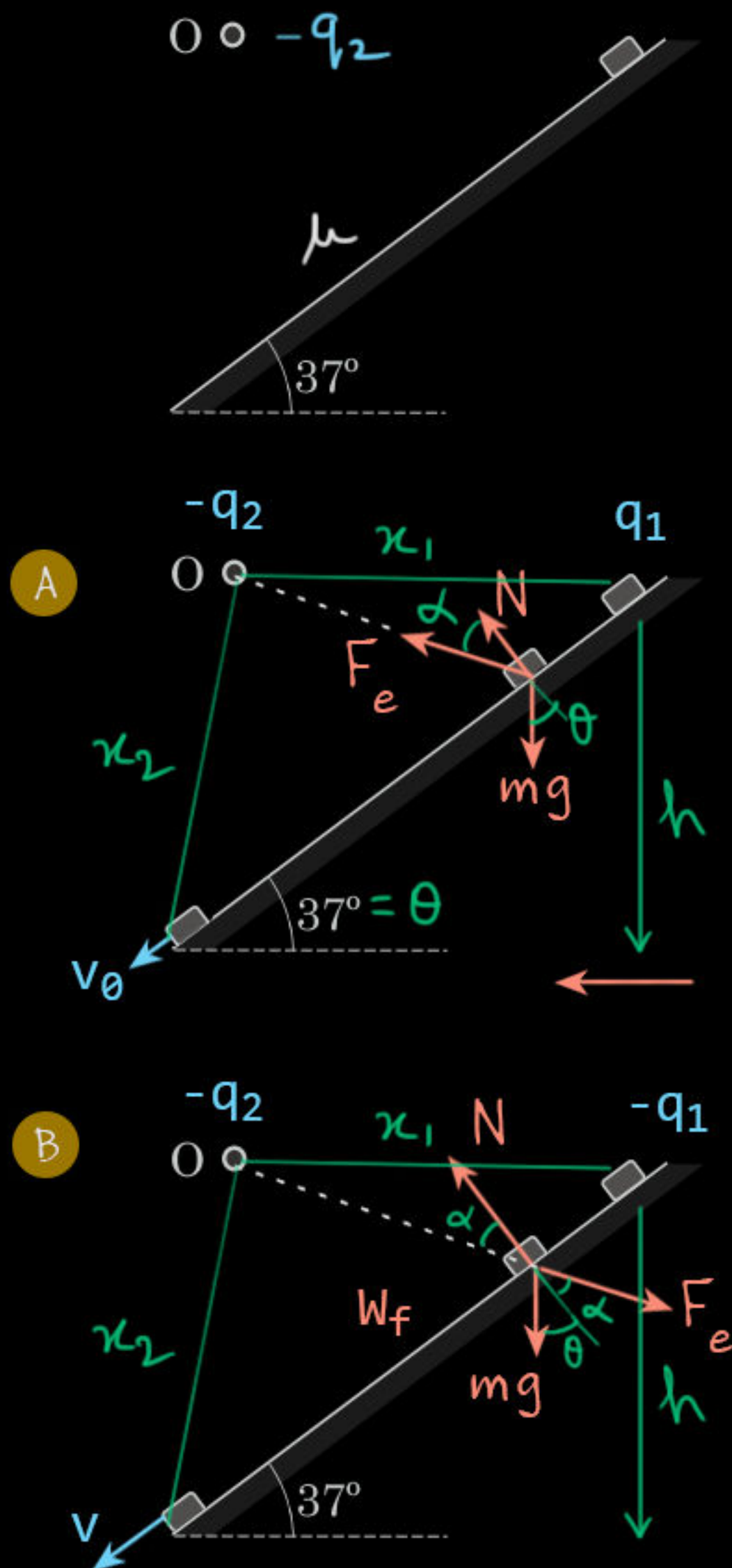
$$W_f = \Delta U + \Delta K$$

$$\text{AB: } W_f = kq^2 \left(\frac{1}{d} - \frac{1}{l} \right) + \frac{1}{2} m (v_1^2 - v^2)$$

$$\text{BC: } W_f = kq^2 \left(\frac{1}{l} - \frac{1}{d} \right) + \frac{1}{2} m (v_2^2 - v_1^2)$$

Subtracting: $v = \sqrt{\frac{4kq^2}{m} \left(\frac{1}{d} - \frac{1}{l} \right) + 2v_1^2 - v_2^2}$

47. A small block carrying a positive charge q_1 is held at rest on an inclined plane. A negative point charge q_2 is fixed somewhere at point O in the vertical plane containing the block. Coefficient of friction between the block and the inclined plane is $\mu = 0.375$. When released, the block acquires a speed $v_0 = 3.0$ m/s after sliding down a height $h = 1.25$ m. If the block has negative charge of the same magnitude (i.e. $-q_1$), what speed will it acquire after sliding down from rest between the same initial and final points. Acceleration due to gravity is $g = 10$ m/s².



$$W_e + W_g + W_f = \Delta KE$$

$$\downarrow = -\Delta U_e$$

Vertical height

$$A - \left[k(-q_2)q_1 \left(\frac{1}{x_2} - \frac{1}{x_1} \right) \right] + mgh - \mu \int (mg \cos \theta - f_e \cos \alpha) dx = \frac{1}{2} m v_0^2$$

$$B - \left[k(-q_2)(-q_1) \left(\frac{1}{x_2} - \frac{1}{x_1} \right) \right] + mgh - \mu \int (mg \cos \theta + f_e \cos \alpha) dx = \frac{1}{2} m v^2$$

$$\text{add: } 2mgh - 2\mu \int mg \cos \theta dx = \frac{1}{2} m (v_0^2 + v^2)$$

$$\Rightarrow 2mgh - 2\mu mg \cos \theta \int dx = \frac{1}{2} m (v_0^2 + v^2)$$

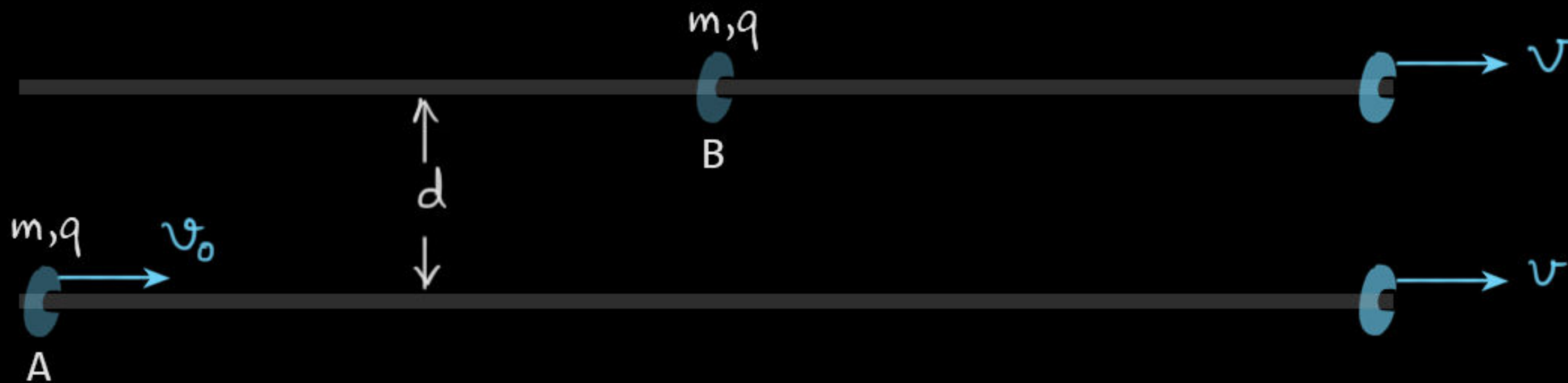
$$\Rightarrow v = \sqrt{4gh(1 - \mu \cot \theta) - v_0^2}$$

$h/\sin \theta$

by Ankit Singhvi

Youtube / Arihant Senior

48. Two identical beads each of mass m and charge q can slide on two long parallel insulating frictionless rigid rods that are a distance d apart. Initially the beads are so far from each other that no electrostatic interaction exists between them. With what speed must one of the beads be projected so that it passes over the other bead? Assume radii of the beads negligibly small as compared to the distance between the rods. Neglect effects of gravity.



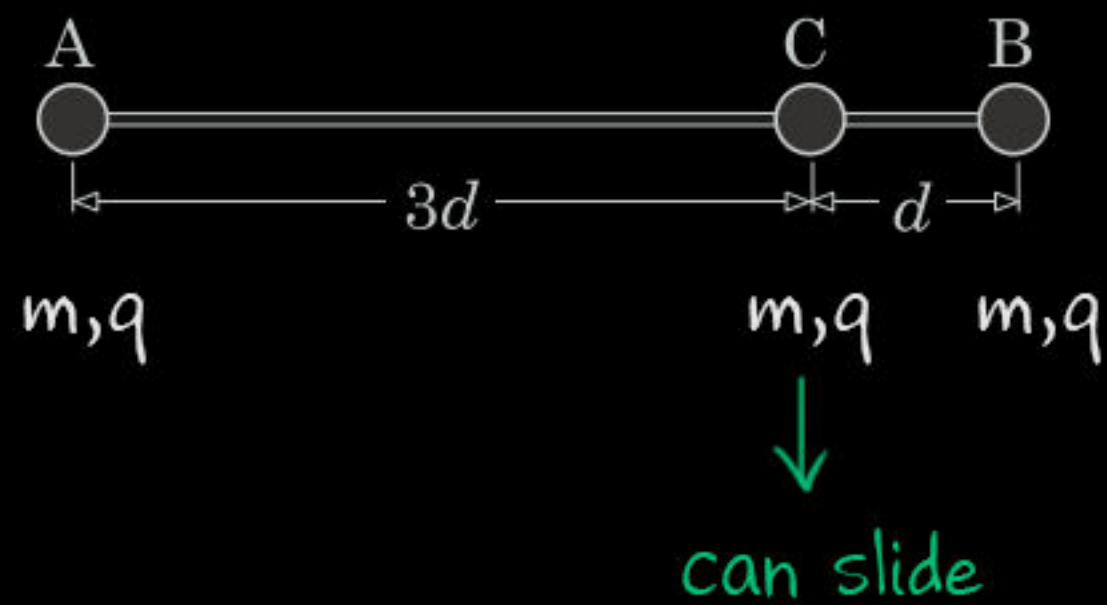
Eventually, let A and B will move together with same velocity v .

v_0 be the minimum velocity required.

$$\text{Momentum: } mv_0 = 2mv$$

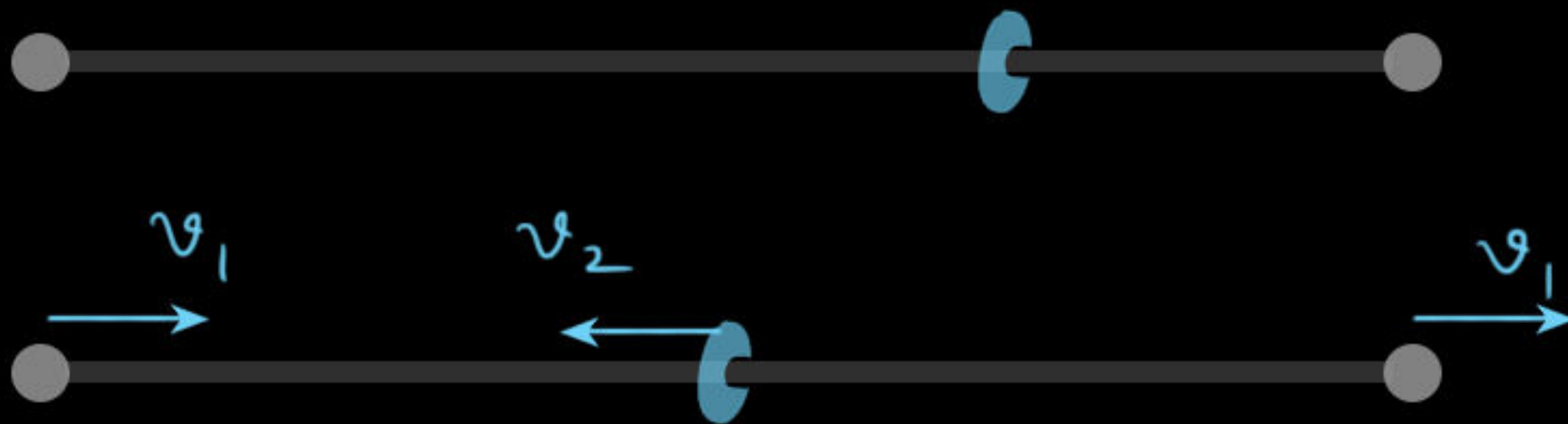
$$\text{Energy: } \frac{1}{2}mv_0^2 = 2\left(\frac{1}{2}mv^2\right) + \frac{kq^2}{d}$$

$$\text{From 1,2: } v_0 = 2q\sqrt{\frac{k}{md}}$$



v_C is maximum, when it crosses the center of rod.

49. Two small non-conducting balls A and B are rigidly attached at the ends of a light frictionless non-conducting rod. A third non-conducting ball C can slide on the rod like a threaded bead between the balls A and B. All the three balls are of equal mass m and have equal charges q . Initially the balls are held at rest in free space with centre-to-centre spacing as shown in the figure. All the balls are simultaneously released. Find maximum speed of the ball C in subsequent motion. Neglect radii of the balls as compared to the distance d .



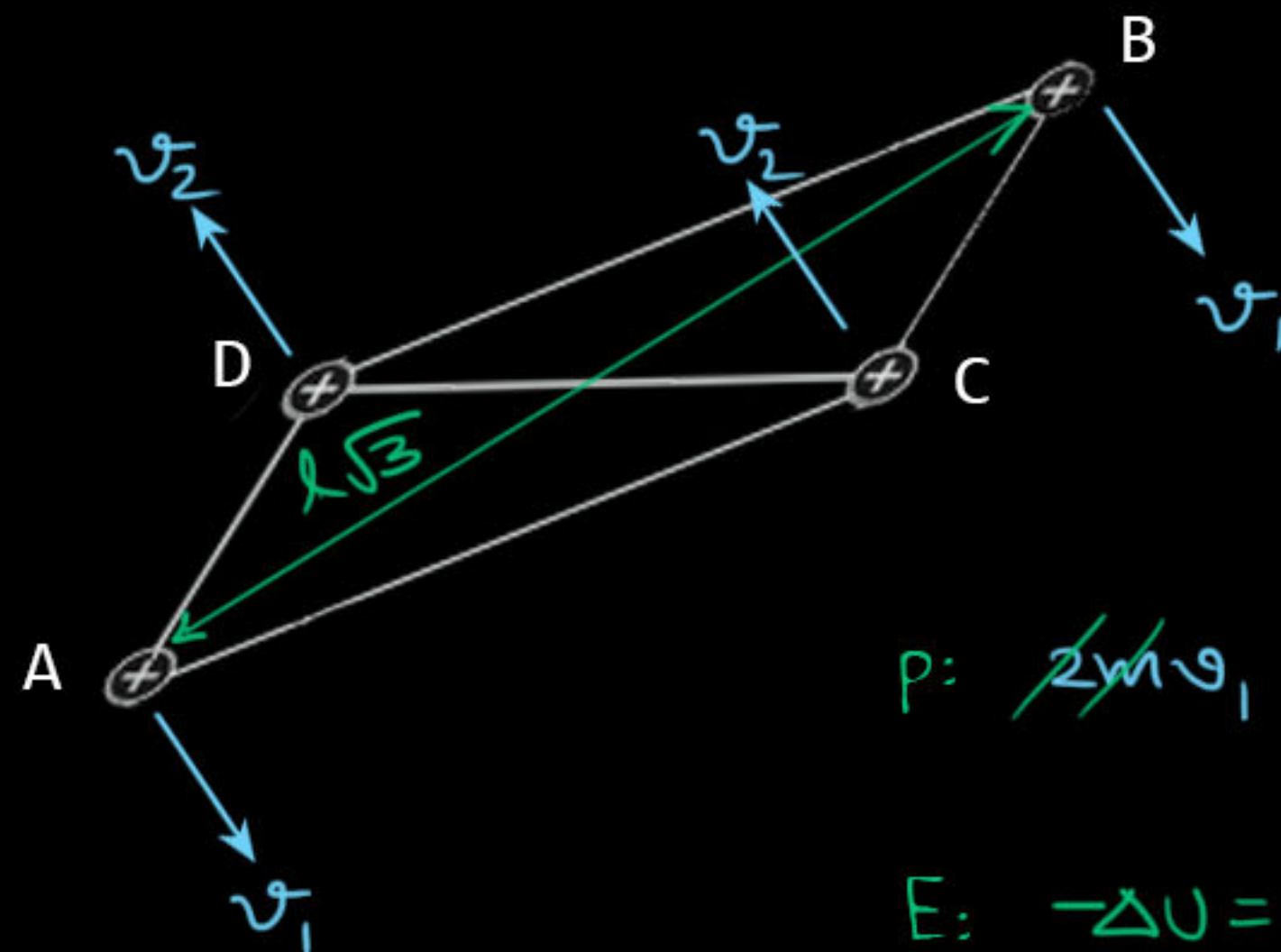
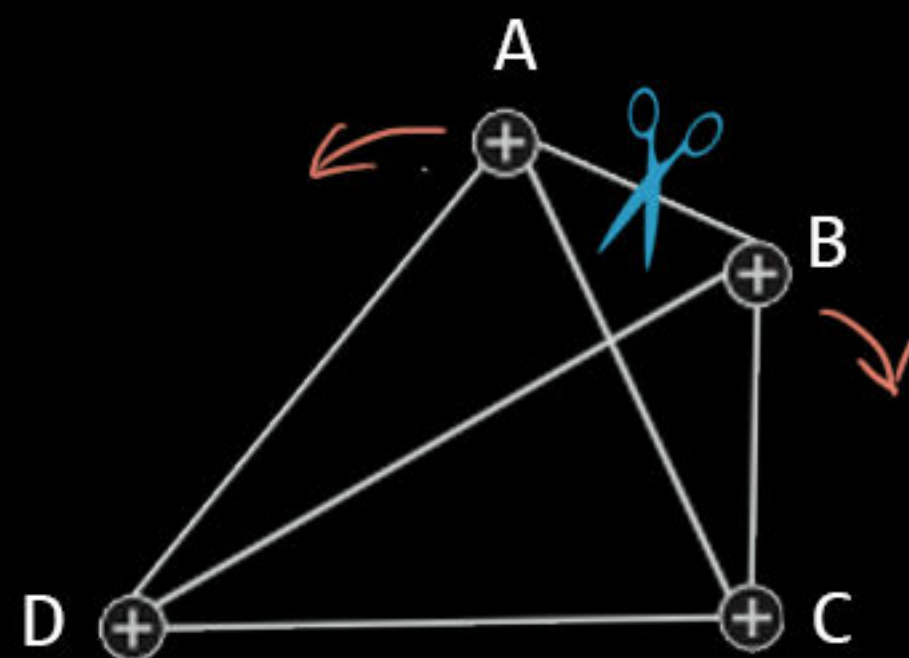
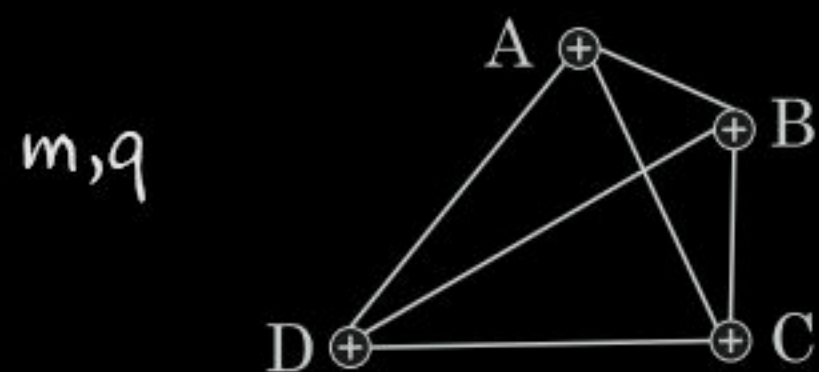
P: $m v_2 = 2 m v_1$ (1)

E: $\left(\frac{kq^2}{3d} + \frac{kq^2}{d} \right) - 2 \left(\frac{kq^2}{2d} \right) = \frac{1}{2} m v_2^2 + 2 \left(\frac{1}{2} m v_1^2 \right)$ (2)

From 1,2:

$$v_2 = \sqrt{\frac{4kq^2}{9dm}}$$

Youtube / Arihant Senior



At maximum kinetic energy of the system, A and B will be at extremes, as potential energy of system will be minimum then.

50. Four identical particles A, B, C and D each of mass m and charge q are connected by insulating light inextensible threads of equal length l to make a tetrahedron as shown in the figure.

The thread connecting particles A and B is cut. Find maximum speed acquired by every particle in subsequent motion.

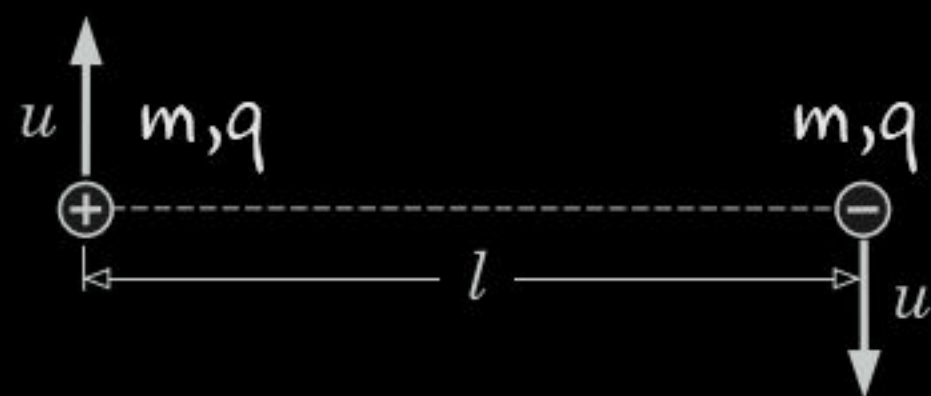
$$P: \cancel{2mv_1} = \cancel{2mv_2} \Rightarrow v_1 = v_2$$

$$E: -\Delta U = \Delta KE$$

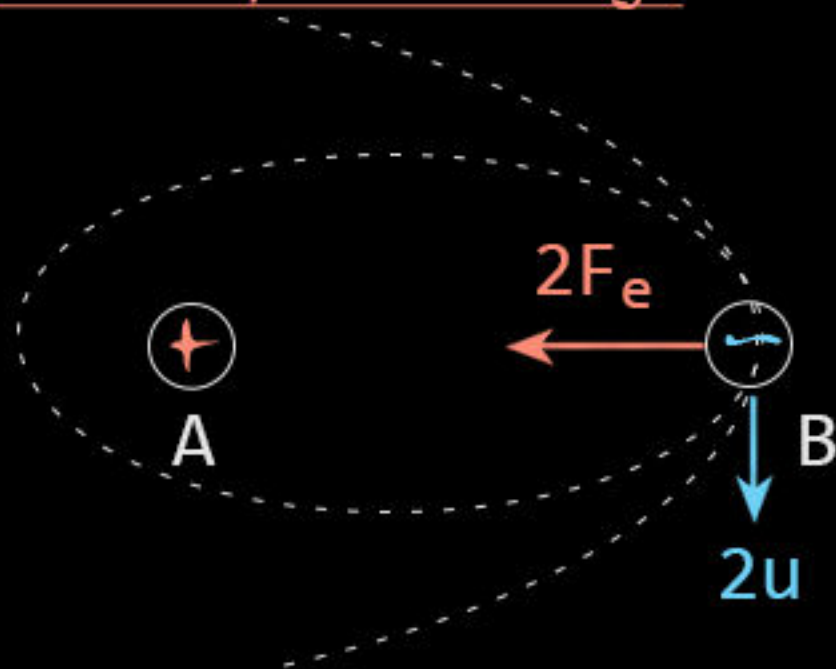
$$\Rightarrow \frac{\frac{kq^2}{l}}{\sqrt{3}} - \frac{kq^2}{l} = 2\left(\frac{1}{2}mv_1^2\right) + 2\left(\frac{1}{2}mv_2^2\right)$$

$$\Rightarrow v_1 = v_2 = v = \sqrt{\frac{\frac{kq^2}{2ml} \left(1 - \frac{1}{\sqrt{3}}\right)}{1}}$$

Youtube / Arihant Senior



W.r.t first charge



- A** Reasoning 1: B shall go in elliptical or parabolic orbit. Since we have point charges they will therefore never collide, unless $u=0$.

Reasoning 2: If B collides, its angular momentum about A becomes zero. Which means its initial angular momentum has to be zero too. Its possible only if $u=0$.

51. Two particles each of mass m and charges $+q$ and $-q$ separated by a distance l are given initial velocities u perpendicular to the line joining them as shown in the figure. Neglect gravitational and magnetic forces between the particles in comparison to Coulombian forces between them.

- Under what conditions will the masses eventually collide?
- Under what conditions will the masses follow circular orbits of diameter l ?
- Under what conditions will the masses follow closed orbits?
- X** What is the minimum distance between the masses in their subsequent motion?

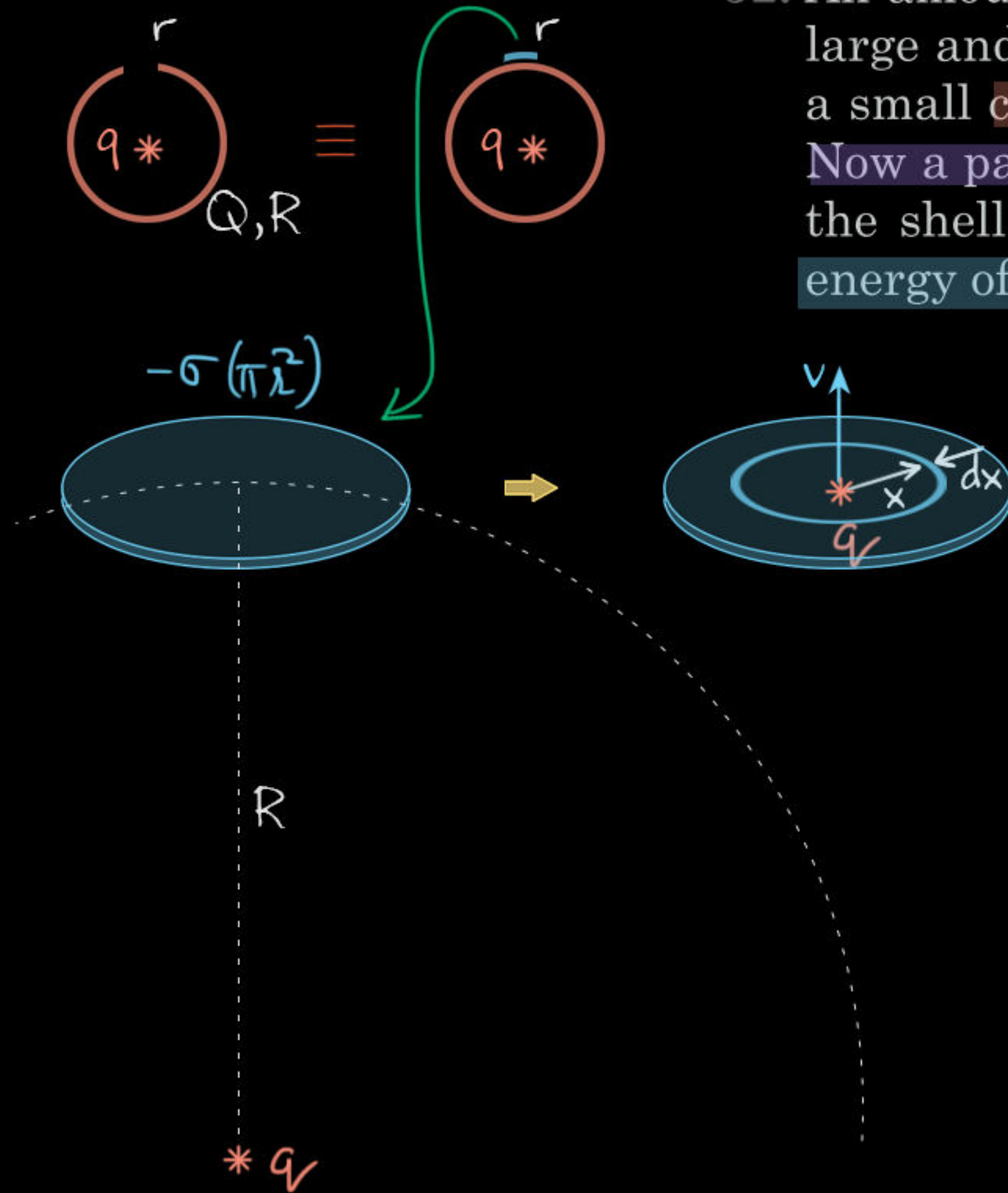
B For circular orbit: $a = \frac{v^2}{r}$

$$\Rightarrow \frac{2 \frac{kq^2}{l^2}}{m} = \frac{(2u)^2}{l} \Rightarrow \frac{kq^2}{m u^2 l} = 2 //$$

C For closed orbits: $E < 0$

$$\Rightarrow 2 \left(\frac{1}{2} m u^2 \right) + \frac{kq(-q)}{l} < 0 \Rightarrow \frac{kq^2}{m l u^2} > 1 //$$

52. An amount Q of positive charge is sprinkled uniformly on surface of a large and massive non-conducting spherical shell of radius R and then a small circular portion of radius r ($r \ll R$) is removed from the shell. Now a particle carrying positive charge q is released from the centre of the shell. Considering only electrostatic interactions, find the kinetic energy of the charged particle when it comes out of the hole.



$$E_i = E_f$$

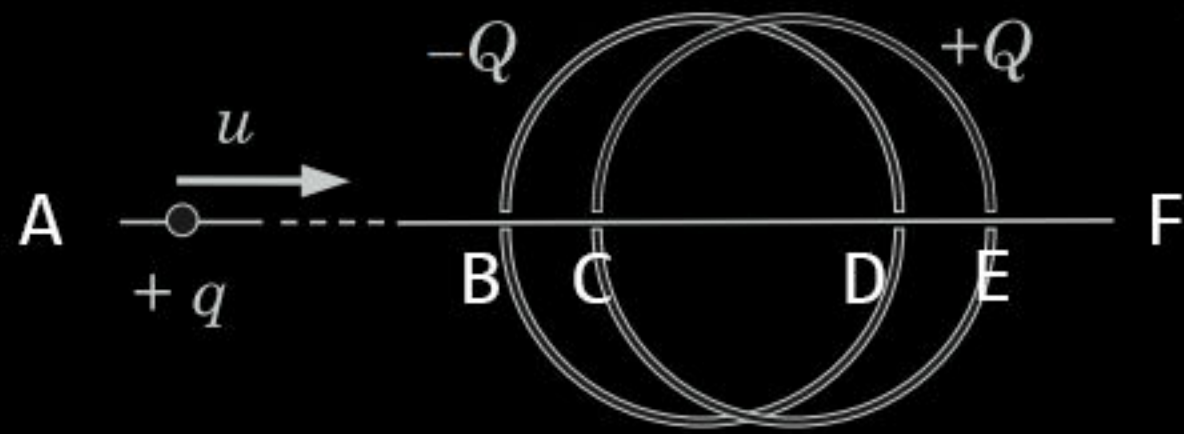
$$\Rightarrow \frac{kq(-\sigma\pi r^2)}{R} = \int_0^r \frac{kq(-\sigma 2\pi x dx)}{x} + \frac{1}{2}mv^2$$

$$\Rightarrow -\frac{kq\sigma\pi r^2}{R} = -kq\sigma 2\pi r + \frac{1}{2}mv^2$$

$$\Rightarrow KE = \frac{1}{2}mv^2 = kq\sigma\pi r \left[2 - \frac{r}{R} \right] \approx 0$$

$$\frac{1}{4\pi\epsilon_0} \quad \frac{Q}{4\pi R^2}$$

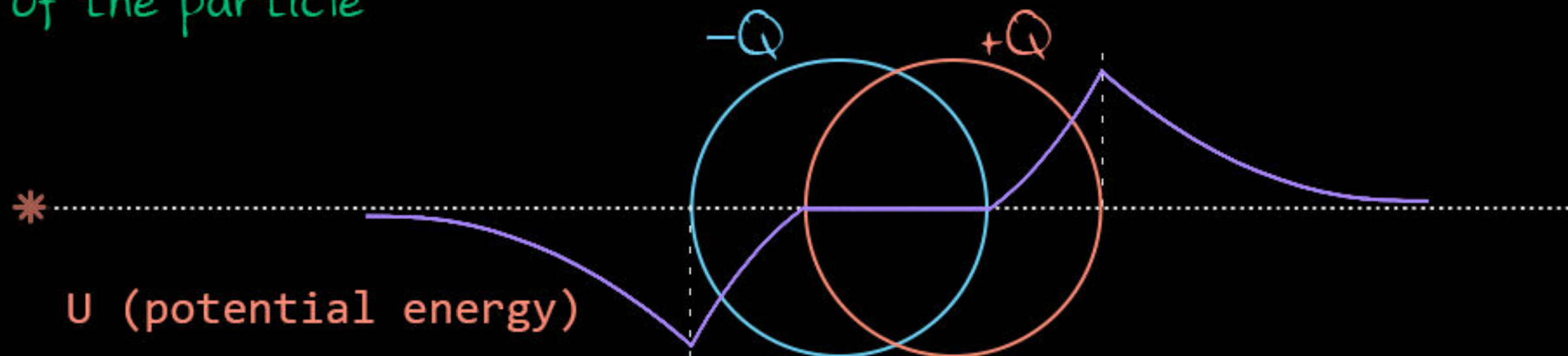
$$= \frac{qQr}{8\pi\epsilon_0 R^2}$$



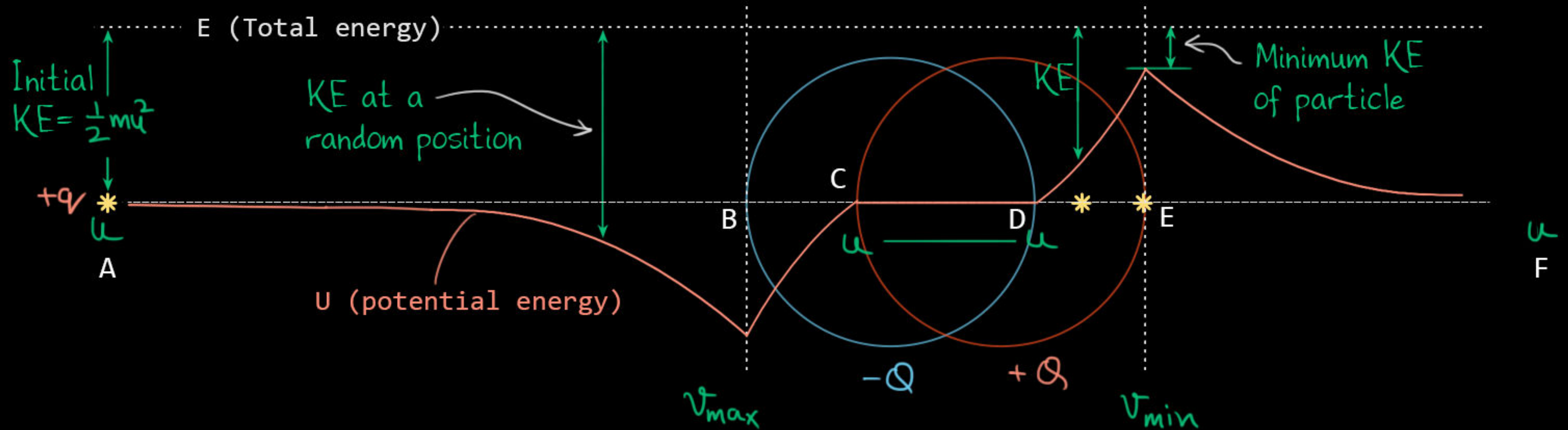
53. Two identical dielectric spherical shells each of radius R carrying uniformly distributed charges $+Q$ and $-Q$ are fixed intersecting each other with their centres at separation $x_0 < R$. A straight thin uncharged insulating rod is fixed along the line joining centres of the shells, on which a small bead of mass m carrying a positive charge q can slide without friction. The holes made in the shells to pass the rod and the bead are so small that their effect on the electric field of the charges of the shells can be ignored. The rod is made of such a material that no charge accumulates on it due to induction or polarization. The bead is projected with an initial velocity u towards the shells from a point that is far away from the shells as shown in the figure. If the bead stops sliding on the rod, where must it stop and what must be the range of its initial velocity u ?

To specify your answer, assume centre of the positively charged dielectric sphere at the origin of a coordinate system whose x -axis coincides with the rod.

Plotting U for all positions of the particle

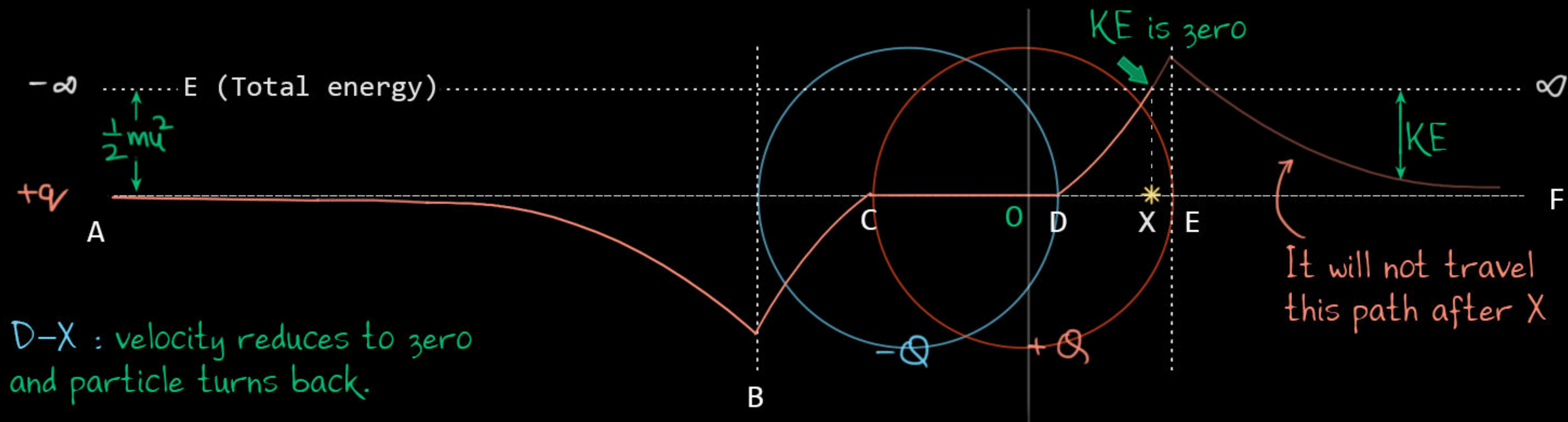


When it passes through: $KE_{\max}$ at B. $KE_{\min}$ at E



Till B: Velocity increases to a maximum.
B-C : velocity reduces back to u .
C-D : velocity remains u .
D-E : velocity reduces to a minimum.
E-infinity: velocity increases back to u .

When its unable to pass through It will stop at X (between D and E)

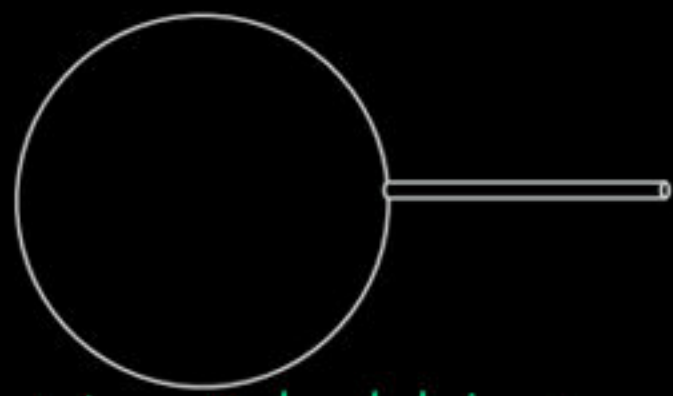


$\therefore$ As it can only stop between D and E. Condition for it to happen is: $KE_{-\infty} < U_E$

ie. $R-x_0 < x < R$

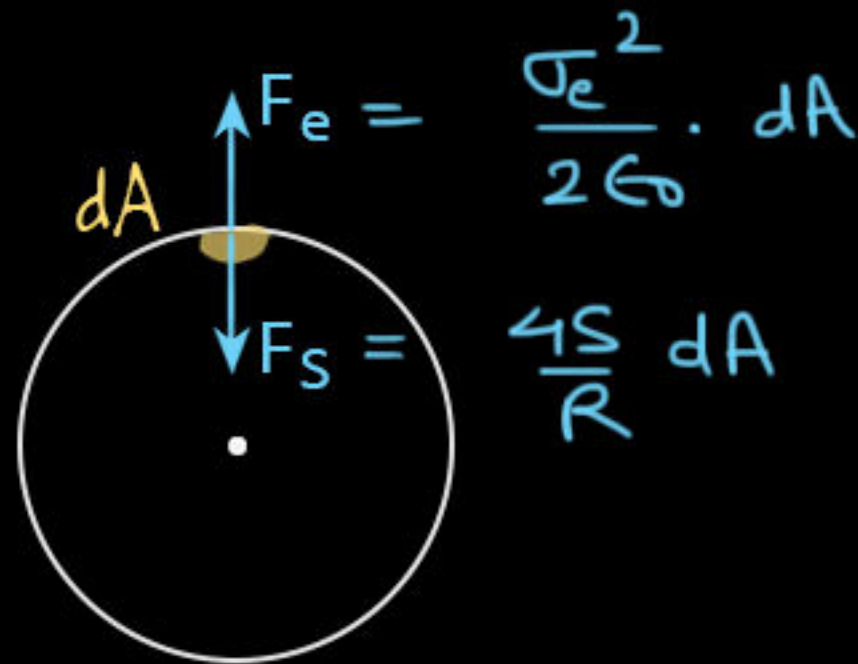
$$\Rightarrow \frac{1}{2}mu^2 < \frac{k(-Q)q}{(R+x_0)} + \frac{kQq}{R}$$

$$\Rightarrow u < \sqrt{\frac{2kQqx_0}{mR(R+x_0)}}$$



So inside of bubble is always at atmospheric pressure.

Now only forces left on a bubble element are due to surface tension and electrostatic force.

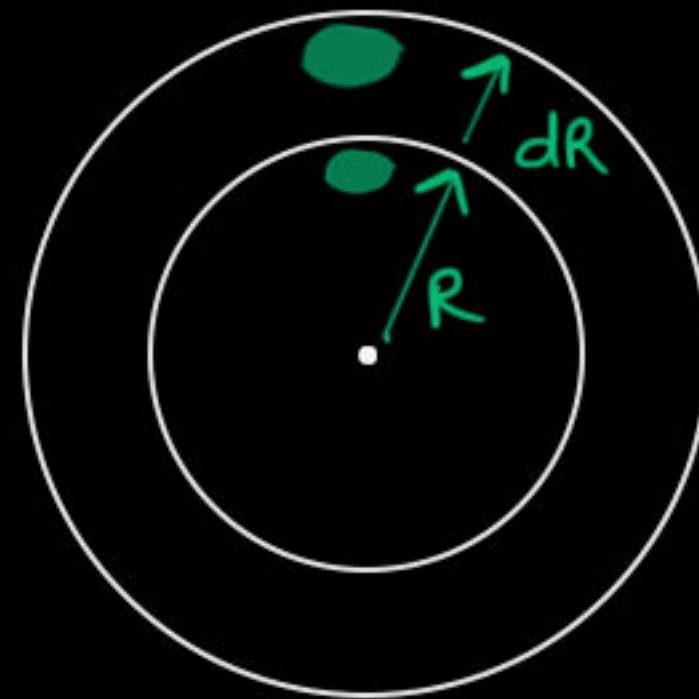


At equilibrium:

$$F_e = F_s \Rightarrow \frac{\sigma_e^2}{2\epsilon_0} = \frac{4S}{R} \quad (1)$$

54. A soap bubble of mass m (excluding air inside) blown with the help of a capillary tube is given an unknown amount of charge. Surface tension of the soap solution is σ . When the bubble acquires a stable radius, an additional small amount of air is blown into the bubble from the tube and then the tube is left open. Because of this operation, the soap bubble starts oscillations. If the soap bubble retains its spherical shape during the oscillations, find expression for period of these small amplitude oscillations.

$$\text{Restoring force} = F_s' - F_e'$$



$$= \frac{4S}{R+dR} dA - \left[\frac{Q}{4\pi(R+dR)^2} \right]^2 \frac{dA}{2\epsilon_0}$$

↓ From 1

$$\approx \frac{4S}{R} dA \left[\left(1 - \frac{dR}{R}\right) - \left(1 - \frac{dR}{R}\right)^4 \right]$$

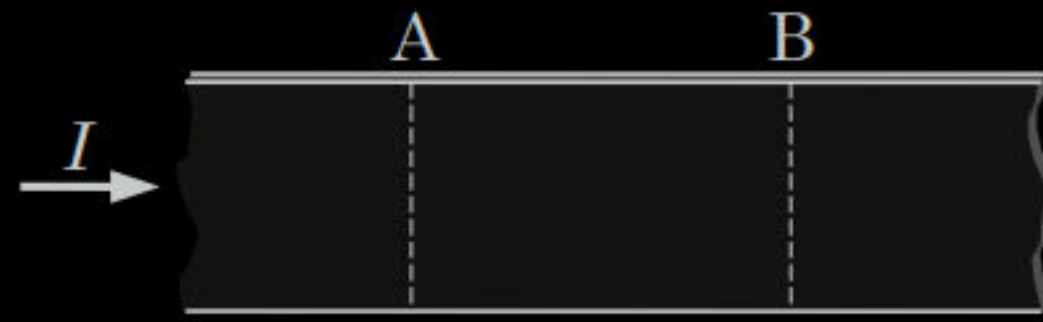
$$\approx \frac{4S}{R} dA \cdot 3 \frac{dR}{R} = \underbrace{\frac{12S dA}{R^2}}_{k \text{ of SHM}} dR$$

$$\frac{dA}{4\pi R^2} \cdot m$$

$$\therefore T = 2\pi \sqrt{\frac{dm}{k}} = \sqrt{\frac{4\pi^2 \cancel{dA} m}{\cancel{4\pi R^2} \cdot \frac{12S \cancel{dA}}{R^2}}} = \sqrt{\frac{m\pi}{12S}}$$

by Ankit Singhvi

Youtube / Arihant Senior



Concept: Difference in electric field at A and B will be caused by excess charge.

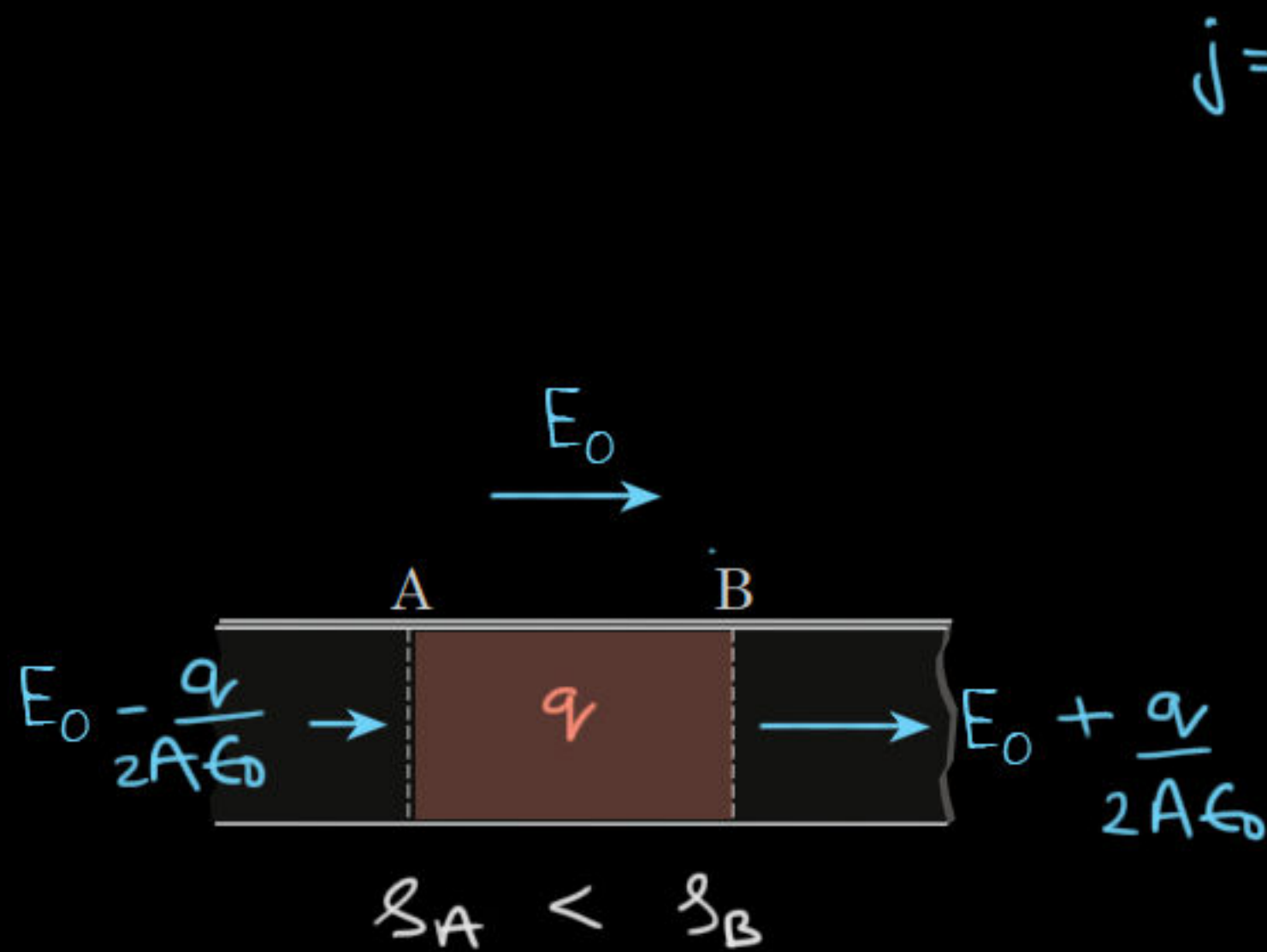
1. A constant current I flows in a strip-line conductor, resistivity of which increases monotonically in the direction of the current flow. If resistivities at cross-sections A and B are ρ_A and ρ_B , find excess charge accumulated in the section AB. Permittivity of free space is ϵ_0 and relative permittivity of material of the strip is ϵ_r everywhere in the strip.

✓(a) $\epsilon_0 (\rho_B - \rho_A) I$

(b) $\epsilon_0 \epsilon_r (\rho_B - \rho_A) I$

(c) $\epsilon_0 (\epsilon_r - 1) (\rho_B - \rho_A) I$

(d) $\epsilon_0 (\epsilon_r^2 - 1) (\rho_B - \rho_A) I$



$$j = \frac{I}{A} = \sigma E$$

$$E = \frac{I}{\sigma A} = \frac{\rho I}{A}$$

$$E_B - E_A = \frac{q}{A\epsilon_0}$$

$$\Rightarrow (\rho_B - \rho_A) \frac{I}{A} = \frac{q}{A\epsilon_0}$$

$$\Rightarrow q = \epsilon_0 I (\rho_B - \rho_A)$$

Coefficient of Aerial expansion is **twice** the coefficient of linear expansion

2. Under which of the following conditions, resistance of a cylindrical specimen of an Ohmic material is least affected by small temperature variations.

- (a) Temperature coefficient of resistivity is greater than that of linear expansion.
- (b) Temperature coefficient of resistivity is lesser than that of linear expansion.
- ✓ (c) Temperature coefficient of resistivity is equal to that of linear expansion.
- (d) Temperature coefficient of resistivity is negligible as compared to that of linear expansion.

$$R_1 = \frac{\rho l}{A}$$

$$R_2 = \rho(1 + \alpha_1 \Delta T) \cdot \frac{l(1 + \alpha_2 \Delta T)}{A(1 + 2\alpha_2 \Delta T)}$$

$$\approx \frac{\rho l}{A} (1 + \alpha_1 \Delta T) (1 + \alpha_2 \Delta T) (1 - 2\alpha_2 \Delta T)$$

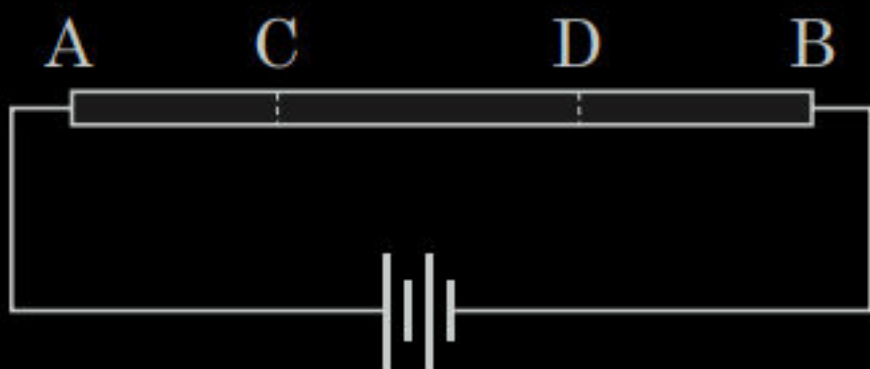
$$\approx \frac{\rho l}{A} (1 + \alpha_1 \Delta T + \alpha_2 \Delta T - 2\alpha_2 \Delta T + \underbrace{\dots}_{\text{Higher orders of } \Delta T})$$

$$\therefore \nexists R_1 \approx R_2$$

$$\Rightarrow \alpha_1 + \alpha_2 - 2\alpha_2 = 0$$

$$\Rightarrow \alpha_1 = \alpha_2 //$$

Youtube / Arihant Senior



v_d decreases with temperature

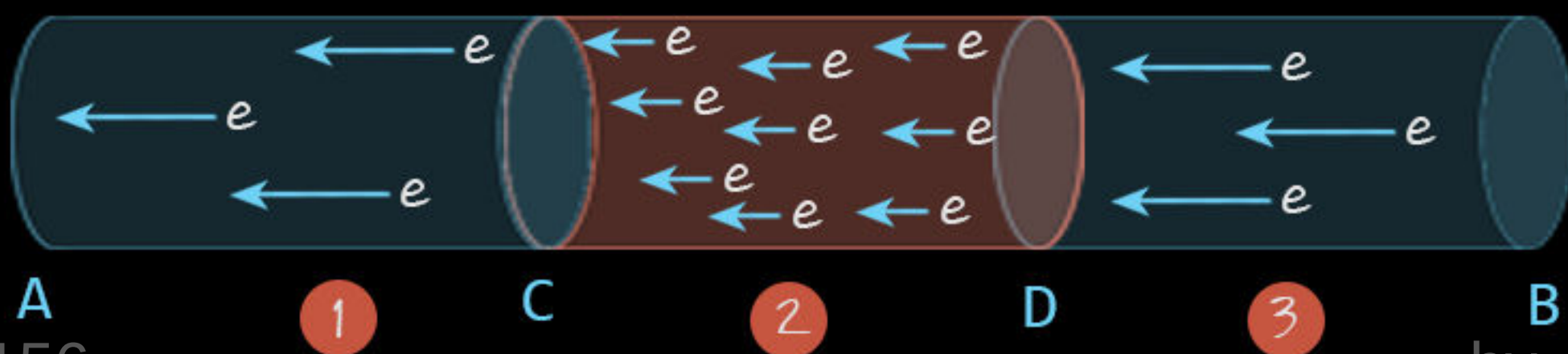
As n increases with temperature v_d will fall.

$$I = neAv_d$$

3. A homogeneous rod AB of uniform cross-section is connected across a battery. By some arrangement (not shown in the figure), the section CD of the rod is being uniformly heated. What can you conclude regarding temperature distribution in the rod?

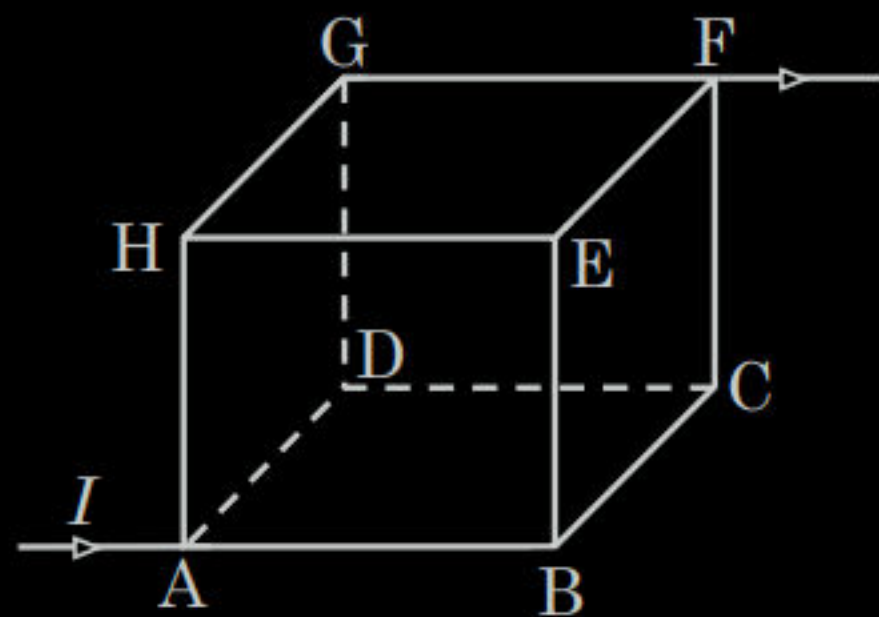
- (a) Temperatures in all the three sections will be uniform with the highest value in the section CD and equal lower values in the sections AC and DB.
- (b) Temperatures in all the three sections will be uniform with the highest value in the section AC, the lowest value in the section DB and an intermediate value in the section CD.
- (c) Temperatures in all the three sections will be uniform with the highest value in the section CD, the lowest value in the section DB and an intermediate value in the section AC.
- ✓ (d) Temperature distribution in all the three sections will be non-uniform with location of the highest temperature nearer to D in the section CD.

$$\therefore v_{d2} < v_{d1} \& v_{d3}$$



∴ At junction D, high speed electrons coming from B will loose KE to slow down.

As a result D will heat up even further.



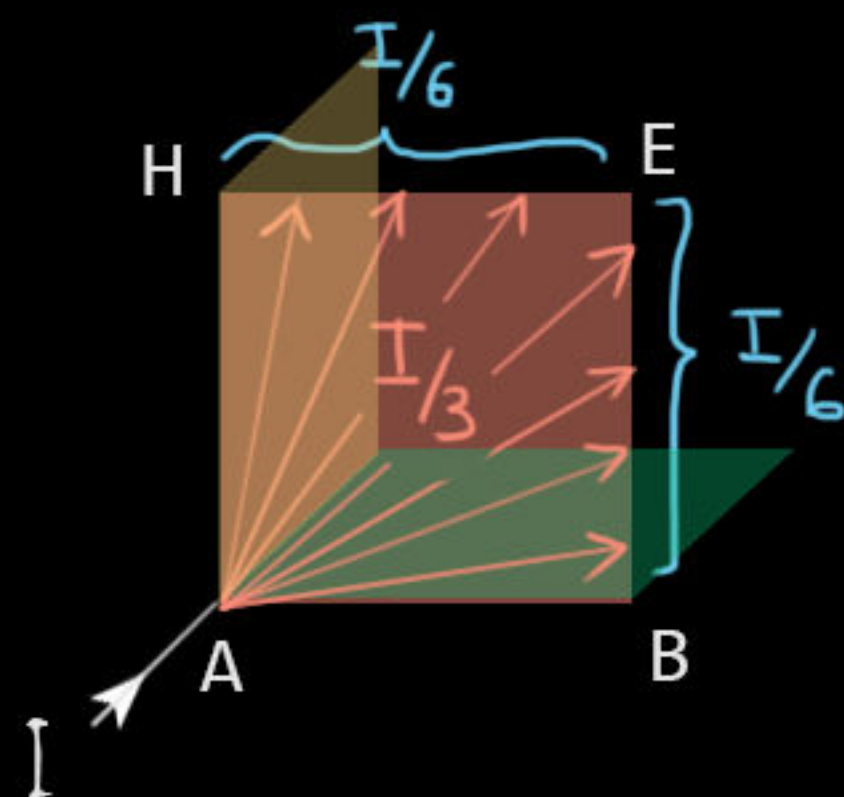
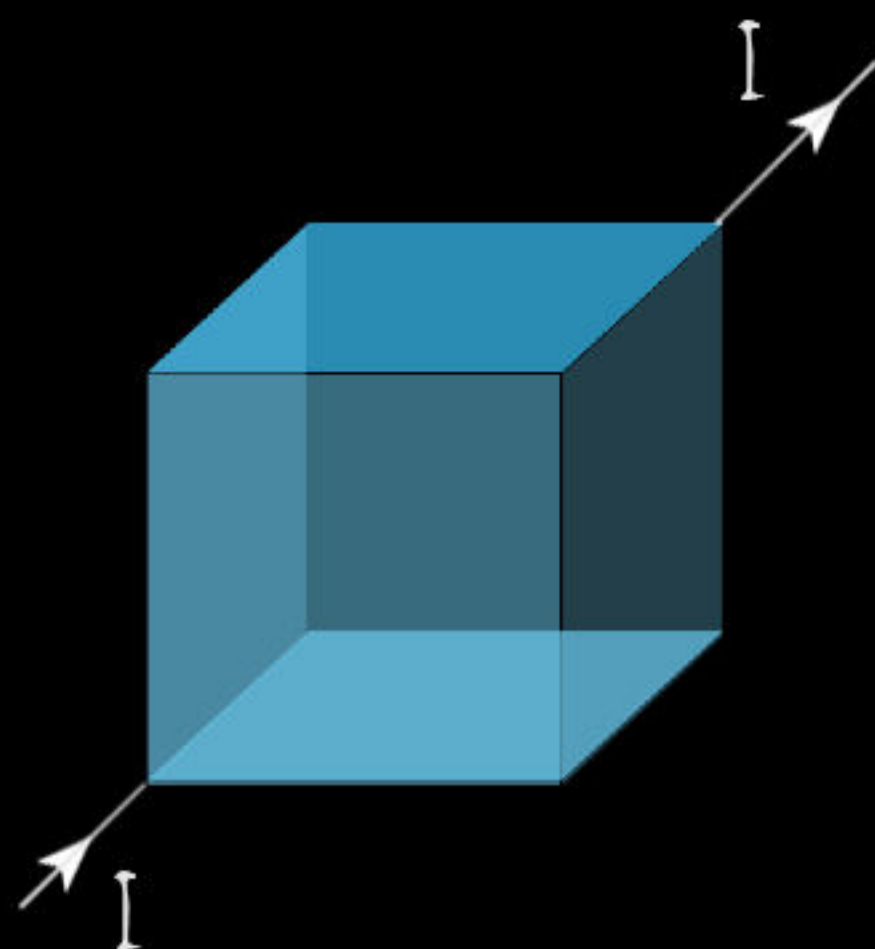
4. A closed cubical box is made by a thin conducting sheet. When its diagonally opposite corners are connected across a battery, it draw a current I from the battery. Find current flowing across the edge BE.

(a) $\frac{I}{3}$

✓ (b) $\frac{I}{6}$

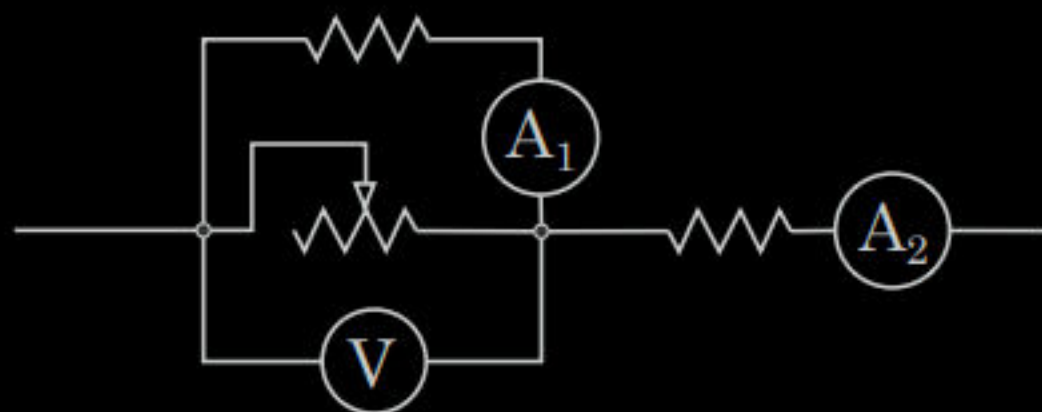
(c) $\frac{I}{9}$

(d) Between $\frac{I}{6}$ and $\frac{I}{3}$



From A, I gets divided in three symmetric faces equally as $I/3$

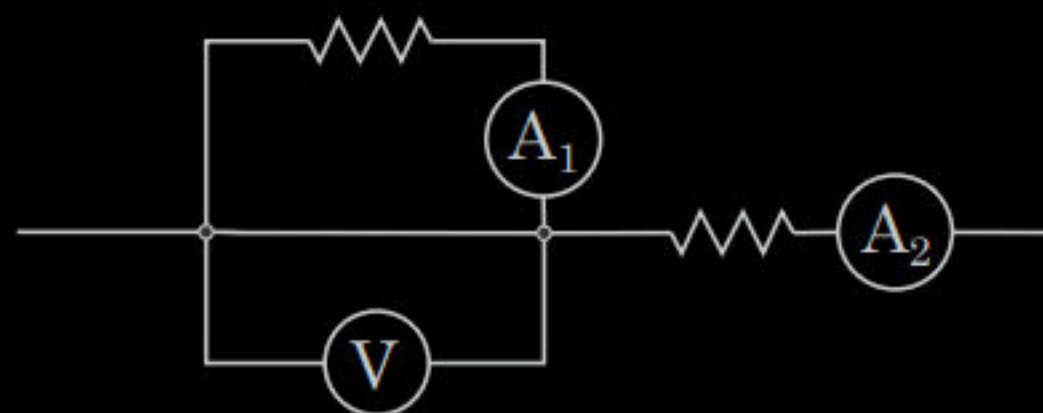
That $I/3$ crosses symmetric lines BE and EH equally as $I/6$.



5. In the figure is shown a part of a large network. If resistance of the rheostat is increased, how will the readings I_1 and I_2 of the ammeters A_1 and A_2 respectively and the reading of the voltmeter V change?

- (a) I_1 and I_2 will decrease and V increase.
- (b) I_1 will decrease and I_2 and V increase.
- ✓ (c) I_2 will decrease and I_1 and V increase.
- (d) It is sure that I_2 will decrease but nothing can be predicted about I_1 and V from the given information.

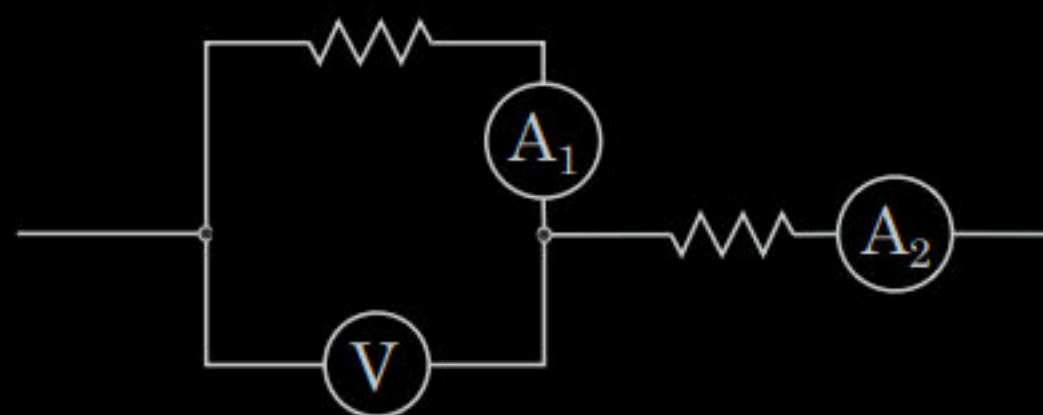
Hint: Take extreme cases.
That is, let's say resistance increases from zero to infinity.



$$V = 0$$

$$I_1 = 0$$

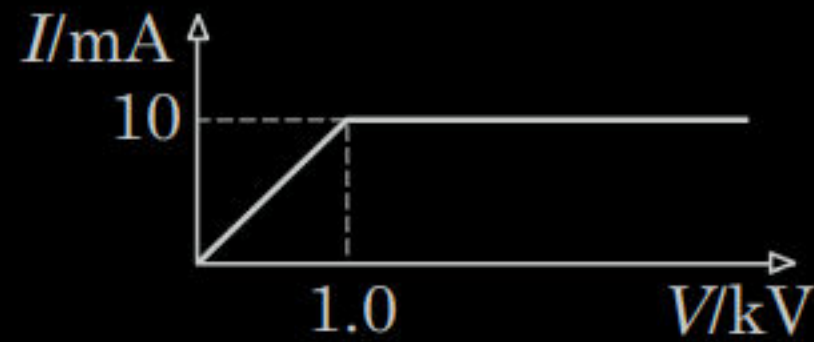
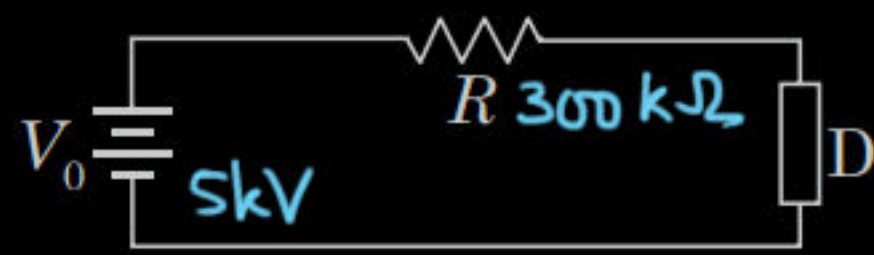
$$I_2$$



$$V \uparrow$$

$$I_1 \uparrow$$

$$I_2 \downarrow \text{ (As overall resistance increases)}$$

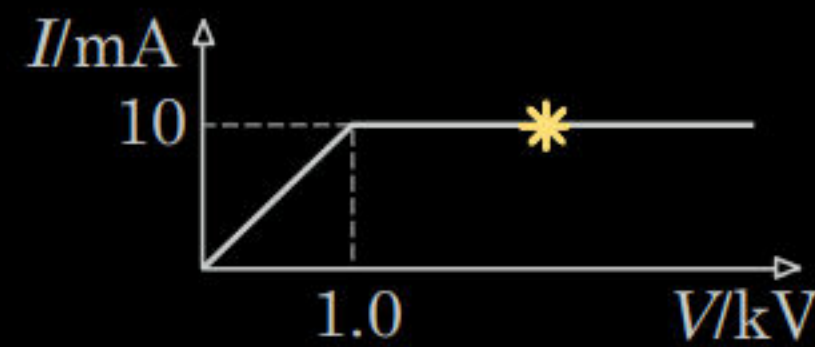


6. Volt-ampere (V-I) characteristics of an unknown device D connected in a circuit in series with a resistance and a battery is shown in the figure. If value of the resistance is $R = 300 \text{ k}\Omega$ and terminal voltage of the battery is $V_0 = 5 \text{ kV}$, find potential drop across the device.

(a) 1.0 kV
(c) 3.0 kV

✓(b) 2.0 kV
(d) 4.0 kV

Case A Lets say breakdown happened



$$\text{KVL: } 5000 - 10(300) - V = 0$$

$$\Rightarrow V = 2 \text{ kV} //$$

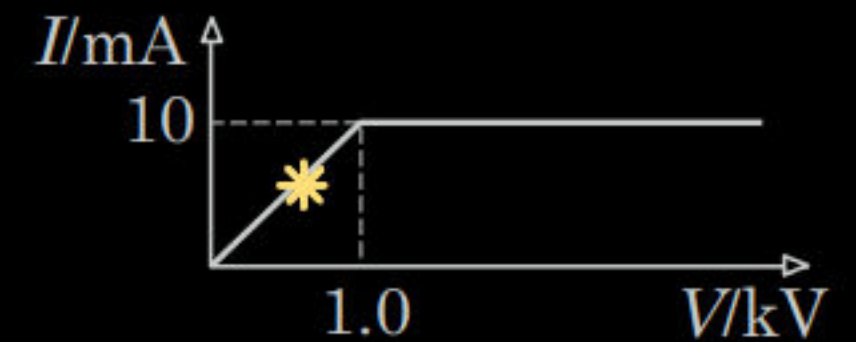
feasible!

Case B Lets say breakdown did NOT happen

For example at $V_0 = 3.5 \text{ kV}$

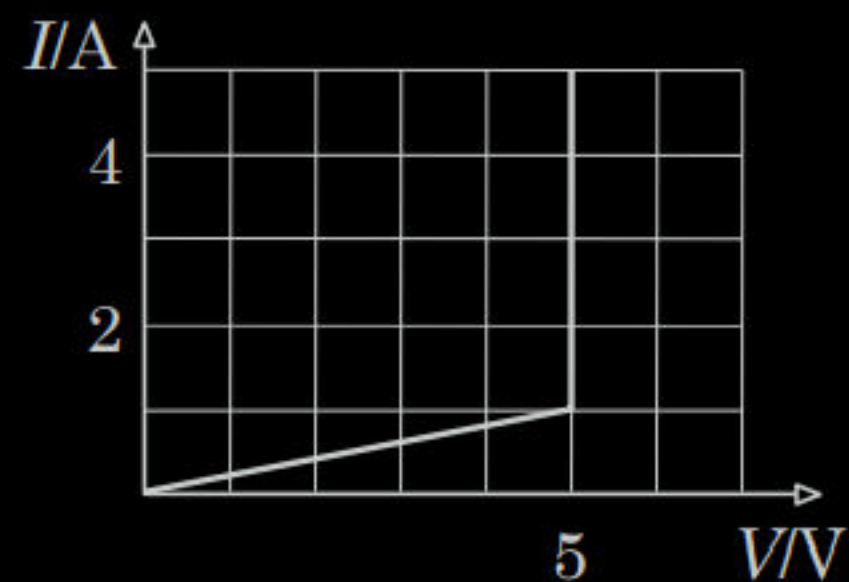
$$\text{KVL: } 3500 - 10(300) - V = 0 \Rightarrow V = 0.5 \text{ kV } \times$$

∴ Breakdown did NOT happen

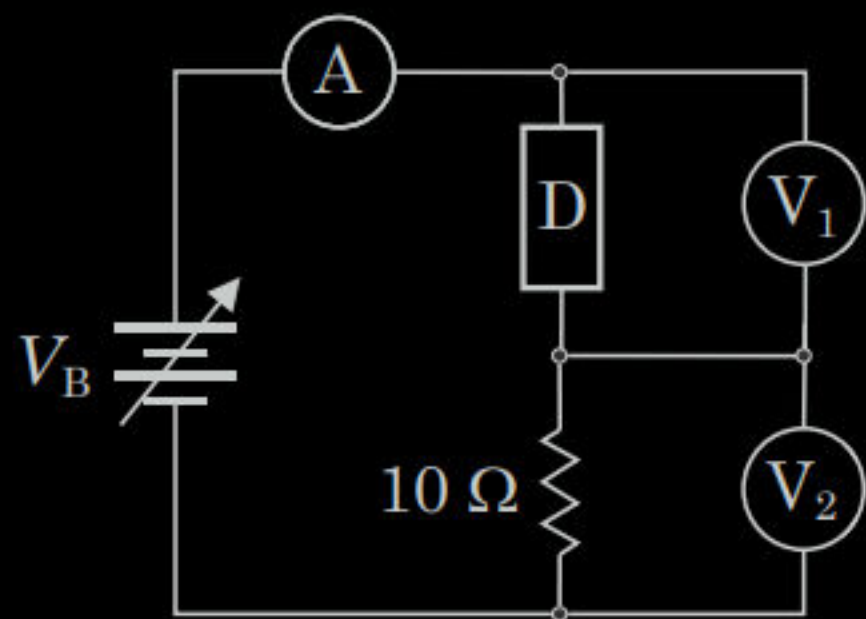


$$R_d = \frac{V}{I} = \frac{1 \text{ kV}}{10 \text{ mA}} = 100 \text{ k}\Omega$$

$$\therefore V = V_0 \cdot \frac{R_d}{R + R_d} = 5 \cdot \frac{100}{300 + 100} = 1.25 \text{ V} //$$



7. Volt-ampere (V - I) characteristic of a nonlinear device D and corresponding circuit used are shown in the adjoining figures. Voltage V_B of the battery is made to vary from zero to 30 V and voltage V_1 across the device D , voltage V_2 across the $10\ \Omega$ resistor and current I supplied by the battery are measured by ideal voltmeters V_1 and V_2 and an ideal ammeter A . Which of the following graph/graphs represent correct relationship?



Before breakdown, $R = 5\ \Omega$ ①

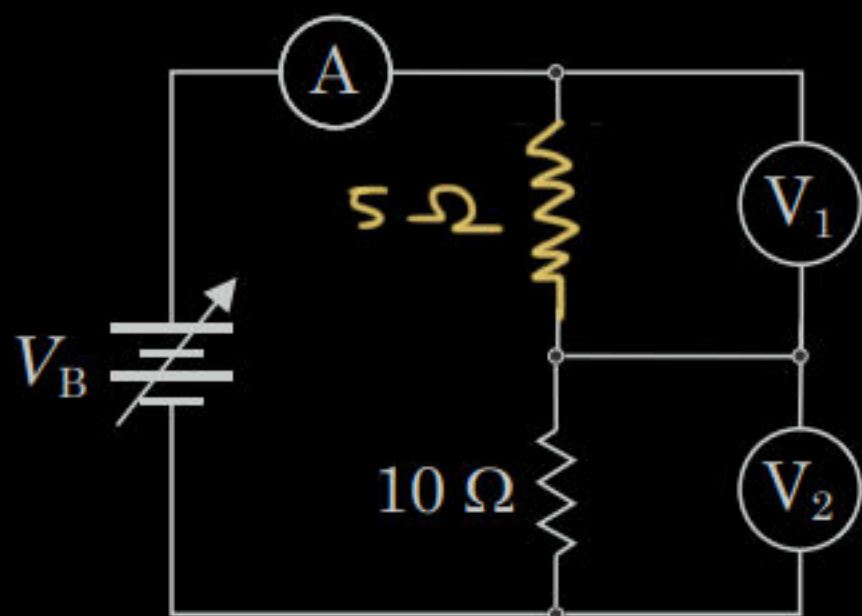


Breakdown occurs @ $V_B = 15\text{ V}$ ②

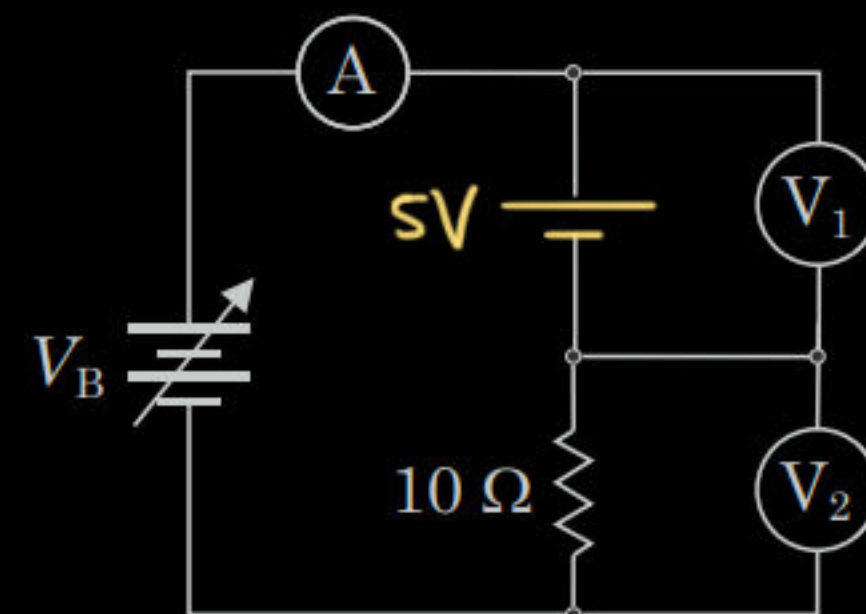
Also, after breakdown device becomes a battery of 5V. ③

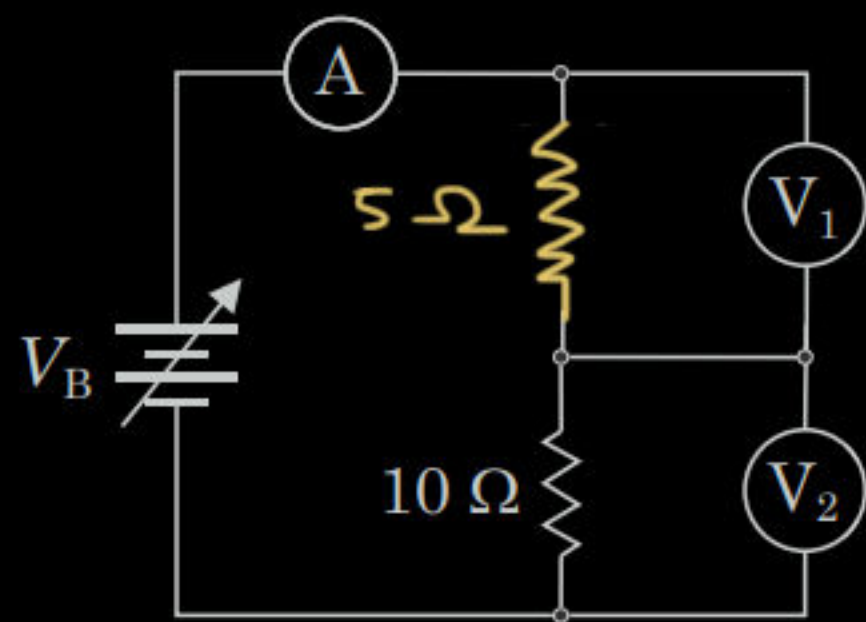
From 1,2,3:

Before breakdown:



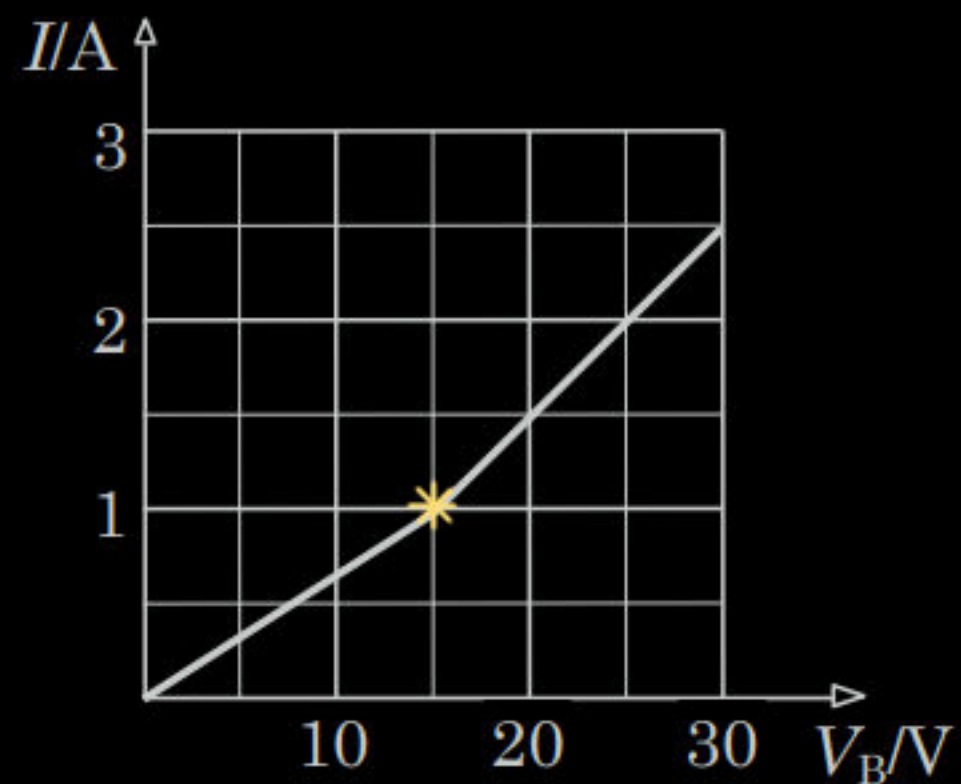
After breakdown:



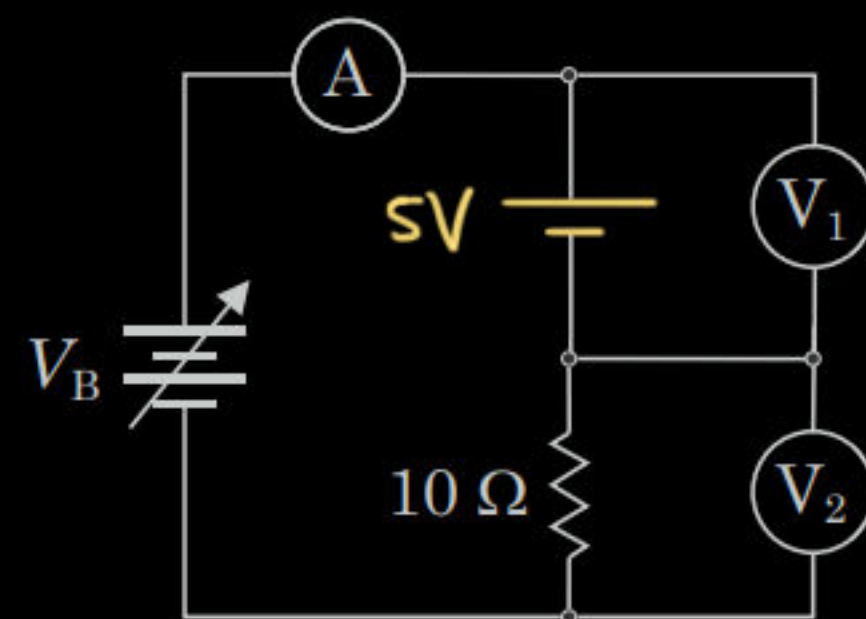
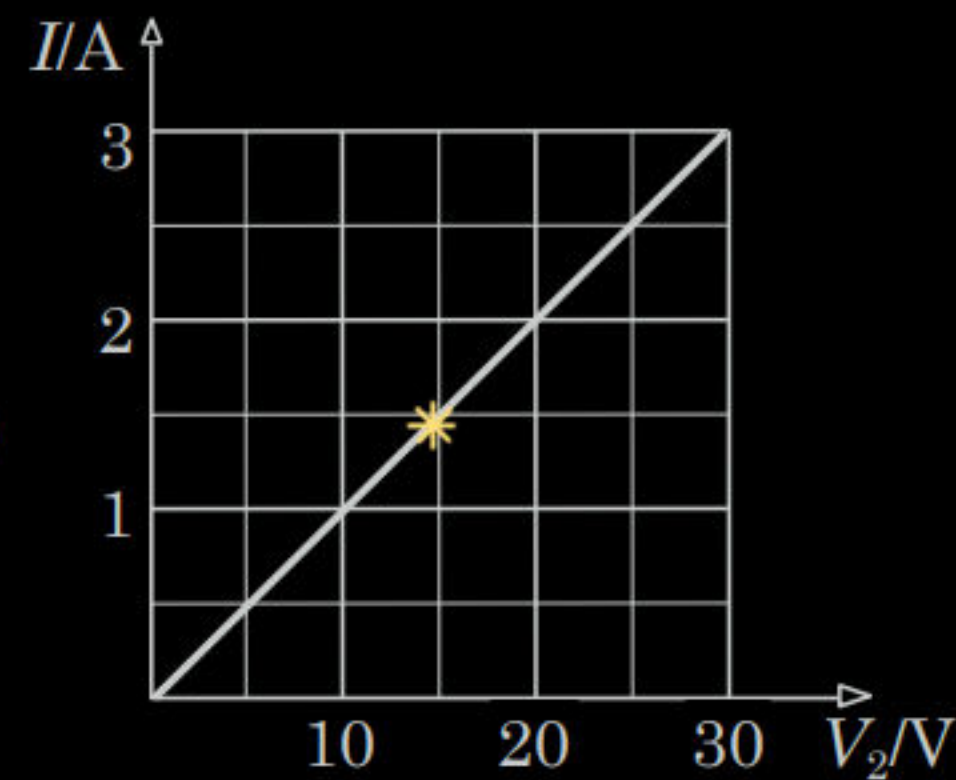


$V_B < 15$

✓ (a)

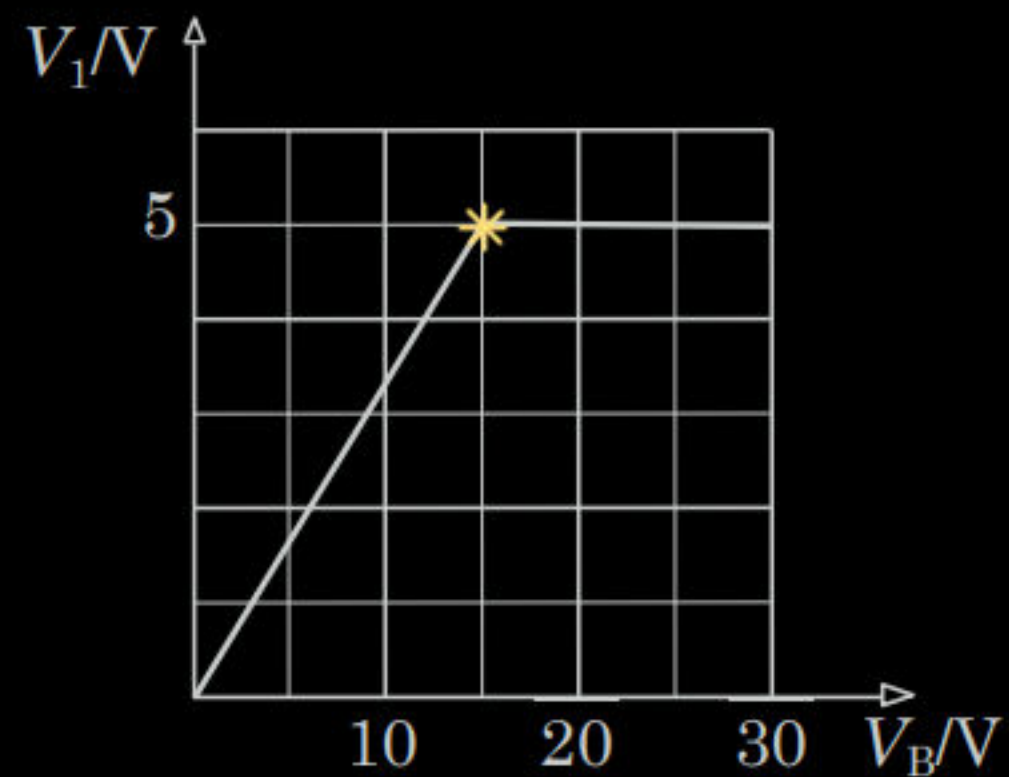


✓ (b)

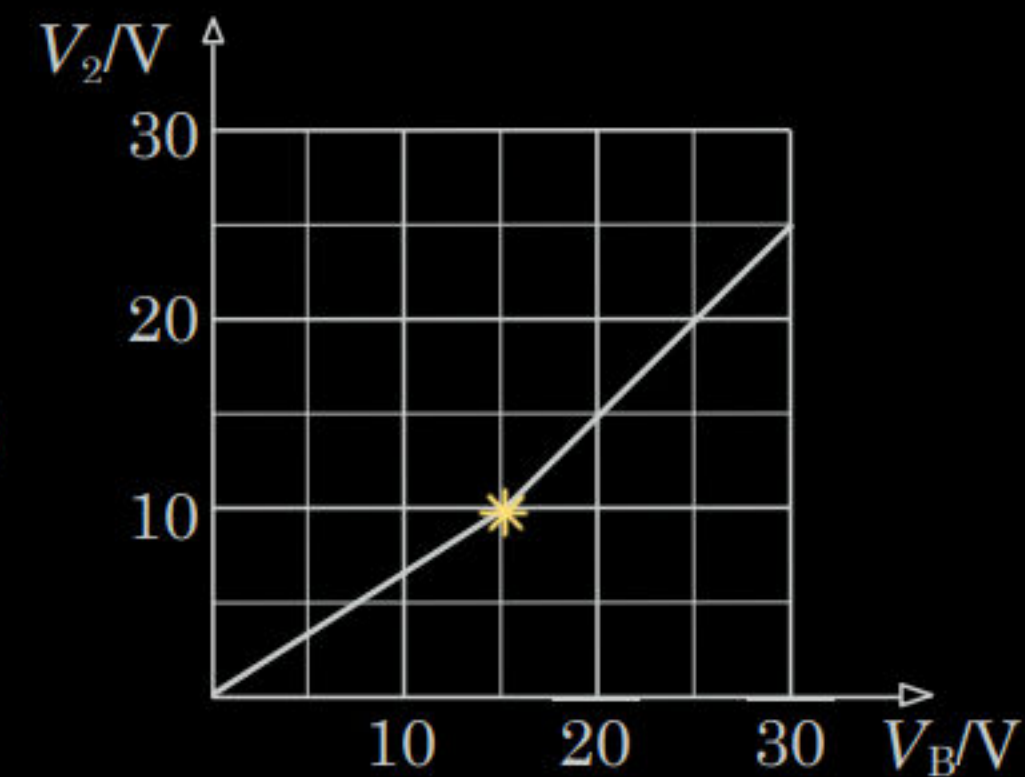


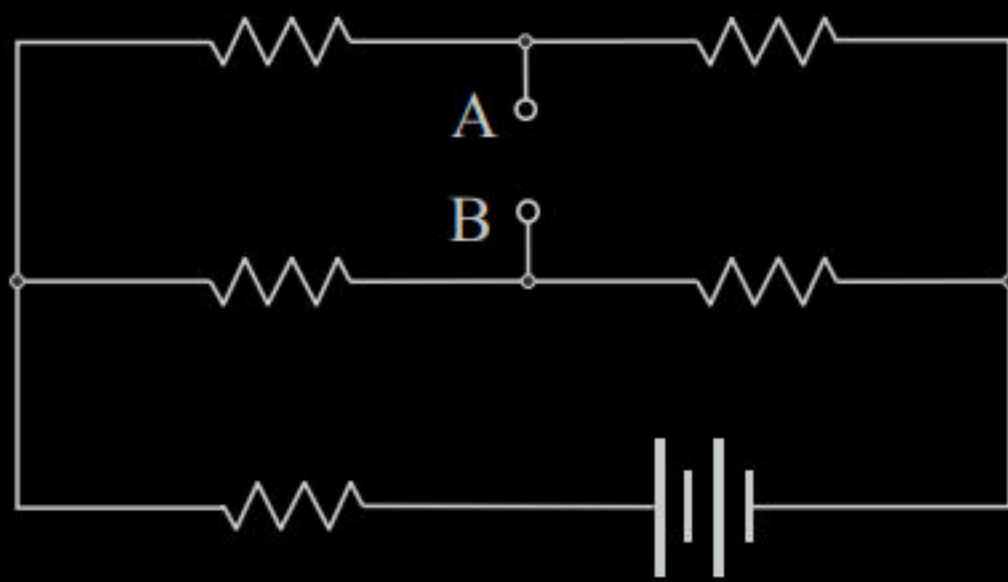
$V_B \geq 15$

✓ (c)



✓ (d)



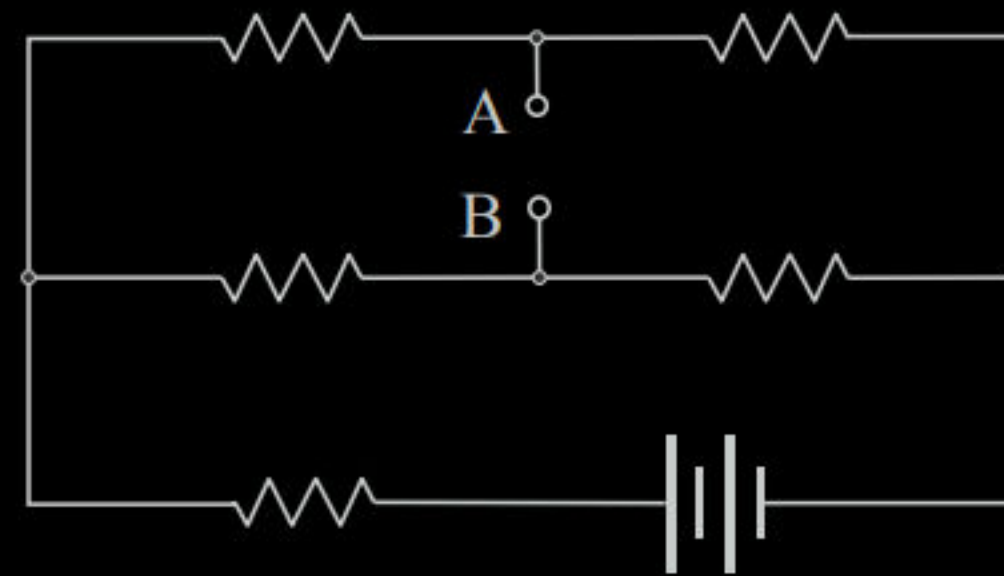


8. A network shown in the figure consists of a battery and five unknown resistors. When an ideal ammeter is connected between the terminals A and B, its reading is 4 A and when a resistance of $3\ \Omega$ is connected in series with the ammeter its reading becomes 2 A. Now the ammeter and the resistance are disconnected and an ideal voltmeter is connected between the terminals A and B. What would the voltmeter read?

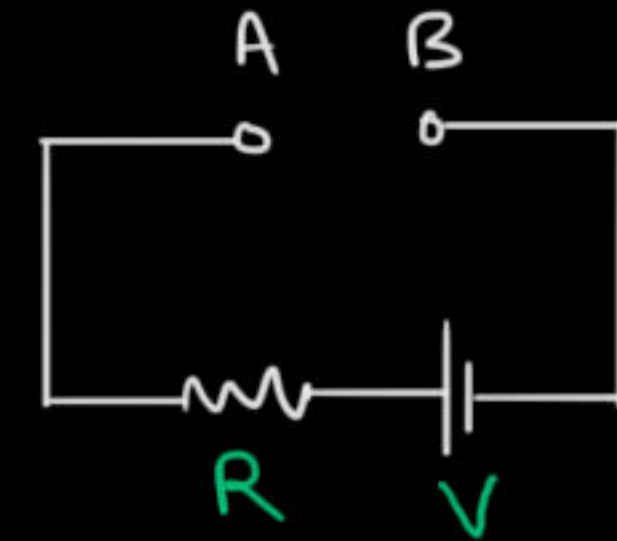
(a) 6 V
 ✓(c) 12 V

(b) 10 V
 (d) 18 V

Thevenins theorem: It is possible to simplify any circuit, to an equivalent circuit with a single voltage source and a series resistance



$\equiv$



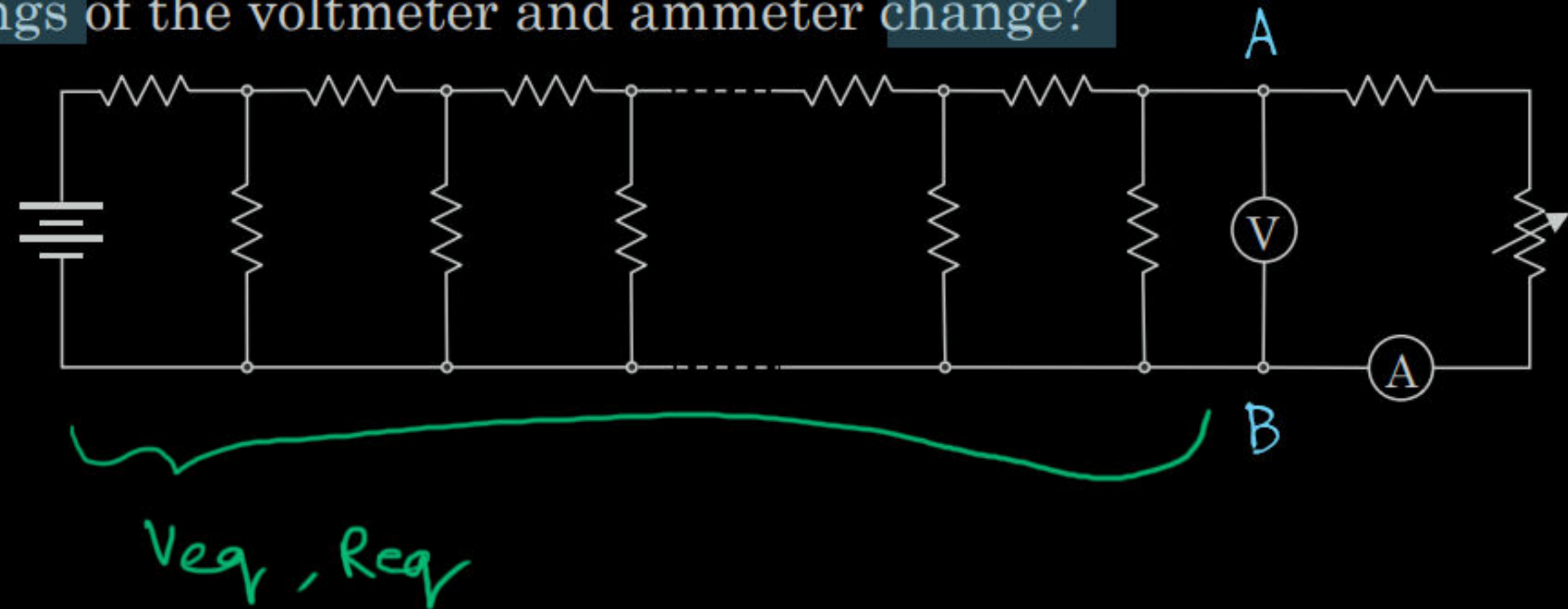
Given: $\frac{V}{R} = 4$

Given: $\frac{V}{R+3} = 2$

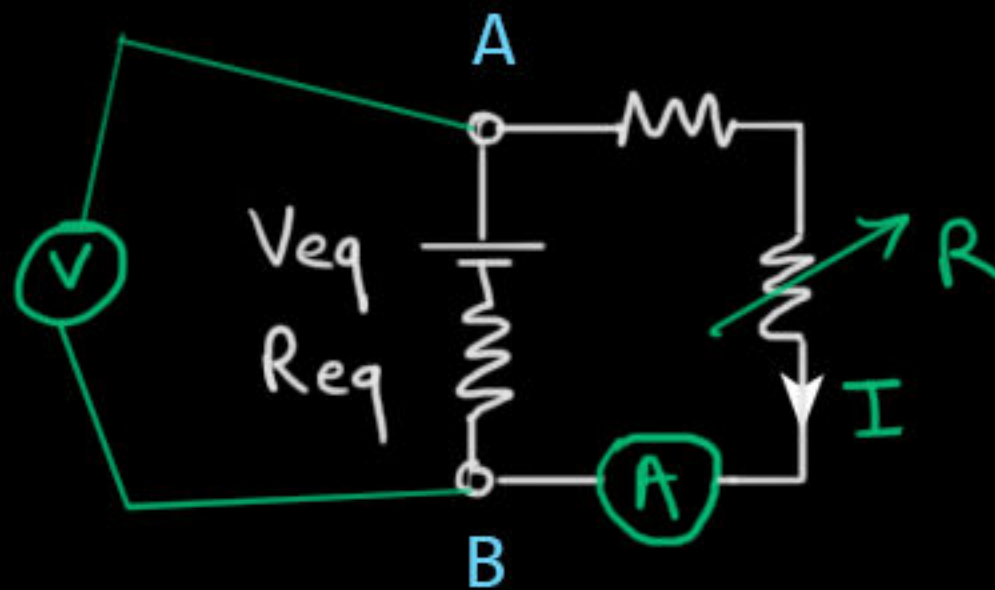
Solving: $V = 12$

Youtube / Arihant Senior

9. The network shown consists of an ideal battery, an ideal voltmeter V , an ideal ammeter A , several resistors of different values and a rheostat. If resistance of the rheostat is increased by a finite amount, how will the readings of the voltmeter and ammeter change?

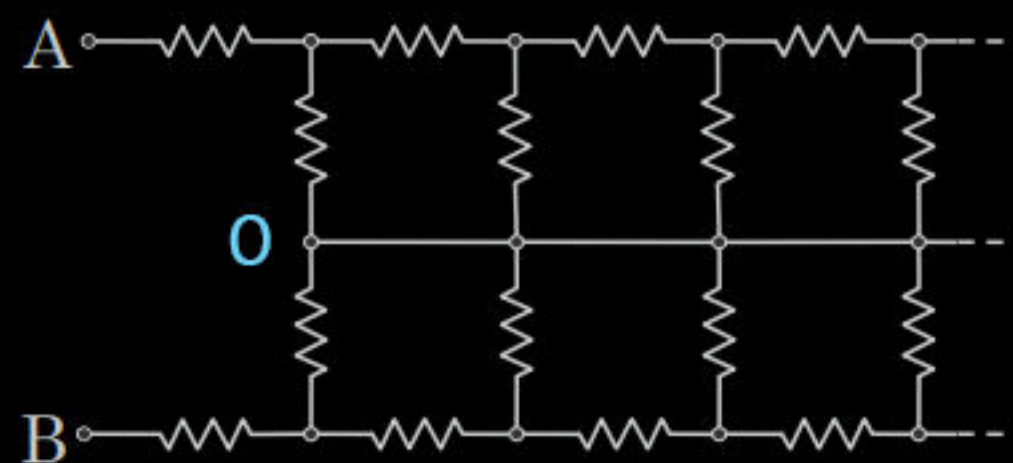


Replacing the left part of circuit with equivalent battery and resistor.



- (a) Both will decrease.
 (b) Voltmeter reading will decrease and ammeter reading will increase.
 ✓ (c) Voltmeter reading will increase and ammeter reading will decrease.
 (d) Voltmeter reading will not change but ammeter reading will decrease.

Clearly, as R is increased, I will decrease and V_{AB} will increase.



10. The circuit shown in the diagram extends to the right into infinity. Each branch resistance is denoted by r . What is the resistance between the terminals A and B?

(a) $\frac{(\sqrt{5} + 1)r}{2}$

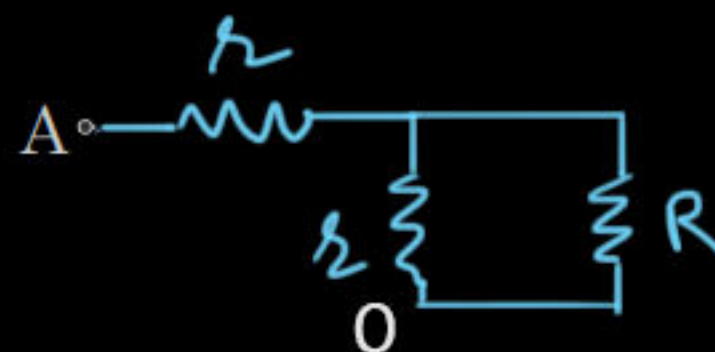
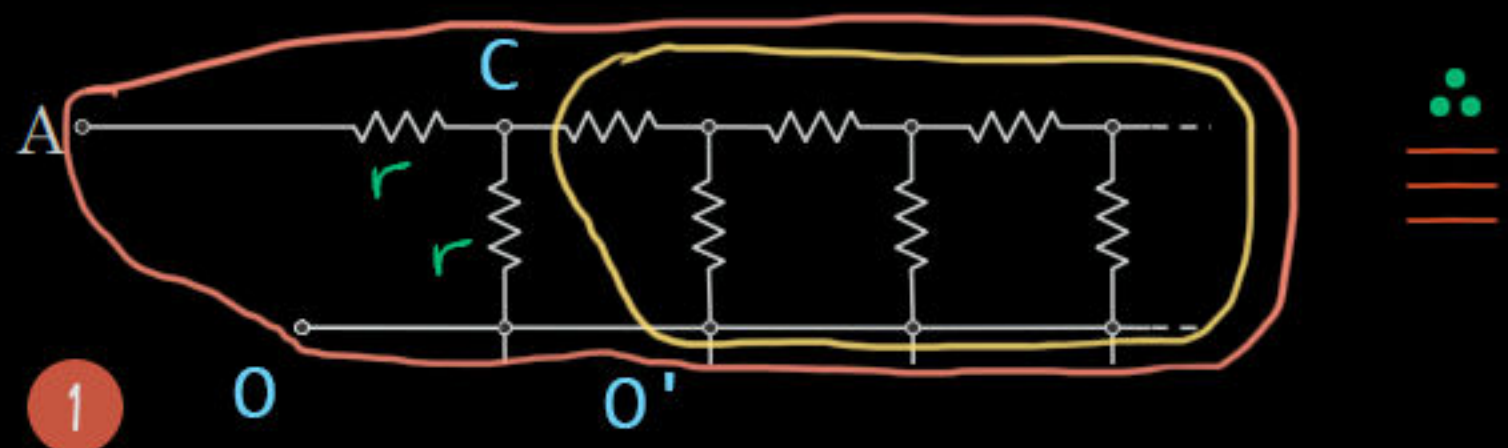
(b) $\frac{(\sqrt{5} - 1)r}{2}$

✓(c) $(\sqrt{5} + 1)r$

(d) $(\sqrt{5} - 1)r$

For top half of circuit:

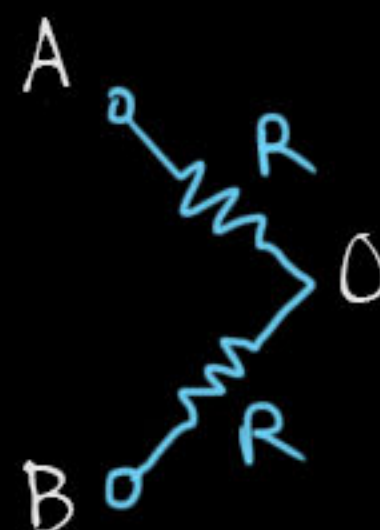
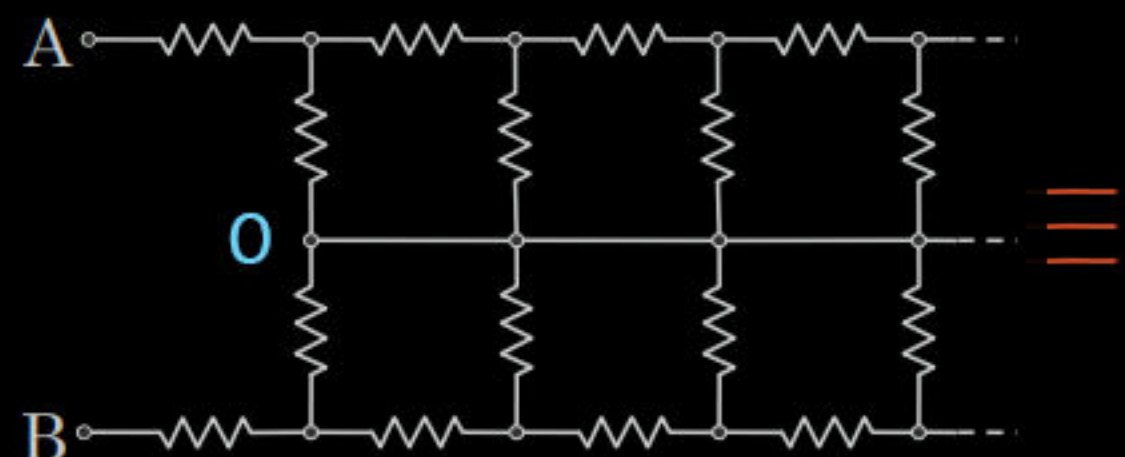
$$R_{AO} = R_{CO} = R$$



$$R_{AO} = R = r + \frac{rR}{r+R}$$

$$\Rightarrow R^2 - rR - r^2 = 0$$

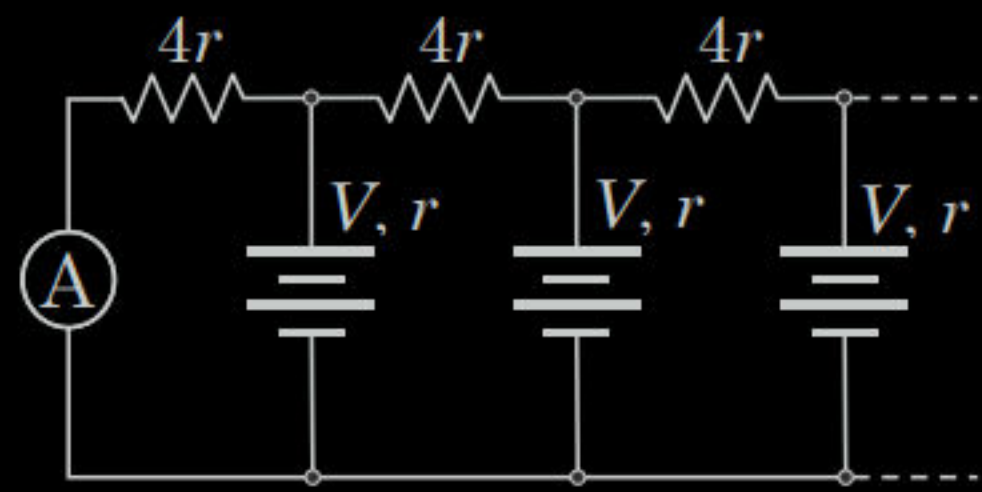
$$\Rightarrow R = r \left(\frac{1 + \sqrt{5}}{2} \right)$$



$$R_{AB} = 2R = r(1 + \sqrt{5})$$

by Ankit Singhvi

Youtube / Arihant Senior



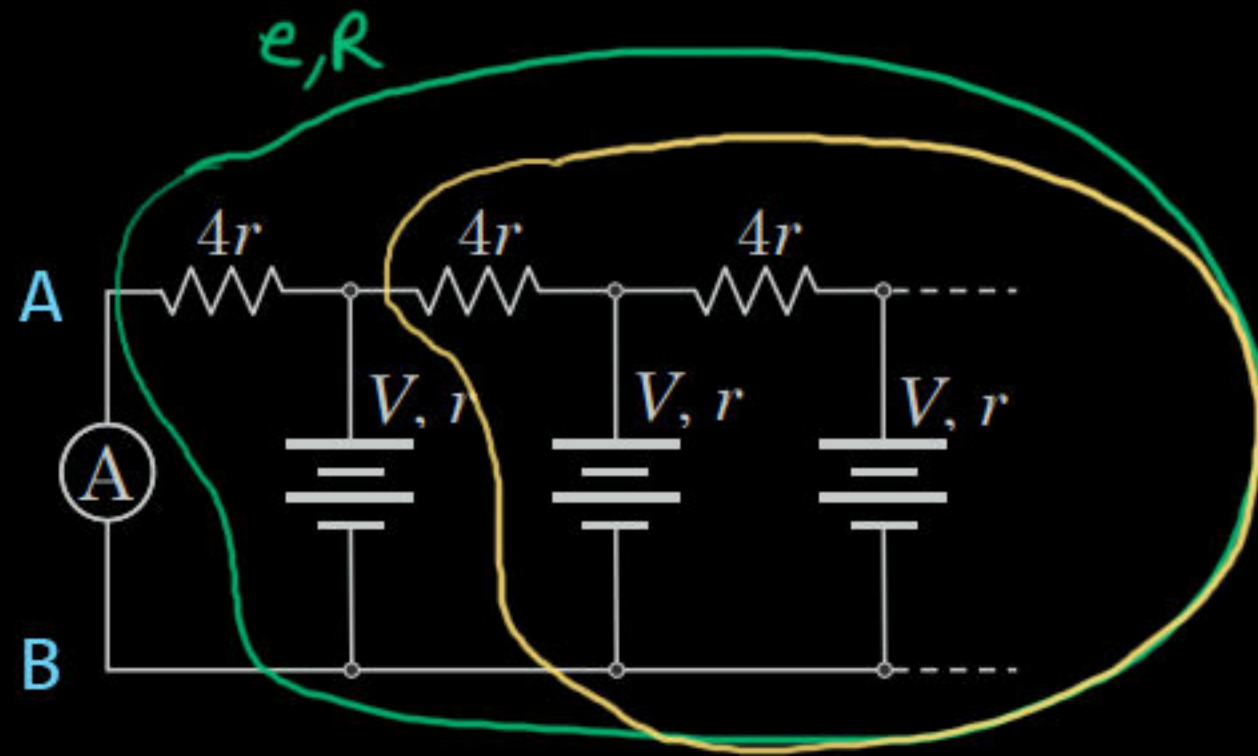
11. The circuit shown in the diagram extends to the right into infinity. Each battery has electromotive force V (unknown) and internal resistance r (known). Each resistor has resistance $4r$. Reading of the ideal ammeter shown in the diagram is I . Find the value of V in terms of I and r .

(a) $(2 - 2\sqrt{2})Ir$

✓(b) $(2 + 2\sqrt{2})Ir$

(c) $4.5Ir$

(d) $3.0Ir$



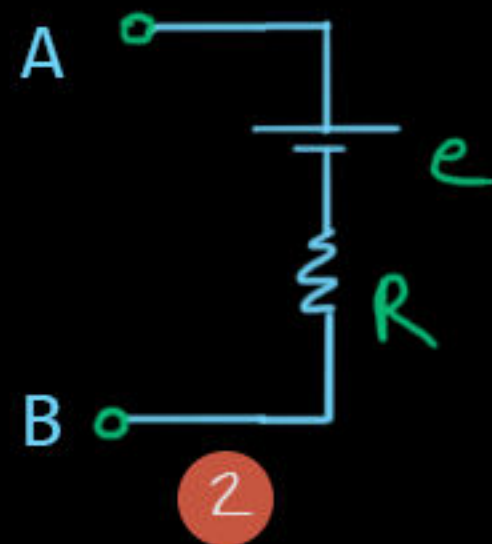
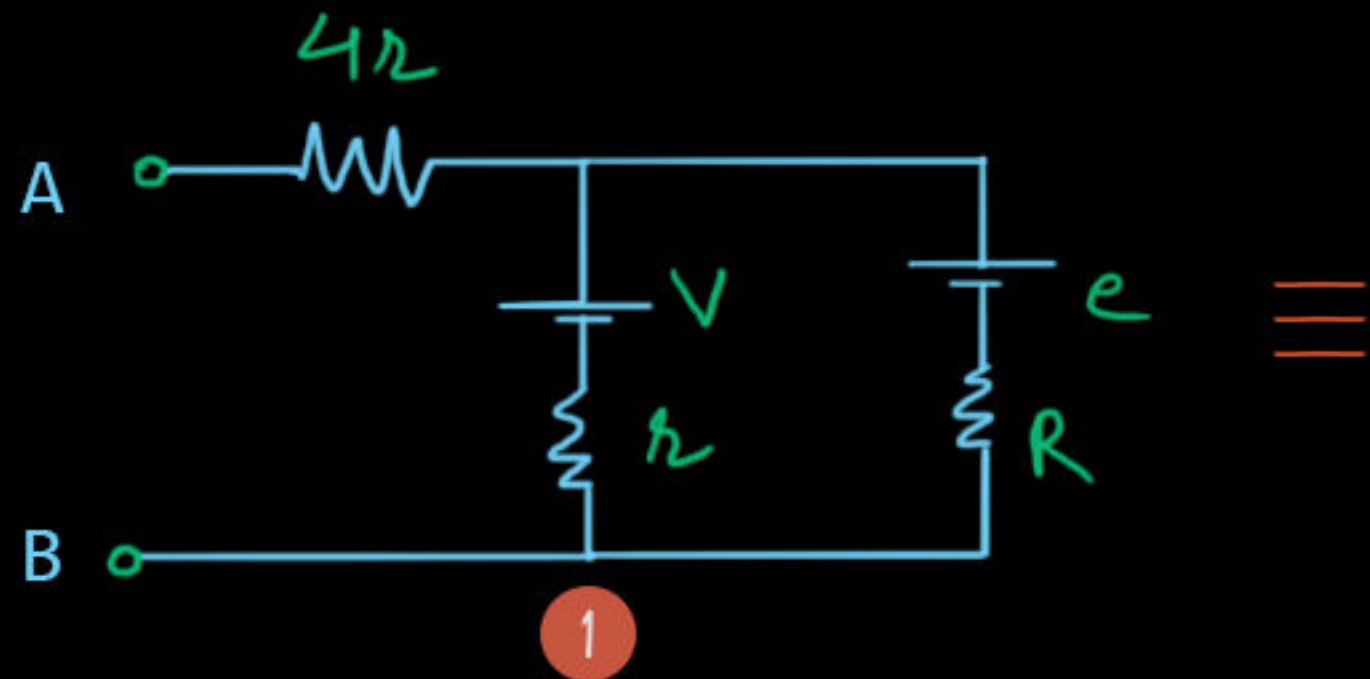
Using thevenins analysis on 1 and comparing 1 and 2:

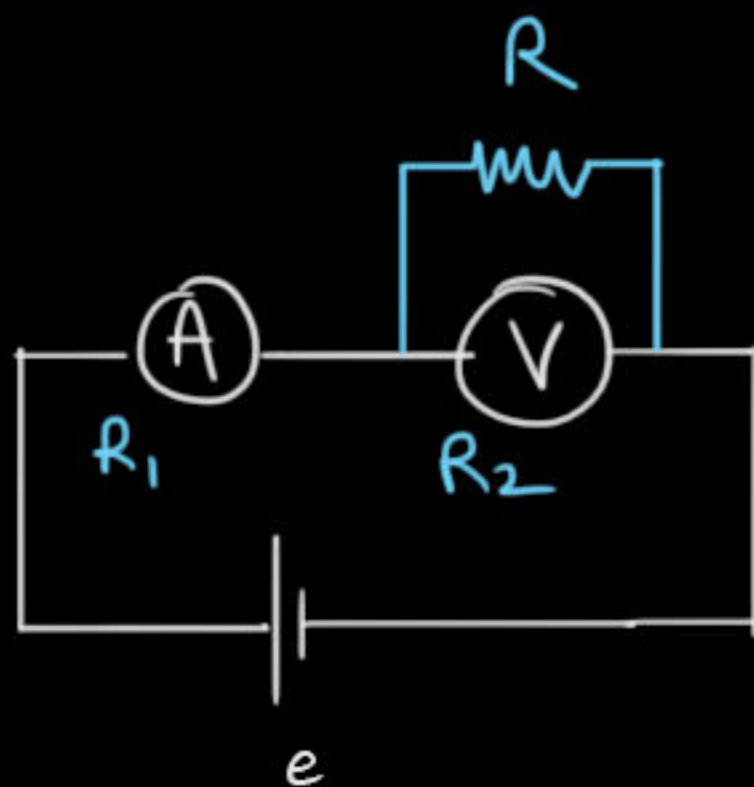
$$\frac{\frac{V}{r} + \frac{e}{R}}{\frac{1}{r} + \frac{1}{R}} = e \Rightarrow e = V \quad (1)$$

$$4r + \frac{rR}{r+R} = R \Rightarrow R = r(2 + 2\sqrt{2}) \quad (2)$$

Given: $\frac{e}{R} = I \quad (3)$

From 1, 2, 3: $V = IR = Ir(2 + 2\sqrt{2}) //$





12. An ideal battery of electromotive force $\mathcal{E}$ is connected in series with an ammeter and a voltmeter of unknown internal resistances. If a certain resistance is connected in parallel with the voltmeter, the voltmeter and the ammeter readings becomes $1/\eta$ and η times of their respective initial readings. What is the initial reading of the voltmeter? #beautiful maths

(a) $\frac{\mathcal{E}}{(\eta + 1)}$

(c) $\frac{\eta \mathcal{E}}{(\eta - 1)}$

✓ (b) $\frac{\eta \mathcal{E}}{(\eta + 1)}$

(d) $\frac{(\eta + 1) \mathcal{E}}{\eta}$

Without R

$$\frac{e}{R_1 + R_2} = i \quad (1)$$

$$i R_2 = V \quad (2)$$

With R

$$\frac{e}{R_1 + \frac{R R_2}{R + R_2}} = \eta i \quad (3)$$

$$e - \eta i R_1 = \frac{V}{\eta} \quad (4)$$

Lets not bother with this equation as there is un-necessary variable R.

In other three equations reduce one variable by substituting: $R_1 i \rightarrow x$

$$R_2 i \rightarrow y$$

$$e = x + y \quad (1)$$

$$y = V \quad (2)$$

$$e - \eta x = \frac{V}{\eta} \quad (3)$$

3 equations, 3 variables

Solving: $V = \frac{\eta e}{\eta + 1} //$

13. Two identical conducting spheres each of radius a are placed at centre to centre separation d ($d \gg a$). They are kept in a homogeneous medium of permittivity ϵ and resistivity ρ . Which one of the following is a correct expression of resistance between them?

✓(a) $\frac{\rho}{2\pi a}$

(b) $\frac{\rho}{4\pi a}$

(c) $\frac{\rho d}{2\pi a^2}$

(d) $\frac{\rho d}{4\pi a^2}$

We know,

$$C_{eq} = 2\pi\epsilon_0\epsilon_r a \quad (1) \text{ (Proved in irodov 3.110)}$$

$$\text{And, } C_{eq} \cdot R_{eq} = \frac{\rho}{\epsilon} \quad (2) \text{ (Proved in irodov 3.157 method 2)}$$

From 1,2:

$$\begin{aligned} 2\pi\cancel{\epsilon} a \cdot R &= \frac{\rho}{\cancel{\epsilon}} \\ \Rightarrow R &= \frac{\rho}{2\pi a} // \end{aligned}$$

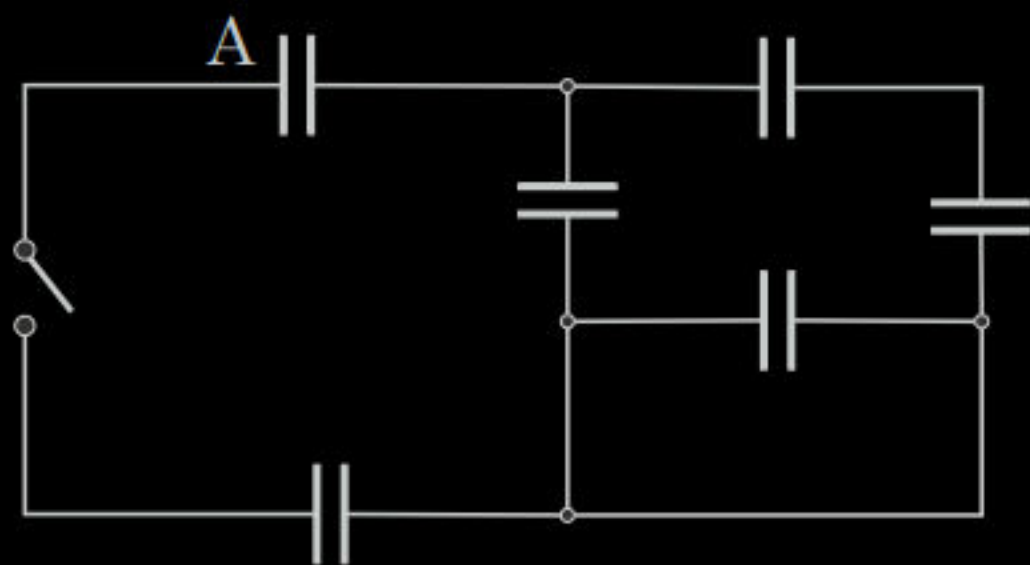
14. An air-filled parallel-plate capacitor with the plate area A is connected to a battery of electromotive force V and negligible internal resistance. One of the plates is made to vibrate so that the distance between the plates varies as $d = d_0 + a \cos(\omega t)$, where $a \ll d_0$. If instantaneous current in the circuit reaches a maximum value of I_0 , the maximum possible amplitude of vibrations a is

(a) $\frac{aI_0}{VA\omega\epsilon_0}$ (b) $\frac{I_0 d_0}{VA\omega\epsilon_0}$ ✓ (c) $\frac{I_0 d_0^2}{VA\omega\epsilon_0}$ (d) $\frac{I_0 d_0}{V\sqrt{A}\omega\epsilon_0}$

$$i = \frac{dq}{dt} = V \frac{dC}{dt} = VA\epsilon_0 \frac{d}{dt} \left(\frac{1}{d_0 + a \cos \omega t} \right) = \frac{-VA\epsilon_0 - a\omega \sin \omega t}{(d_0 + a \cos \omega t)^2} = \frac{VA\epsilon_0 a \omega \sin \omega t}{(d_0 + a \cos \omega t)^2}$$

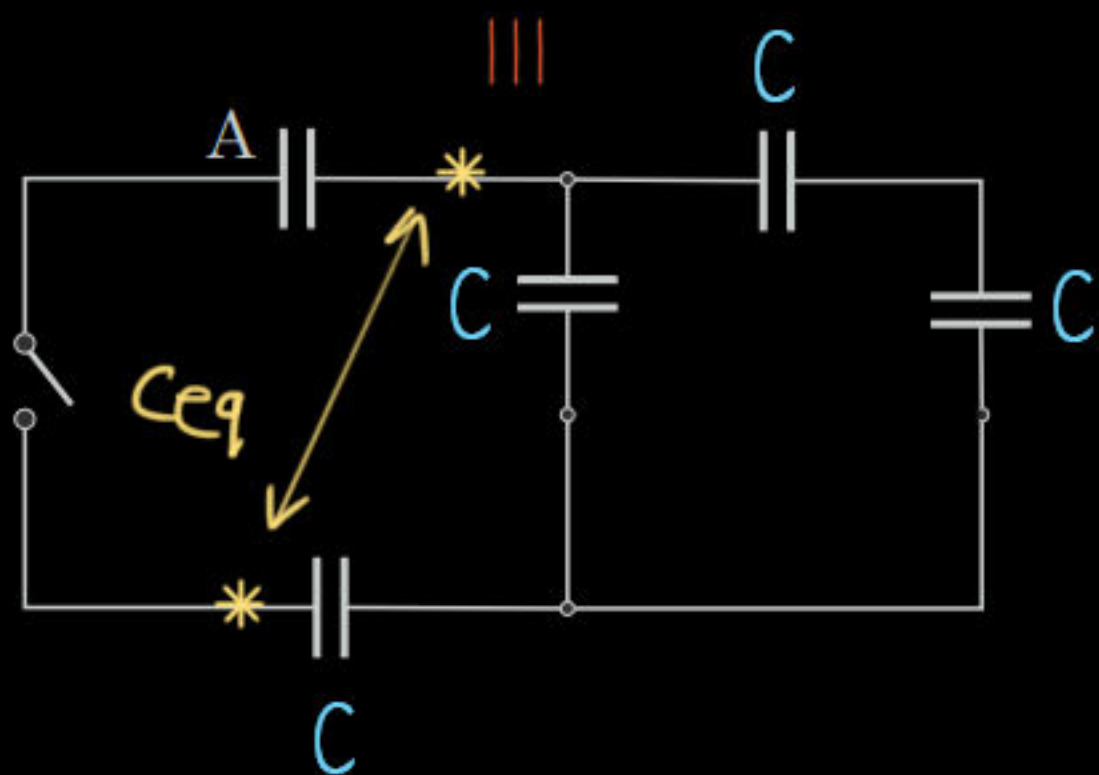
maximum @ $\sin \omega t = 1$
as $d_0 + a \cos \omega t \approx d_0$

$$\therefore I_0 = \frac{VA\epsilon_0 a \omega}{d_0^2} \Rightarrow a = \frac{I_0 d_0^2}{VA\epsilon_0 \omega}$$

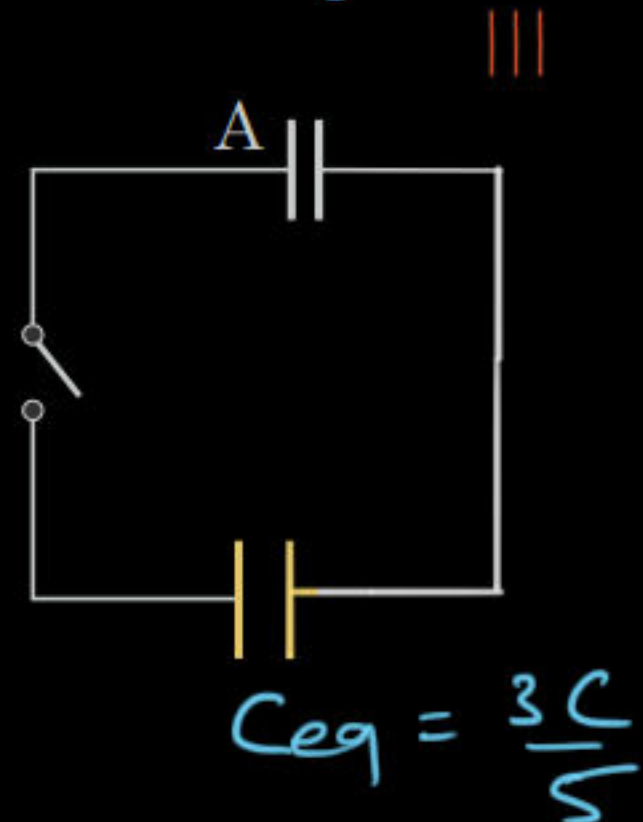


15. In the circuit shown, all capacitors are identical, initially the switch is open and only the capacitor A is charged. After the switch is closed and a steady-state is established, charge on the capacitor A becomes $5.0 \mu\text{C}$. Initial charge on this capacitor is closest to

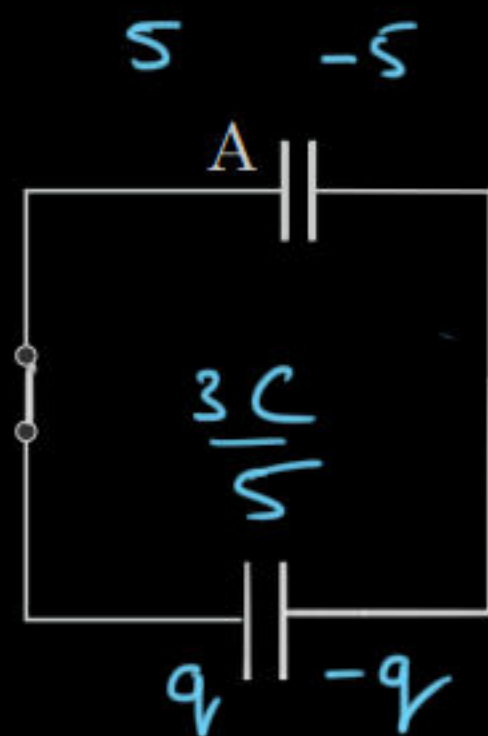
- (a) $7.5 \mu\text{C}$ (b) $8.0 \mu\text{C}$ ✓
(c) $13.3 \mu\text{C}$ (d) $15.0 \mu\text{C}$



$$C_{eq} = \frac{\left(C + \frac{C}{2}\right)C}{\left(C + \frac{C}{2}\right) + C} = \frac{3C}{5}$$



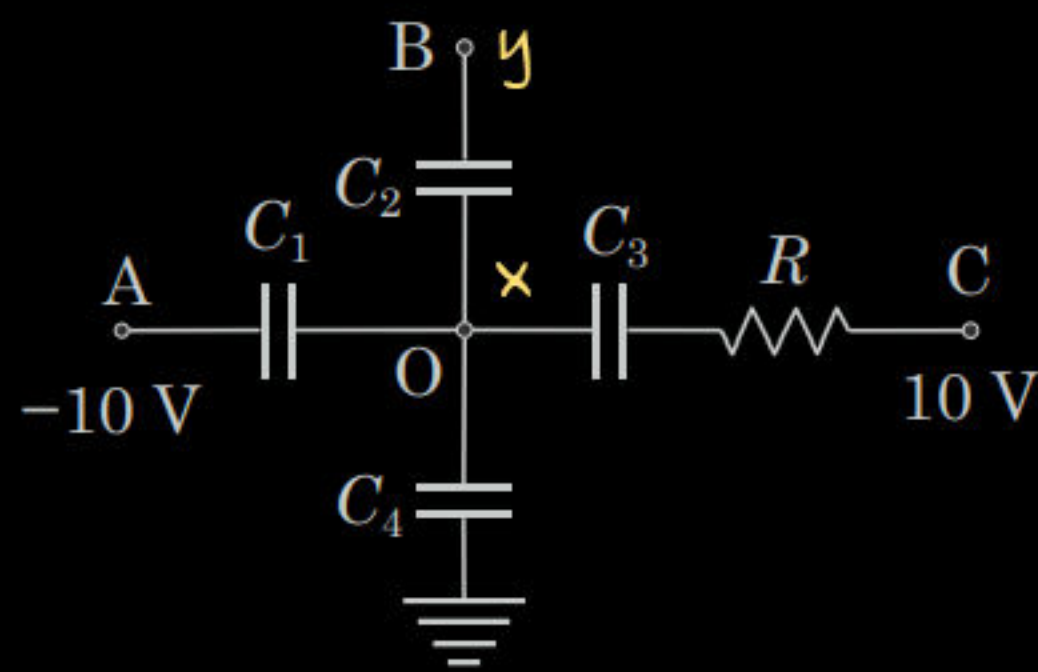
on closing switch



KVL: $-\frac{5}{C} + \frac{q}{3C/5} = 0$

$\Rightarrow q = 3$

$\Rightarrow \text{Initial charge on A} = 5 + q = 8 \mu\text{C}$



16. The figure shows part of a large network in which potentials of some of the points are shown. Each capacitor has a capacitance $5 \mu\text{F}$. Which of the following statements is/are true?

- (a) From the given information, potential of point O can be determined but that of B cannot be determined.
- ✓ (b) From the given information, potential of both the points O and B cannot be determined.
- (c) If charge on capacitor C_2 were also specified, potential of point O can be determined but that of B cannot be determined.
- ✓ (d) If charge on capacitor C_2 were also specified, potentials of both the points O and B can be determined.

B Net charge at junction must be zero.

$$\therefore \cancel{C_1}(x + 10) + C_2(x - y) + \cancel{C_3}(x - 10) + \cancel{C_4}(x - 0) = 0$$

$$\Rightarrow 4x - y = 0 \quad (1)$$

One equation two variables. Cant be solved.

D If charge q_2 is given $\Rightarrow C_2(x - y) = q_2 \quad (2)$ Two equations, two variables. Can be solved.

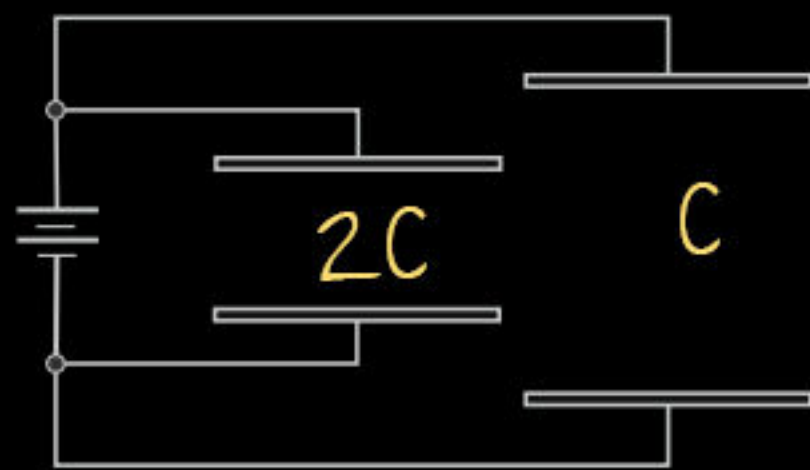


Figure-I

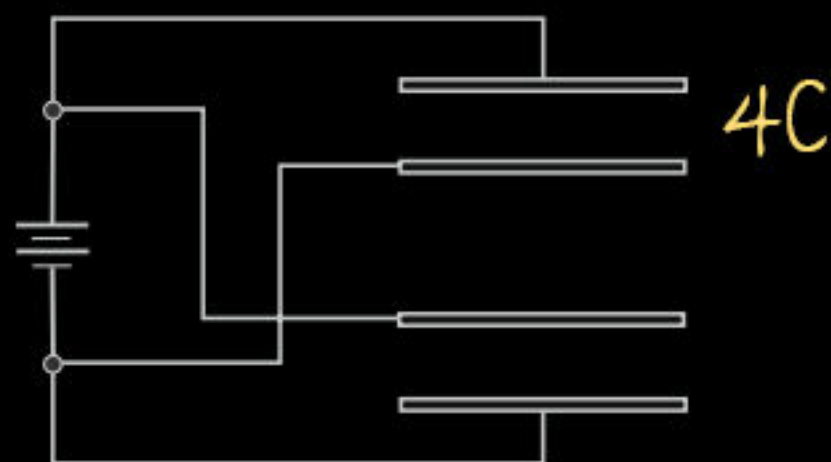


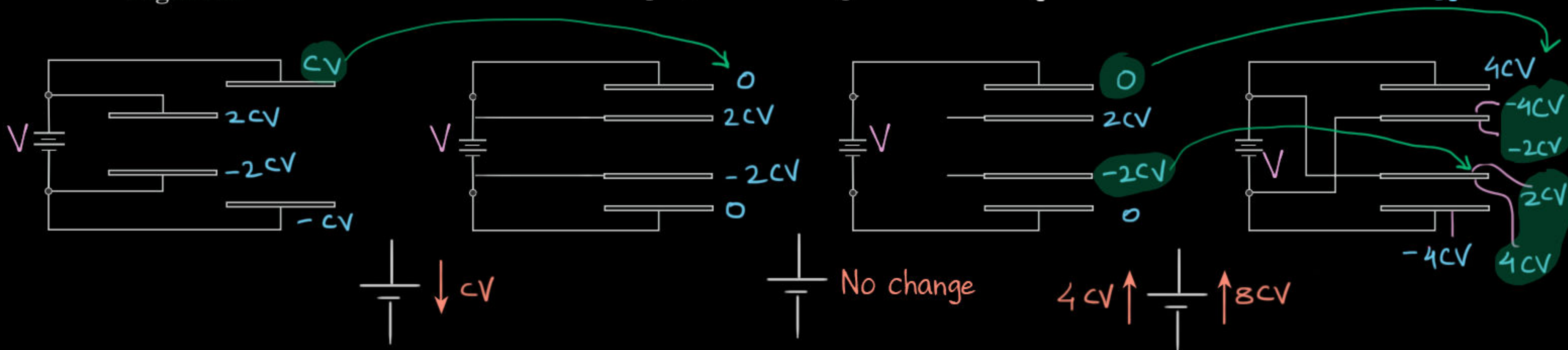
Figure-II

17. Two parallel plate capacitors of capacitances C and $2C$ consisting of plates of identical dimensions are connected with an ideal battery of terminal voltage V as shown in the figure-I. Now the capacitor smaller in volume is completely inserted into the larger capacitor and kept in symmetric position and then connection polarities of the plates are reversed as shown in the figure-II. Which one of the following statements is correct?

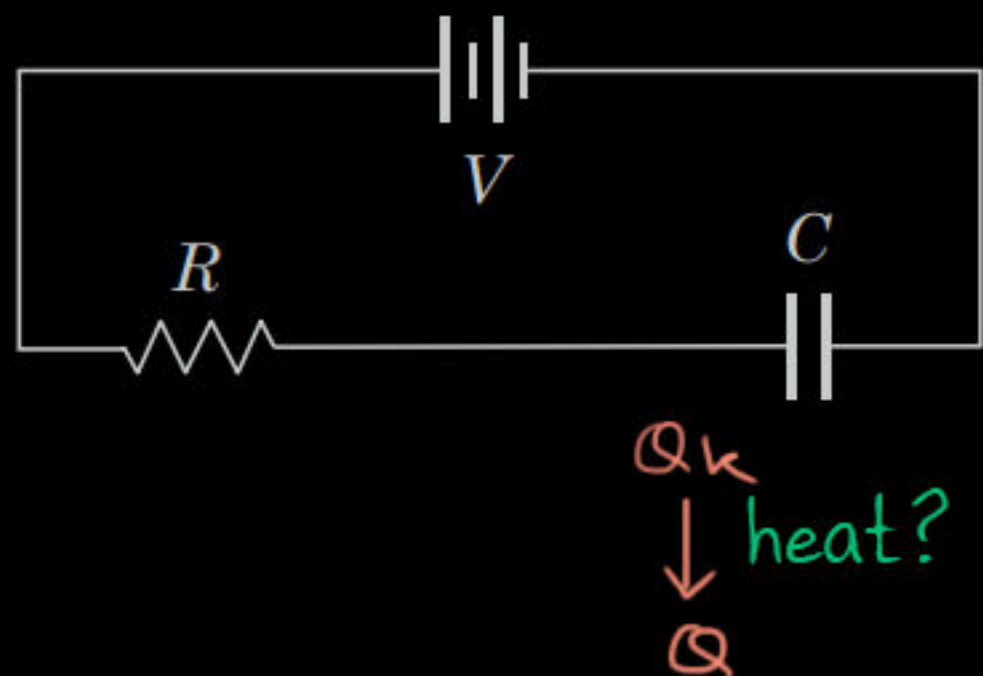
- (a) Charge flown through the battery is $8CV$ and it absorbs energy.
- (b) Charge flown through the battery is $8CV$ and it delivers energy.
- (c) Charge flown through the battery is $7CV$ and it absorbs energy.
- (d) Charge flown through the battery is $7CV$ and it delivers energy.
- (e) Charge flown through the battery is $11CV$ and it delivers energy.

In book: ✓

Mine: ✓



∴ Net charge flow: $-CV + 4CV + 8CV = 11CV$



18. An air filled parallel plate capacitor of capacitance C is connected through a resistance R to an ideal voltage source of electromotive force V . A dielectric plate of dielectric constant k is inserted in the capacitor to occupy whole space between the plates. After a steady state is reached, the plate is quickly pulled out. Which of the following is correct expression for heat generated in the resistance until a steady state is reached again?

- (a) $CV^2(k-1)$ (b) $\frac{1}{2}CV^2(k-1)$ (c) $CV^2(k-1)^2$ (d) $\frac{1}{2}CV^2(k-1)^2$ ✓

Let the charge on capacitor with dielectric be Q_k and at eventual steady state be Q .

Just after dielectric is quickly pulled, charge on capacitor is still Q_k but its capacitance has reduced to C again.

We need to find heat generated between these two events:

a) just after dielectric plate is quickly pulled, and b) steady state later.

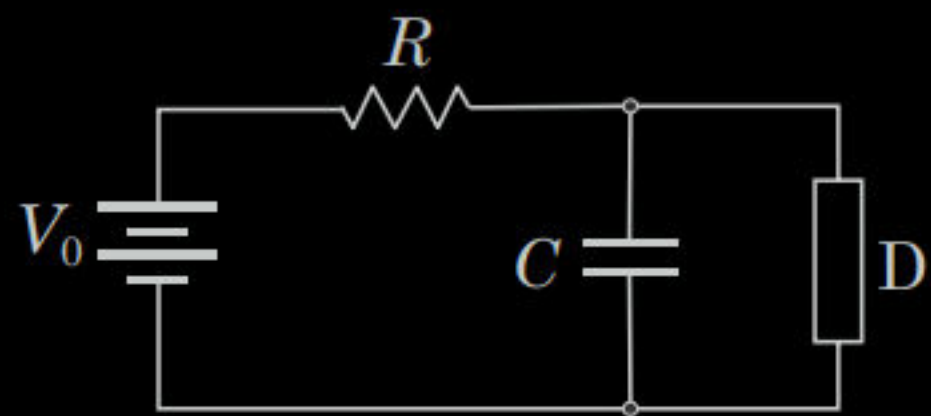
$$W_{\text{Battery}} = \Delta U + \text{Heat}$$

$$\Rightarrow V(Q - Q_k) = \frac{Q^2}{2C} - \frac{Q_k^2}{2C} + \text{Heat}$$

$$\Rightarrow V(CV - kCV) = \frac{(CV)^2}{2C} - \frac{(kCV)^2}{2C} + \text{Heat}$$

$$\Rightarrow CV^2(1-k) = \frac{CV^2}{2}(1-k^2) + \text{Heat}$$

$$\Rightarrow \text{Heat} = \frac{CV^2(1-k)^2}{2}$$



19. In the circuit shown, the device D has a property that, if initially non-conducting, it remains non-conducting until voltage across it rises to a value V_1 ($< V_0$). It then rapidly discharges the capacitor until the voltage across it drops to a negligibly small value, where upon it returns to a non-conducting state. The voltage developed across the capacitor is

Basically capacitor is shorted at V_1 periodically..

So we need to find the time it takes to charge the capacitor to V_1 potential.

$$V = V_0 (1 - e^{-t/RC})$$

$$\therefore V_1 = V_0 (1 - e^{-T/RC})$$

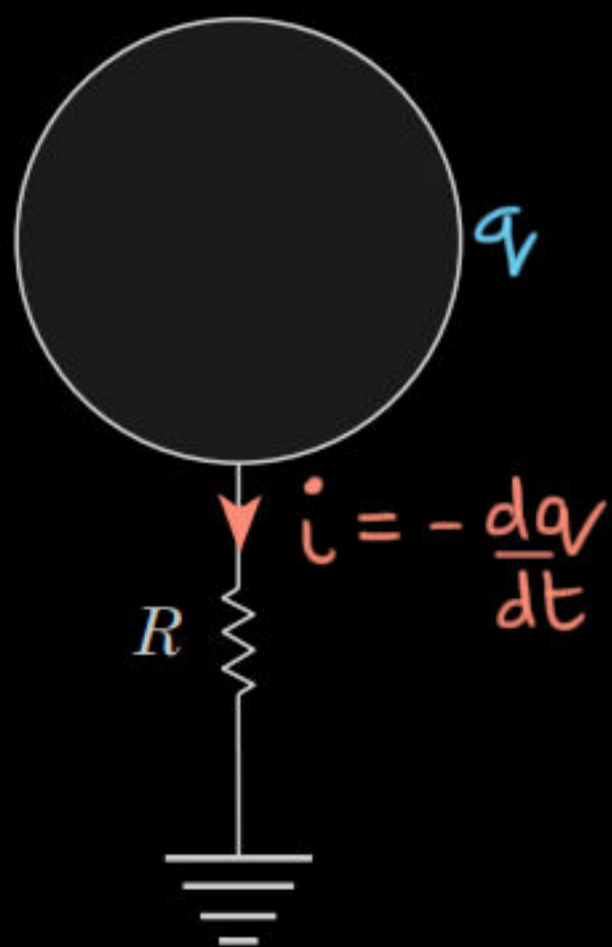
$$\Rightarrow T = RC \ln \frac{V_0}{V_0 - V_1} //$$

(a) periodic with time period RC .

(b) periodic with time period $RC \ln \left(\frac{V_0}{V_1} \right)$.

✓ (c) periodic with time period $RC \ln \left(\frac{V_0}{V_0 - V_1} \right)$.

(d) periodic with time period $RC \ln \left(\frac{V_0^2}{V_1 (V_0 - V_1)} \right)$.



A conducting balloon of radius a is charged to a potential V_0 and held at a large height above the earth surface. The large height of the balloon from the earth ensures that charge distribution on the surface of the balloon remains unaffected by the presence of the earth. It is earthed through a resistance R and a valve is opened. The gas inside the balloon escapes through the valve and the size of the balloon decreases. The rate of decrease in radius of the balloon is controlled in such a manner that potential of the balloon remains constant. Assume electric permittivity of the surrounding air equal to that of free space (ϵ_0) and charge does not leak to the surrounding air.

20. Time rate at which radius r of the balloon changes is best represented by the expression

✓ (b) $-\frac{1}{4\pi\epsilon_0 R}$

21. How much heat is dissipated in the resistance R until radius of the balloon becomes half?

✓ (c) $2\pi\epsilon_0 a V_0^2$

21 All charge is lost from V_0 potential.

$$\therefore \text{Heat} = W_e = -V_0 \Delta q = \left(\frac{V_0 \epsilon_0}{k} - \frac{V_0 \epsilon_0}{k} \right) V_0$$

$$= \left(\frac{V_0}{k} \cdot \frac{q}{2} \right) V_0 = 2\pi\epsilon_0 a V_0^2 //$$

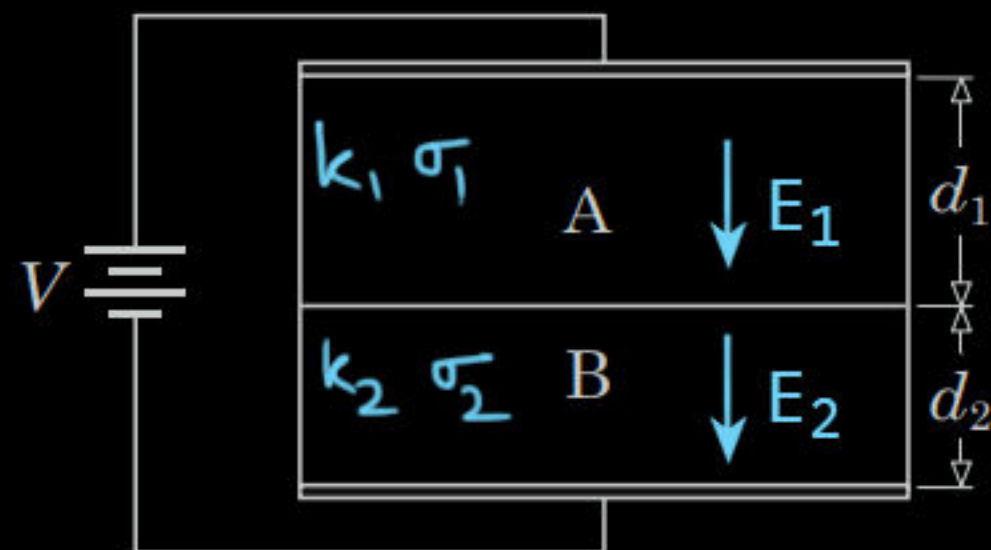
20

$$V_0 = \frac{kq}{r}$$

$$\Rightarrow V_0 r = kq$$

$$\Rightarrow \cancel{V_0} \frac{dr}{dt} = k \frac{dq}{dt} \quad \text{--- } -i = -\frac{V_0}{R}$$

$$\Rightarrow \frac{dr}{dt} = -\frac{k}{R} = -\frac{1}{4\pi\epsilon_0 R} //$$



A parallel plate capacitor is filled with two layers of different materials A and B as shown in the figure on the next page. The material A has dielectric constant k_1 and conductivity σ_1 and the material B has dielectric constant k_2 and conductivity σ_2 . The capacitor is connected across an ideal battery of terminal voltage V . Permittivity of free space is ϵ_0 .

22. Electric field in the material A is

✓(d) $\frac{V\sigma_2}{d_1\sigma_2 + d_2\sigma_1}$

23. What is the total surface charge density on the interface of the two materials?

✓(b) $\frac{\epsilon_0 V(\sigma_1 - \sigma_2)}{d_1\sigma_2 + d_2\sigma_1}$

22

$$V = E_1 d_1 + E_2 d_2 \quad (1)$$

$$j = \sigma_1 E_1 = \sigma_2 E_2 \quad (2)$$

From 1, 2:

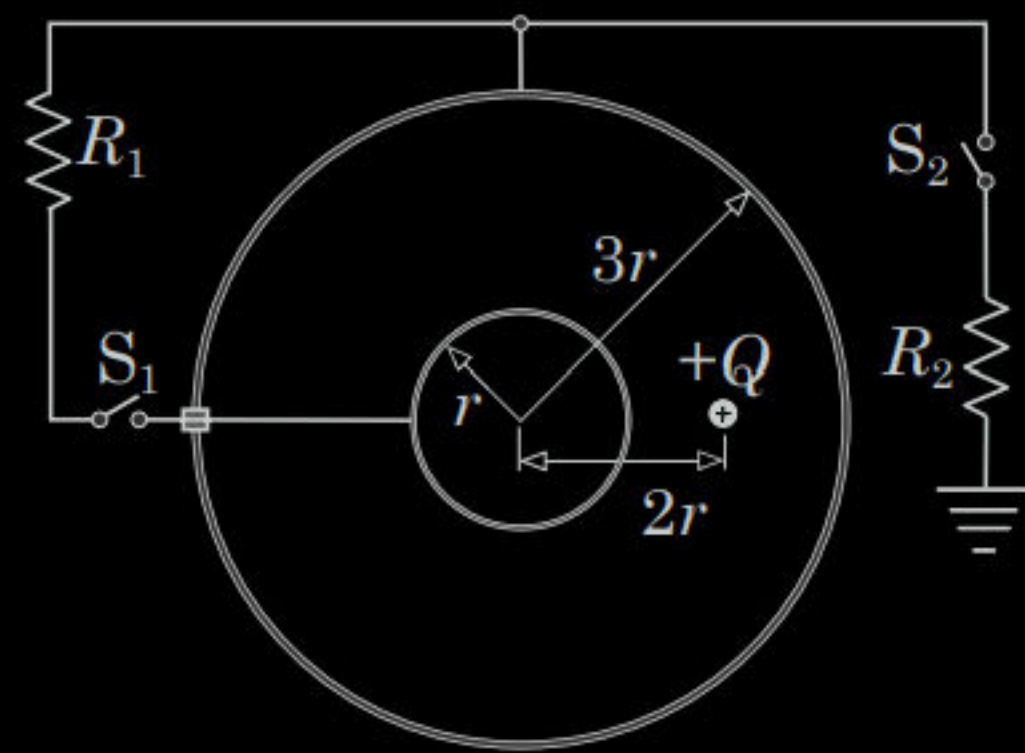
$$E_1 = \frac{V\sigma_2}{\sigma_2 d_1 + \sigma_1 d_2}$$

$$\& E_2 = \frac{V\sigma_1}{\sigma_2 d_1 + \sigma_1 d_2}$$

23 $E_2 - E_1 = \frac{\sigma}{\epsilon_0}$

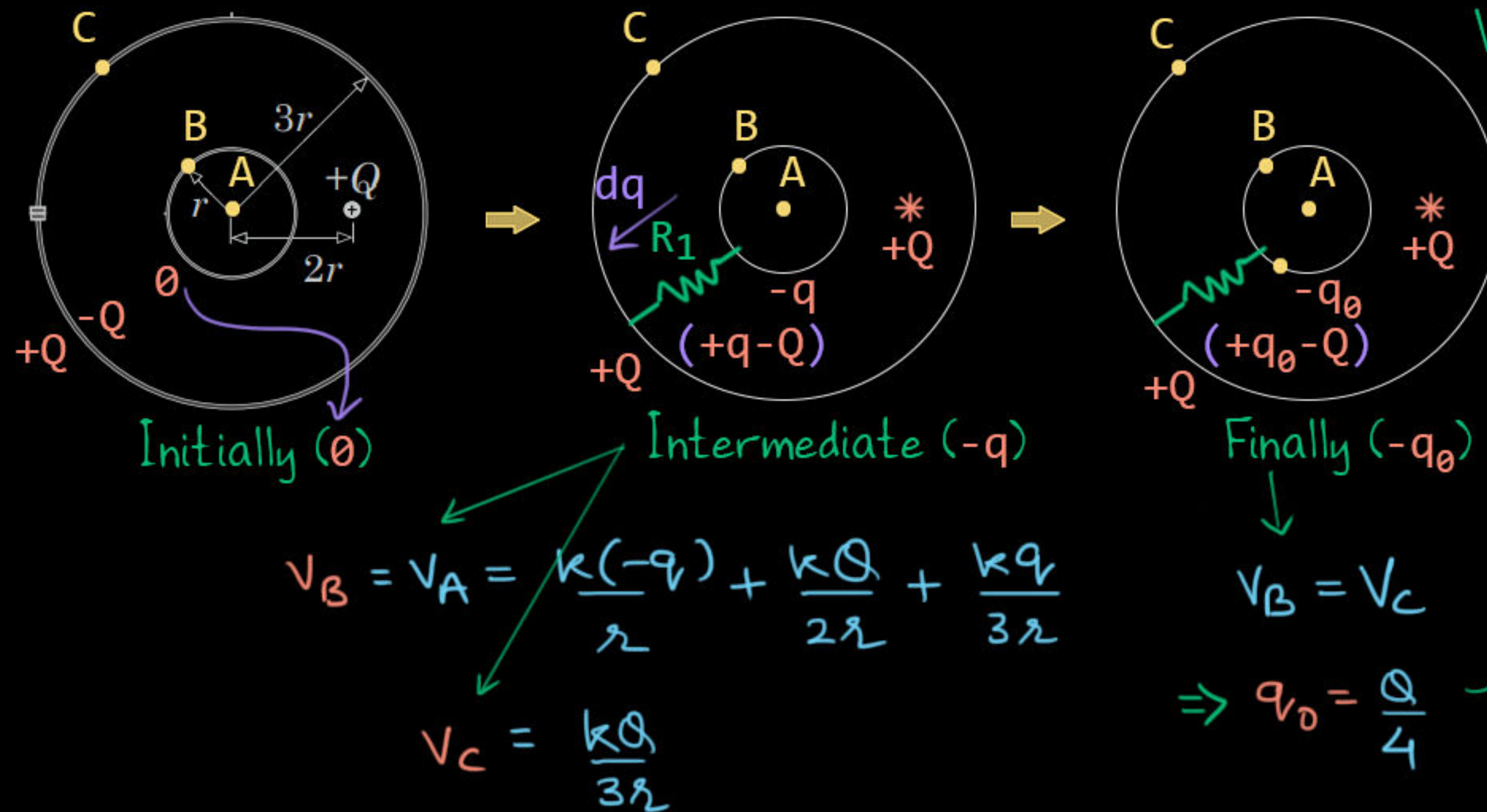
$$\Rightarrow \sigma = (E_2 - E_1)\epsilon_0$$

$$= \frac{V(\sigma_1 - \sigma_2)\epsilon_0}{\sigma_2 d_1 + \sigma_1 d_2}$$



Consider two electrically neutral concentric conducting thin spherical shells of radii r and $3r$. A particle of positive charge Q is fixed at a distance $2r$ from the common centres of the shells. The inner shell can be electrically connected to the outer shell through a resistance R_1 with the help of a switch S_1 and the outer shell can be grounded through a resistance R_2 with the help of another switch S_2 as shown in the figure.

24. How much total heat is dissipated in the resistance R_1 after the switch S_1 is closed keeping the switch S_2 open until a steady state is reached?



We know:

✓ (c) $\frac{Q^2}{192\pi\epsilon_0 r}$

$W_e = \Delta KE + \text{Heat}$

$\Rightarrow \text{Heat} = W_e$

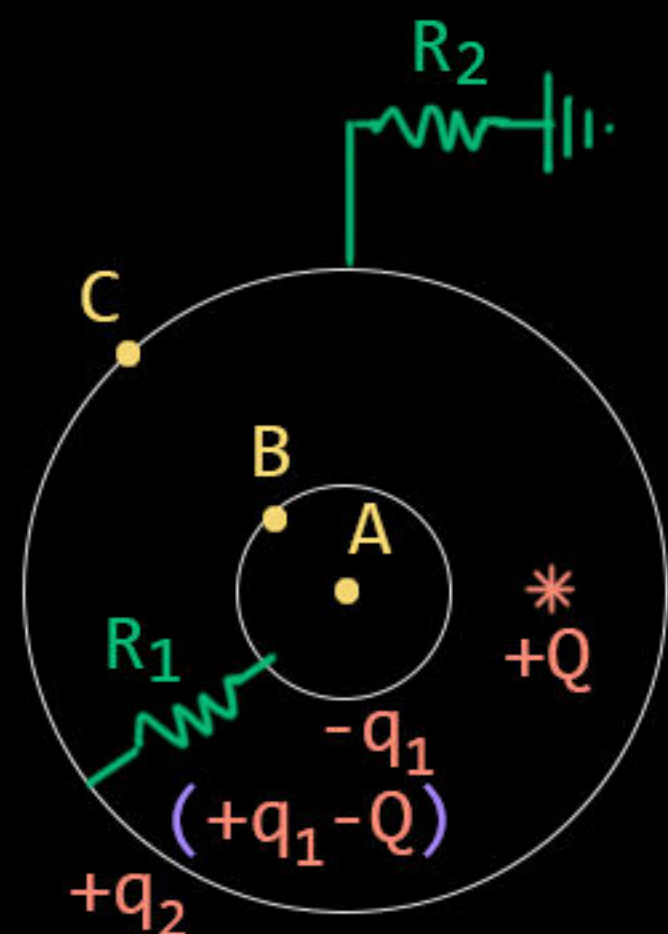
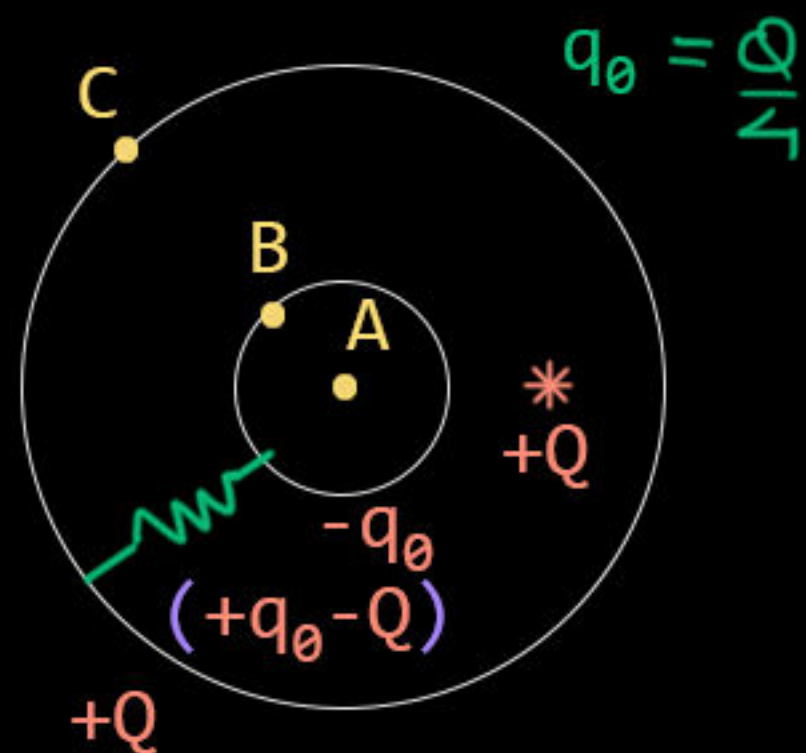
$= \int dq (V_B - V_C)$

$= \int_0^{q_0} \left(\frac{kQ}{6r} - \frac{2}{3} \frac{kq}{r} \right) dq$

$= \frac{kQ^2}{r} \left(\frac{1}{24} - \frac{1}{48} \right)$

$= \frac{Q^2}{192\pi\epsilon_0 r}$

Youtube / Arihant Senior



25. When a steady state is reached after closing the switch S_1 , the other switch S_2 is closed. How much total heat is dissipated after the switch S_2 is closed until a new steady state is reached?

(a) Heat dissipated in each resistor is $\frac{Q^2}{48\pi\epsilon_0 r}$.

(b) Heat dissipated in R_1 is $\frac{Q^2}{24\pi\epsilon_0 r}$ and no heat is dissipated in R_2 .

(c) Heat dissipated in R_2 is $\frac{Q^2}{24\pi\epsilon_0 r}$ and no heat is dissipated in R_1 .

(d) Both the resistors dissipate different amounts of heat, total amount

of which is $\frac{Q^2}{24\pi\epsilon_0 r}$.

Method 1

⇒ No change inside.
⇒ Nothing goes through R_1
⇒ Heat(R_1) = 0 //

$$\Rightarrow \frac{k(-q_1)}{r} + \frac{kQ}{2r} + \frac{k(q_2 + q_1 - Q)}{3r} = \frac{kq_2}{3r} = 0 \Rightarrow q_1 = Q/4, q_2 = 0$$

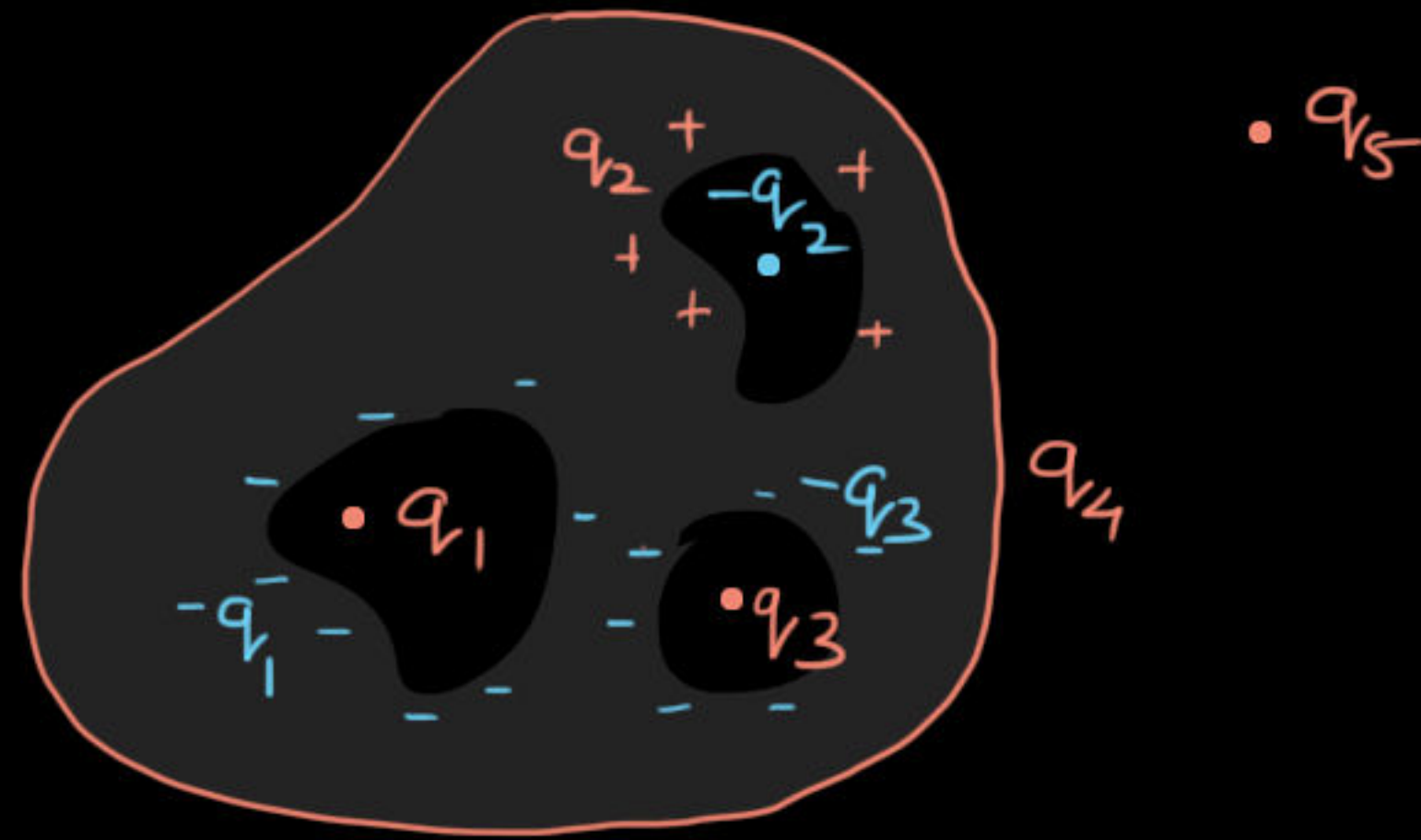
For outer shell: $\Delta U = -\text{Heat}$

$$\Rightarrow 0 - \frac{1}{2} \frac{kQ^2}{(3r)} = -\text{Heat} \Rightarrow \text{Heat}(R_2) = \frac{Q^2}{24\pi\epsilon_0 r}$$

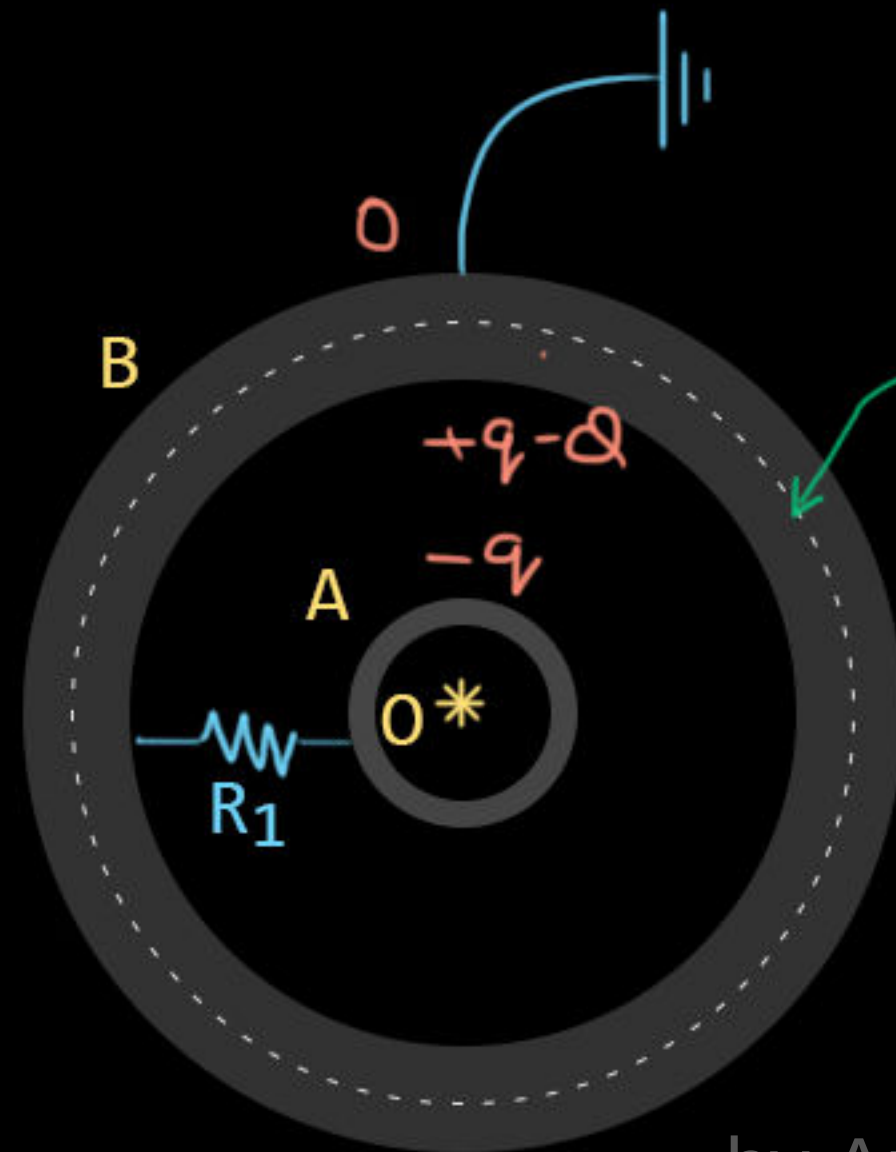
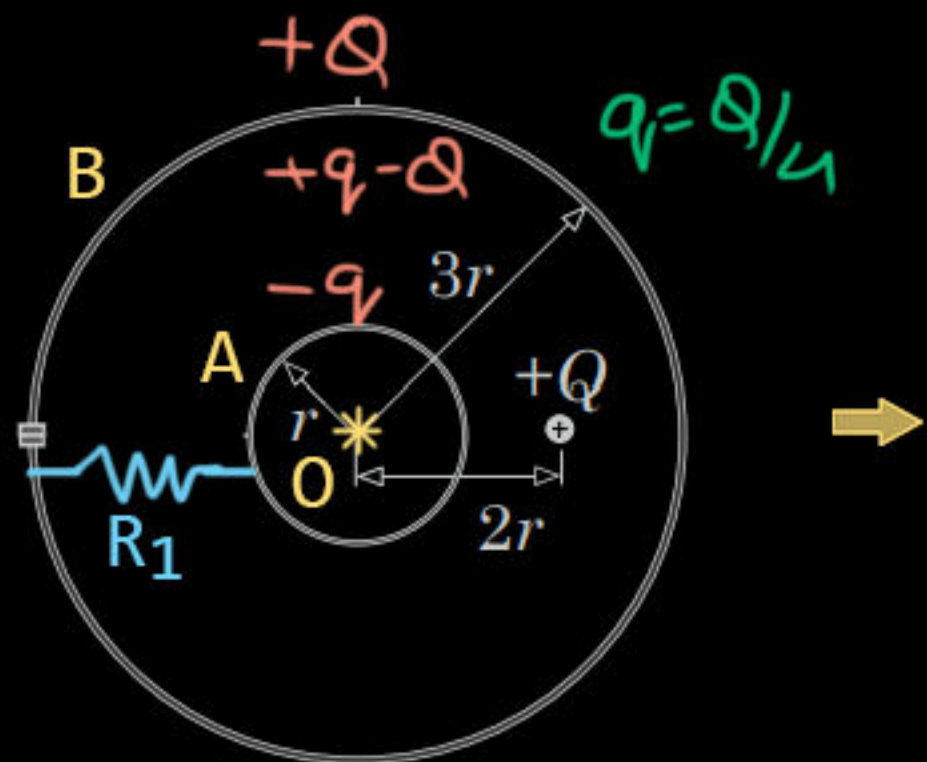
by Ankit Singhvi

Youtube / Arihant Senior

Concept: Once equilibrium is reached:
if we change q_4 or q_5 or turn on any
external electric field, none of the
distribution inside the conductor will
be affected.



Method 2



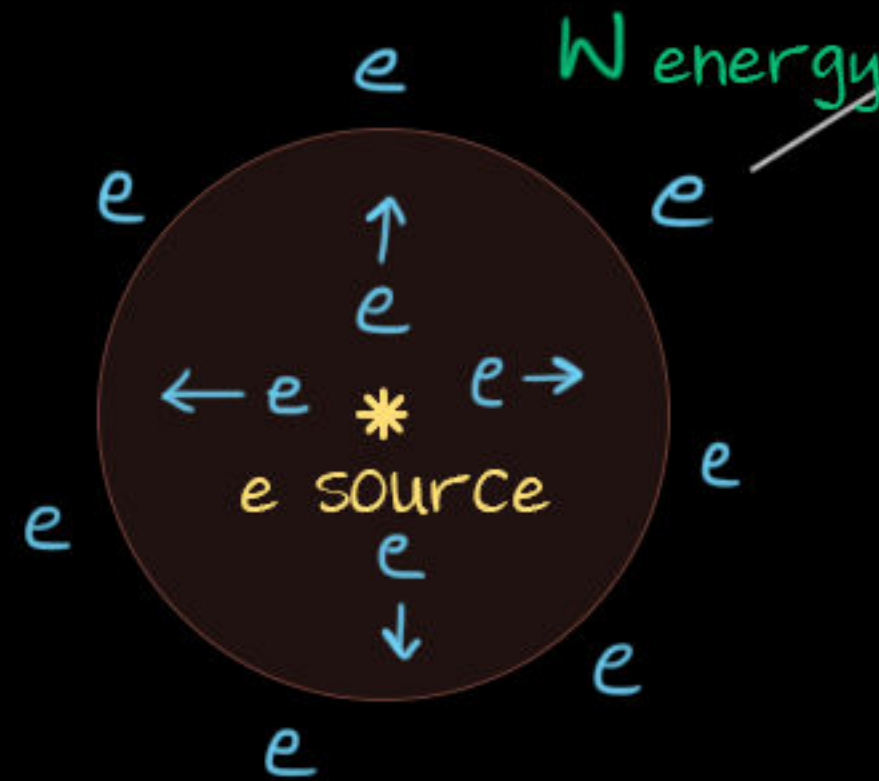
∴ When outer surface is earthed, none of
the inside distribution will change.
So zero charge flows through R_1 and
Heat (R_1) = 0

And vice versa, as when inside distribution is
changed, nothing changes outside.

✓

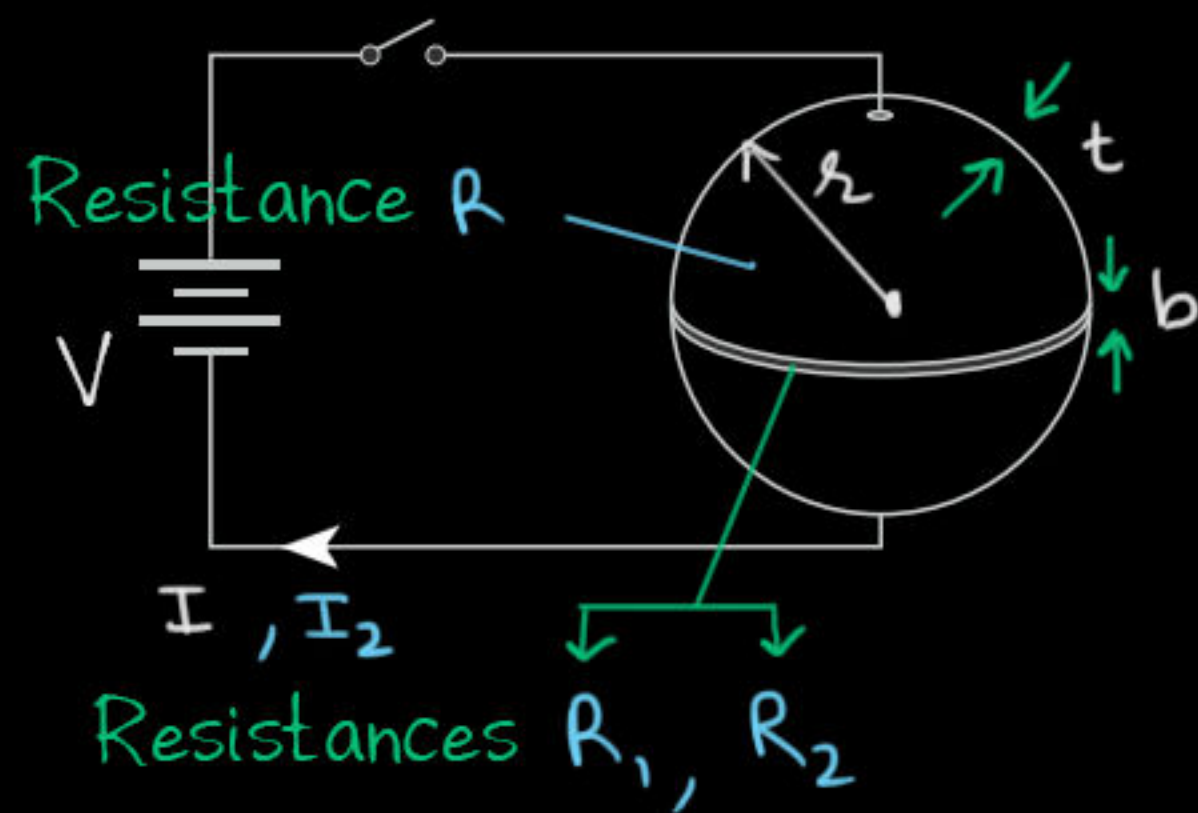
1. An electron gun on a space probe starts emitting an electron beam into the space. Electrons in the beam have energy $W = 9.0 \times 10^4 \text{ eV}$ and the beam is equivalent to electric current $I = 1.0 \text{ mA}$. If the space probe is a metal sphere of radius $r = 1.0 \text{ m}$, find electric field strength at the surface of the space probe 1.0 min after starting the electron gun. Charge on an electron is $e = -1.6 \times 10^{-19} \text{ C}$.

From inside source, electrons are supplied.
They travel to the surface and escape.



$$\begin{aligned}
 W &= |eV| \\
 &= e \frac{kQ}{r} \\
 &= e \left(\frac{kQ}{r^2} \right) r \\
 &= eEr
 \end{aligned}$$

$$\Rightarrow E = \frac{W}{er} //$$



2. A composite spherical shell of radius $r = 10$ cm and wall thickness $t = 0.1$ mm is made of copper except an equatorial strip of width $b = 0.2$ cm, which is made of aluminium. When the poles of the sphere are connected across an ideal battery of terminal voltage $V = 0.1$ mV, a current $I = 5.12$ A flows through it. If the aluminium strip is replaced by an iron strip of identical dimensions and the experiment is repeated with the same battery, what current will flow through the battery? Resistivities of aluminium and iron are $\rho_{Al} = 0.03 \Omega \cdot \text{mm}^2/\text{m}$ and $\rho_{Fe} = 0.10 \Omega \cdot \text{mm}^2/\text{m}$.

$$\frac{V}{R + R_1} = I \quad (1)$$

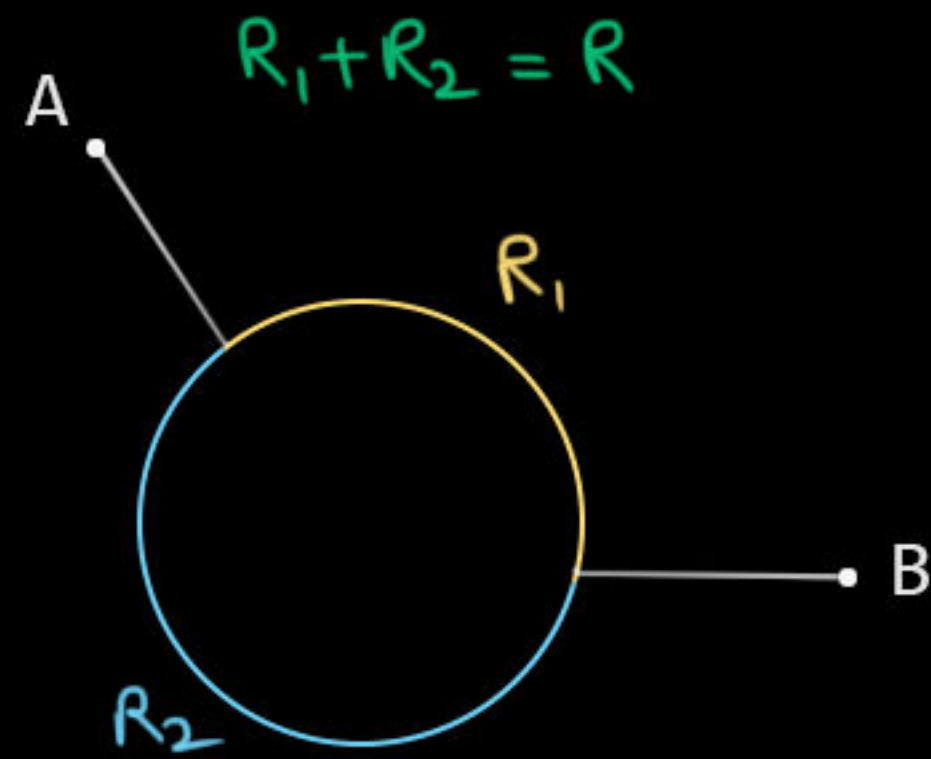
$$R_1 = \frac{\rho_1 b}{2\pi r t}$$

$$\frac{V}{R + R_2} = I_2 \quad (2)$$

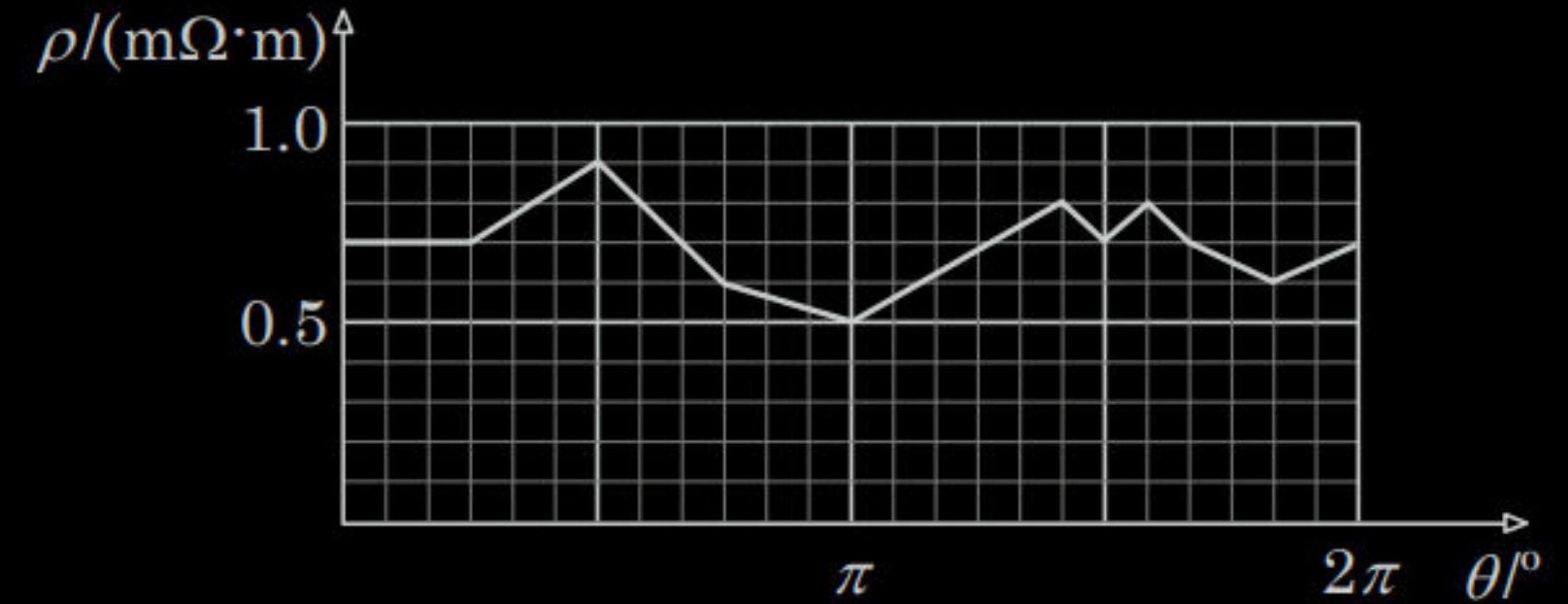
$$R_2 = \frac{\rho_2 b}{2\pi r t}$$

Put R from 1 into 2:

$$I_2 = \frac{V}{\frac{V}{I} - \frac{\rho_1 b}{2\pi r t} + \frac{\rho_2 b}{2\pi r t}}$$



3. A ring of a radius $r = 16$ cm is made of a wire of uniform cross-section of radius $a = 1.0$ mm. Resistivity ρ of the material of the wire is not uniform and varies with angular position θ measured from a fixed radius according to the given graph. How much maximum resistance between a pair of points on this ring can be obtained?



Maths concept:

If $x + y = C$ (constant)

Then, $\frac{xy}{x+y}$ is maximum when, $x = y = \frac{C}{2}$

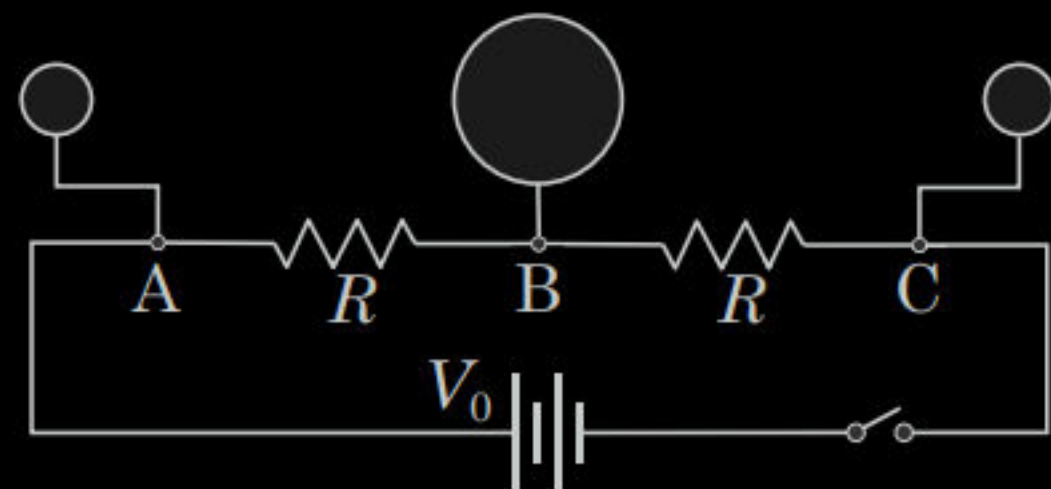
maximum value $\rightarrow = \frac{\frac{C}{2} \cdot \frac{C}{2}}{\frac{C}{2} + \frac{C}{2}} = \frac{C}{4}$

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\Rightarrow R_{\max} = R/4$$

$$R = \int \frac{\rho (r d\theta)}{\pi a^2}$$

$$= \frac{1}{4} \frac{R}{\pi a^2} \int_0^{2\pi} \rho d\theta$$



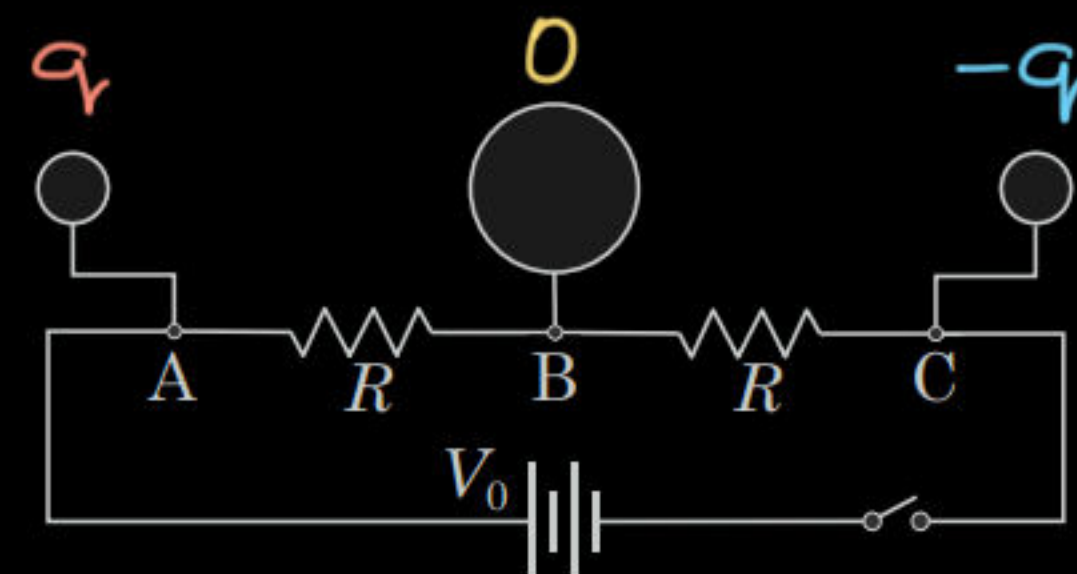
4. Three uncharged metallic balls of radii a , b and a respectively are connected to terminals A, B and C with the help of long thin conductors as shown in the circuit. Find charges established on each of the balls, when a steady state is reached after the switch is closed. Consider the balls to be at great distances from each other as well from the circuit and neglect internal resistance of the battery.

Can NOT comment on absolute potential of spheres. But we can find the potential difference.

$$\frac{kq}{a} - \frac{k(-q)}{a} = V_0$$

$$\Rightarrow q = \frac{aV_0}{2k}$$

$$= 2\pi\epsilon_0 aV_0 //$$

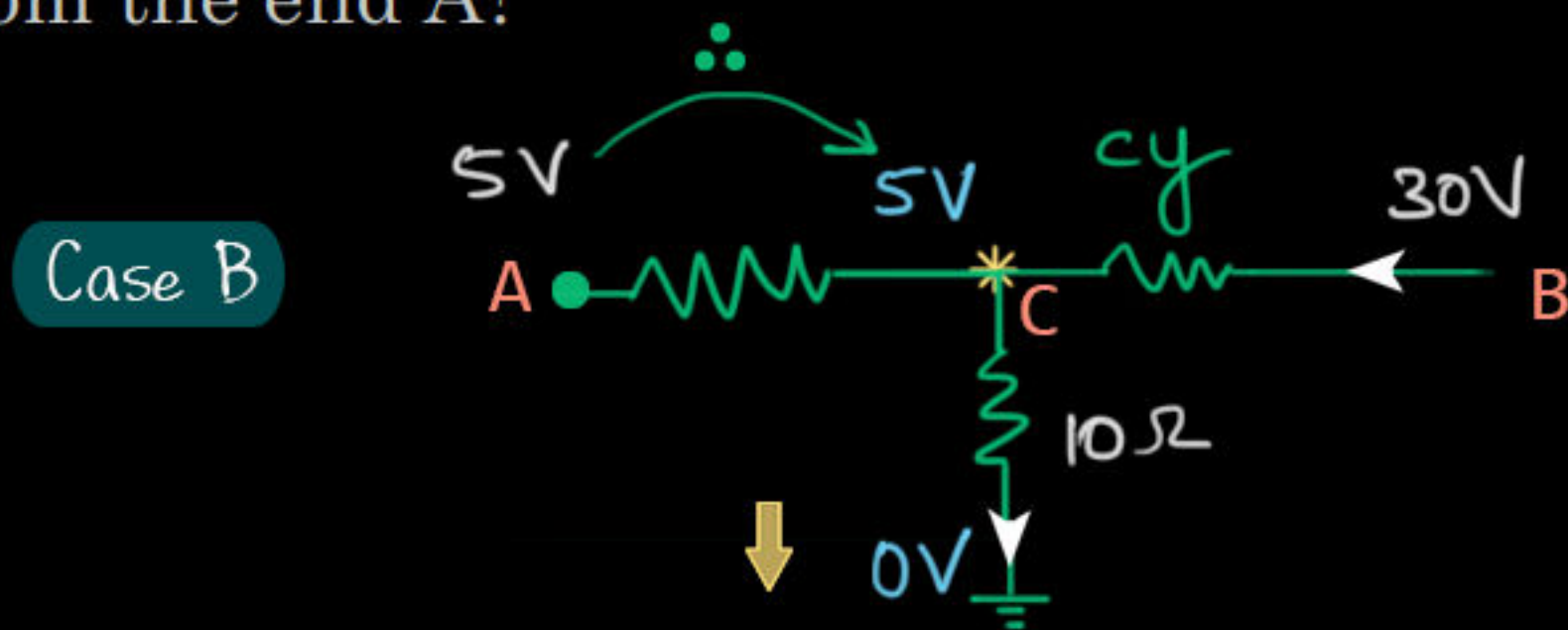
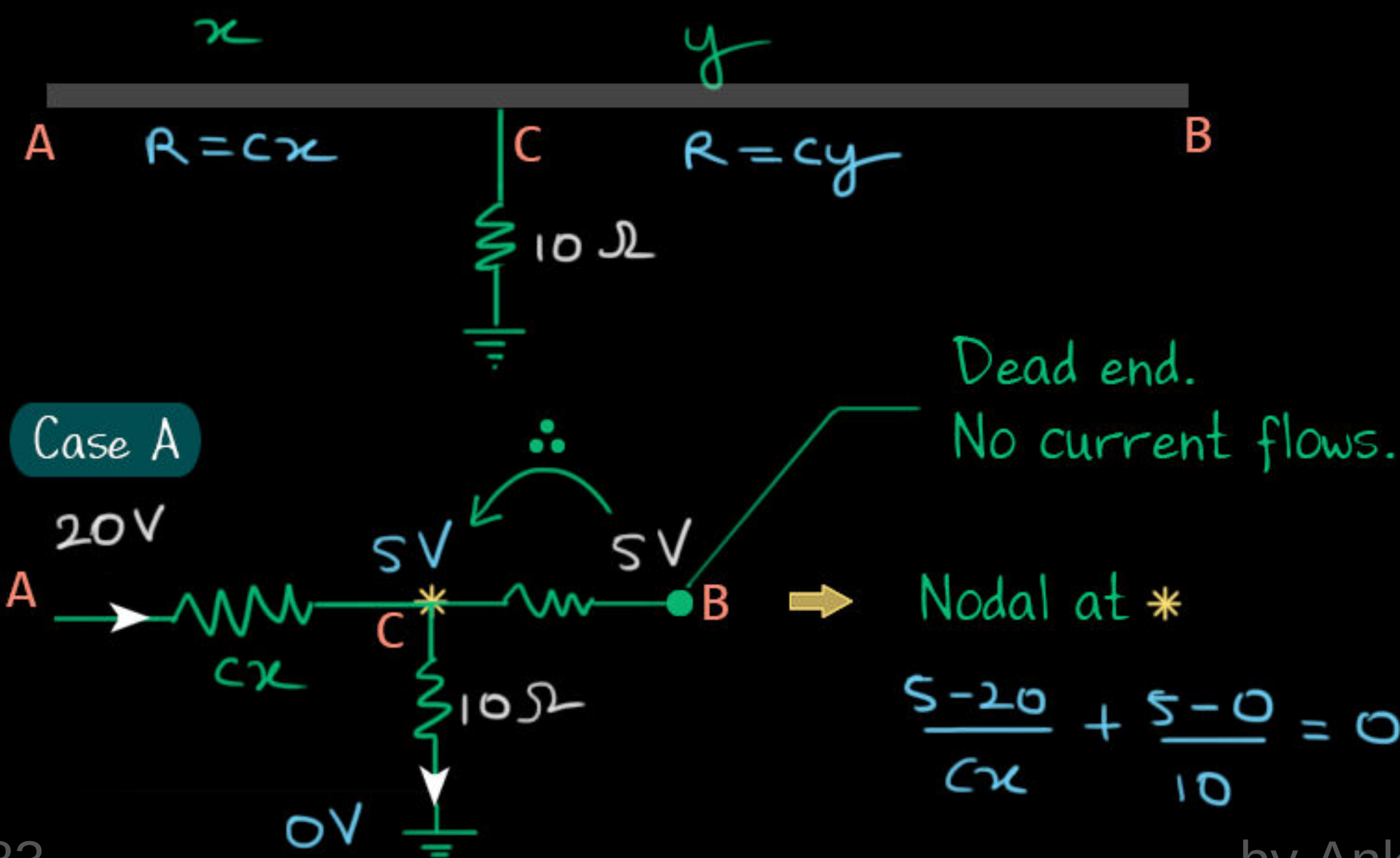


By symmetry equal amount of positive and negative charge will go to A and C respectively.
And B will stay neutral.



Testing method: One end is maintained at a known potential (20/30V). And potential is measured at other end (5V).

5. A uniform underground cable AB connects two telegraph offices, which are 8 km apart. Due to a fault at a place C, the conductor of the cable comes in contact with the earth. Current flowing to the earth at fault location faces $10\ \Omega$ resistance. To locate the fault when an engineer applies 20 V between the end A and the earth, his colleague measures 5 V between the end B and the earth. Moreover, when his colleague applies 30 V between the end B and the earth, the engineer measures 5 V between the end A and the earth. What is the distance of the fault location along the cable from the end A?

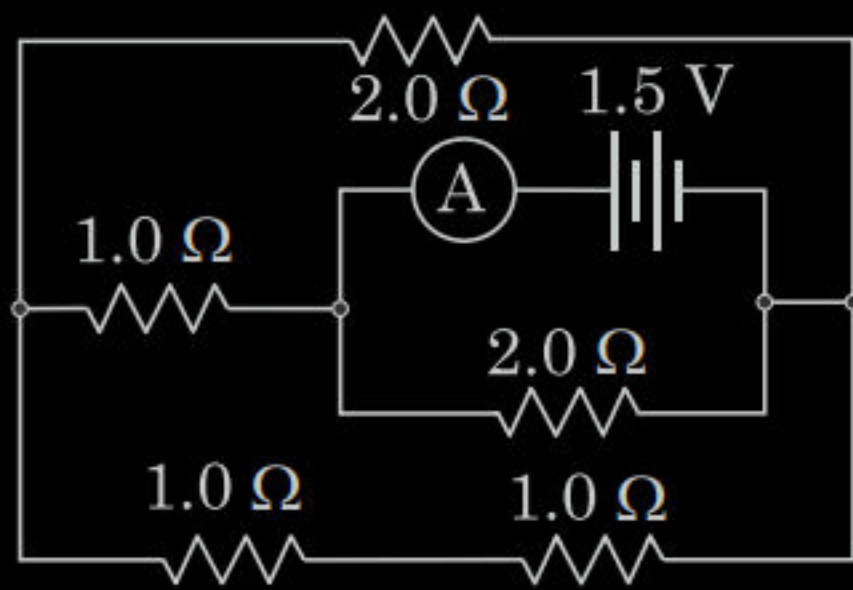


$$\frac{5-30}{cy} + \frac{5-0}{10} = 0 \quad (2)$$

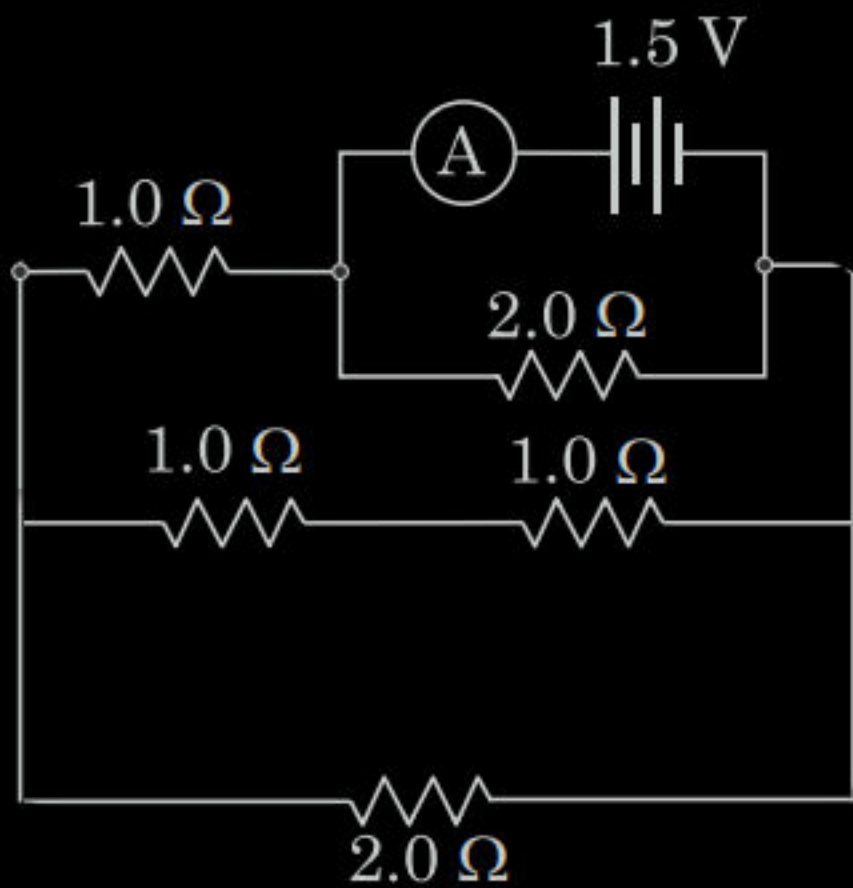
From 1,2: $\frac{x}{y} = \frac{3}{5}$

$\Rightarrow$ Location of C = $8 \left(\frac{x}{x+y} \right) = 8 \left(\frac{1}{1+5/3} \right) = 3 //$

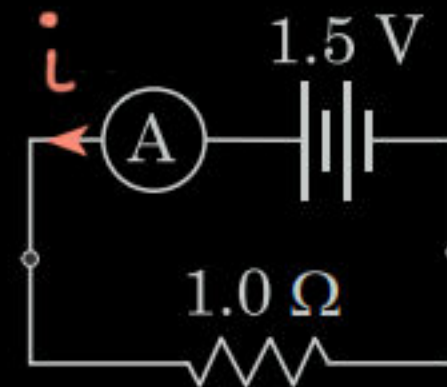
6. The network shown consists of an ideal voltage source, an ideal ammeter and five resistances. Determine reading of the ammeter.



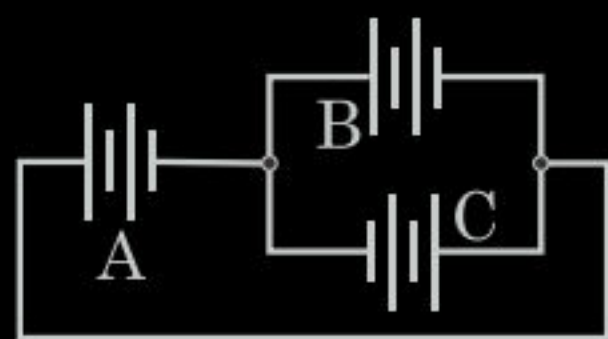
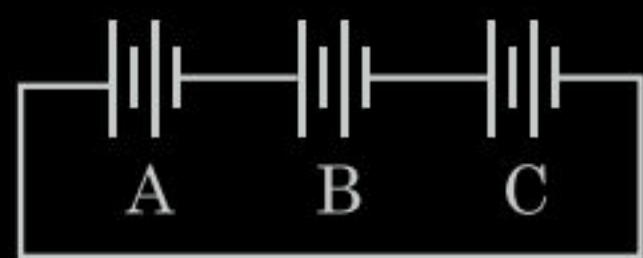
|||



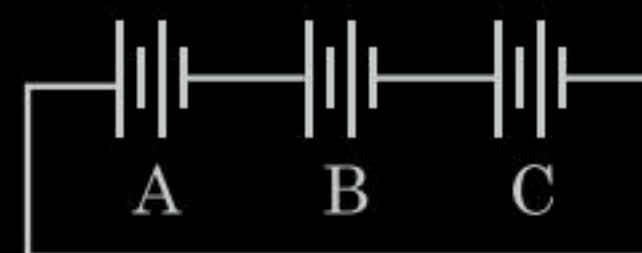
==



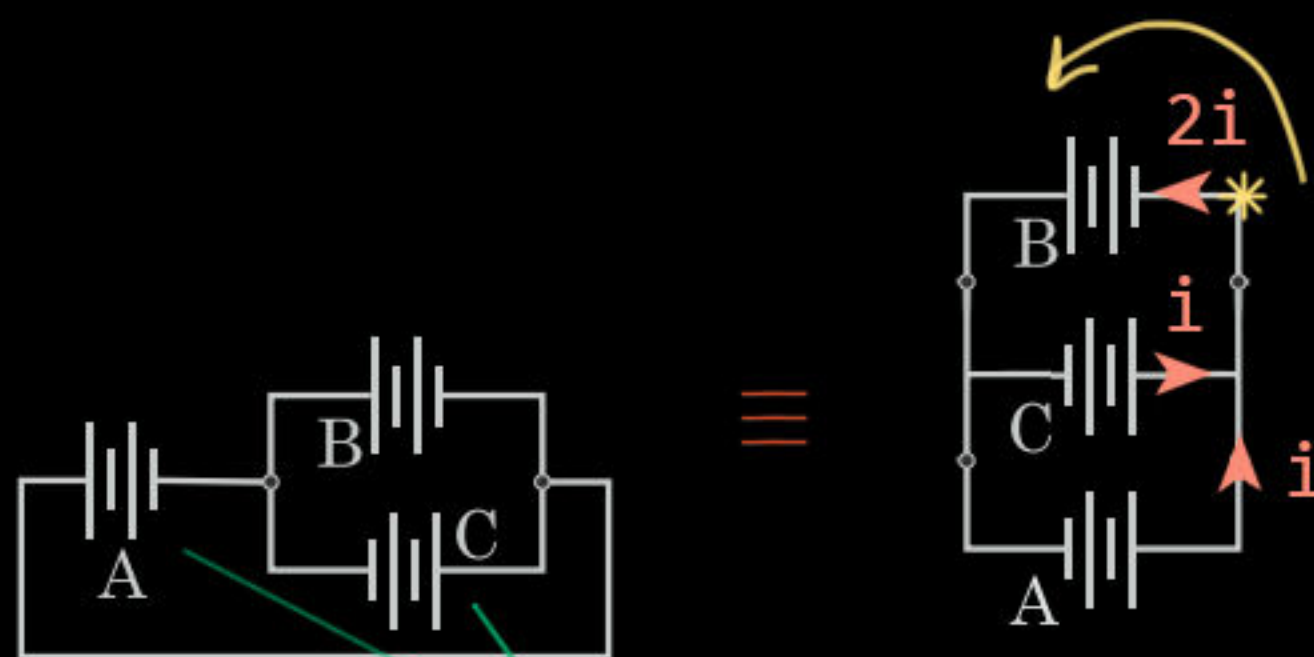
$$\therefore i = \frac{1.5}{1} = 1.5 \text{ Amp}$$



7. An **actual battery** can be represented as a constant voltage source in series with a resistor, known as internal resistance. Three identical actual batteries A, B and C connected as shown in **first figure**, cause a current of **3.0 A** to flow in the circuit. If the batteries were connected as shown in the **second figure**, what would the **current through batteries A, B and C** be?



$$\frac{3V}{3r} = 3 \Rightarrow V/r = 3 \quad (1)$$



Similarly aligned

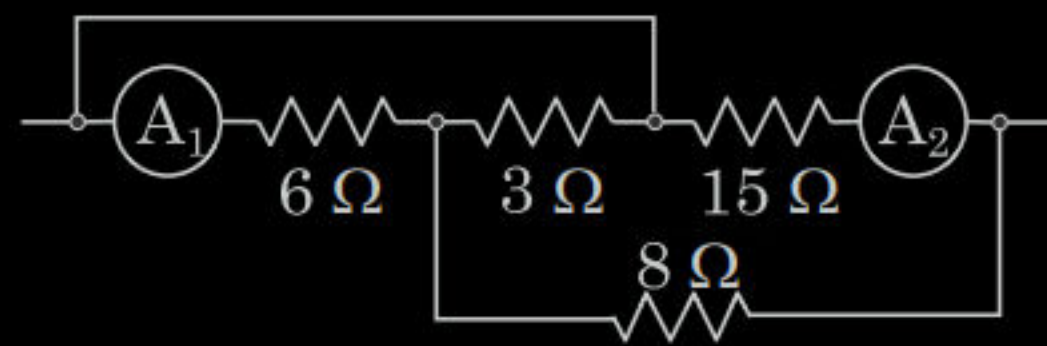
KVL in top loop:

$$V - 2ir + V - ir = 0$$

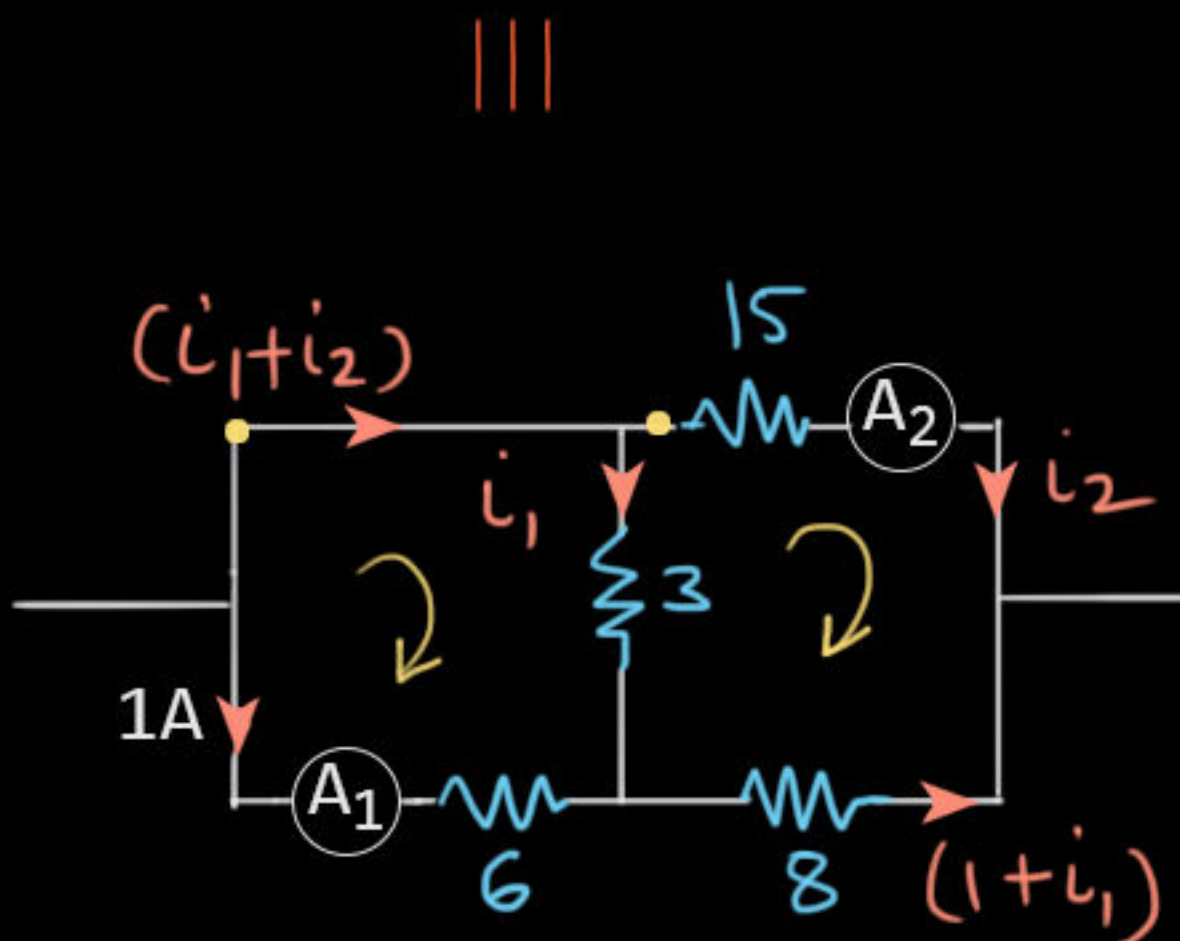
$$2V = 3ir \quad (2)$$

From 1,2: $i = 2$

$$\Rightarrow i_A = i_C = 2, \quad i_B = 4$$



8. A portion of a large network is shown in the figure. Values of the resistances are indicated in the figure and both the ammeters A_1 and A_2 are ideal. If reading of the ammeter A_1 is 1.0 A, how much will the ammeter A_2 read?



Tip: Avoid negative values when deciding upon variables.

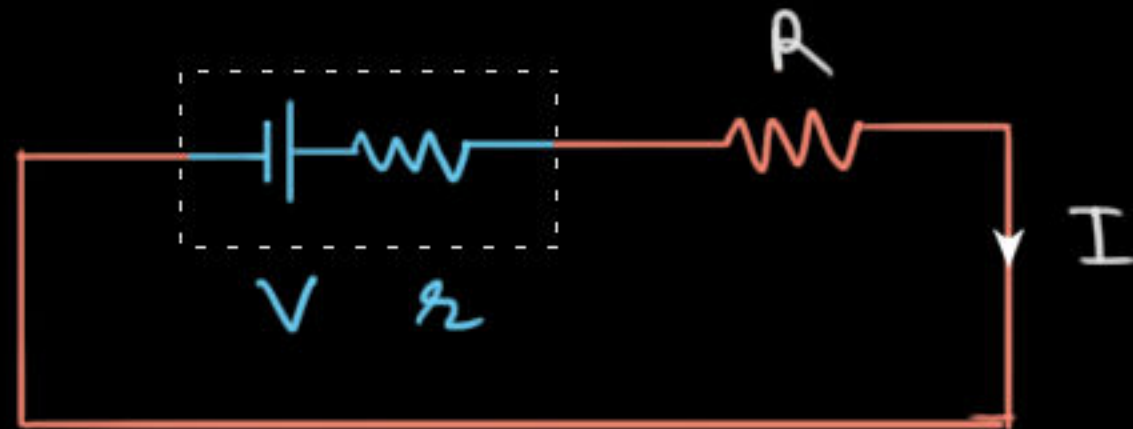
KVL in both loops:

$$-3i_1 + 6(1) = 0 \Rightarrow i_1 = 2$$

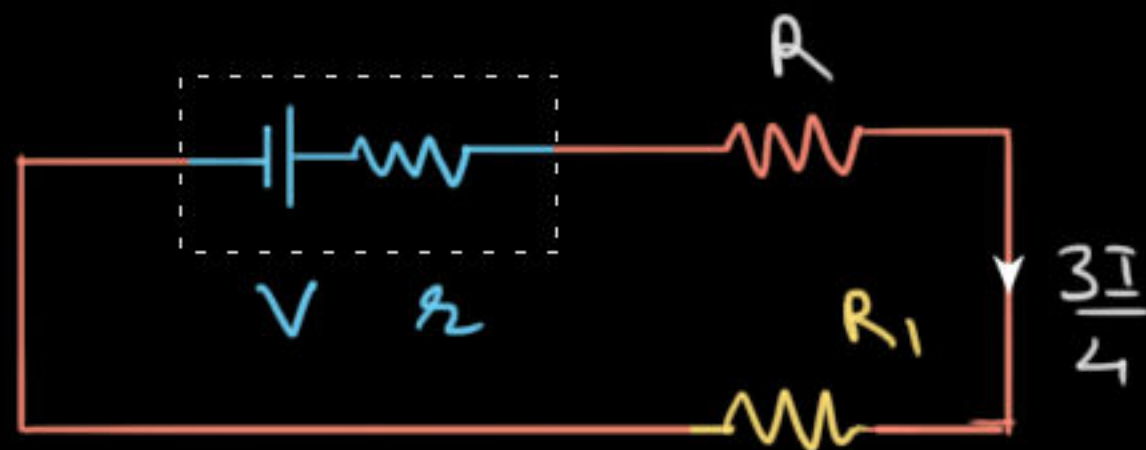
$$-15i_2 + 8(1+i_1) + 3i_1 = 0 \Rightarrow i_2 = 2 //$$

Note: If we don't consider internal resistance we get incompatible equations.

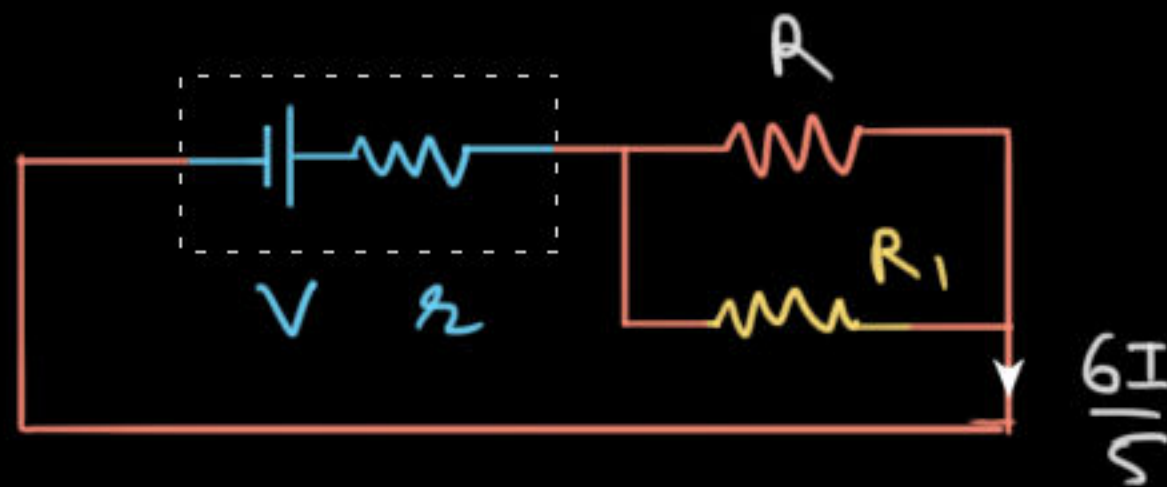
Battery V , with internal resistance r .



$$\frac{V}{r+R} = I \quad (1)$$



$$\frac{V}{r+R+R_1} = \frac{3I}{4} \quad (2)$$



$$\frac{V}{r + \frac{RR_1}{R+R_1}} = \frac{6I}{5} \quad (3)$$

9. When a resistance $R = 100 \Omega$ is connected across terminals of a battery, a current I flows through it and when an additional unknown resistance is connected in series with R , a current $3I/4$ flows through the battery. If the unknown resistance is connected in parallel with R , the battery current becomes $6I/5$. Find value of the unknown resistance.

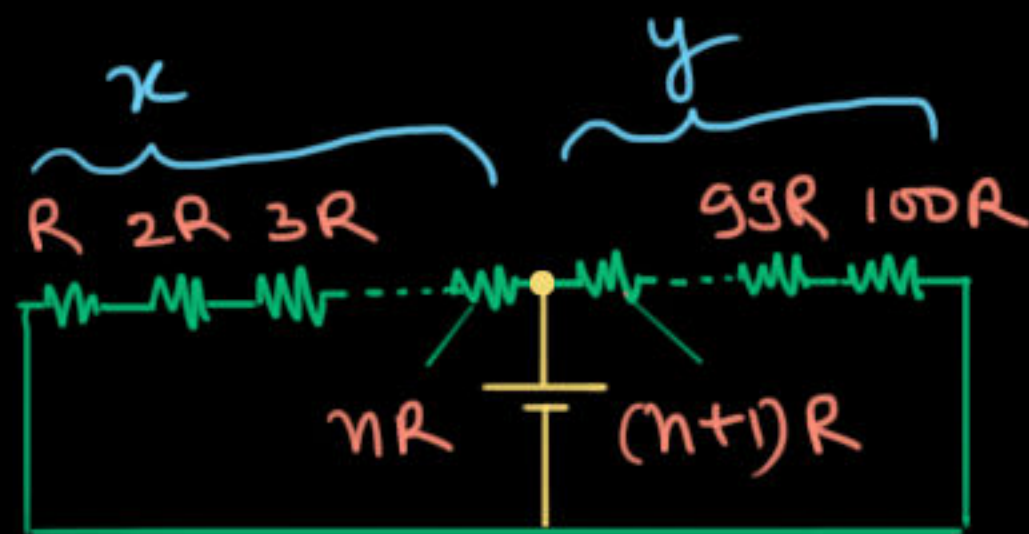
From 1,2,3:

3 equations, 3 variables:

$$\frac{V}{I}, r, R_1$$

Reject

Solving: $R_1 = R_{||} = -2R$
 Youtube / Arihant Senior



10. Resistors $R, 2R, 3R, \dots, 100R$ are connected in series and free ends of this series are connected to make a closed loop. An ideal battery is connected between the junction of resistor R and $100R$ and the junction of resistors nR and $(n+1)R$. Find value of n for which a minimum current flows through the battery?

Concept:

Sum of all the resistors is constant.

So, when broken in two parallel resistors, the equivalent resistance is maximum when both halves are equal (or as close as possible in our case)

Mathematically, if: $x + y = \text{Constant}$

then: $\frac{xy}{x+y}$ is maximum @ $x = y = \frac{x+y}{2}$

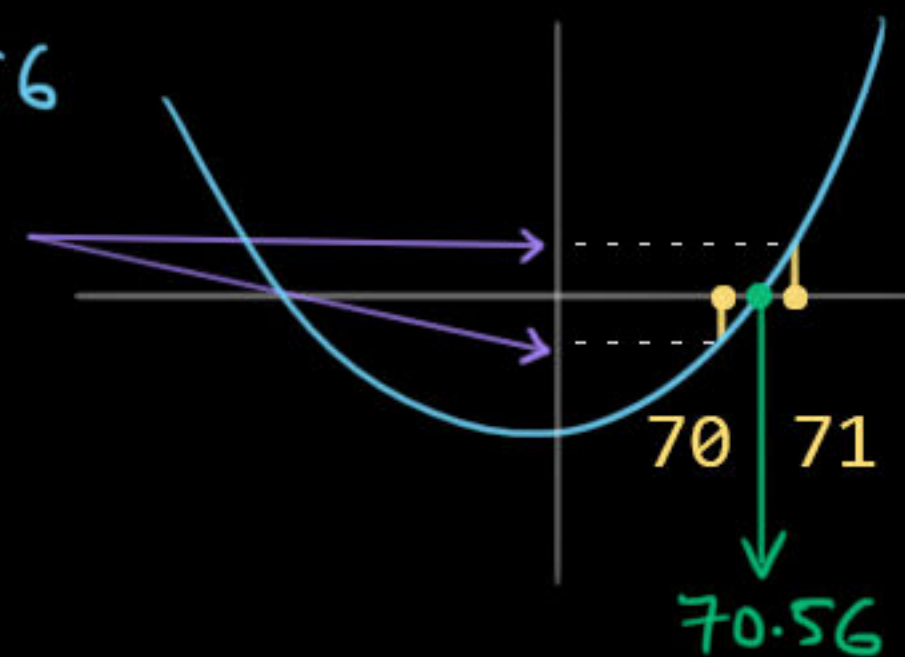
Sol: Junction is between $(n$ and $n+1)$

Resistance of first n resistors should be as close to half of total resistance as possible.

$$\therefore \left[\frac{n(n+1)}{2} \right] R - \frac{1}{2} \left[\frac{100 \times 101}{2} \right] R \approx 0 \quad (1)$$

$$\Rightarrow n = 70.56, -71.56$$

Can not say for sure, which one is closer to zero.
So must calculate further

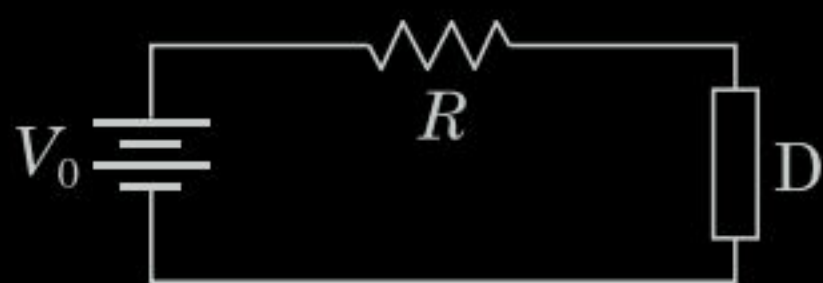


With, $n = 70$, (1) is -80

With, $n = 71$, (1) is $+62$ ✓

Clearly, (1) is nearer to 0 with $n = 71$. So that's the answer.
by Ankit Singhvi

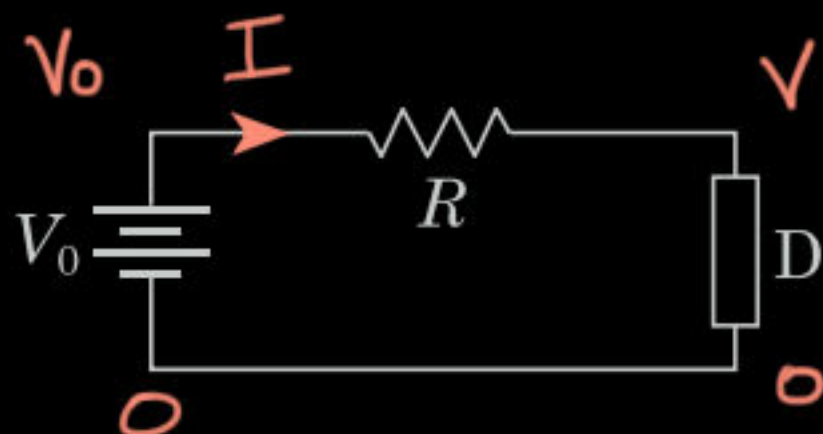
Youtube / Arihant Senior



11. A resistance R and a nonlinear device D are connected with an ideal battery as shown in the given network. Current I through the nonlinear device varies with voltage V across the device according to equation $I = kV^2$. Here k is a positive constant. Find expression for the current in the circuit.

Let voltage across device be V .

Current I .



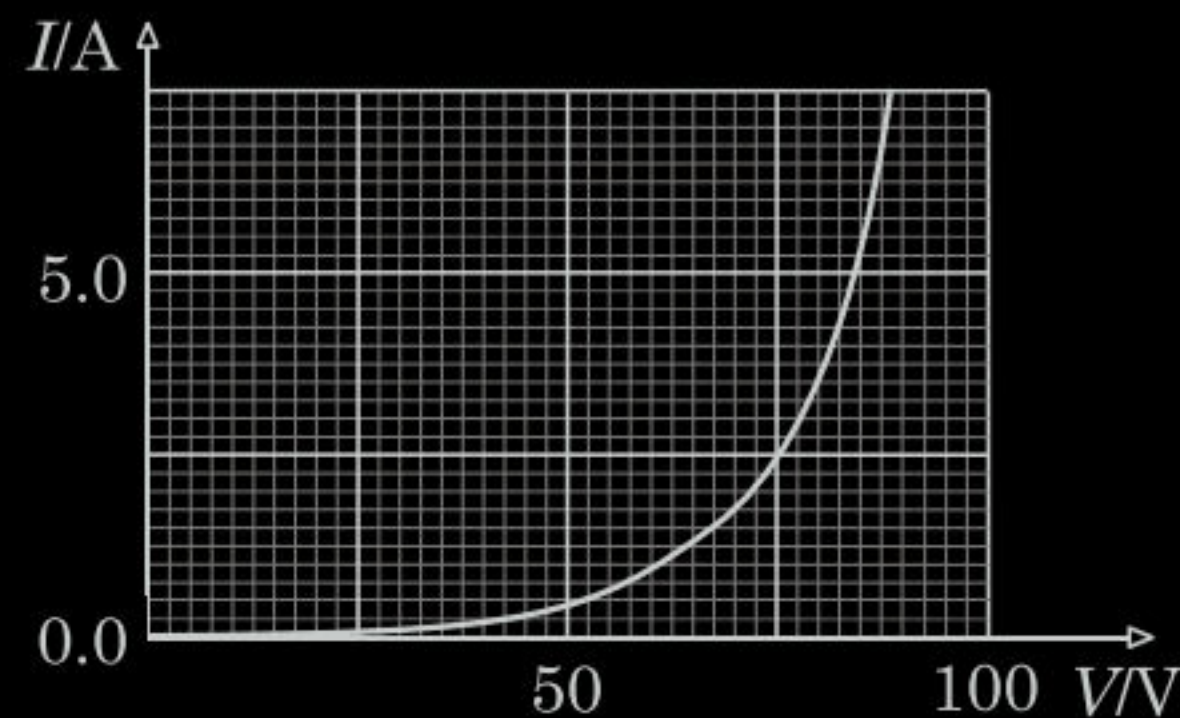
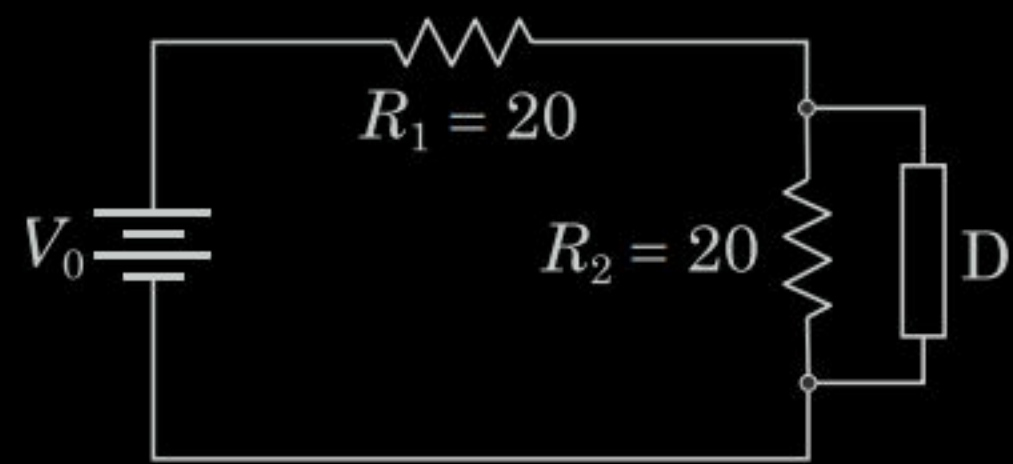
$$\text{KVL: } V_0 - IR - V = 0$$

$$\Rightarrow V_0 - IR - \frac{\sqrt{I}}{\sqrt{k}} = 0$$

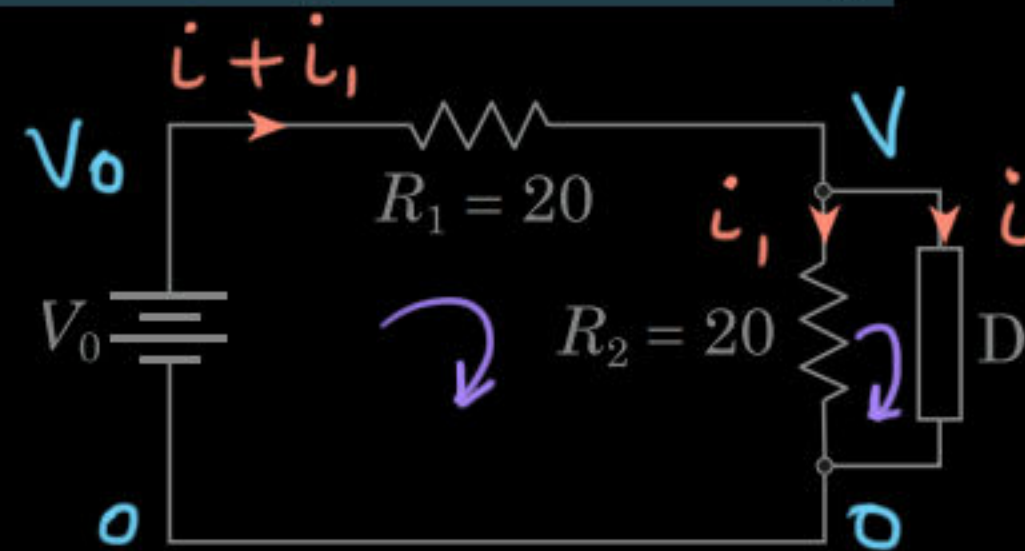
$$\Rightarrow \sqrt{k}RI + \sqrt{I} - V_0\sqrt{k} = 0$$

$$\Rightarrow \sqrt{I} = \frac{-1 + \sqrt{1 + 4kV_0R}}{2\sqrt{k}R}$$

$$\Rightarrow I = \frac{[\sqrt{1 + 4kV_0R} - 1]^2}{4kR^2}$$



12. In the given circuit, an ideal battery of terminal voltage $V_0 = 200$ V is connected across a network consisting of resistors $R_1 = 20 \Omega$ and $R_2 = 20 \Omega$ and a nonlinear device D. Volt-ampere (V-I) characteristic of the nonlinear device D is also shown in the adjoining figure. Find voltage across the resistance R_2 .



KVL in two loops:

$$V_0 - (i + i_1)R_1 - i_1R_2 = 0 \quad (1)$$

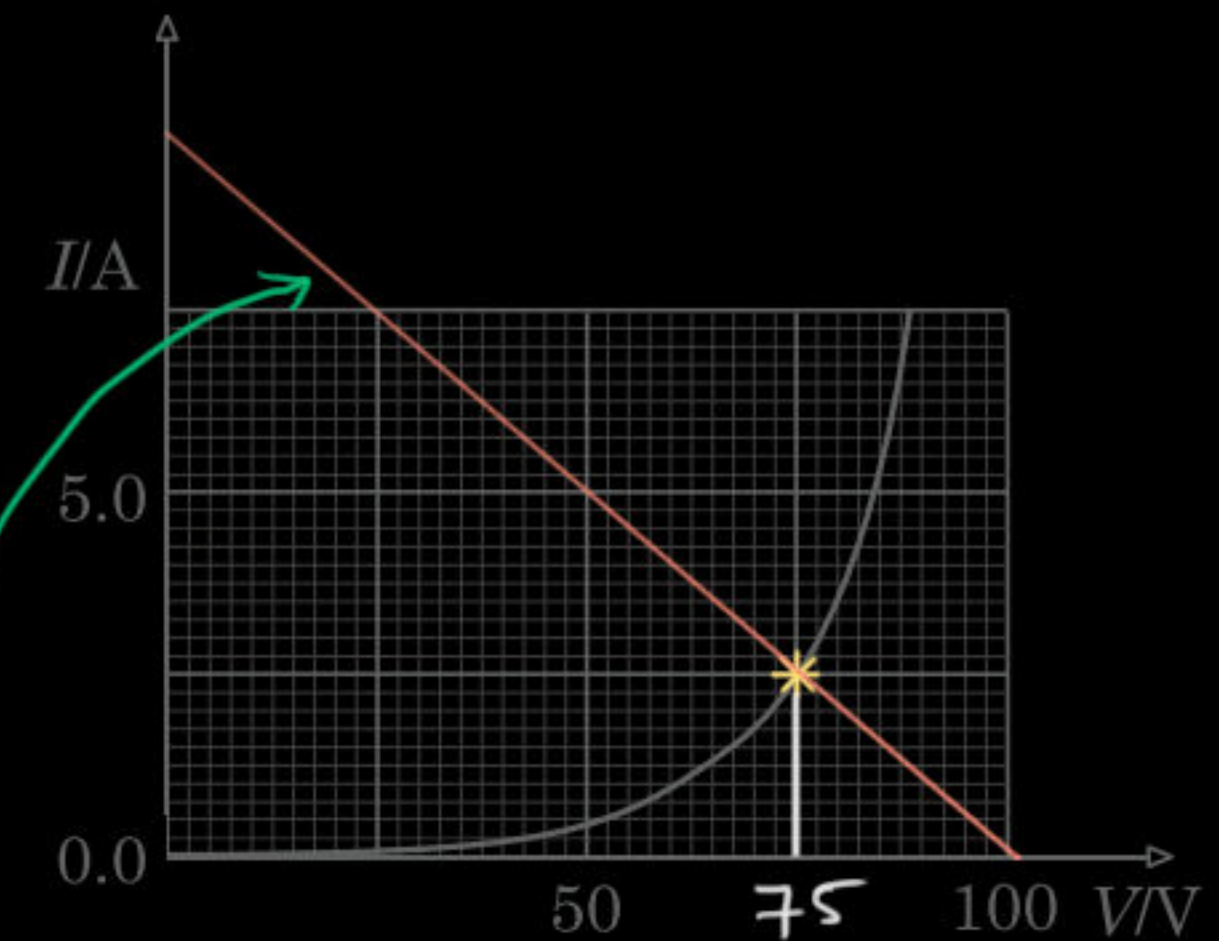
$$0 + i_1R_2 = V \quad (2)$$

Eliminating i_1 and putting values:

$$200 - \left(i + \frac{V}{20}\right)20 - V = 0$$

$$\Rightarrow V = 10i - 100$$

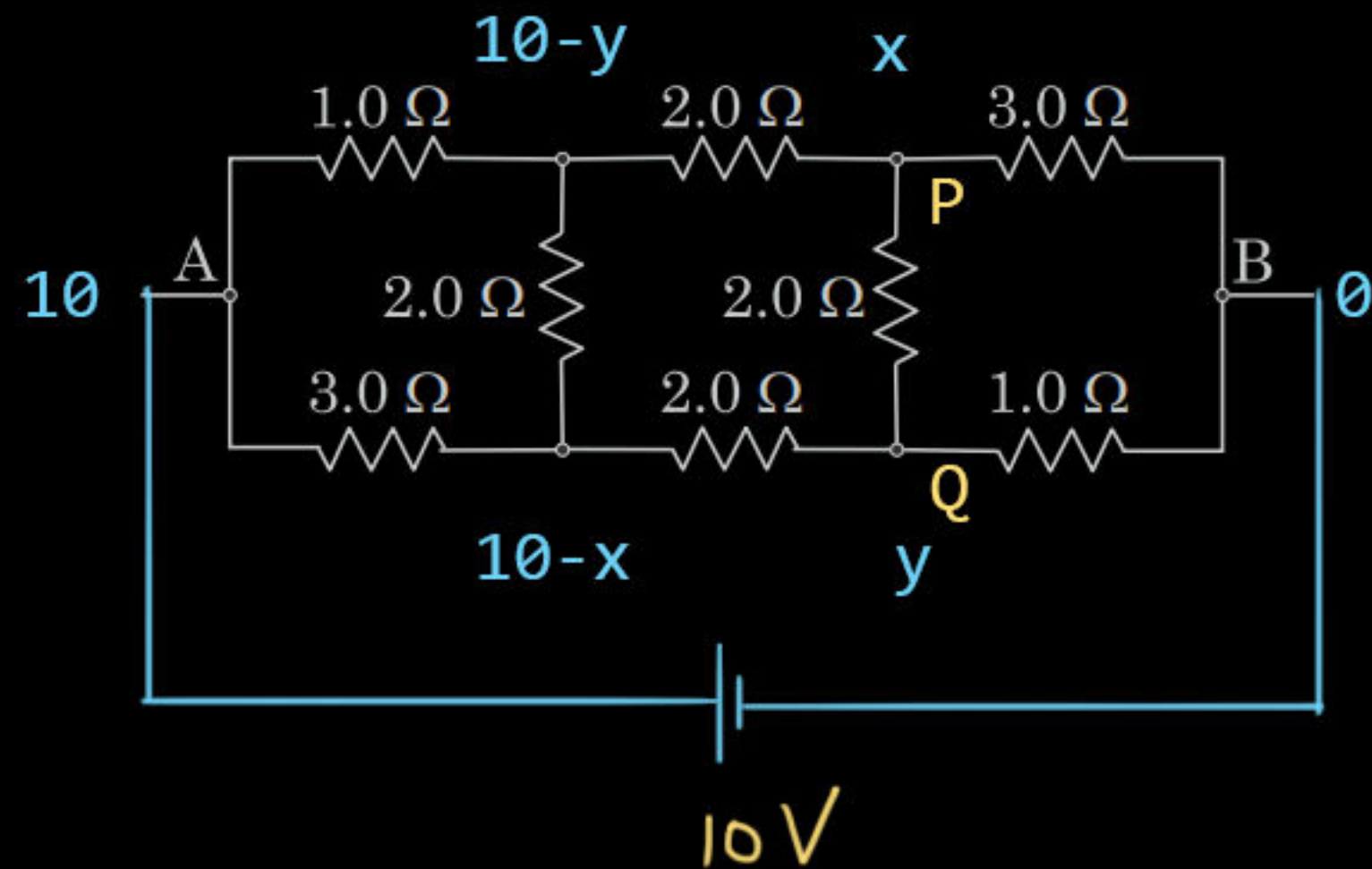
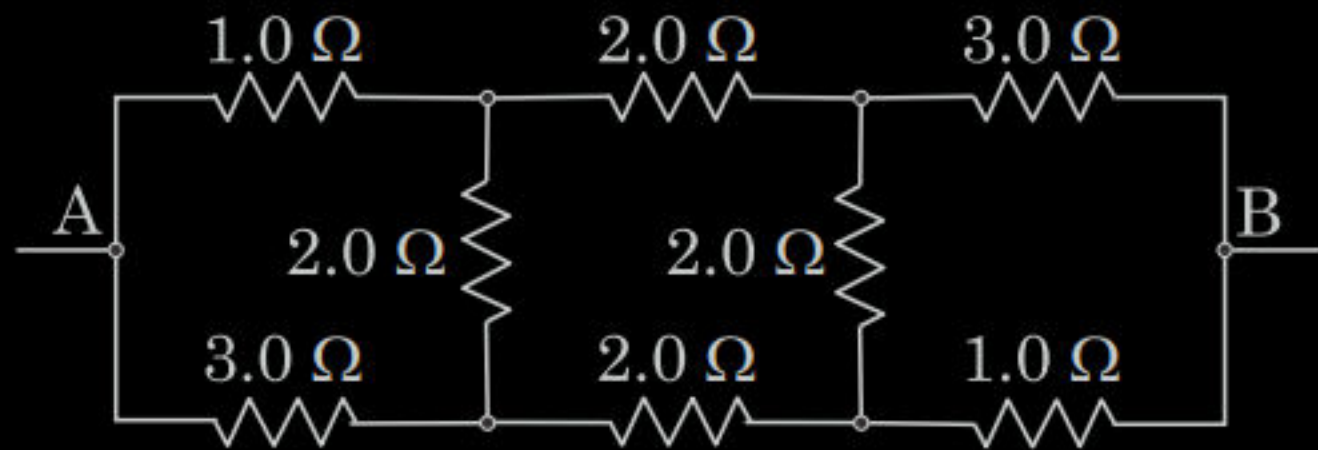
by Ankit Singhvi



$$\therefore V = 75$$

Youtube / Arihant Senior

13. Determine equivalent resistance between terminals A and B of the network shown.



$$P: \frac{x - (10 - y)}{2} + \frac{x - y}{2} + \frac{x}{3} = 0 \quad (1)$$

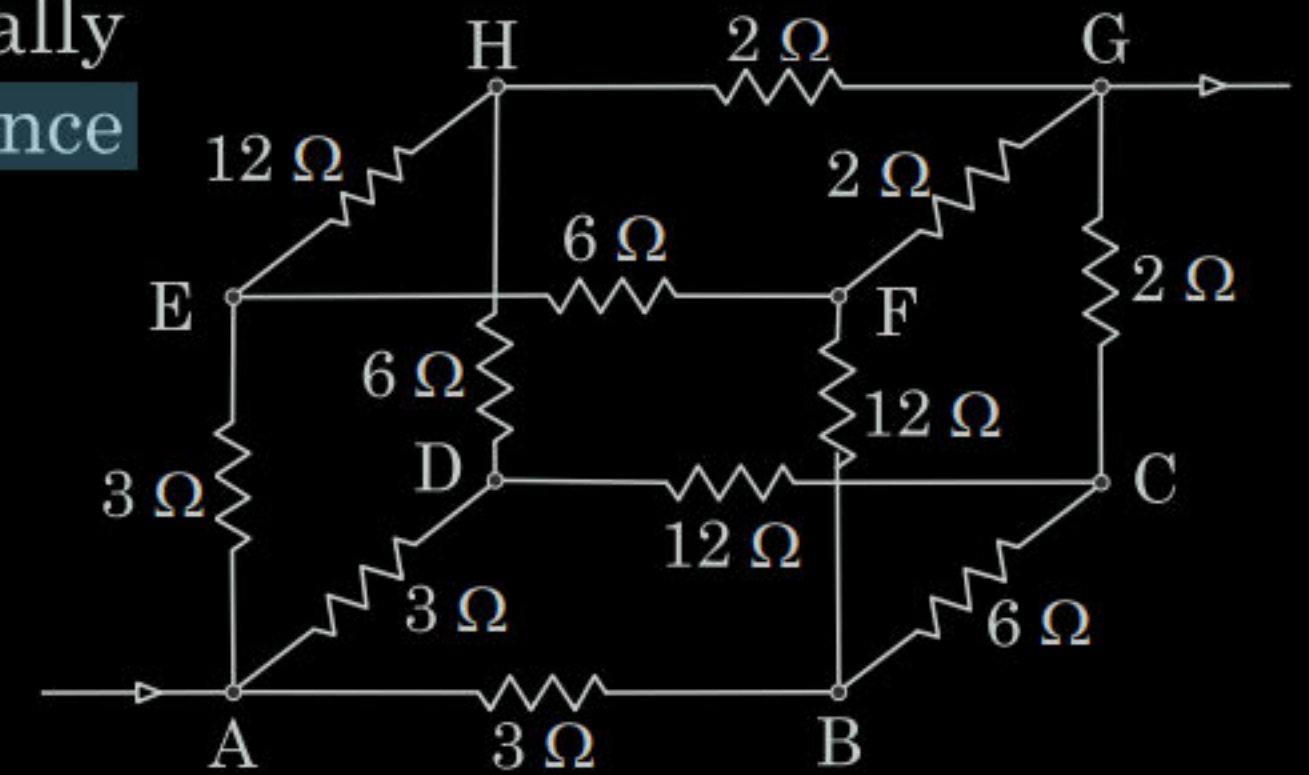
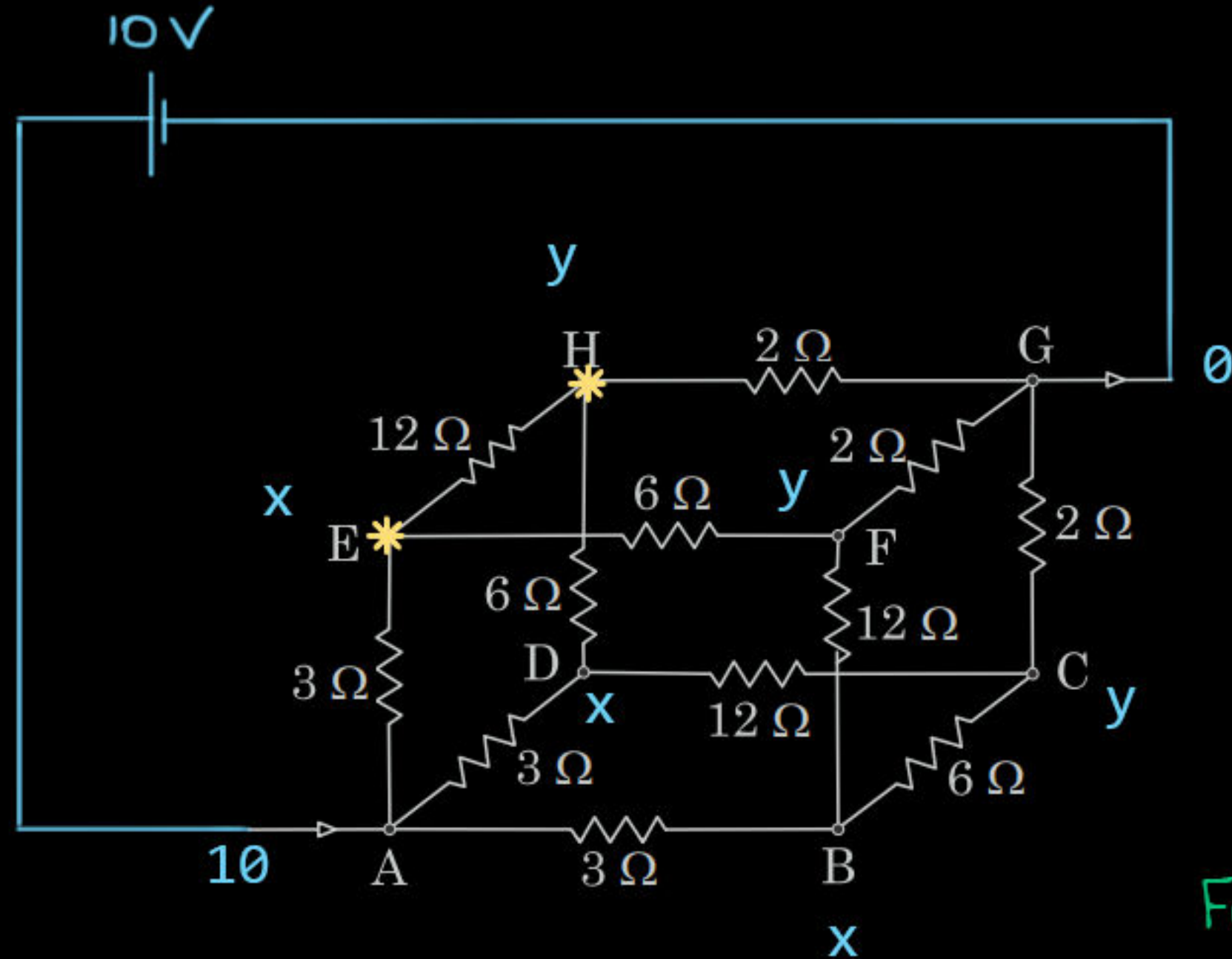
$$Q: \frac{y - (10 - x)}{2} + \frac{y - x}{2} + \frac{y}{1} = 0 \quad (2)$$

$$\text{From 1, 2: } x = \frac{15}{4}, y = \frac{5}{2}$$

$$\Rightarrow R = \frac{V_0}{i} = \frac{10}{\frac{x}{3} + \frac{y}{1}} = \frac{8}{3} \Omega$$

Youtube / Arihant Senior

14. Twelve resistances are arranged in the edges of a cube and electrically connected only at the corners of the cube. Find equivalent resistance between the corners A and G.



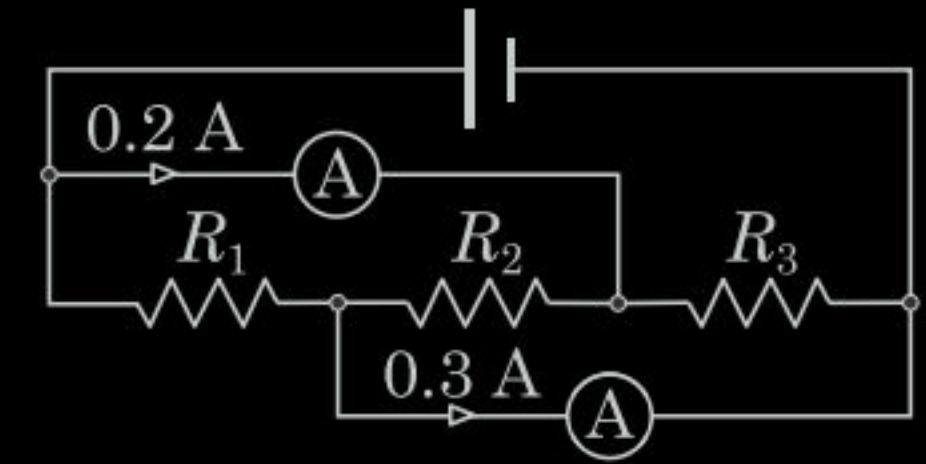
$$E: \frac{x-10}{3} + \frac{x-y}{12} + \frac{x-y}{6} = 0 \quad (1)$$

$$H: \frac{y-x}{12} + \frac{y-x}{6} + \frac{y}{2} = 0 \quad (2)$$

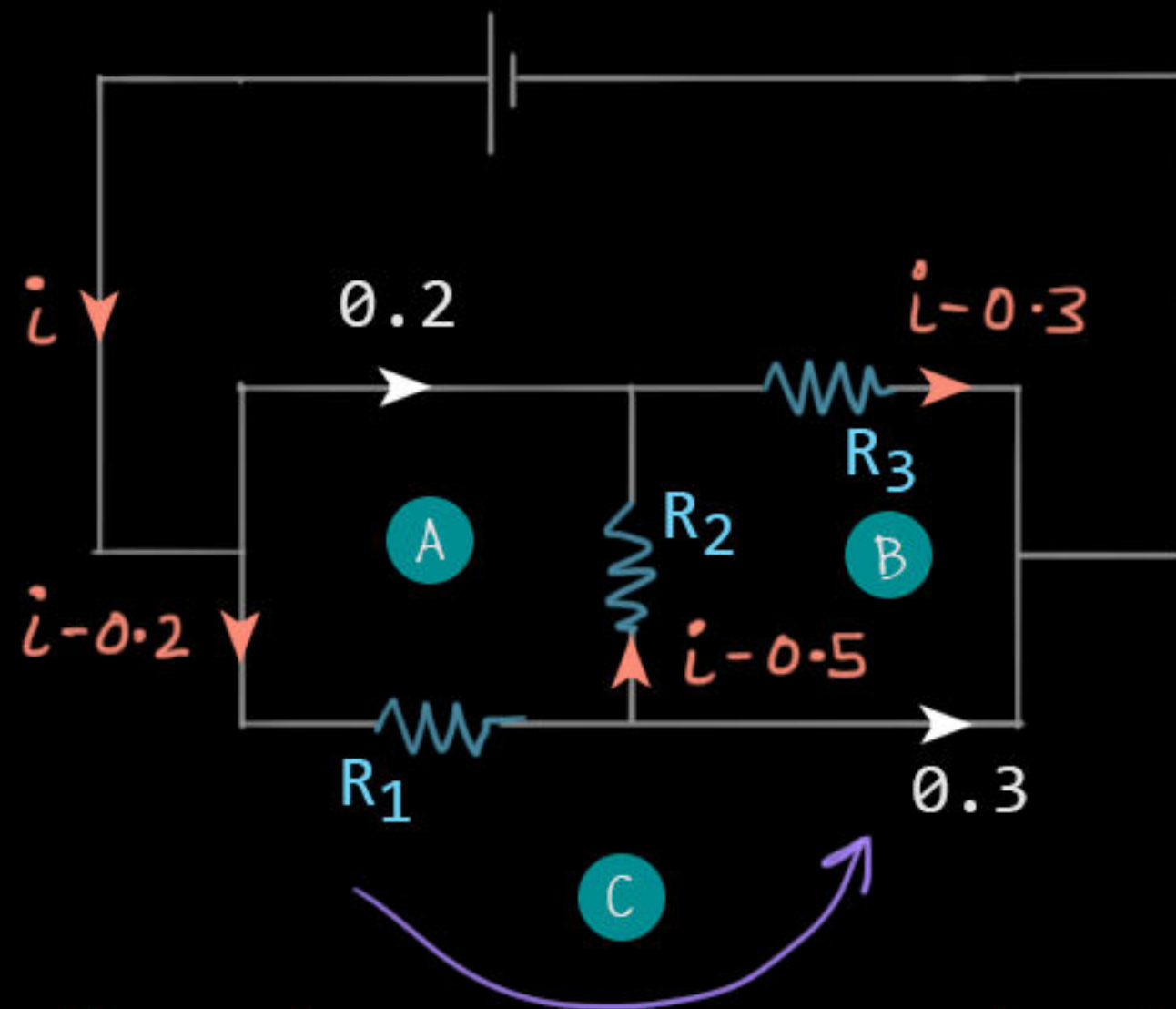
From 1,2: $x = 20/3, y = 20/9$

$$R = \frac{10}{i} = \frac{10}{\frac{5y}{2} + \frac{y}{2} + \frac{y}{2}} = \frac{10}{\frac{3 \cdot 20}{2 \cdot 9}} = 3 \Omega //$$

15. A circuit shown in the figure consists of an ideal battery, three resistances R_1 , R_2 and R_3 and two ideal ammeters. At present the ammeters read 0.2 A and 0.3 A as shown. If on interchanging places of any two of these resistances, the readings of the ammeters remain unaffected, how much current is the battery supplying?



- a) If R_1 and R_2 are interchanged.
- a) If R_2 and R_3 are interchanged.
- c) If R_1 and R_3 are interchanged.



Current, i won't change on interchanging any two resistors !!

Loop A $(i-0.5)R_2 + (i-0.2)R_1 = 0$ ①

$R_1 \leftrightarrow R_2 : (i-0.5)R_1 + (i-0.2)R_2 = 0$ ②

From 1,2: $\frac{R_2}{R_1} = \frac{R_1}{R_2} \Rightarrow R_1 = R_2 \Rightarrow i = 0.35 //$

Similarly:

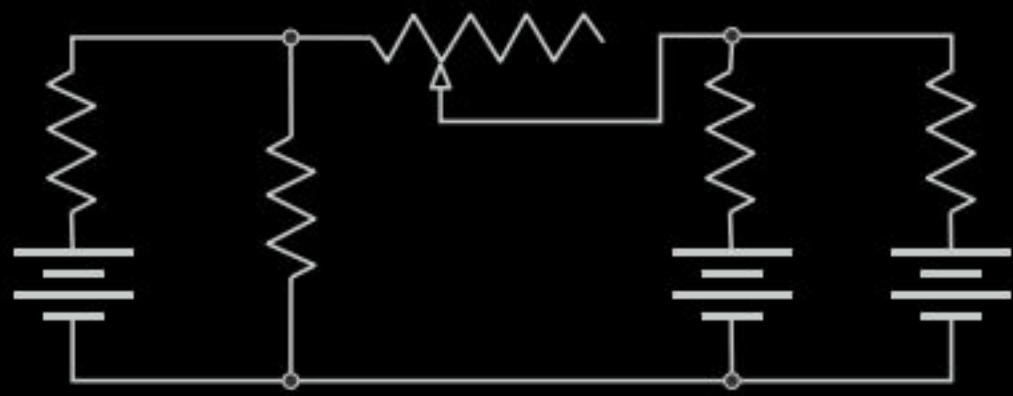
Loop B $(i-0.5)R_2 + (i-0.3)R_3 = 0 \Rightarrow R_1 = R_2$
 $R_2 \leftrightarrow R_3 : (i-0.5)R_3 + (i-0.3)R_2 = 0 \Rightarrow i = 0.4 //$

Big Loop C $(i-0.2)R_1 - (i-0.3)R_3 = 0 \Rightarrow R_1 = R_3$
 $R_1 \leftrightarrow R_3 : (i-0.2)R_3 - (i-0.3)R_1 = 0 \Rightarrow 0.1 = 0 \times$

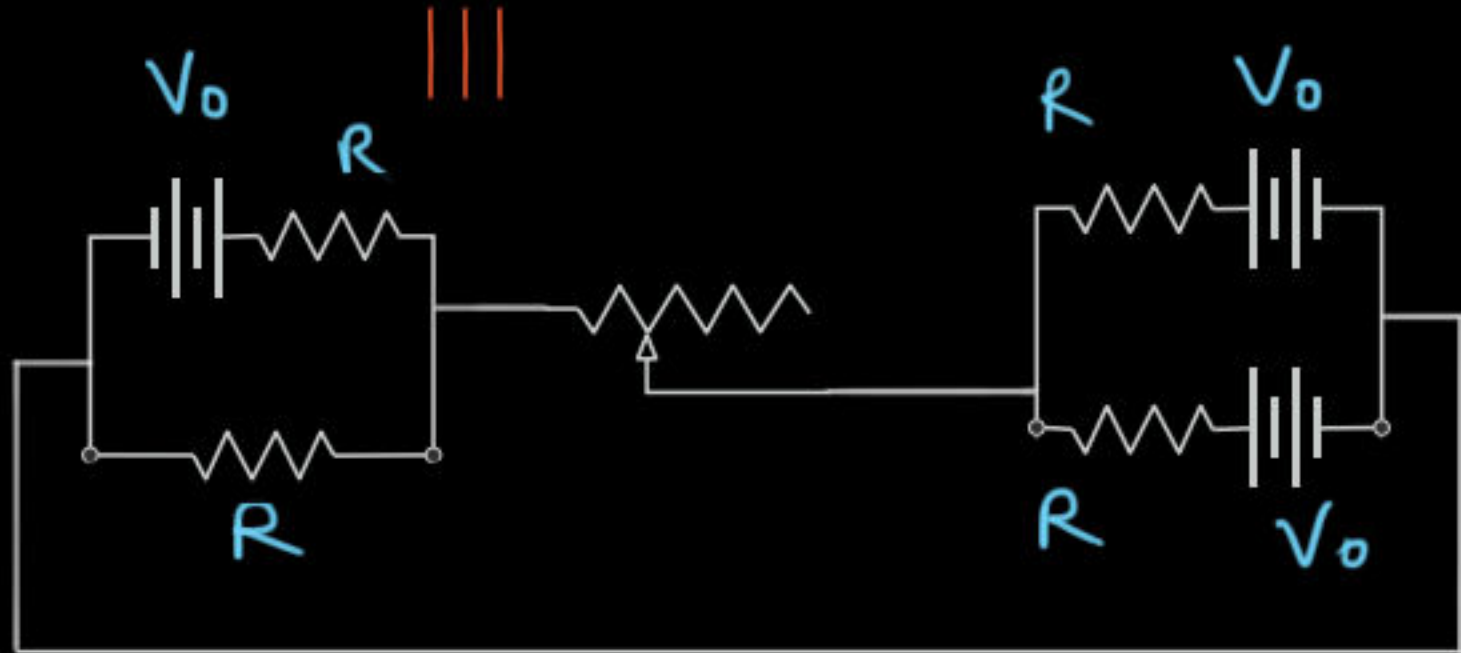
Not possible

Youtube / Arihant Senior

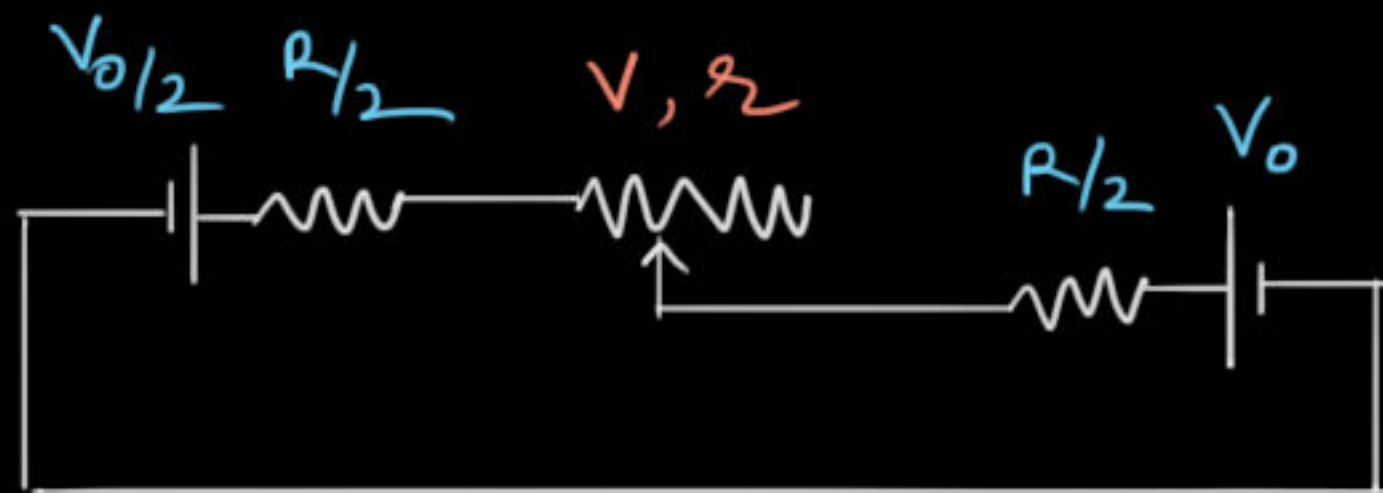
by Ankit Singhvi



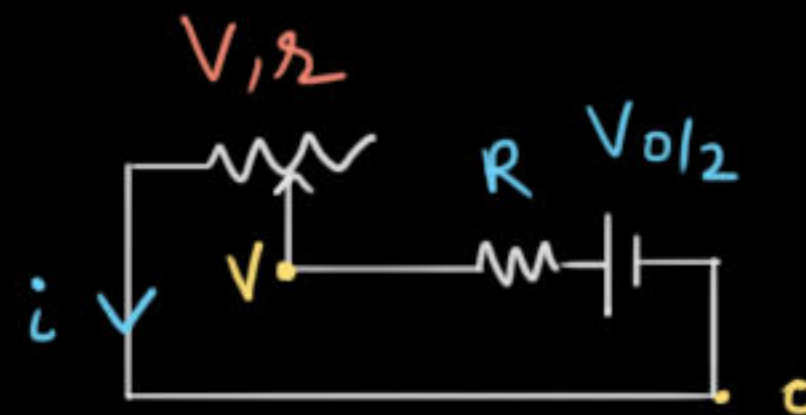
16. A network shown in the figure consists of three ideal batteries each of terminal voltage 100 V , four resistances each of value $4\ \Omega$ and a rheostat. Find relationship between current I through the rheostat and voltage V across the rheostat for all values of resistance of the rheostat.



||| Thevenins theorem



==



$$\therefore \frac{V_0}{2} - IR - V = 0$$

$$\Rightarrow 50 - 4I - V = 0 //$$

Assumption: For both voltmeters, top point is at higher potential.

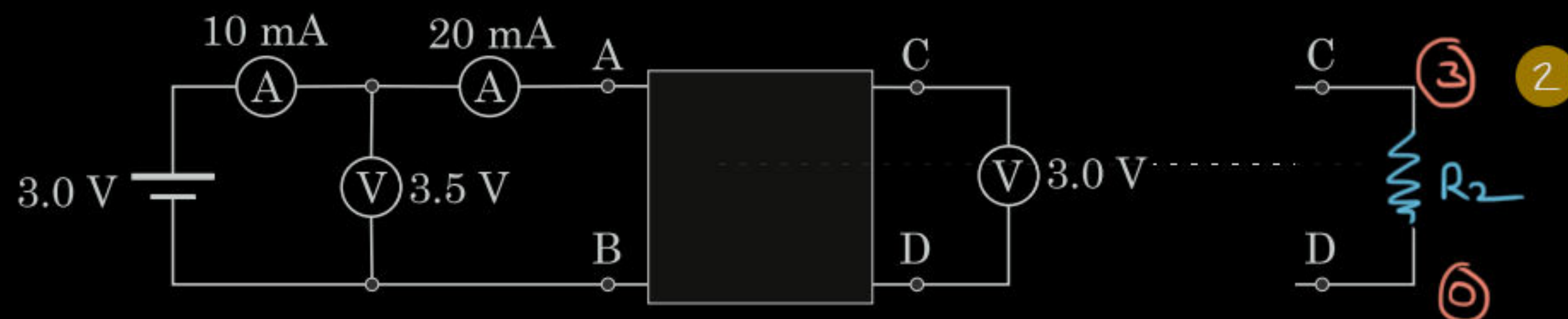
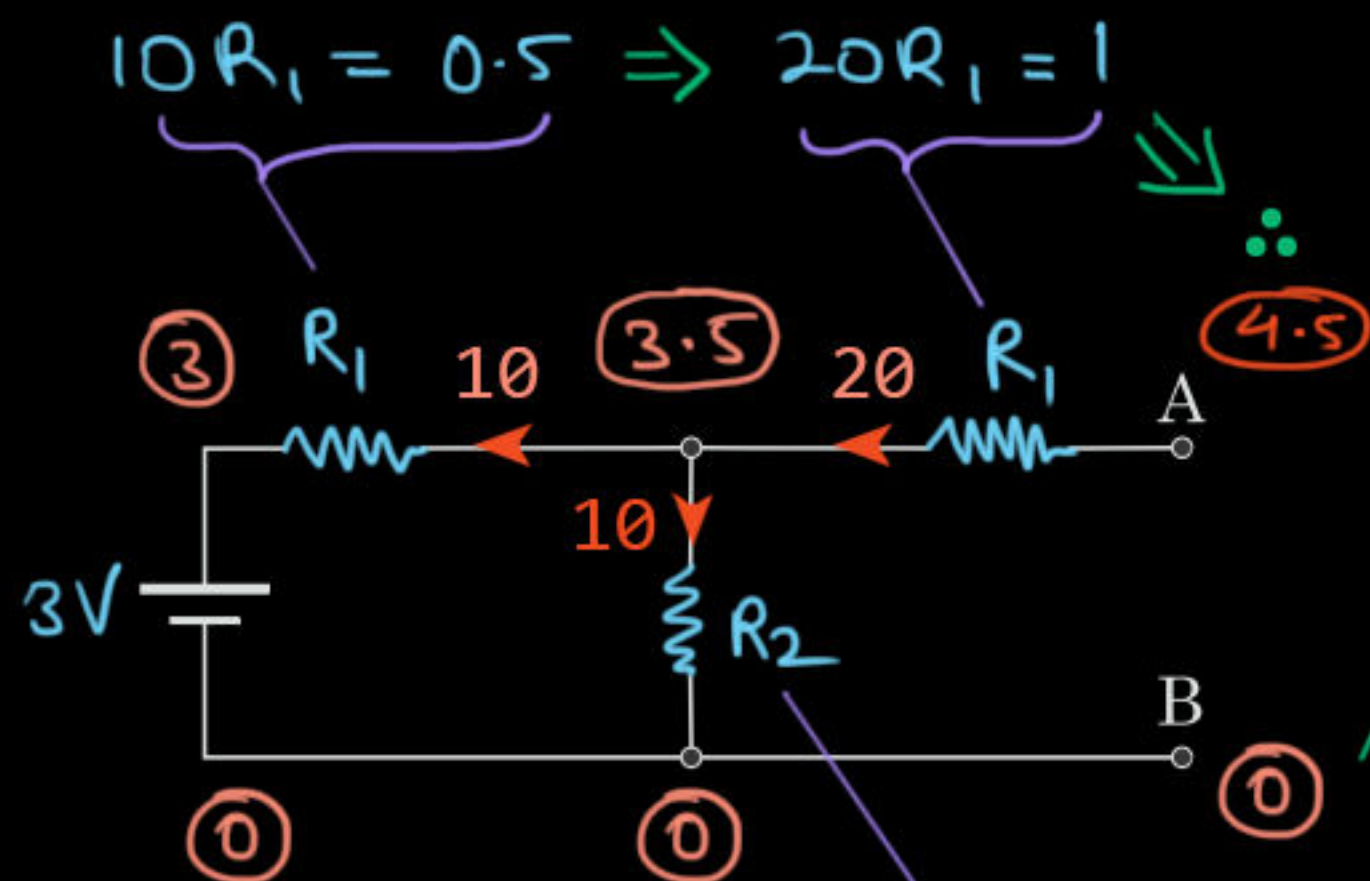
Let, (A) $\rightarrow R_1$

(V) $\rightarrow R_2$

On the left part:

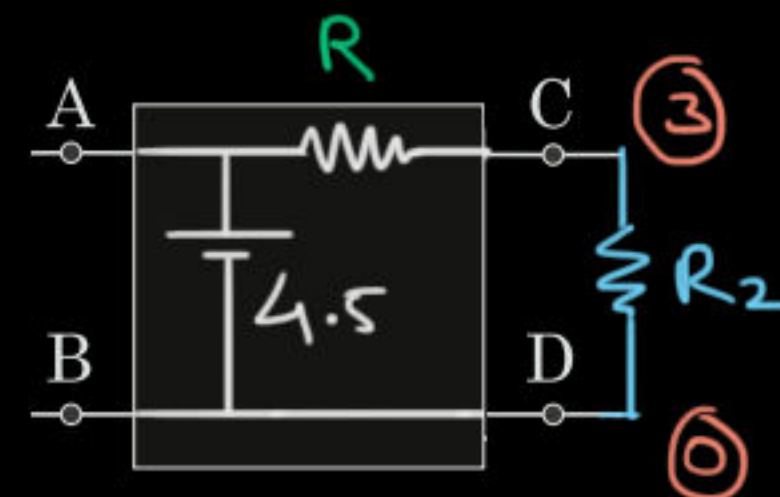
● Given

● Figured out



A 4.5V battery will take care of the left part of circuit. 1

From 1,2:



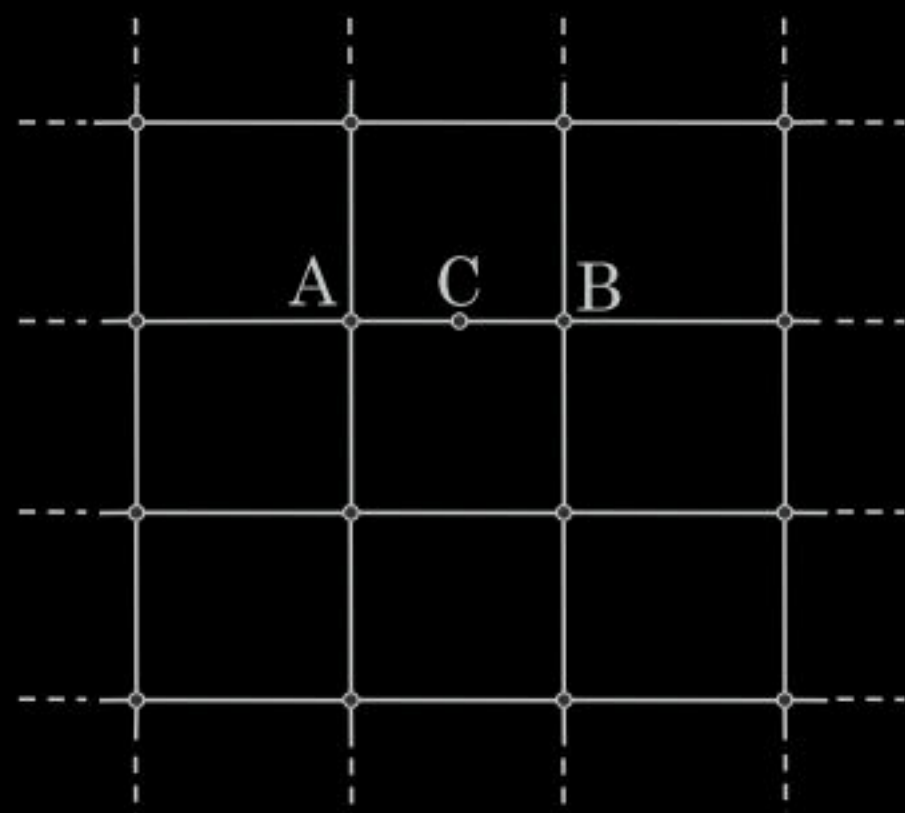
$$\frac{R_2}{R+R_2} \times 4.5 = 3 \Rightarrow R = 175 \Omega$$

$$10 R_2 = 3.5 \Rightarrow R_2 = 350 \Omega$$

by Ankit Singhvi

Youtube / Arihant Senior

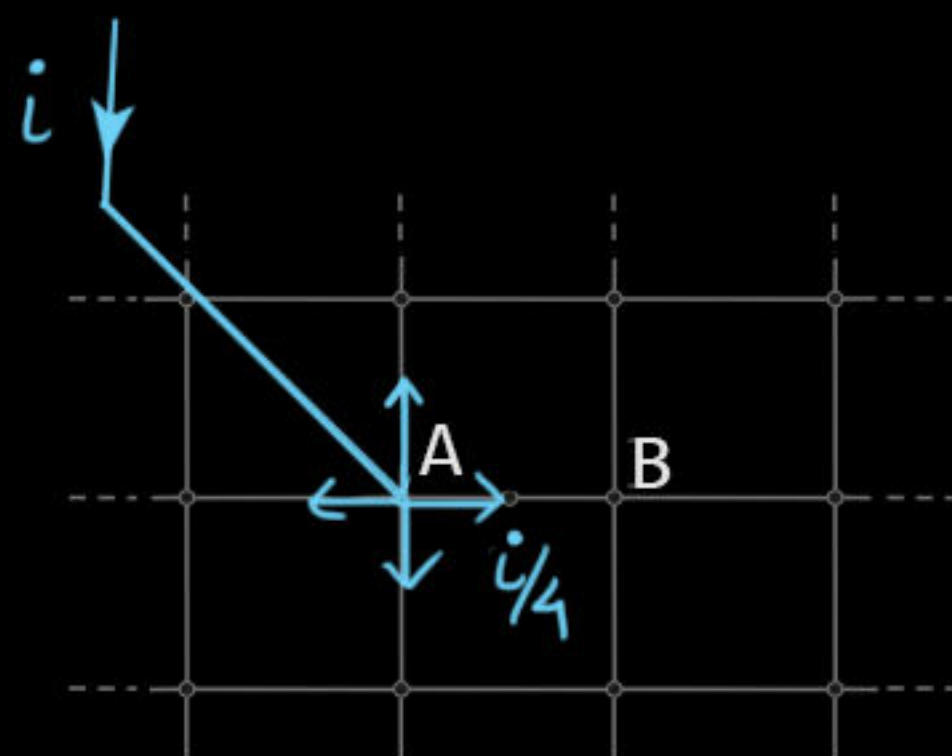
17. A black-box (you cannot see what is inside the box) with an unknown electric circuit inside has four terminals A, B, C and D. To know the contents inside the black-box, following circuit consisting of an ideal battery, two non-ideal identical voltmeters and two non-ideal identical ammeters is arranged. Terminal voltage of the battery and readings of the instruments are shown in the figure. Suggest the simplest possible circuit in the black-box and find its parameters.



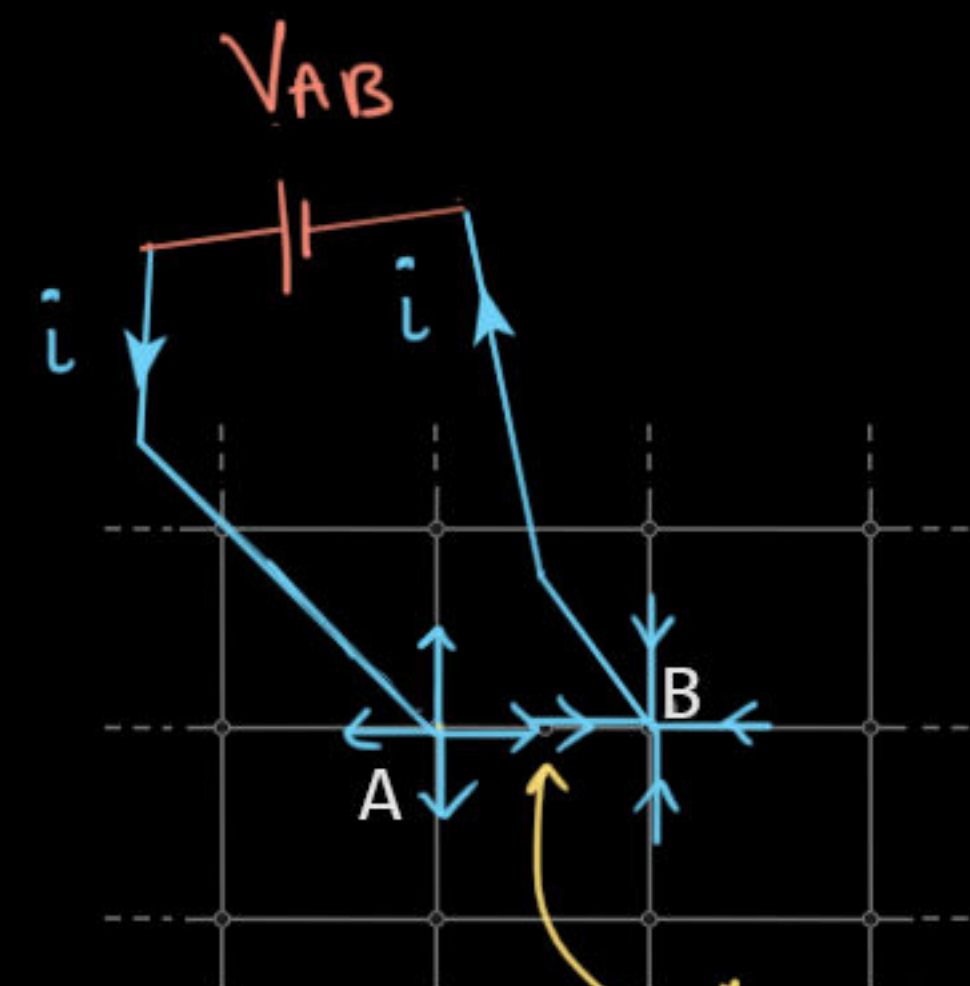
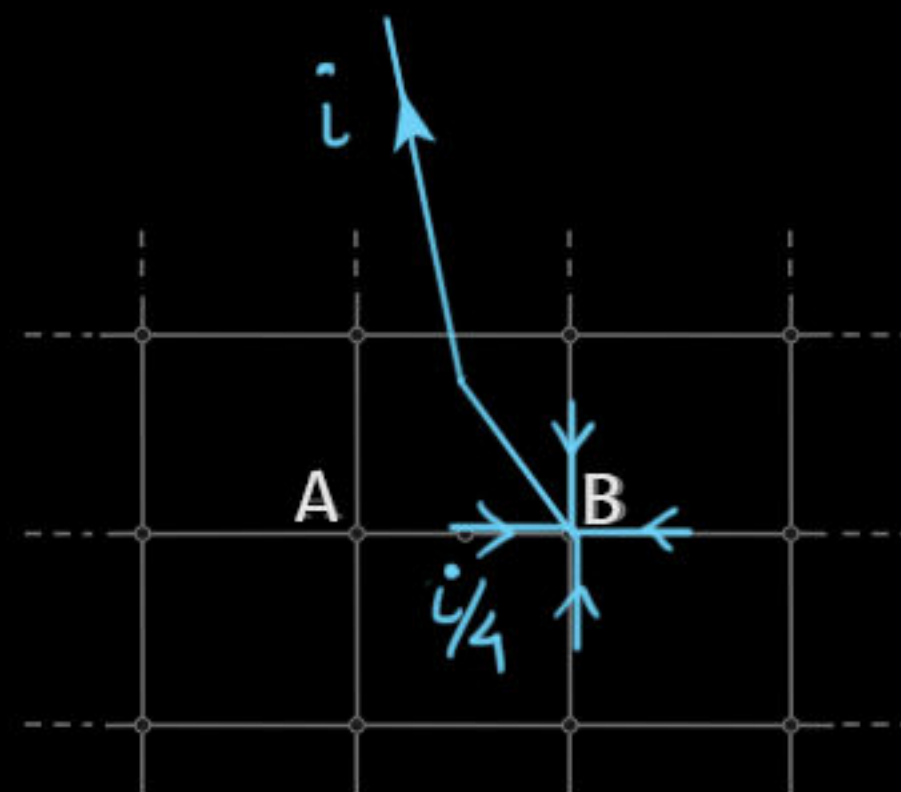
18. In an infinite grid of square cells made of conducting wires, resistance of an edge of a square cell is R . In the figure, point C is midpoint of the edge AB. Find resistance of the entire grid as measured between the terminal pairs A-B and A-C.

irodov 3.153: Using same Superposition principle

1 AB



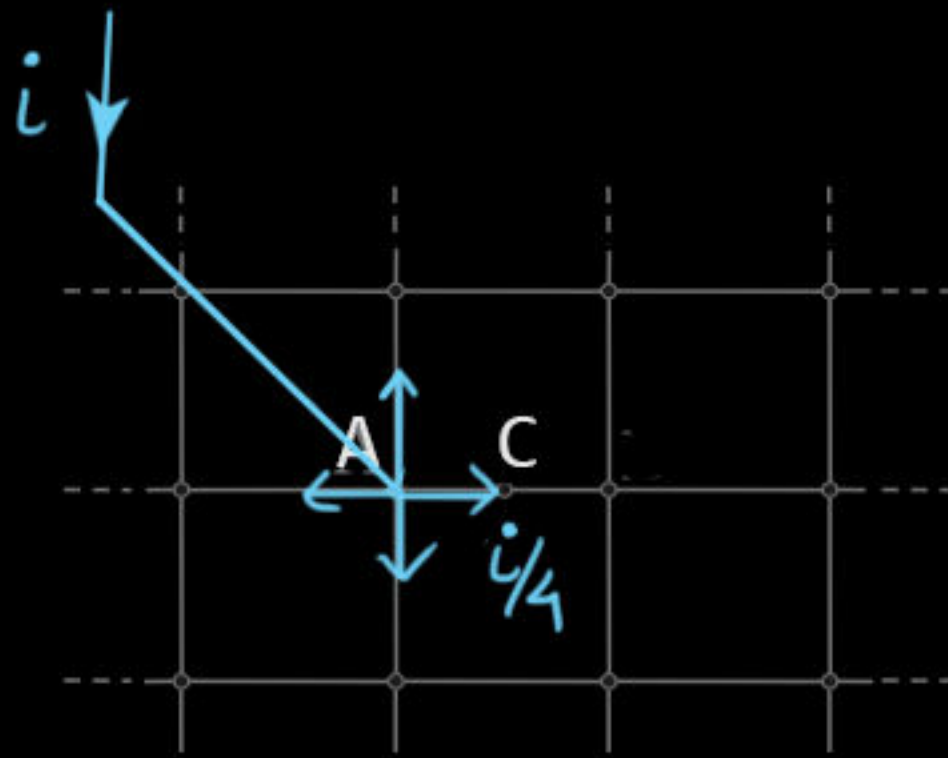
+



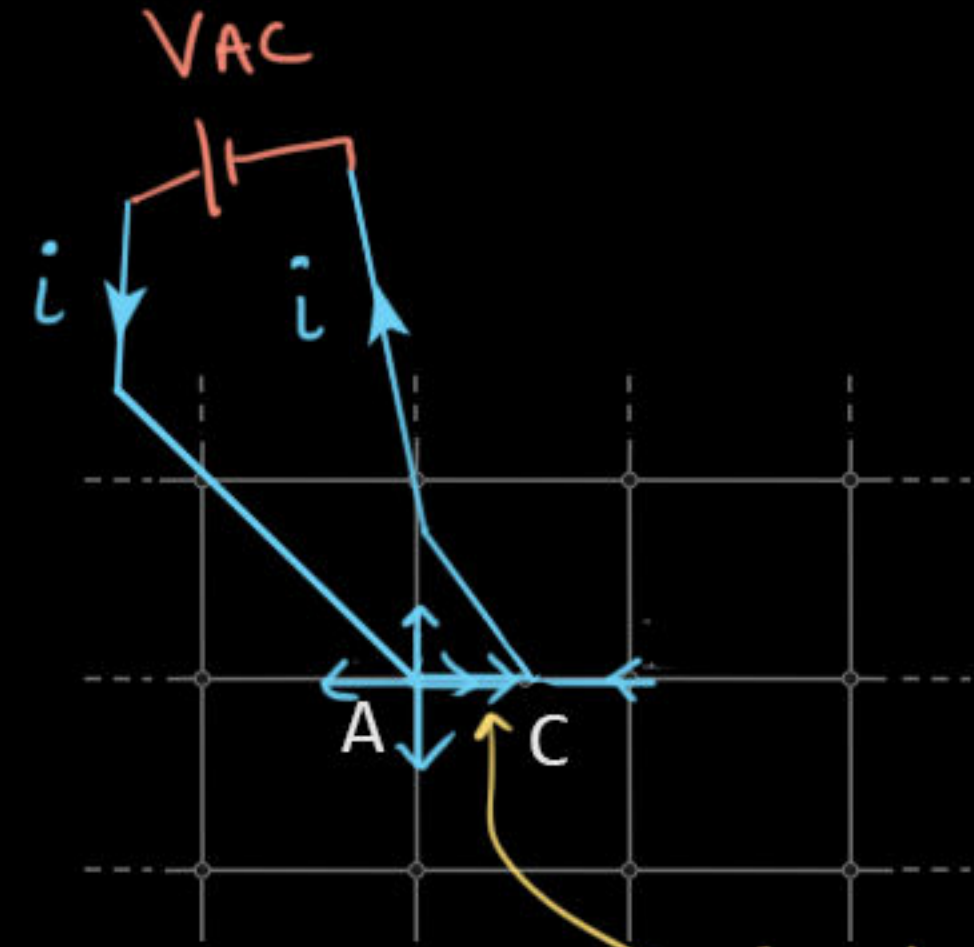
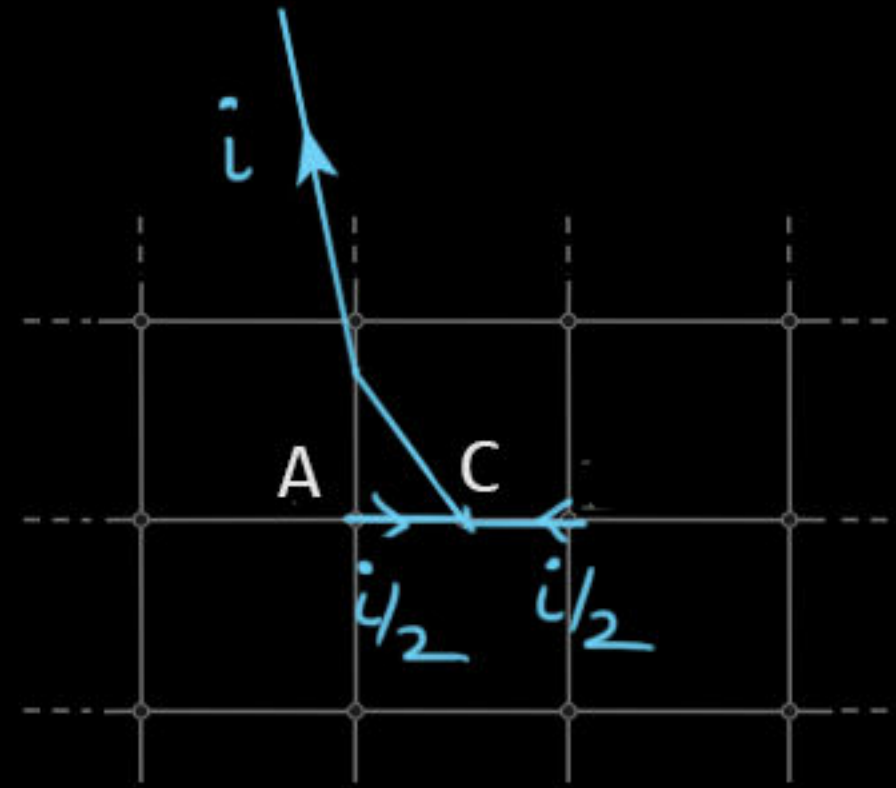
$$i/4 + i/4 = i/2$$

$$V_{AB} = \frac{i}{2} R \Rightarrow R_{eq} = \frac{V_{AB}}{i} = R/2 //$$

2 AC



+



$$i/4 + i/2 = 3i/4$$

$$V_{AC} = \frac{3i}{4} \cdot \frac{R}{2} \quad \because R_{AC} = R/2$$

$$\Rightarrow R_{eq} = \frac{V_{AC}}{i} = \frac{3R}{8} //$$

19. Find equivalent resistance between the terminals A and B of the following infinite ladder networks.

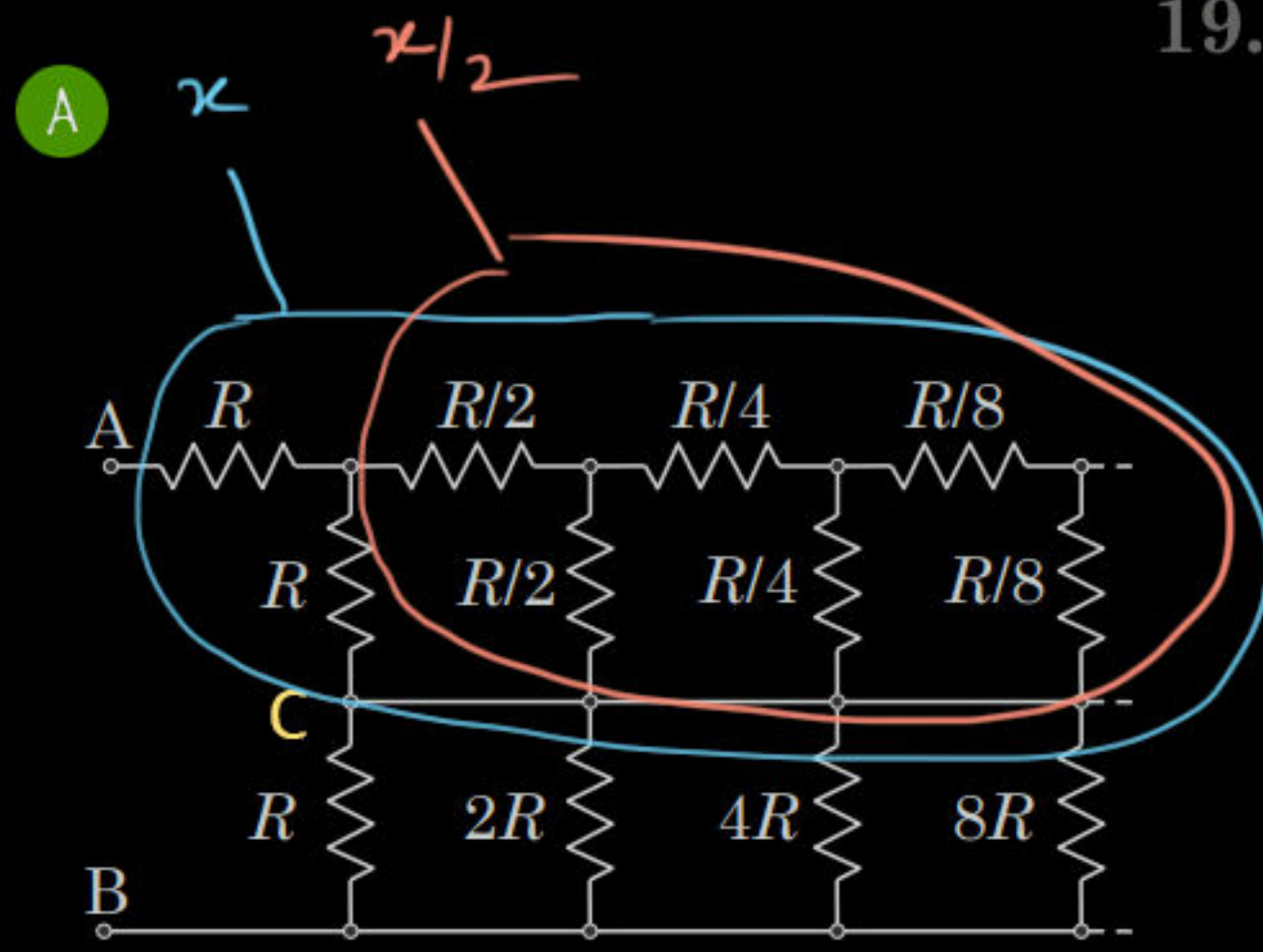
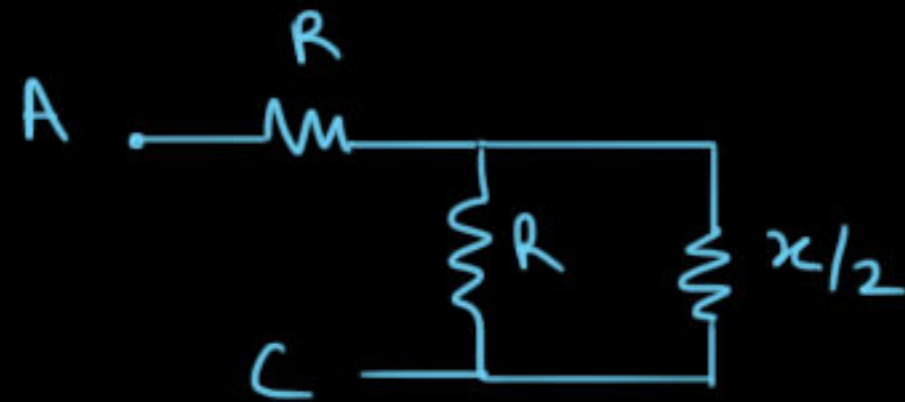


Figure-I



$$R_{AC} = x = R + \frac{R \cdot x/2}{R + x/2}$$

$$R_{AC} = x = \sqrt{2}R \quad (1)$$

Also

$$\frac{1}{R_{BC}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \dots$$

$$\Rightarrow \frac{1}{R_{BC}} = \frac{1}{R} \quad (2)$$

$$\Rightarrow R_{BC} = R/2 \quad (2)$$

From 1,2:

$$\begin{aligned} R_{AB} &= R_{AC} + R_{BC} \\ &= R(\sqrt{2} + \frac{1}{2}) \end{aligned}$$

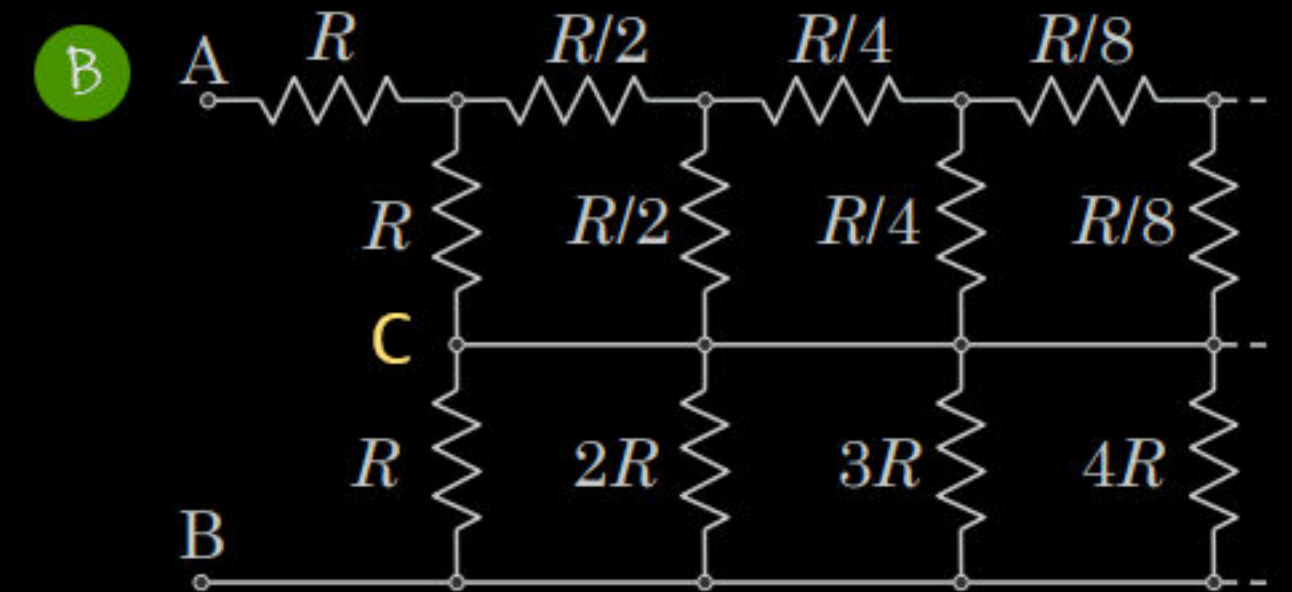


Figure-II

$$R_{AC} = \sqrt{2}R \quad (\text{from 1}) \quad (1)$$

$$\frac{1}{R_{BC}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{3R} + \dots$$

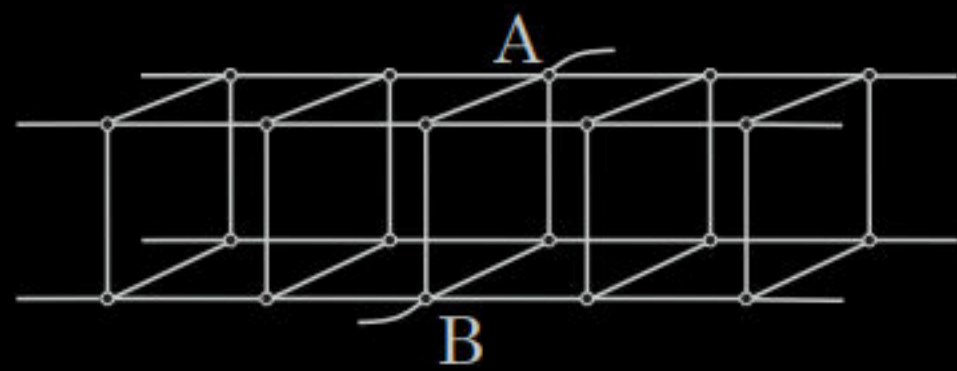
$$\Rightarrow \frac{1}{R_{BC}} = \infty$$

$$\Rightarrow R_{BC} = 0 \quad (2)$$

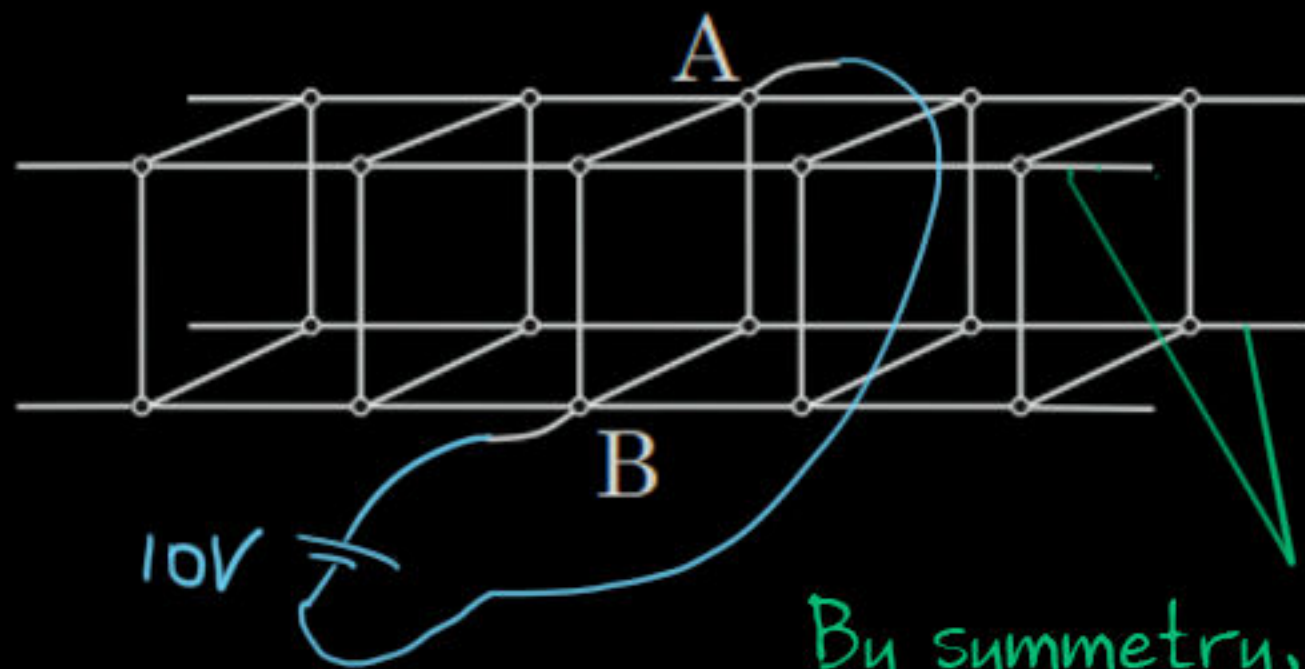
From 1,2:

$$R_{AB} = R_{AC} + R_{BC} = \sqrt{2}R //$$

Youtube / Arihant Senior

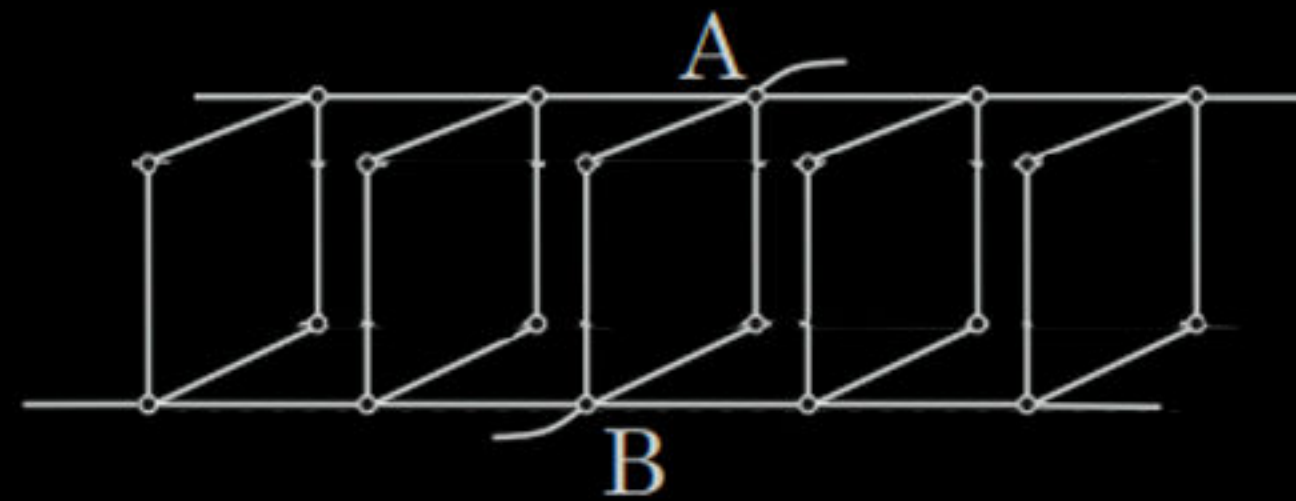


20. The network shown consists of an infinitely large number of identical sections. Resistance of each branch is R . Find resistance of the whole network measured between the terminals A and B.

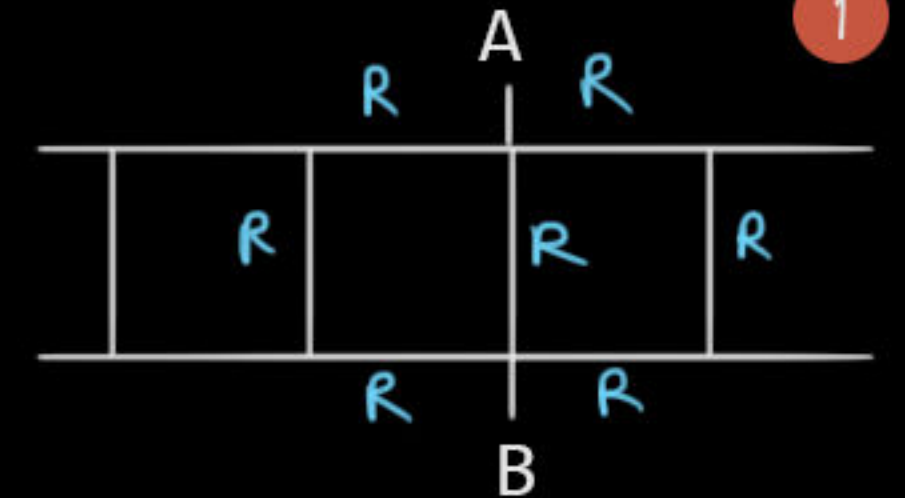


By symmetry, these lines will always be at 5V potential.

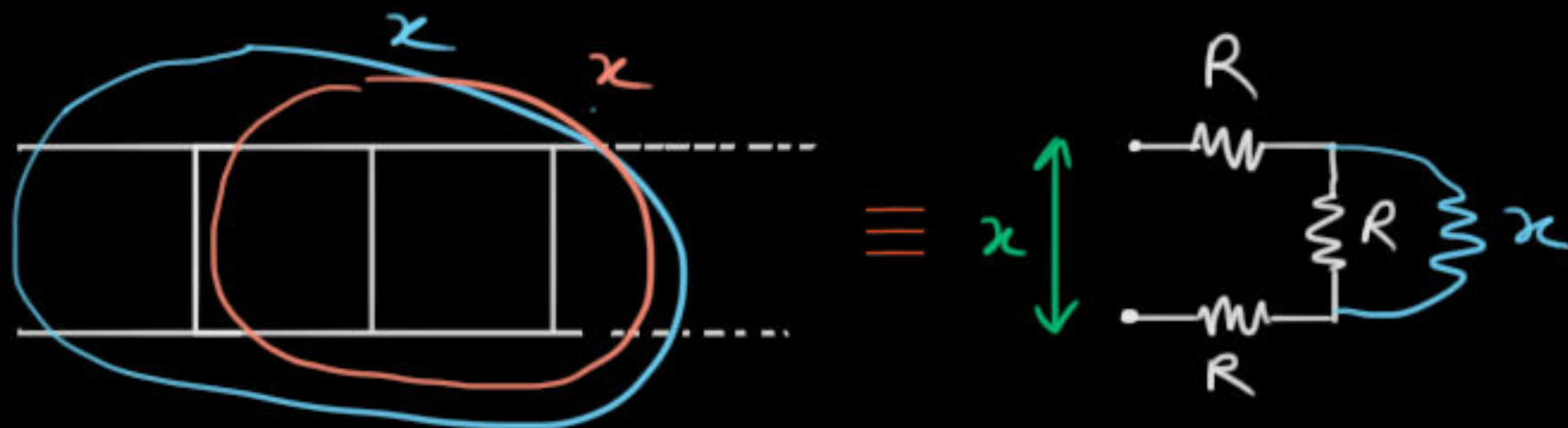
$\therefore$
 $\equiv$



$\equiv$

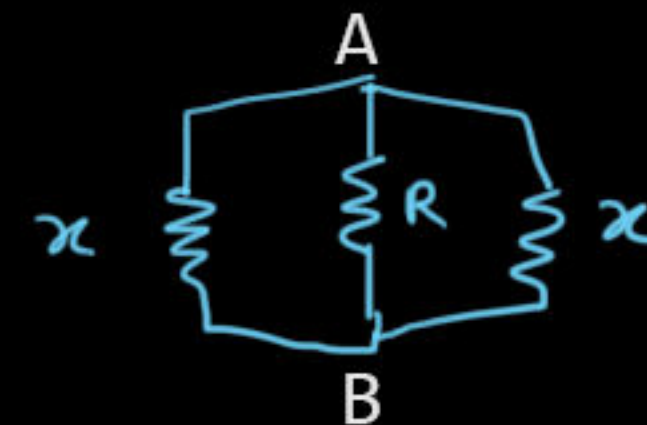


Lets consider right half of AB

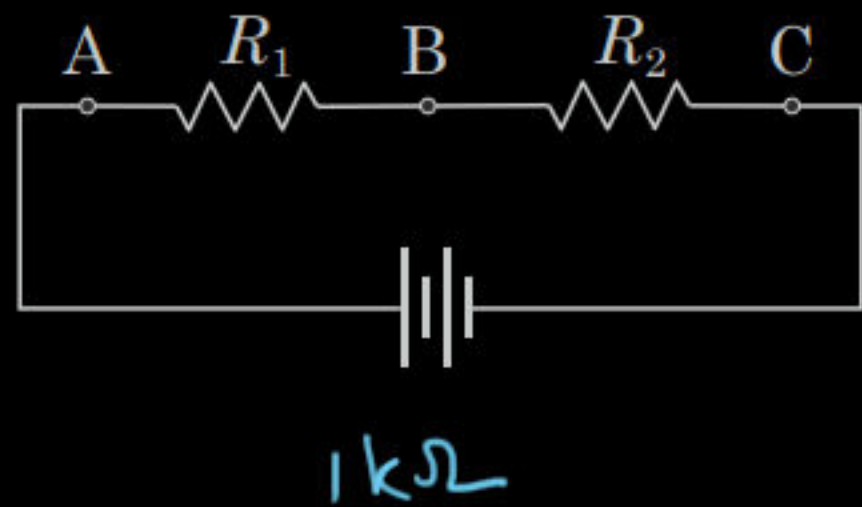


$$\therefore x = 2R + \frac{Rx}{R+x} \Rightarrow x = (\sqrt{3} + 1)R \quad (2)$$

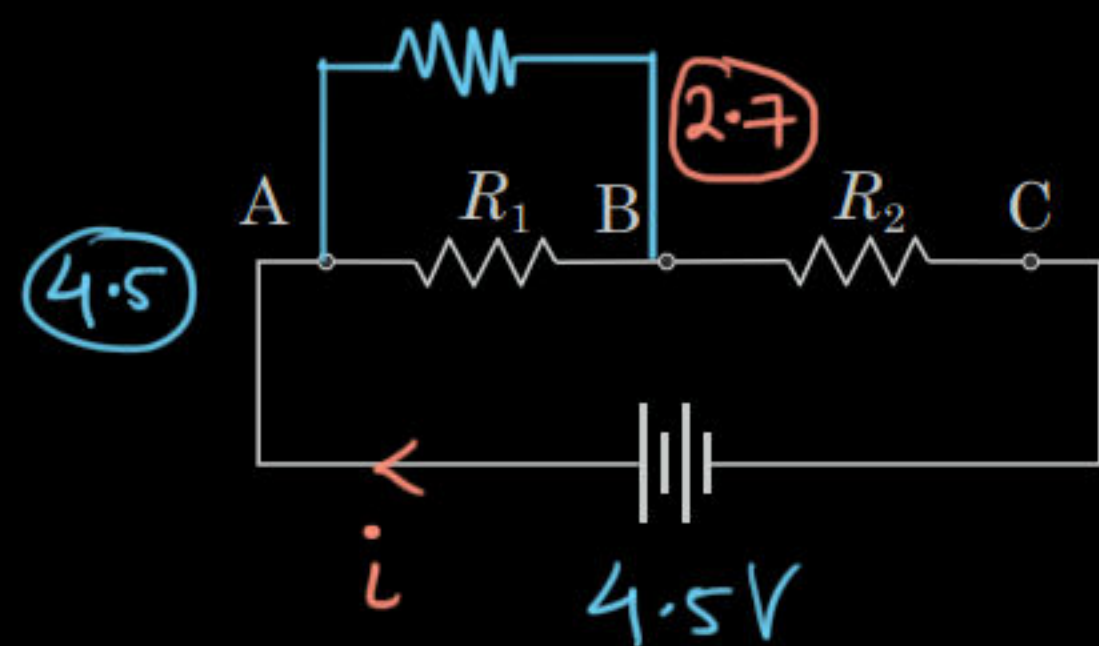
From 1,2, circuit simplifies to:



$$\therefore R_{eq} = \frac{Rx/2}{R+x/2} = \frac{\sqrt{3}+1}{\sqrt{3}+3} R = \frac{R}{\sqrt{3}}$$



21. A voltmeter of internal resistance $1.0 \text{ k}\Omega$ reads voltages 1.8 V , 1.8 V and 4.5 V between the terminal pairs A-B, B-C and A-C respectively of a circuit consisting of an ideal battery and two unknown resistances R_1 and R_2 as shown. What will another voltmeter of internal resistance $2.0 \text{ k}\Omega$ read between these terminal pairs?



When Voltmeter is connected to R_1

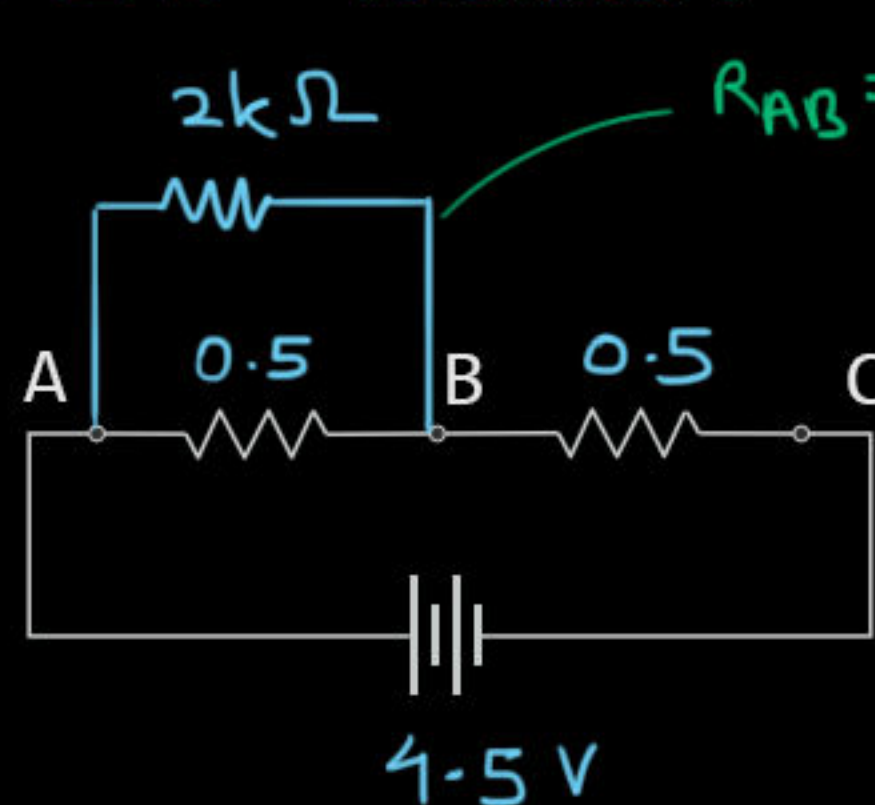
$$i = \frac{2.7}{R_2} = \frac{1.8(1+R_1)}{R_1(1)}$$

Similarly, when connected to R_2

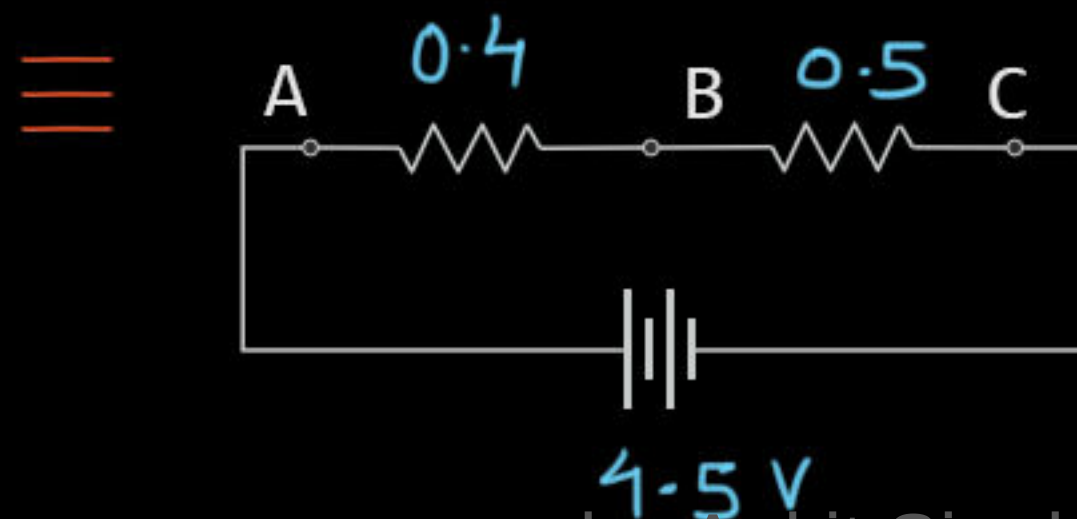
$$i = \frac{2.7}{R_1} = \frac{1.8(1+R_2)}{R_2(1)}$$

$$\Rightarrow R_1 = R_2 = \frac{1}{2}$$

$\therefore$ @ $2 \text{ k}\Omega$ Voltmeter:

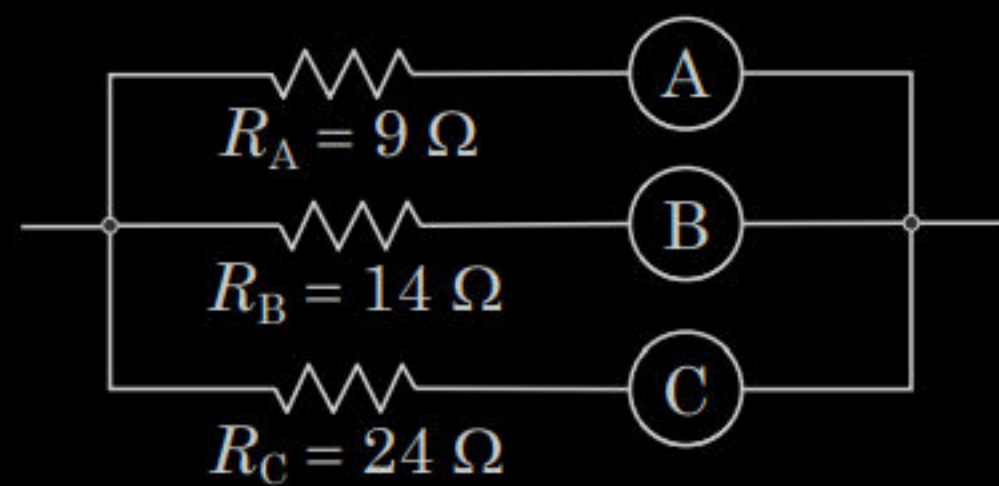


$$R_{AB} = \frac{2(0.5)}{2+0.5} = 0.4$$



$$V_{AB} = \left(\frac{0.4}{0.4+0.5} \right) 4.5 = 2 \text{ V} //$$

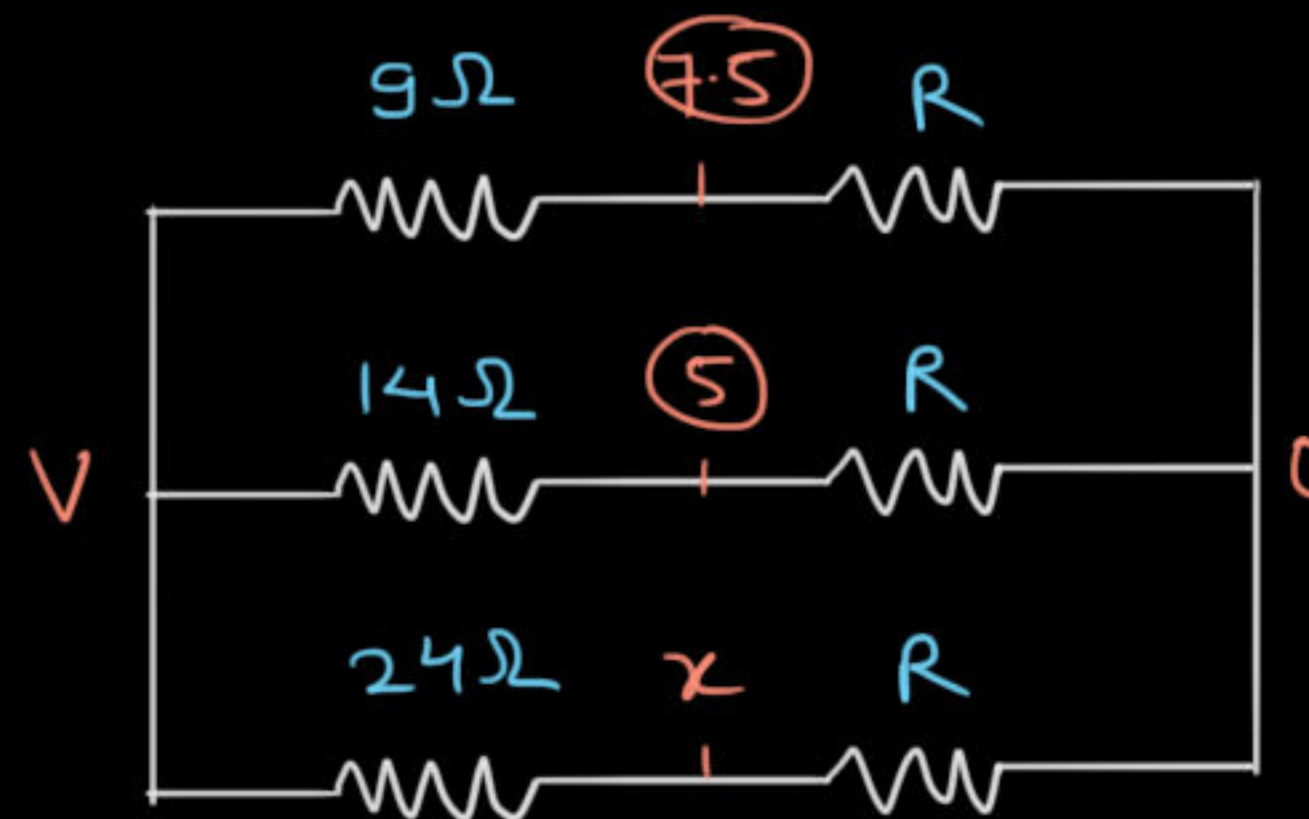
$$V_{BC} = 2 \text{ V} //, \quad V_{AC} = 4.5 \text{ V} //$$



22. The figure shows a portion of an electric circuit. All the resistances R_A , R_B and R_C are known and their values are indicated in the figure. All the voltmeters A, B and C are identical. If the readings of the voltmeters A and B are $V_A = 7.5 \text{ V}$ and $V_B = 5.0 \text{ V}$ respectively, find the reading of the voltmeter C.

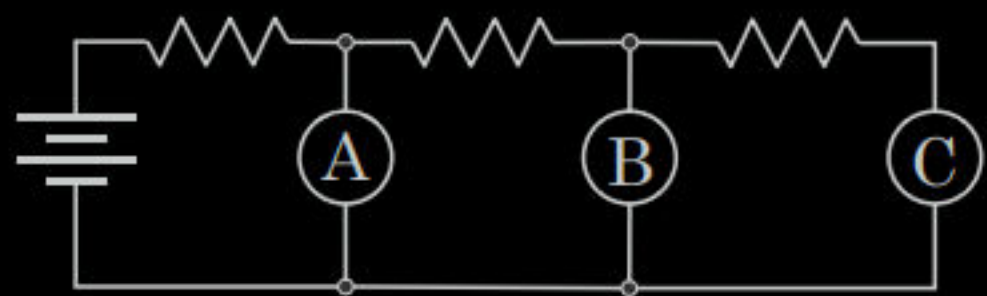
Voltmeters are giving different readings.

So they have internal resistance.
Let it be R .



$$\begin{aligned} \left(\frac{R}{9+R} \right) V &= 7.5 \\ \left(\frac{R}{14+R} \right) V &= 5 \end{aligned} \quad \begin{aligned} &R = 1 \\ &V = 75 \end{aligned}$$

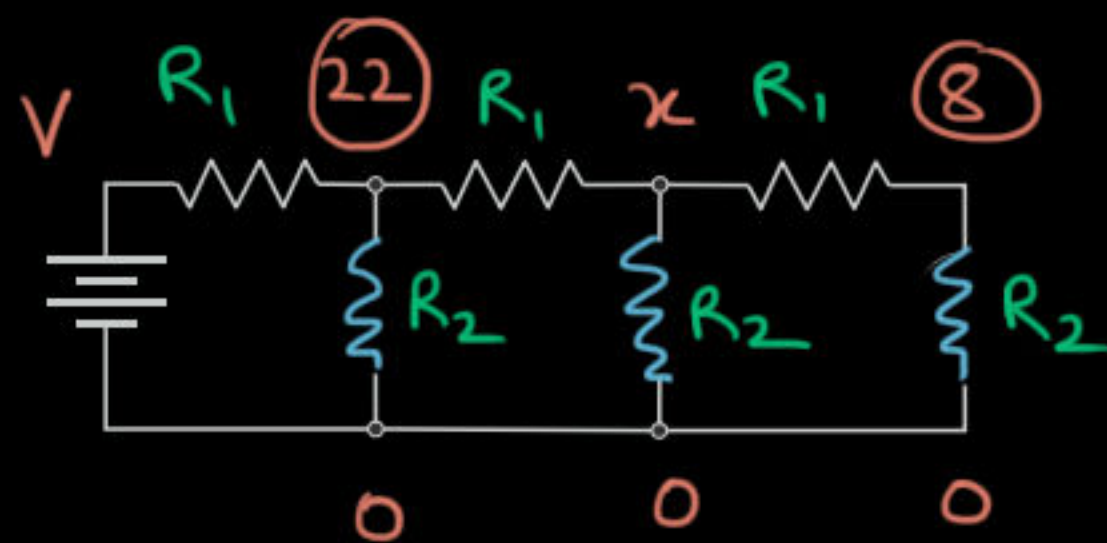
$$\left(\frac{R}{24+R} \right) V = ? = 3 \text{ V} //$$



23. The circuit shown consists of three identical resistances and three identical voltmeters A, B and C. If the first voltmeter A reads $V_A = 22 \text{ V}$ and the third voltmeter C reads $V_C = 8 \text{ V}$, calculate reading of the second voltmeter B.

Voltmeters are giving different readings.

So they have internal resistance.
Let it be R_2 .



$$\frac{22-V}{R_1} + \frac{22}{R_2} + \frac{22-x}{R_1} = 0$$

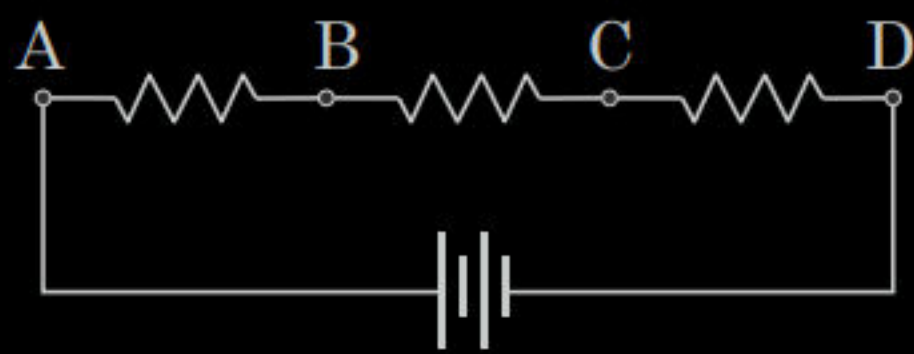
$$\frac{x-22}{R_1} + \frac{x}{R_2} + \frac{x-8}{R_1} = 0$$

$$\frac{8-x}{R_1} + \frac{8}{R_2} = 0$$

3 equations, 3 variables:

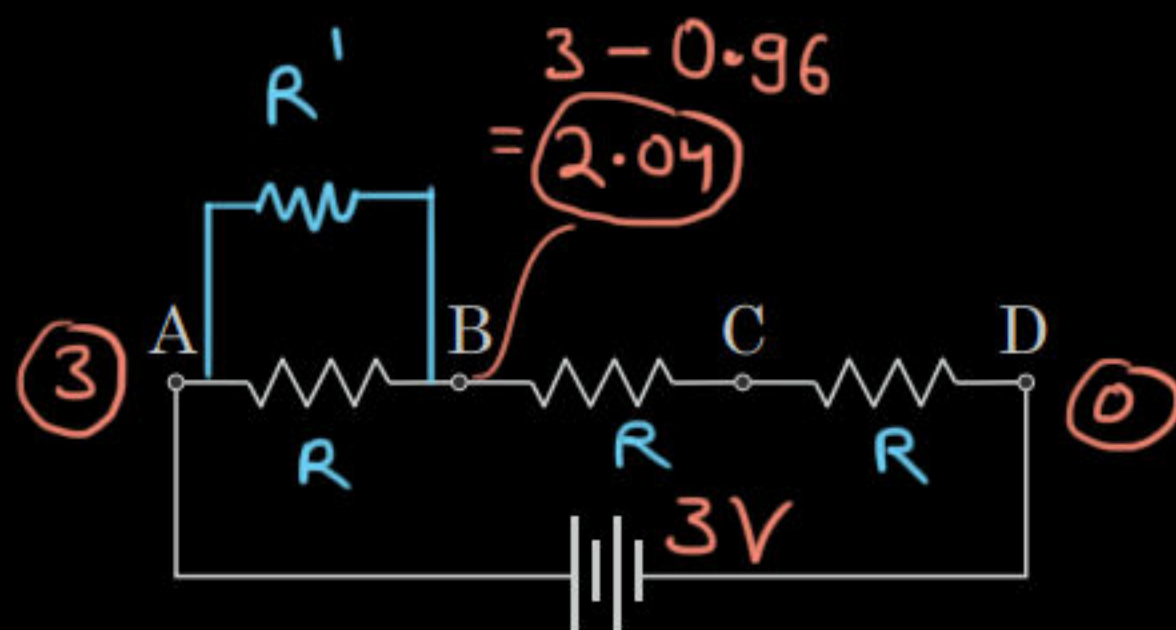
$$\frac{R_1}{R_2}, V, x$$

$$\Rightarrow x = 12, \frac{R_1}{R_2} = \frac{1}{2}, V = 43$$

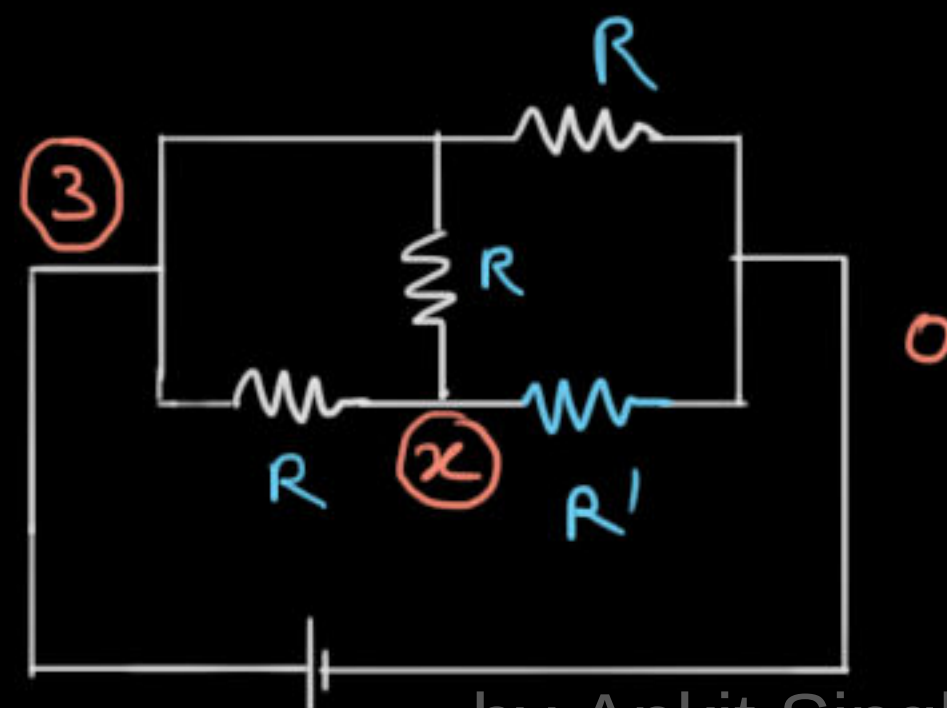
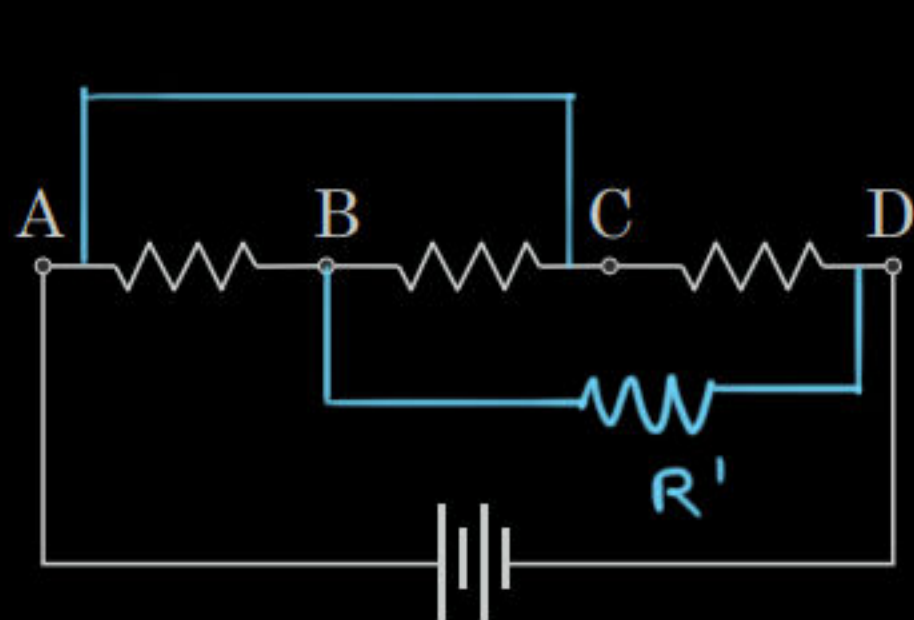


Voltmeters have internal resistance, as: $V_{AB} \neq \frac{V_{AD}}{3}$
Let it be R'

24. Three identical resistances are connected in series and their combination is connected across an ideal battery. If you connect a voltmeter between the terminals A and D, the voltmeter reads $V_1 = 3.00$ V and if you connect the voltmeter between the terminals A and B, the voltmeter reads $V_2 = 0.96$ V. If you short-circuit the terminals A and C and then connect the voltmeter between the terminals B and D, how much will it read?

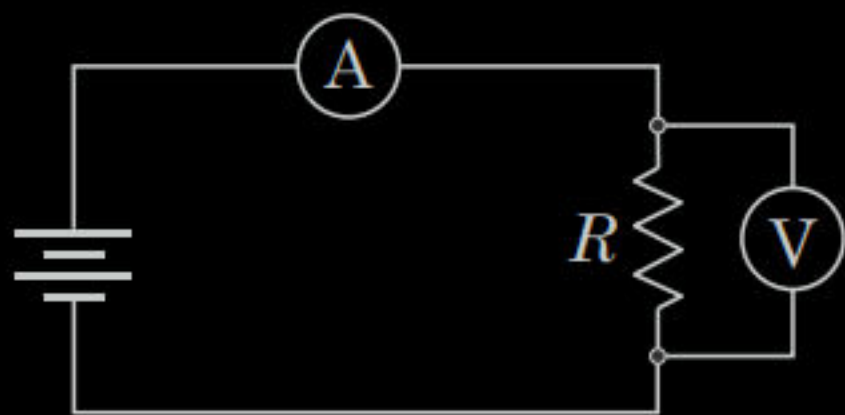


$$\Rightarrow i = \frac{2.04}{2R} = \frac{0.96(R+R')}{R R'} \Rightarrow \frac{R}{R'} = \frac{1}{16} \quad (1)$$

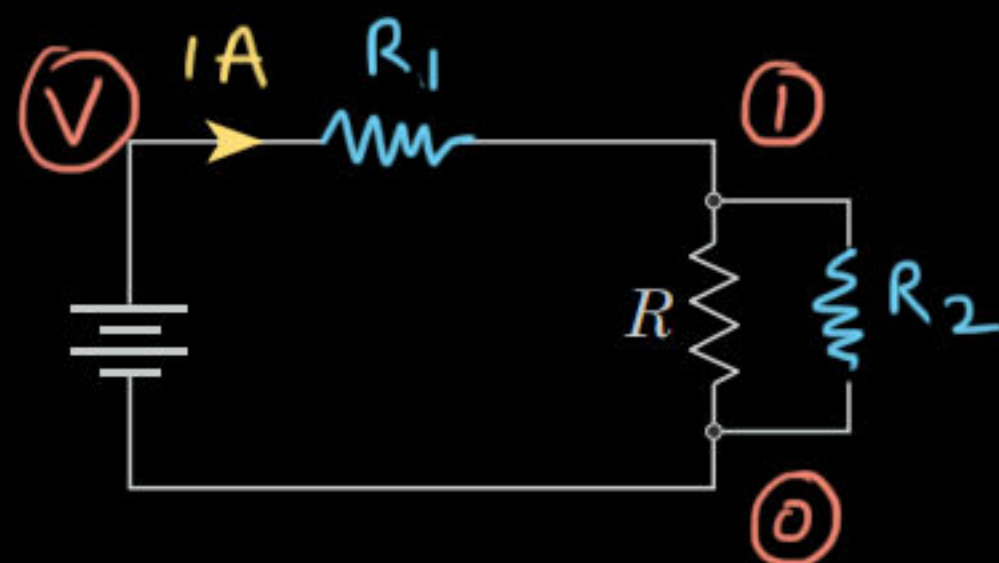


$$\Rightarrow \frac{x-3}{R} + \frac{x-3}{R} + \frac{x-0}{R'} = 0 \quad (2)$$

From 1,2: $x = \frac{32}{11} = 2.9 //$

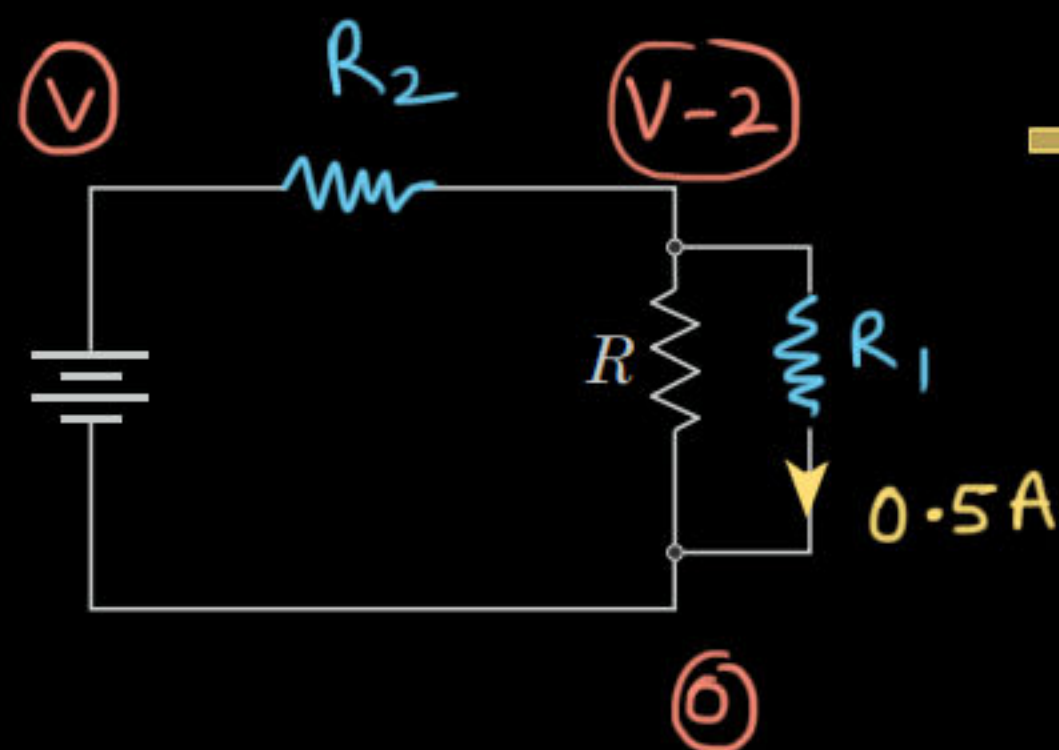


25. In the circuit shown, the voltmeter reads $V_1 = 1.0 \text{ V}$ and the ammeter reads $I_1 = 1.0 \text{ A}$. On changing the places of the ammeter and the voltmeter, the voltmeter will read $V_2 = 2.0 \text{ V}$ and the ammeter will read $I_2 = 0.5 \text{ A}$. Find resistance R , resistances R_V of the voltmeter and R_A of the ammeter.



$$V - 1 = (1) R_1 \Rightarrow V - 1 = R_1 \quad (1)$$

$$(1) \left(\frac{R R_2}{R + R_2} \right) = 1 - 0 \Rightarrow R R_2 = R + R_2 \quad (2)$$



$$(V - 2) - 0 = (0.5) R_1 \Rightarrow 2V - 4 = R_1 \quad (3)$$

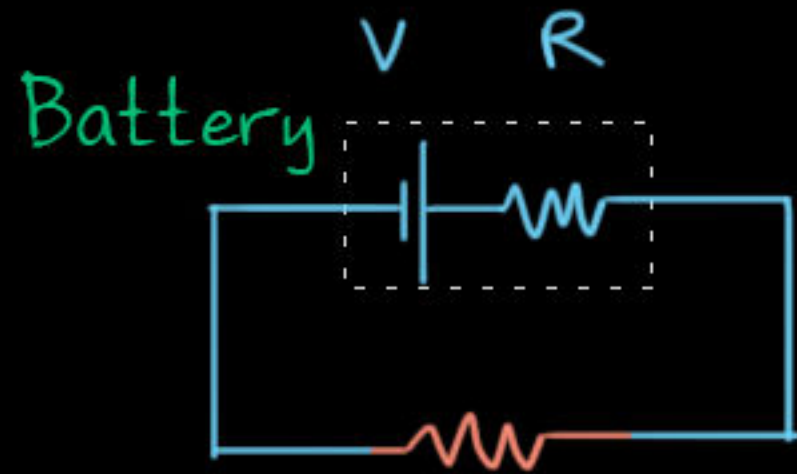
$$\frac{V - (V - 2)}{R_2} = \frac{V - 2}{\frac{R R_1}{R + R_1}} \Rightarrow 2 R R_1 = (V - 2) R_2 (R + R_1) \quad (4)$$

From 1, 2, 3, 4: $V = 3, R = R_1 = R_2 = 2$

by Ankit Singhvi

Youtube // Arihant Senior

26. To measure electromotive force of a wristwatch battery, you are given two voltmeters. Each of them when connected alone across the battery, read $V_1 = 0.9 \text{ V}$ and $V_2 = 0.6 \text{ V}$. If the voltmeters are simultaneously connected across the battery, both of them read $V_3 = 0.45 \text{ V}$. Find the electromotive force of the battery.



$$\left(\frac{R_1}{R_1 + R} \right) V = V_1 \quad (1)$$

3 equations, 3 variables: $\frac{R_1}{R}, \frac{R_2}{R}, V$

$$\left(\frac{R_2}{R_2 + R} \right) V = V_2 \quad (2)$$

$$\Rightarrow R_1 = \frac{R V_1}{V - V_1}, \quad R_2 = \frac{R V_2}{V - V_2}$$

Put in 3

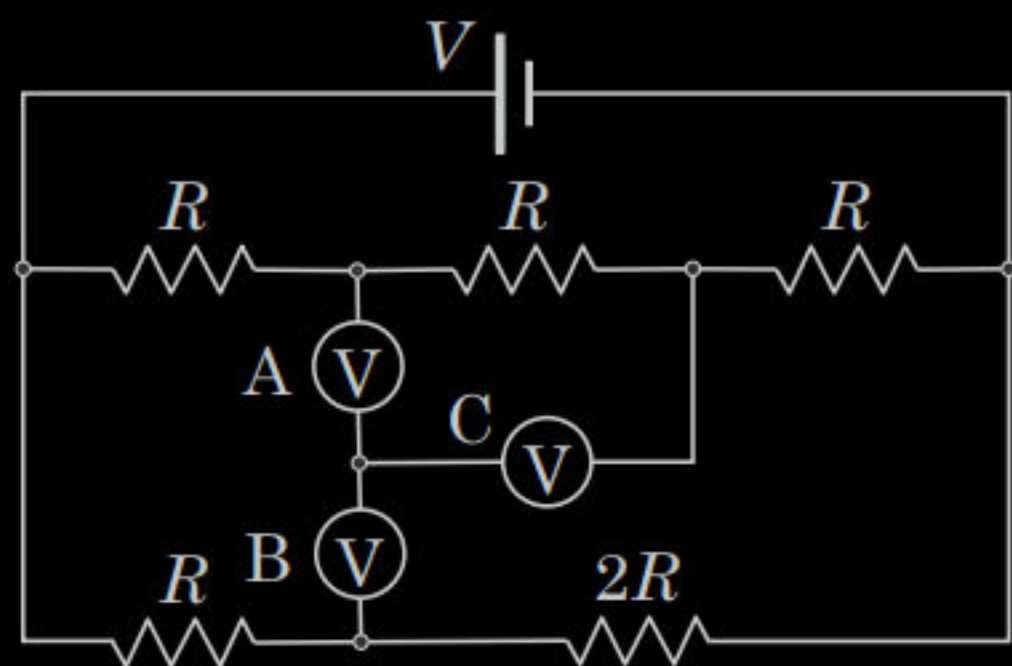
$$\frac{V}{V_3} = \left(\frac{V - V_1}{V_1} + \frac{V - V_2}{V_2} + 1 \right)$$

$$\Rightarrow 1 = V \left(\frac{1}{V_1} + \frac{1}{V_2} - \frac{1}{V_3} \right)$$

$$\Rightarrow V = \frac{V_1 V_2 V_3}{V_2 V_3 + V_1 V_3 - V_1 V_2}$$

Youtube / Arihant Senior

by Ankit Singhvi

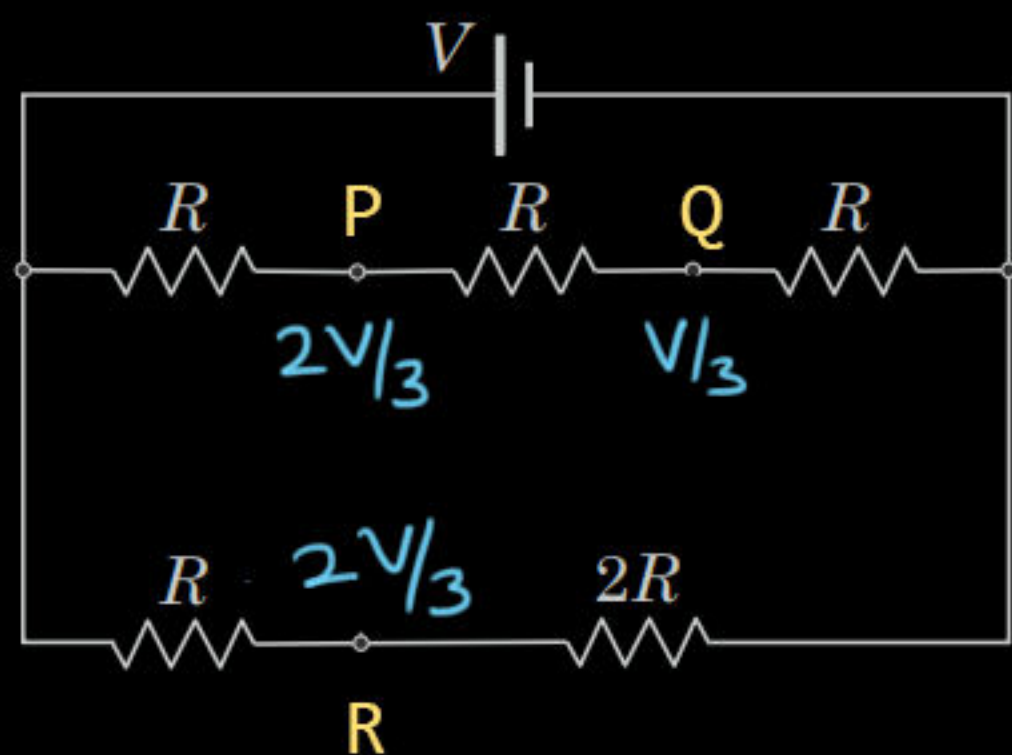


27. The circuit shown consists of three identical ideal voltmeters A, B and C, five resistors and an ideal battery. Find readings of all the voltmeters in terms of the terminal voltage V of the battery.

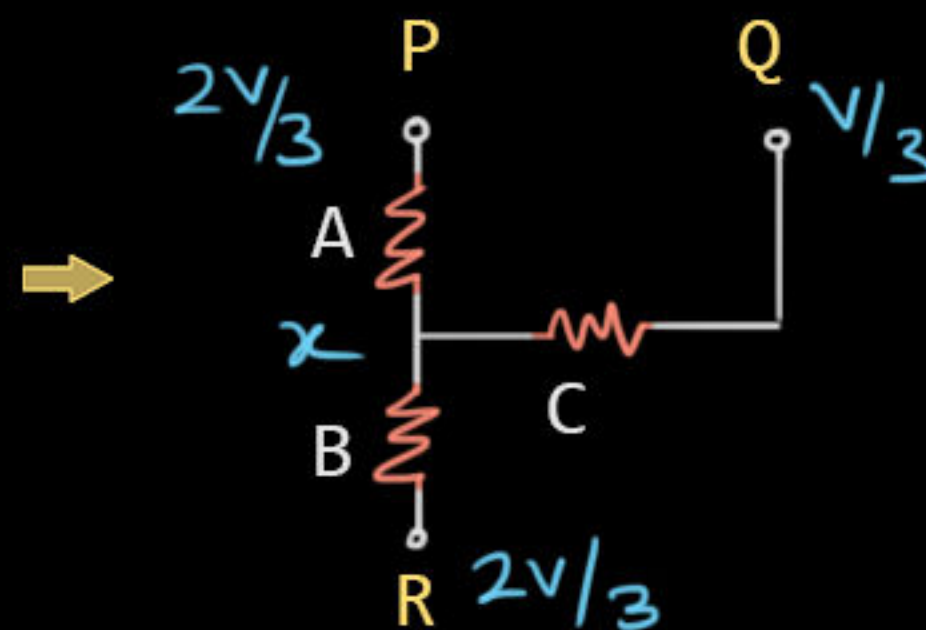
Ideal voltmeter has really high resistance and therefore draws a negligible current.

We need to consider that current when dealing with Voltmeters alone, but ignore in the bigger circuit.

Bigger circuit



Voltmeters

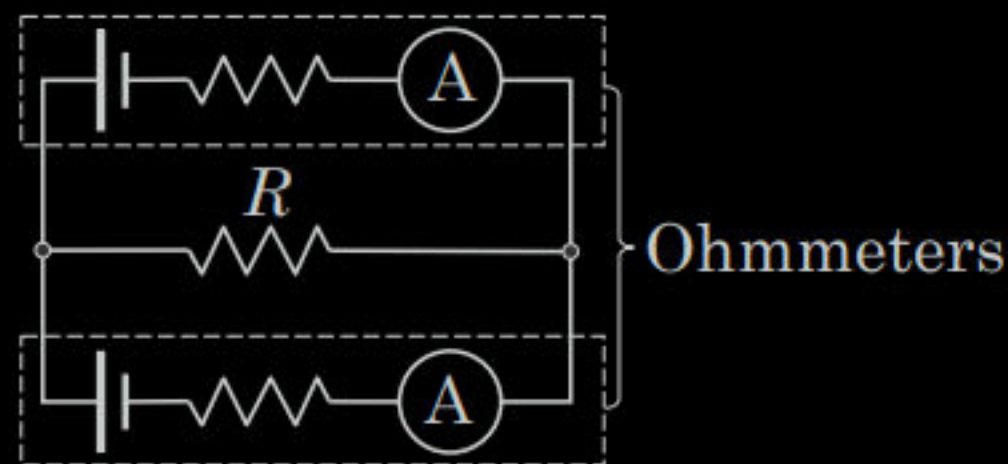


$$\left(x - \frac{2V}{3}\right) + \left(x - \frac{2V}{3}\right) + \left(x - \frac{V}{3}\right) = 0$$

$$\Rightarrow x = \frac{5V}{9}$$

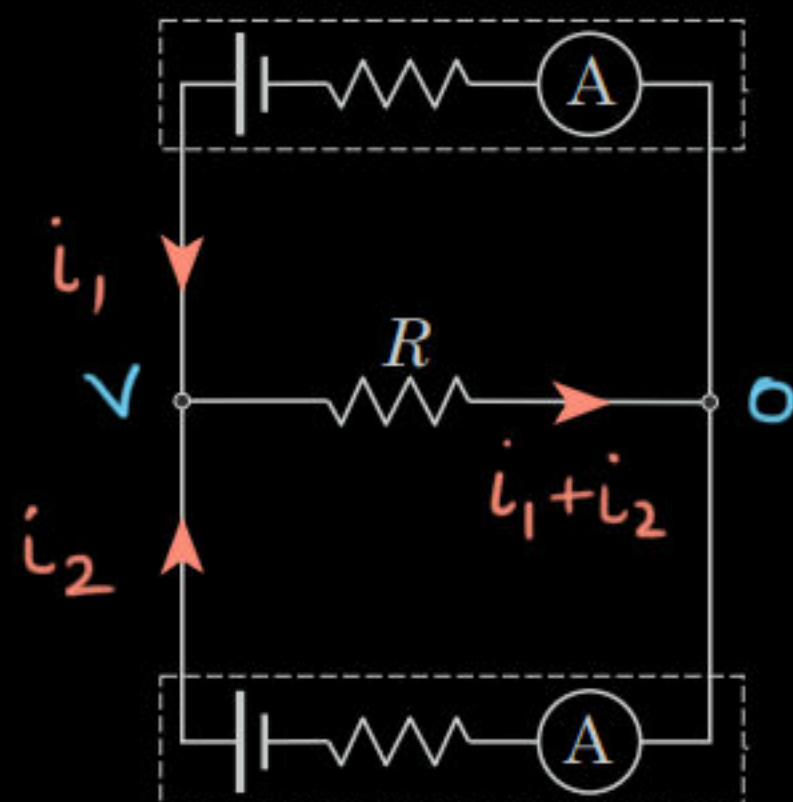
$$\therefore V_A = V_B = \left|x - \frac{2V}{3}\right| = \frac{V}{9}$$

$$V_C = \left|x - \frac{V}{3}\right| = \frac{2V}{9}$$



29. A homemade Ohmmeter consists of a series connected battery, a resistance and an ammeter. A student constructs two such ohmmeters using non-identical components and calibrates them by measuring the same set of known resistances. These two Ohmmeters are together connected in parallel with an unknown resistance R as shown in the figure. If one of the ohmmeter reads $R_1 = 30\ \Omega$ and other reads $R_2 = 60\ \Omega$, find true value of the unknown resistance R .

Ohmmeter measures resistance $\Rightarrow$ Resistance is ratio of V and I $\Rightarrow$ Ohmmeter measures ratio of V and I across two points and declares it as resistance.



Reading $R_1 = \frac{V}{i_1}$

Reading $R_2 = \frac{V}{i_2}$

Actual $R = \frac{V}{i_1 + i_2}$

$$R = \frac{R_1 R_2}{R_1 + R_2} //$$

30. When a 50 W bulb is inserted in a table lamp holder, the connecting wires dissipate 1.0 mW. How much power will be dissipated in the connecting wires, if you insert a 100 W bulb in the table lamp holder?



Wire resistance is negligible compared to bulb.



Current in circuit will be determined by bulb alone.

$$50 = V i_1$$
$$100 = V i_2$$

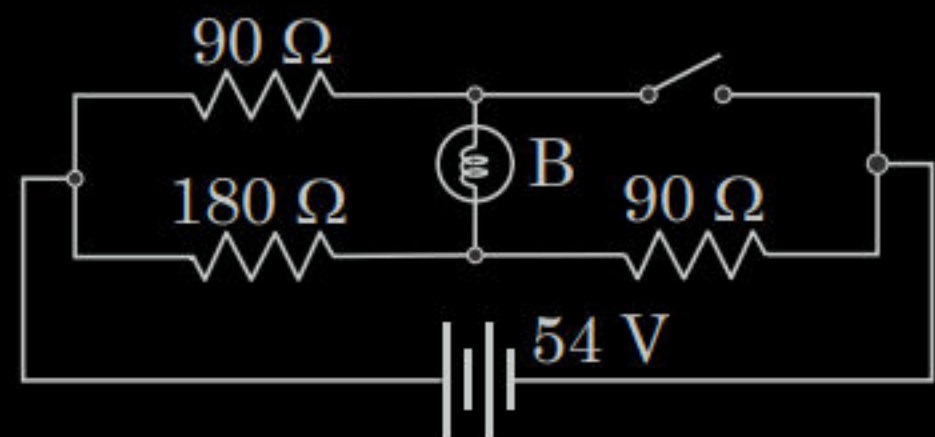
$$i_2 = 2 i_1$$

current doubles

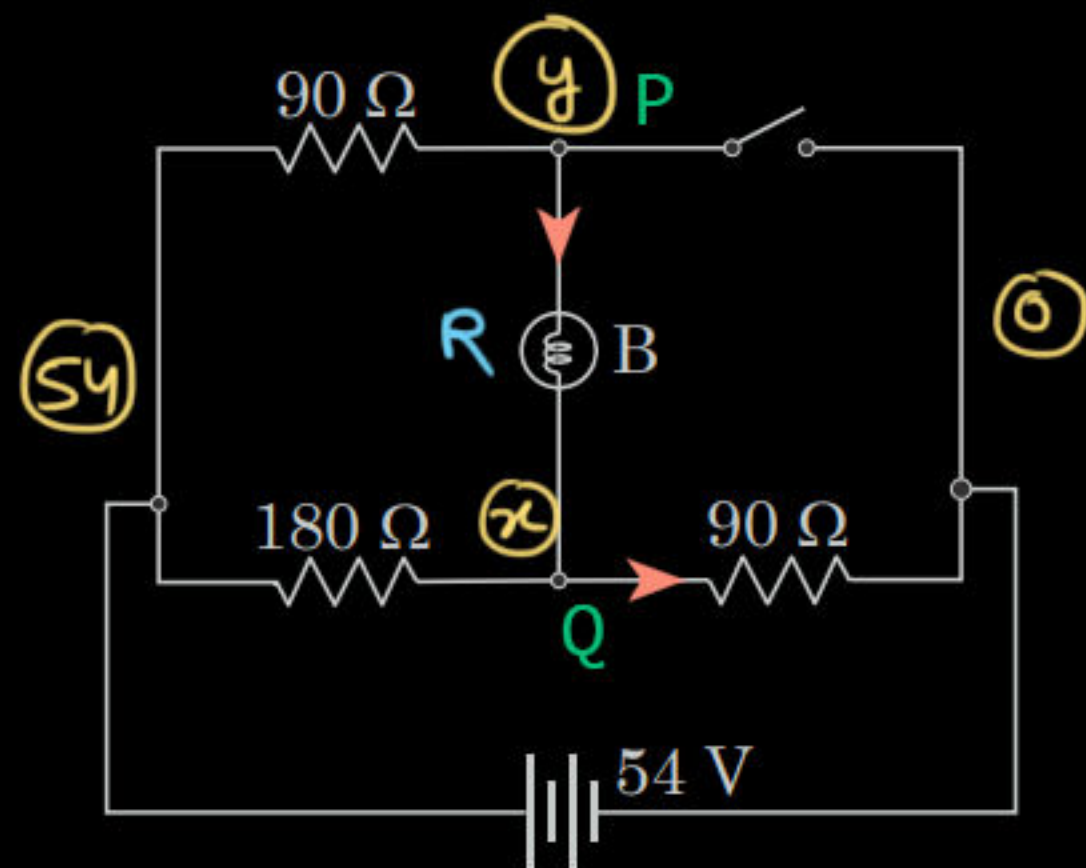
$$\text{Power in wire} = i^2 R$$



Heat dissipated is 4 times. i.e 4 mW //



31. The circuit shown, consists of three resistances, an incandescent bulb, a switch and a battery. If the bulb B glows equally bright whether the switch is closed or open, find resistance of the bulb and the voltage-drop across it.



$$P: \frac{y-54}{90} + \frac{y-x}{R} = 0$$

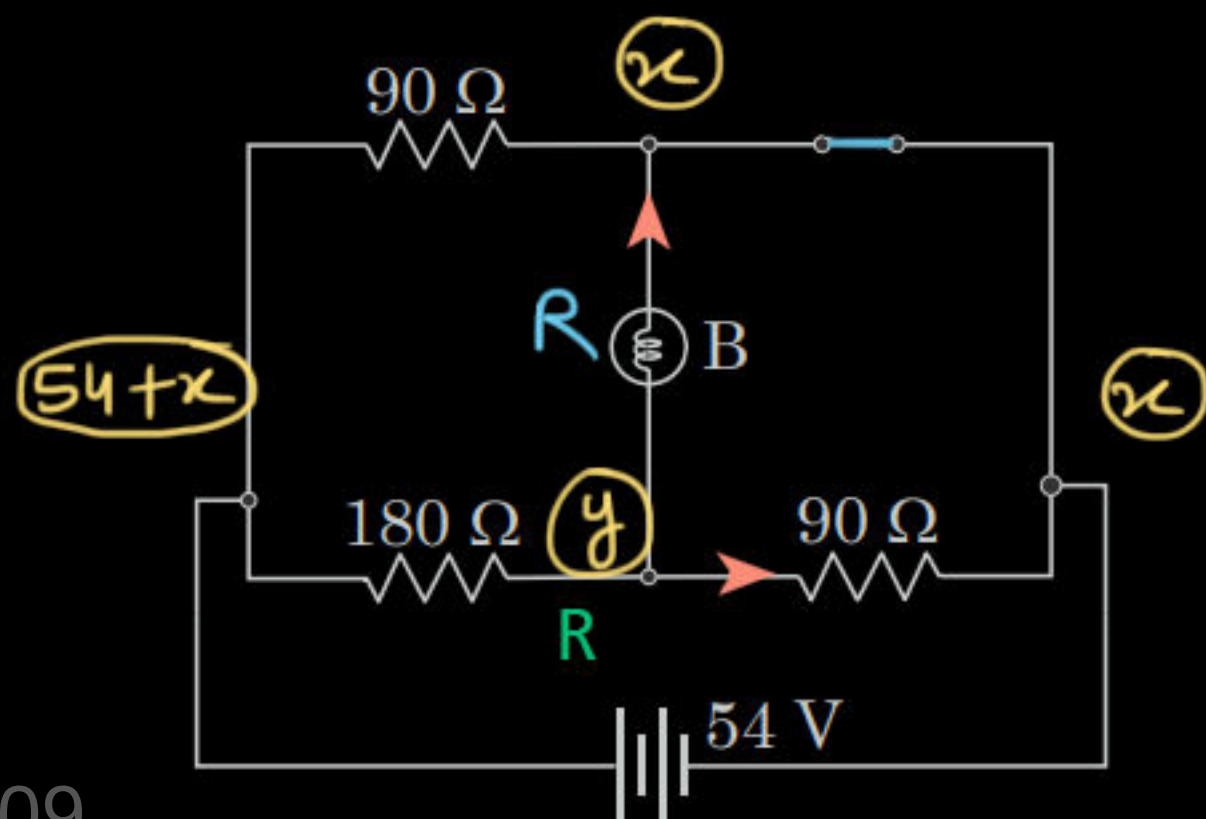
$$Q: \frac{x-54}{180} + \frac{x-y}{R} + \frac{x-0}{90} = 0$$

$$R: \frac{y-(54+x)}{180} + \frac{y-x}{R} + \frac{y-x}{90} = 0$$

3 equations, 3 variables: $x, y, \frac{x-y}{R}$

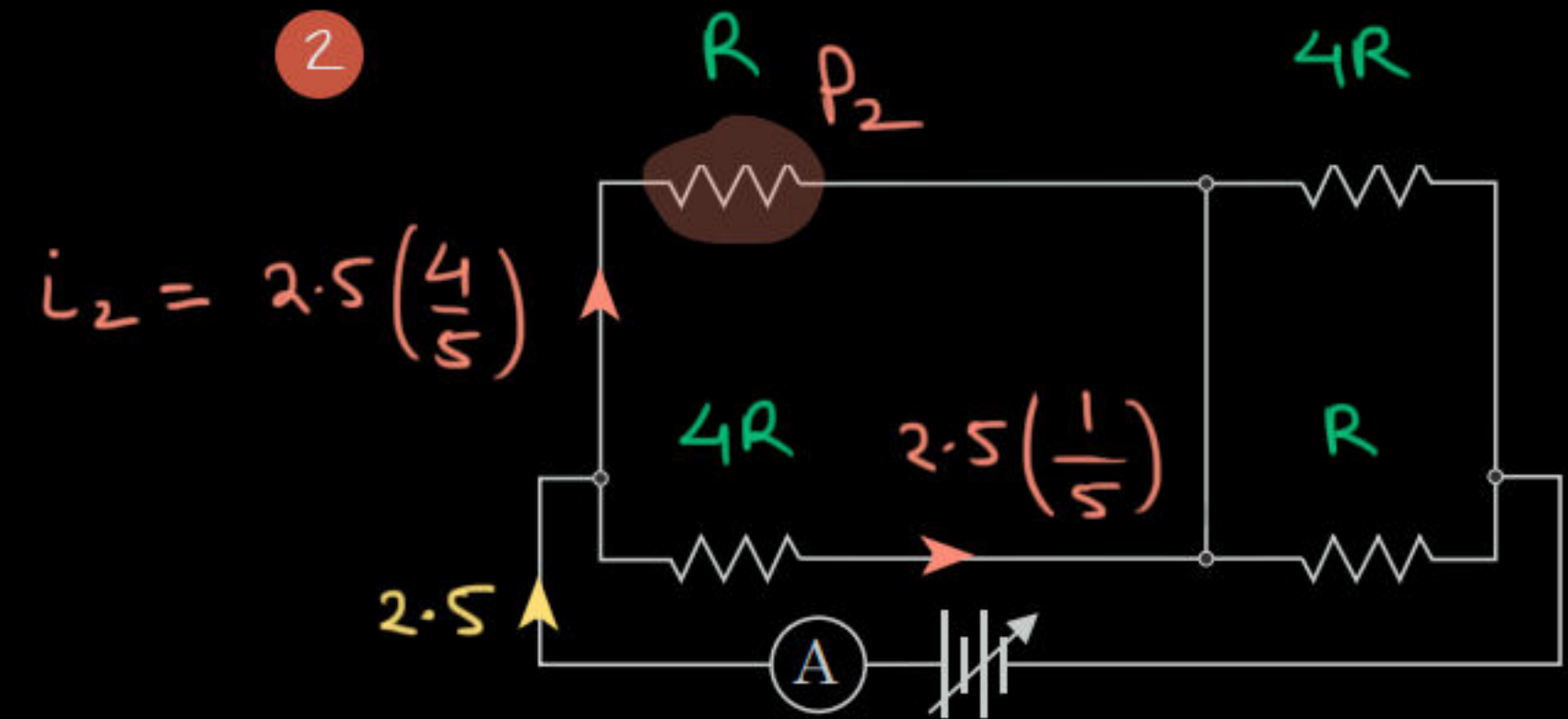
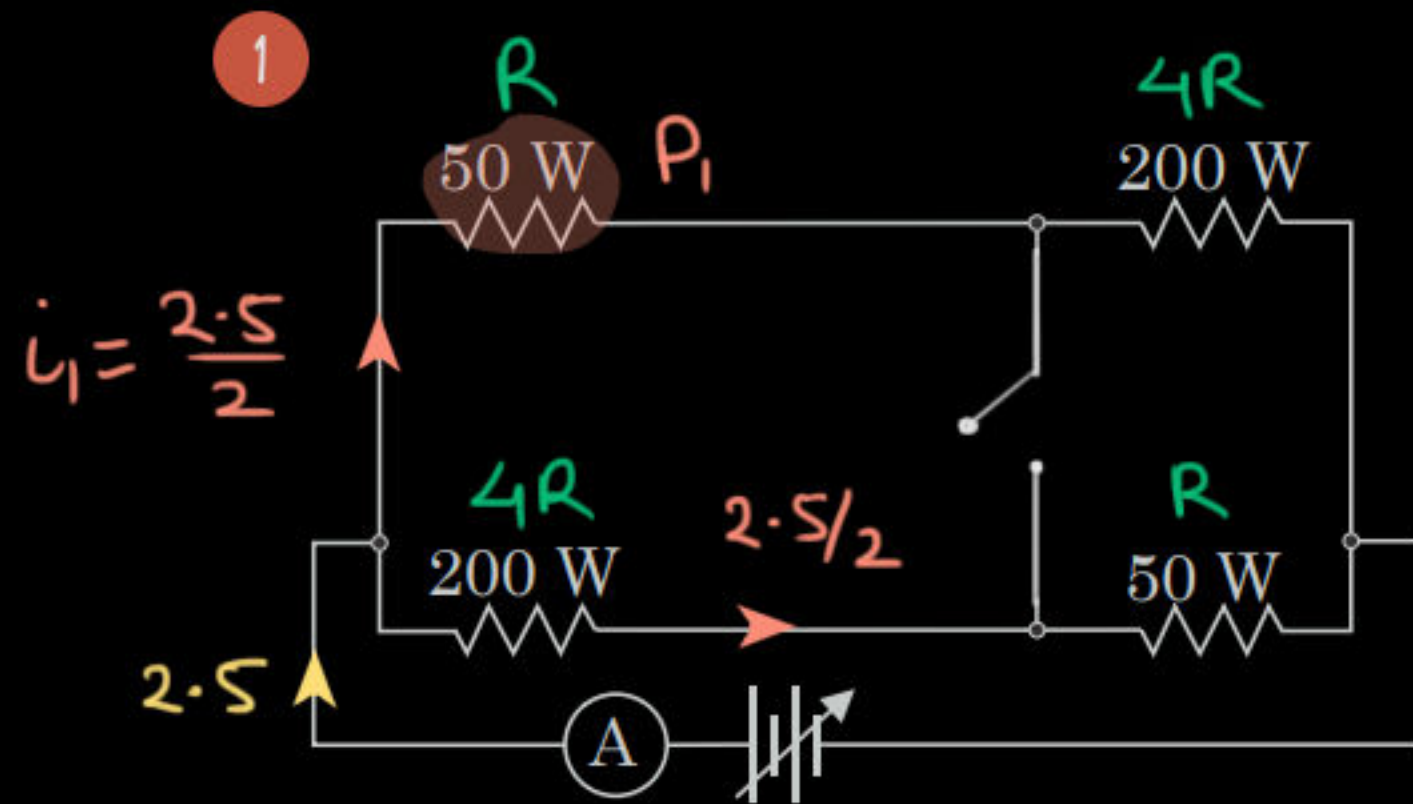
$$\Rightarrow x=30, y=36, R=30 //$$

$$\Rightarrow y-x=6 //$$



Power is proportional to resistance, as same current flows in all of them.

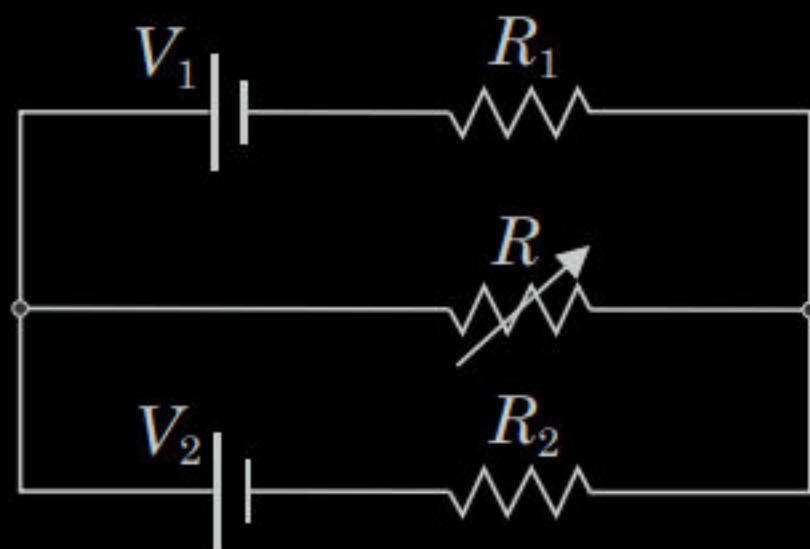
32. In the given circuit when the switch is open, the ammeter A reads 2.5 A, two of the four resistances dissipate 50 W each and other two resistances dissipate 200 W each. If the switch is closed and terminal voltage of the battery is so adjusted that the ammeter reading again becomes 2.5 A, find power dissipation in each of the resistances.



For same resistor in both circuits: $P \propto i^2 \Rightarrow P_2 = P_1 \left(\frac{i_2}{i_1} \right)^2$

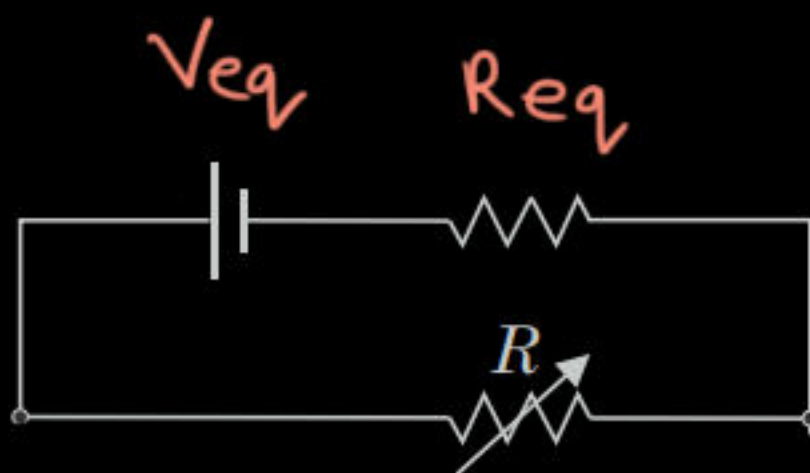
$\therefore P_R = 50 \left(\frac{4/5}{1/2} \right)^2 = 50 \cdot \frac{64}{25} = 128 \text{ W} //$

Similarly, $P_{4R} = 200 \left(\frac{1/5}{1/2} \right)^2 = 200 \cdot \frac{4}{25} = 32 \text{ W} //$



33. The given circuit consists of two ideal batteries, two fixed value resistances and a variable resistance. Terminal voltages V_1 and V_2 of the batteries and resistances R_1 and R_2 are known. Find the resistance R of the variable resistance for which maximum power is developed in it and the corresponding maximum power.

||| Thevenins



For maximum power in R : $R = R_{eq} = \frac{R_1 R_2}{R_1 + R_2} //$

$$V_{eq} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{V_1 R_2 + V_2 R_1}{R_1 + R_2}$$

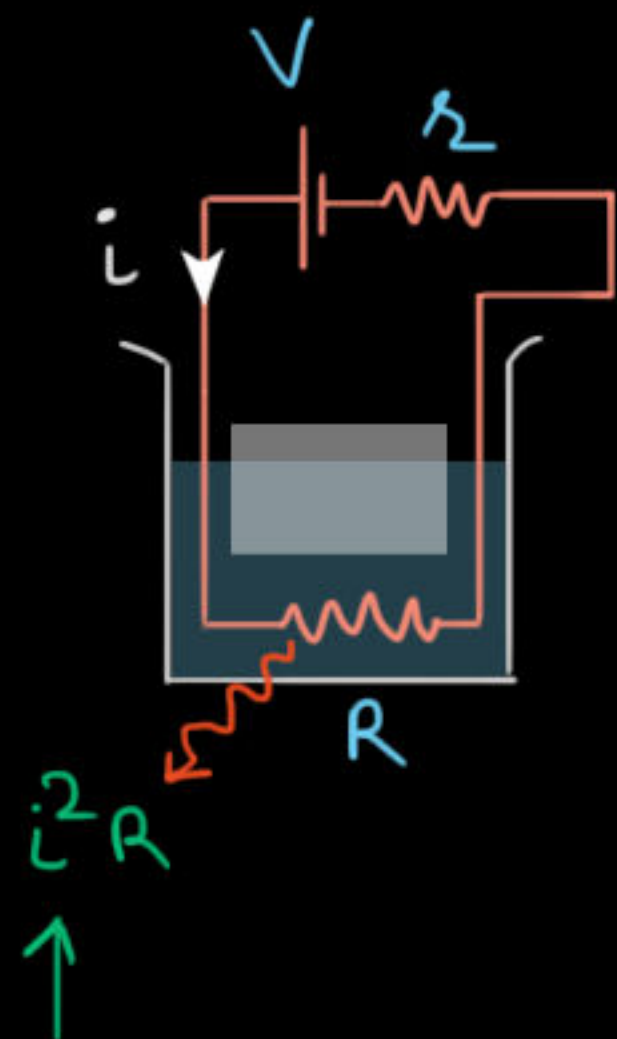
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore \text{Required power: } i^2 R = \left(\frac{V_{eq}}{2R_{eq}} \right)^2 R_{eq}$$

$$= \frac{V_{eq}^2}{4R_{eq}}$$

$$= \frac{(V_1 R_2 + V_2 R_1)^2}{4 R_1 R_2 (R_1 + R_2)}$$

Youtube / Arihant Senior



34. Three identical vessels A, B and C containing equal quantities of ice at 0°C are placed in a room. Identical heating elements placed inside each vessel are connected across batteries of terminal voltages $V_A = 380\text{ V}$, $V_B = 220\text{ V}$ and $V_C = 110\text{ V}$ respectively. If all the ice of vessel A melts in $t_A = 4\text{ min}$ and all the ice of vessel B melts in $t_B = 20\text{ min}$, how long will it take to melt all the ice of vessel C? Assume that the heating element, the ice and vessels remain in thermal equilibrium. $\therefore$ non-ideal battery

$$(P_{\text{electric}})t + \kappa t = \Delta H$$

$$\text{A} \quad \left(\frac{V_A}{R+r}\right)^2 R t_A + \kappa t_A = \Delta H$$

$$\text{B} \quad \left(\frac{V_B}{R+r}\right)^2 R t_B + \kappa t_B = \Delta H$$

$$\text{C} \quad \left(\frac{V_C}{R+r}\right)^2 R t_C + \kappa t_C = \Delta H$$

Room temperature contributes in melting too.

$\therefore$ Let the room supplies the heat @ $\kappa\text{ J/s}$

Also, let the internal resistance of battery be r .

3 equations, 3 variables: t_C , $\frac{\kappa}{R/(R+r)^2}$, $\frac{\Delta H}{R/(R+r)^2}$

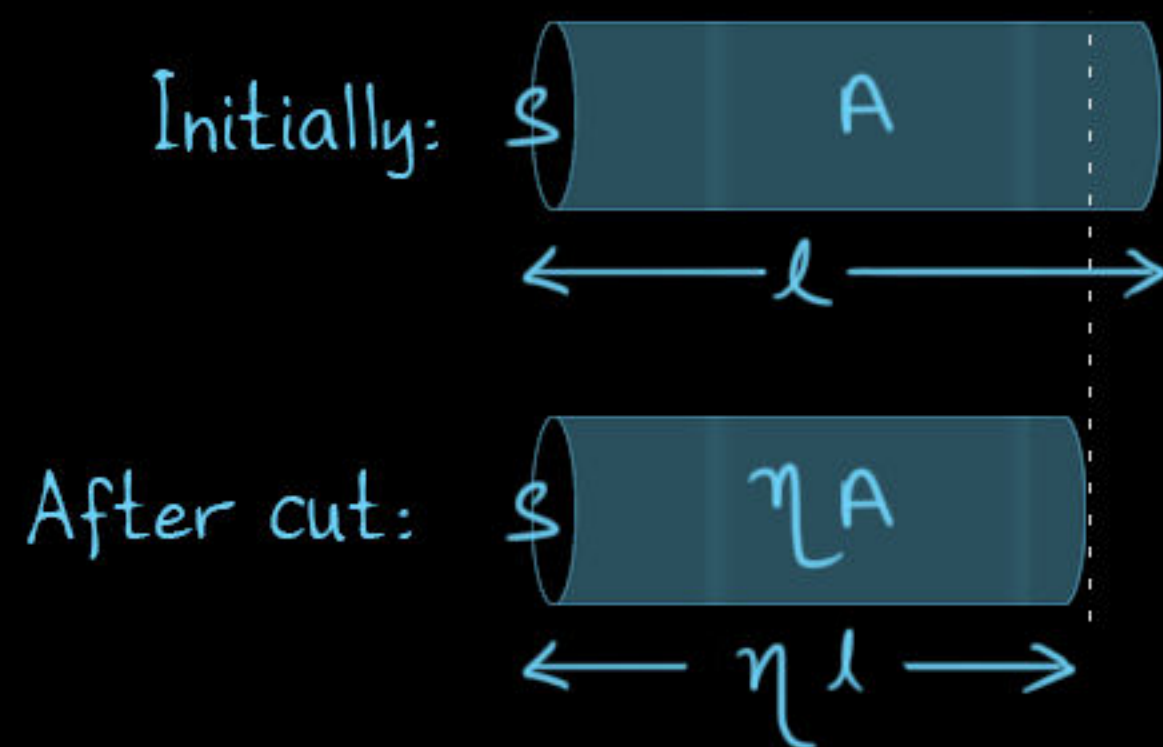
Solving

$t_C \neq \kappa -ve \Rightarrow$ Impossible for 110 W heater.
 $-ve \kappa$ means Room temperature is below zero and won't allow 110 W heater to melt the ice

$$\begin{aligned}
 \text{Radiation heat loss} &= \\
 &= \sigma_b A (T^4 - T_0^4) \\
 &= \sigma_b A [(T_0 + \Delta T)^4 - T_0^4] \\
 &\approx \sigma_b A T_0^3 (4 \Delta T)
 \end{aligned}$$

(When temperature difference with surrounding is less)

⇒ Radiation heat loss $\propto A \Delta T$



35. A cylindrical conductor is connected across a battery and when a steady state is reached, temperature of the conductor exceeds the ambient temperature by $\Delta\theta = 16^\circ\text{C}$. If the length of the conductor is reduced to $\eta = 0.8$ times by cutting off extra length and other conditions are kept unaltered, by what amount will the temperature of the conductor change?

At steady state:

Power gained = Power lost
by electricity by radiation

$$\begin{aligned}
 \text{Power} &= \frac{V^2}{R} \propto A \Delta\theta_1 \\
 &= \frac{V^2}{\frac{8 \eta l}{S}} \propto (\eta A) \Delta\theta_2
 \end{aligned}$$

divide

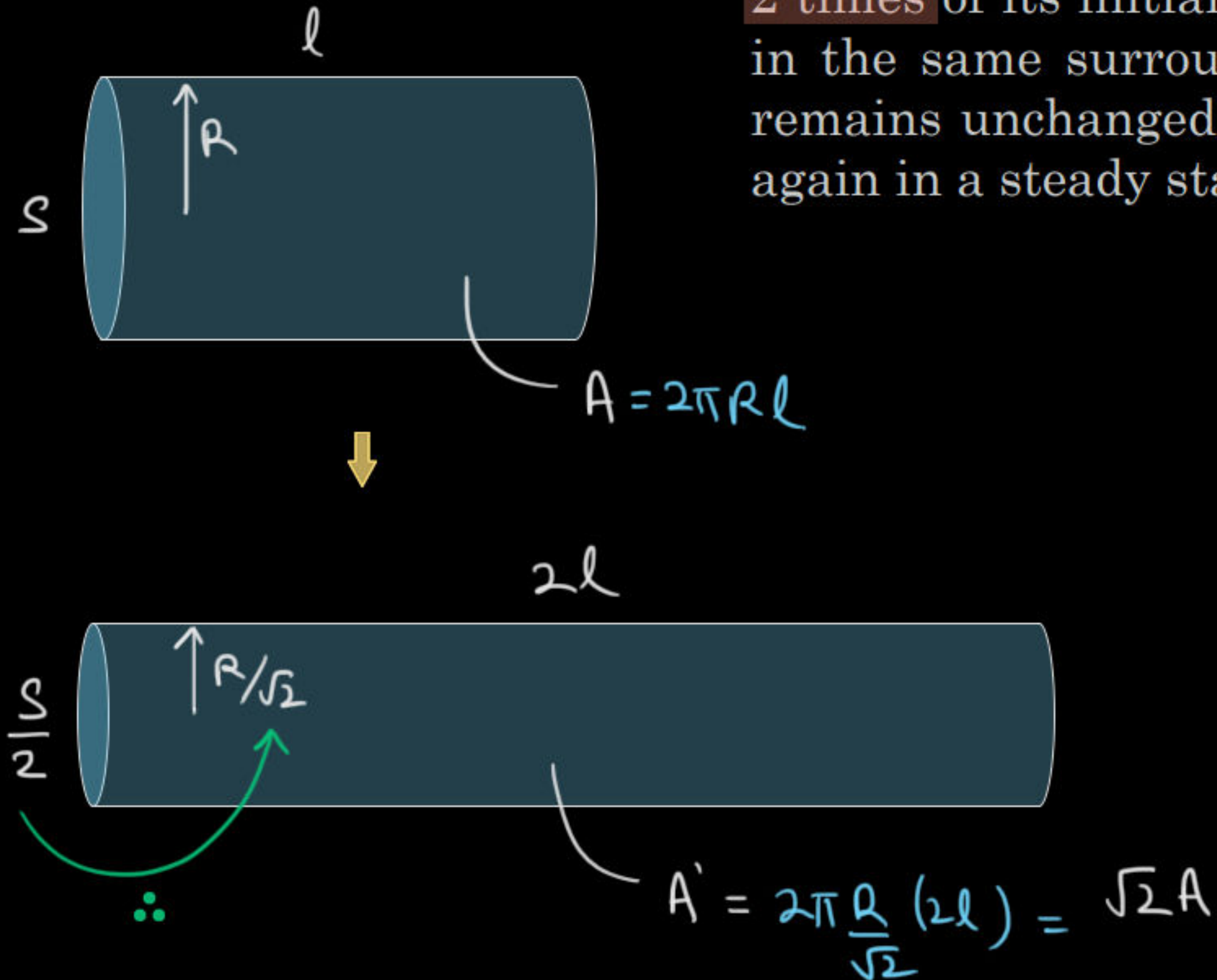
$$\Delta\theta_2 = \frac{\Delta\theta_1}{\eta^2}$$

$$\therefore T_2 - T_1 = (T_2 - T_0) - (T_1 - T_0) = \Delta\theta_2 - \Delta\theta_1 = \left(\frac{1}{\eta^2} - 1\right) \Delta\theta_1$$

by Ankit Singhvi

Youtube / Arihant Senior

Radiation heat loss:
 $\approx \sigma_b A T_0^3 (4 \Delta T)$



36. In a laboratory where room temperature is $\theta_0 = 0^\circ\text{C}$, temperature of a cylindrical conductor connected across a battery reaches a value $\theta = (20\sqrt{2})^\circ\text{C}$ in steady state. Now the cylinder is uniformly stretched to $\eta = 2$ times of its initial length and left connected across the same battery in the same surroundings. If resistivity of material of the conductor remains unchanged on stretching, find its temperature when it comes again in a steady state.

$$P_{in} \propto P_{out}$$

$$\text{i.e.: } \frac{V^2}{R} \propto A \Delta T$$

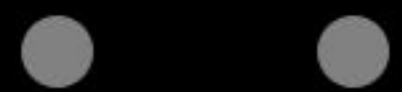
$$\text{Initially: } \frac{V^2}{\left(\frac{8l}{s}\right)} \propto A \Delta\theta, \quad (1)$$

$$\text{Finally: } \frac{V^2}{\left(\frac{8 \cdot 2l}{s/2}\right)} \propto \sqrt{2}A \Delta\theta_2, \quad (2)$$

$$\text{Dividing 1,2: } \Delta\theta_2 = \frac{\Delta\theta_1}{4\sqrt{2}} = \frac{20\sqrt{2}}{4\sqrt{2}} = 5 //$$

Youtube / Arihant Senior

We know,



$$C_{eq} = 2\pi\epsilon_0\epsilon_r a$$

Proved in irodov 3.110 for identical spheres.

But by similar approach, C can be found for dissimilar spheres too.



$$C_{eq} = 4\pi\epsilon_0\epsilon_r \frac{ab}{a+b}$$

And, $C_{eq} \cdot R_{eq} = \frac{\rho}{\epsilon}$

Proved in irodov 3.157 method 2

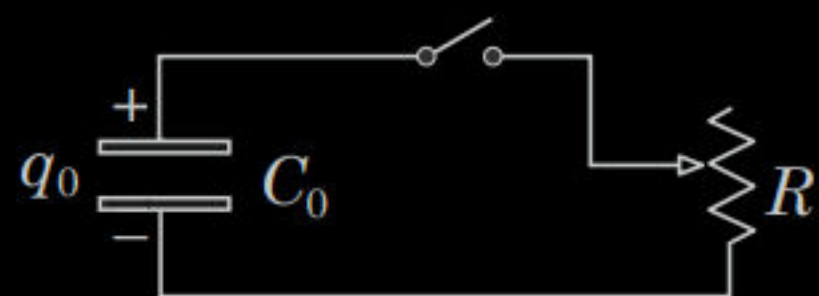
37. Two identical metal spheres are immersed in a homogeneous medium of large extent. Radius of each sphere is negligible as compared to the depth of immersion and the distance between the spheres. When one sphere is connected with the positive terminal of a battery and other with the negative terminal of the battery using thin insulated conductors, current $I_0 = 3.0 \text{ A}$ flows through the battery. What current will flow through the battery, if one of these spheres is replaced by another sphere of radius $\eta = 0.5$ times of that of the previous sphere? Resistance of the conductors and the source can be neglected.

$$R = \frac{\rho}{C_{eq}} = \frac{\frac{\rho}{\epsilon_r}}{2\pi\epsilon_0\epsilon_r a} \quad \text{Initially} \quad \rightarrow R_1$$

$$\frac{\rho}{\frac{4\pi\epsilon_0\epsilon_r (a+b)}{2\eta}} \quad \text{Finally} \quad \rightarrow R_2 = \frac{R_1(1+\eta)}{2\eta}$$

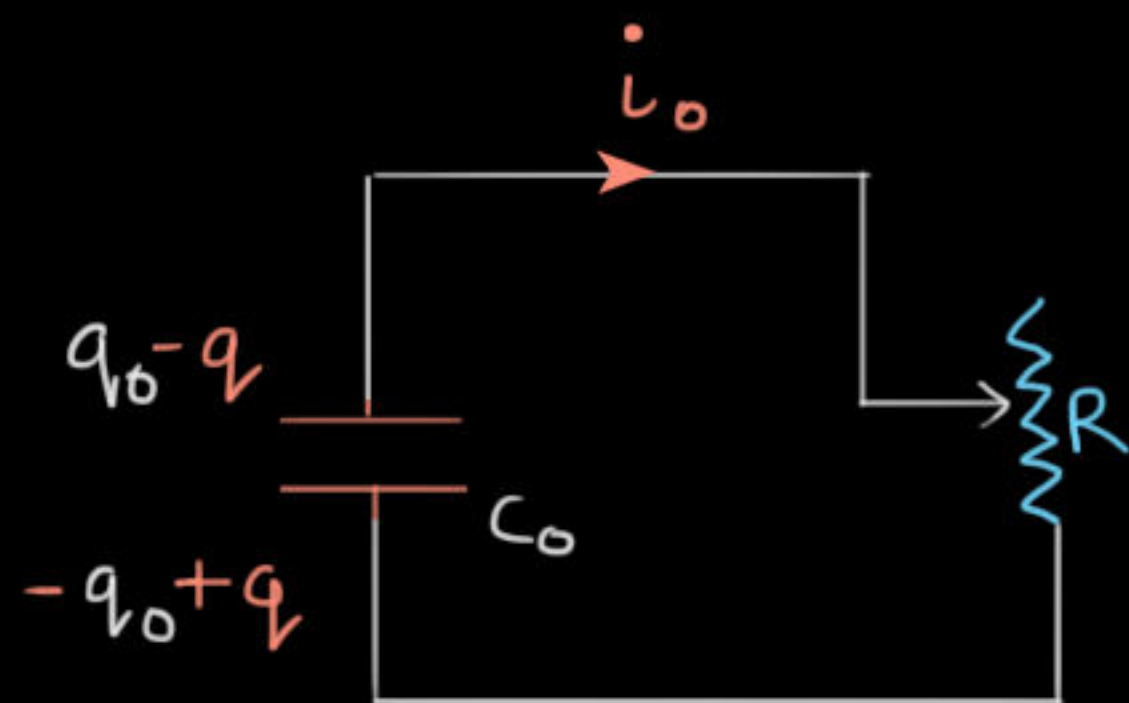
$$I_1 R_1 = I_2 R_2$$

$$\Rightarrow I_2 = I_1 \left(\frac{R_1}{R_2} \right) = \frac{2\eta}{1+\eta} I_1$$



38. It is desired to keep current in discharge process of a capacitor constant. For the purpose, following three schemes (a), (b) and (c) are suggested.

(a) In this scheme, after closing the switch you have to vary resistance of the rheostat. The initial charge on the capacitance C_0 is q_0 and range of values of resistance of the rheostat is zero to R_0 . Find the law $R(t)$ according to which resistance of the rheostat should be changed with time.



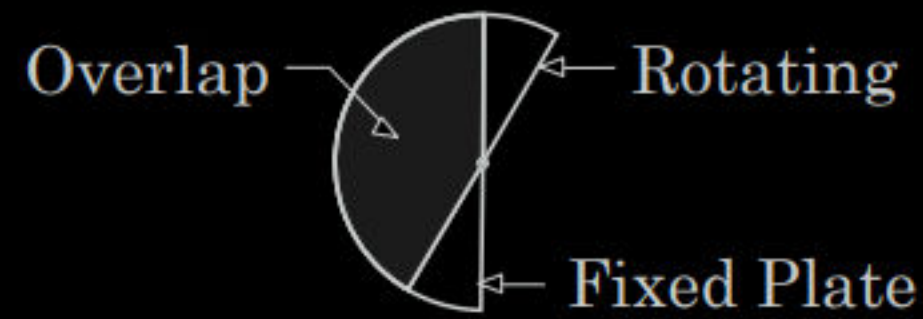
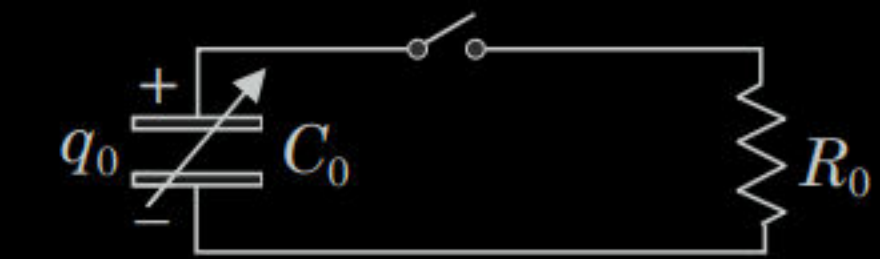
$$\frac{q_0 - q}{C_0} - i_0 R = 0$$

$$\Rightarrow q_0 - q = i_0 C_0 R$$

↓ differentiate

$$-\frac{dq}{dt} = i_0 C_0 \frac{dR}{dt}$$

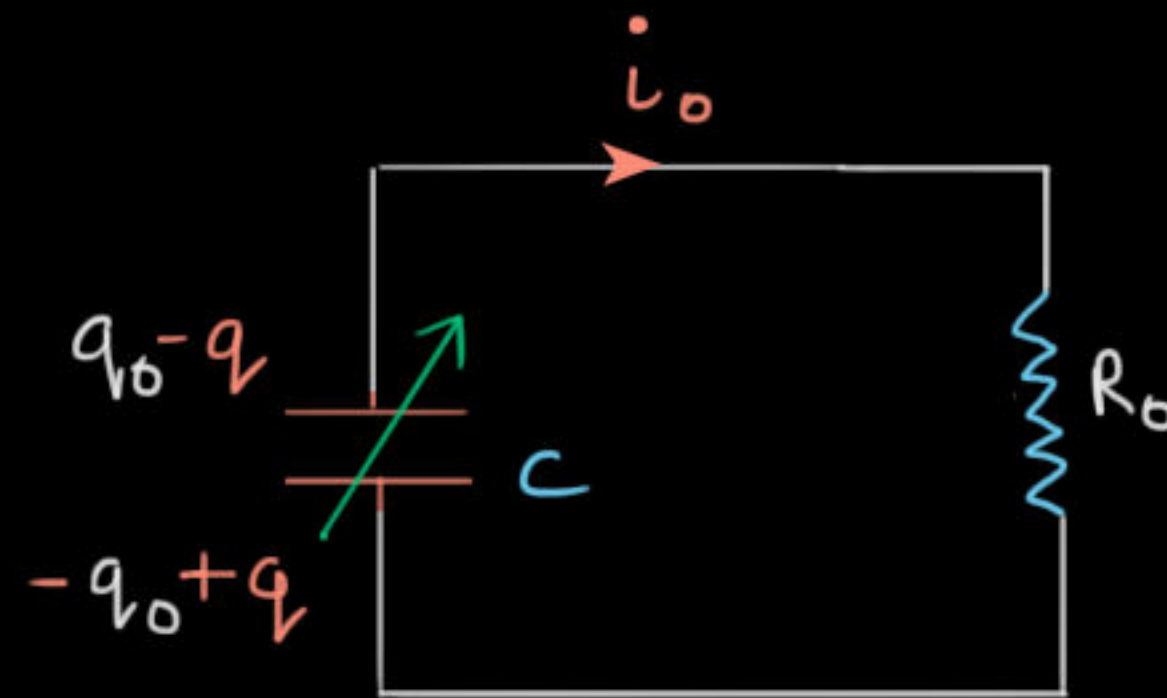
$$\Rightarrow \int_{R_0}^R dR = -\frac{1}{C_0} \int_0^t dt \Rightarrow R = R_0 - \frac{t}{C_0} = R_0 \left(1 - \frac{t}{R_0 C_0}\right) //$$



(b) In this scheme, a **variable disk capacitor** is connected across a fixed resistance R_0 . After closing the switch, you have to **vary the capacitance** by rotating one of its plates. The initial value of capacitance is C_0 and charge on it is q_0 . **Find the law $C(t)$ according to which capacitance should be changed with time and corresponding angular velocity $\omega(t)$ of one plate relative to the other.**

$$C = \frac{A\epsilon_0}{d} = \left(\frac{\pi - \theta}{\pi} \right) \left(\frac{\pi r^2}{2} \right) \frac{\epsilon_0}{d} \rightarrow C_0$$

$$= C_0 \left(1 - \frac{\theta}{\pi} \right)$$



$$\frac{q_0 - q}{C} - i_0 R_0 = 0$$

$$\Rightarrow q_0 - q = i_0 R_0 C$$

↓ differentiate

$$-\frac{dq}{dt} = i_0 R_0 \frac{dC}{dt}$$

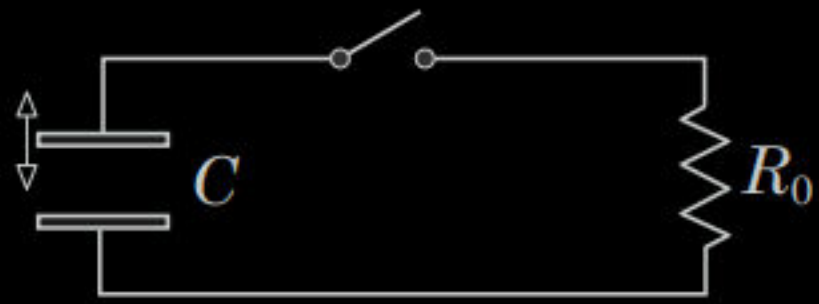
$$\Rightarrow -1 = R_0 \left[-\frac{C_0}{\pi} \frac{d\theta}{dt} \right]$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{\pi}{R_0 C_0}$$

$$\Rightarrow C = C_0 \left(1 - \frac{t}{R_0 C_0} \right)$$

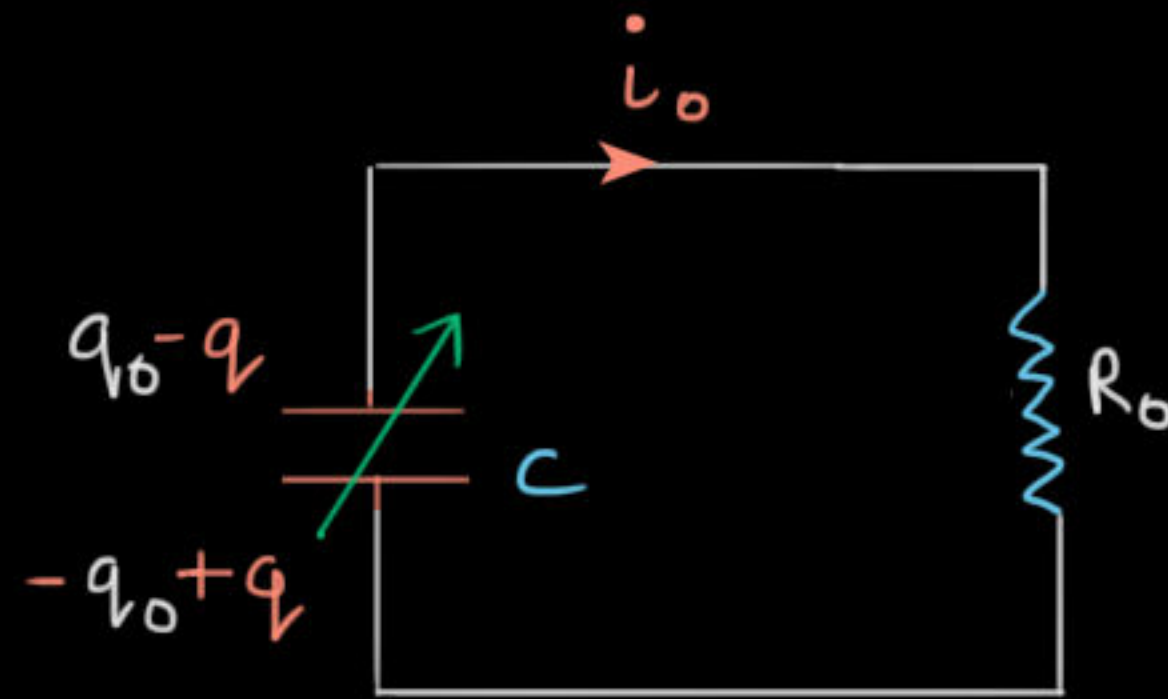
by Ankit Singhvi

Youtube / Arihant Senior



(c) In this scheme, a variable capacitor is used in which one plate can be shifted towards or away from the other fixed plate without changing overlap area. After closing the switch, you have to move the movable plate. The initial value of capacitance is C_0 and charge on it is q_0 . Find the law $C(t)$ according to which capacitance and the distance $x(t)$ between the plates should be changed with time.

Initial gap between plates x_0 is given.



$$C = \frac{A\epsilon_0}{x}$$

$$\frac{q_0 - q}{C} - i_0 R_0 = 0$$

$$\Rightarrow q_0 - q = i_0 R_0 C$$

↓ differentiate

$$-\frac{dq}{dt} = i_0 R_0 \frac{dC}{dt}$$

$$\Rightarrow \int_{C_0}^C dC = -\frac{1}{R_0} \int_0^t dt$$

$$\Rightarrow C = C_0 \left(1 - \frac{t}{R_0 C_0}\right)$$

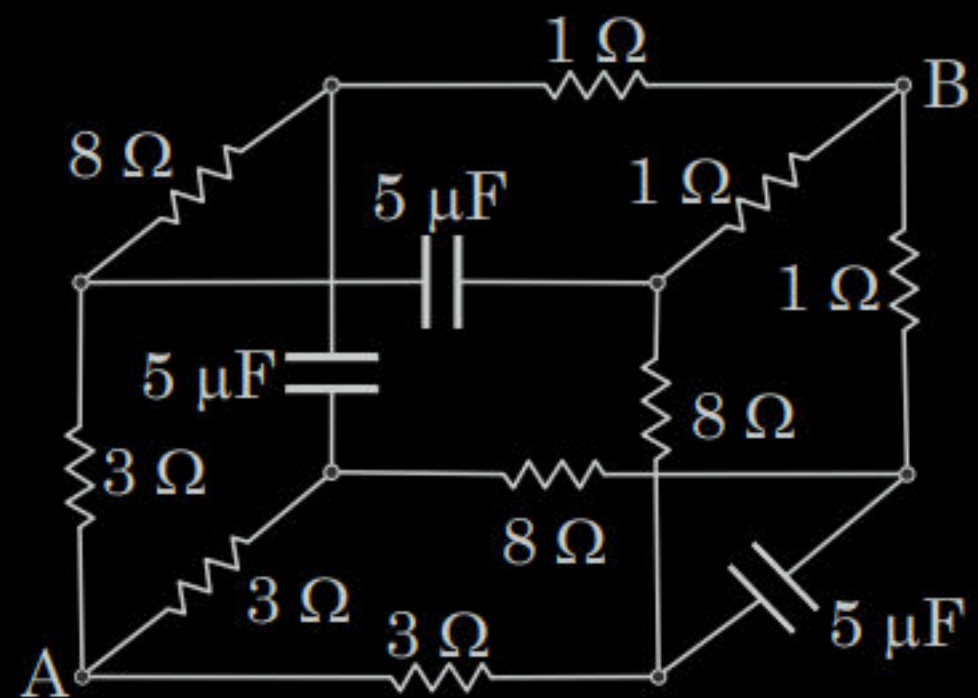
$$\frac{A\epsilon_0}{x} = C_0 \left(1 - \frac{t}{R_0 C_0}\right)$$

$$\Rightarrow \frac{x}{x_0} = \frac{R_0 C_0}{R_0 C_0 - t}$$

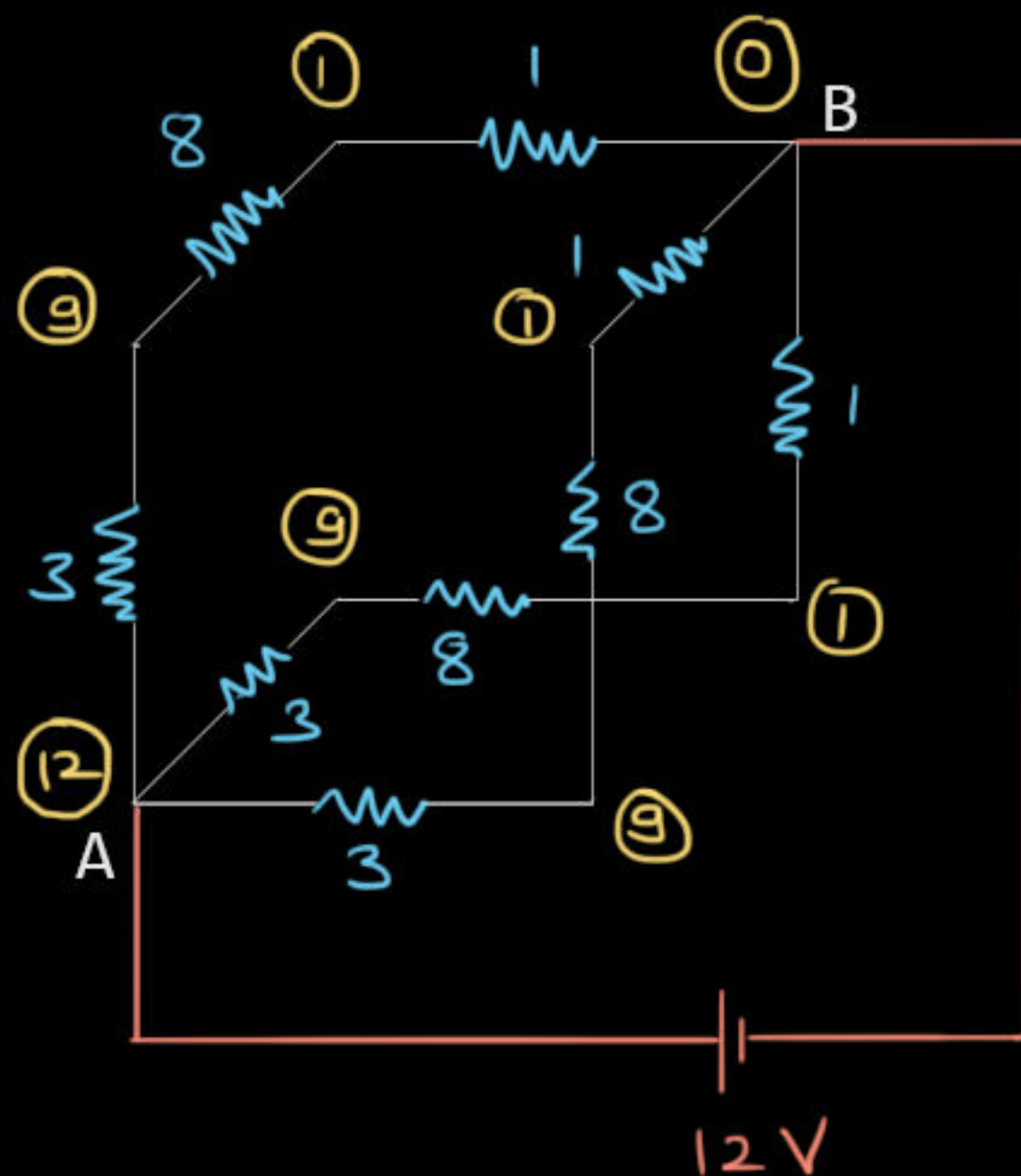
$$\Rightarrow x = \frac{R_0 C_0 x_0}{R_0 C_0 - t}$$

by Ankit Singhvi

Youtube / Arihant Senior

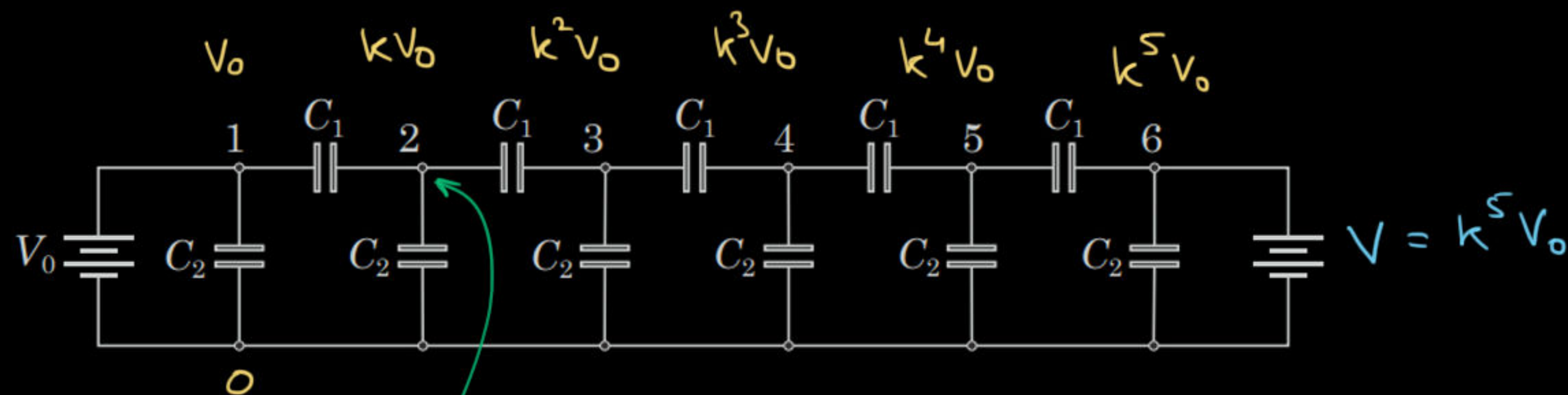


39. Three $1\ \Omega$, three $8\ \Omega$ and three $3\ \Omega$ resistances and three $5\ \mu\text{F}$ capacitors are connected in the edges of a cube constructed by conducting wires. If an ideal battery of electromotive force $12\ \text{V}$ is connected between the two diagonally opposite corners A and B of the cube, find charges on the capacitors.



$$\begin{aligned}
 q &= CV \\
 &= 5 \cdot 10^{-9} (9-1) \\
 &= 40\ \mu\text{C} //
 \end{aligned}$$

40. In the circuit shown, ratio of capacitors is $C_2/C_1 = 4/3$ and terminal voltage of the battery connected at the left end is V_0 . What should be terminal voltage of the battery connected at the right most end so that the potentials of nodes 1, 2, 3, 4, 5 and 6 be in a geometric progression?



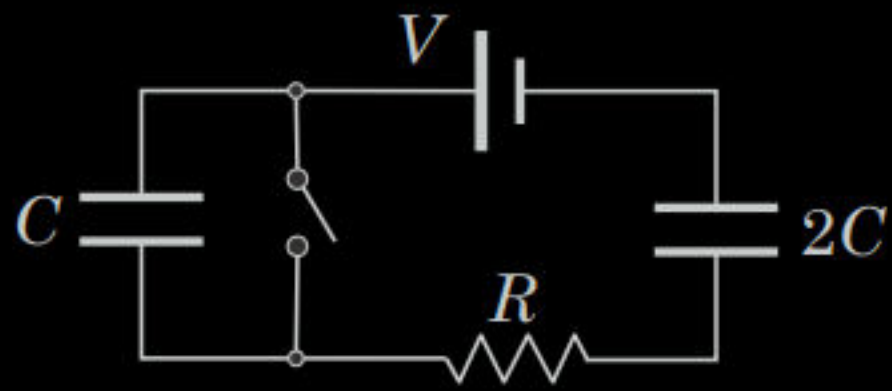
At 2: $C_1(kV_0 - V_0) + C_2(kV_0) + C_1(kV_0 - k^2V_0) = 0$

$$\downarrow \frac{C_2}{C_1} = \frac{4}{3}$$

$$3(k-1) + 4k + 3(k-k^2) = 0$$

$$\Rightarrow 3k^2 - 10k + 3 = 0 \Rightarrow k = \frac{1}{3}, 3$$

$$\therefore V = k^5 V_0 //$$



41. The circuit shown consists of two capacitors of capacitances C and $2C$, a resistance R and an ideal battery of terminal voltage V . Initially the switch is open and the capacitors are fully charged. How much total heat will be dissipated in the circuit after the switch is closed?

$$W_B = \Delta U + \text{Heat}$$

$$\Rightarrow \text{Heat} = W_B - \Delta U$$

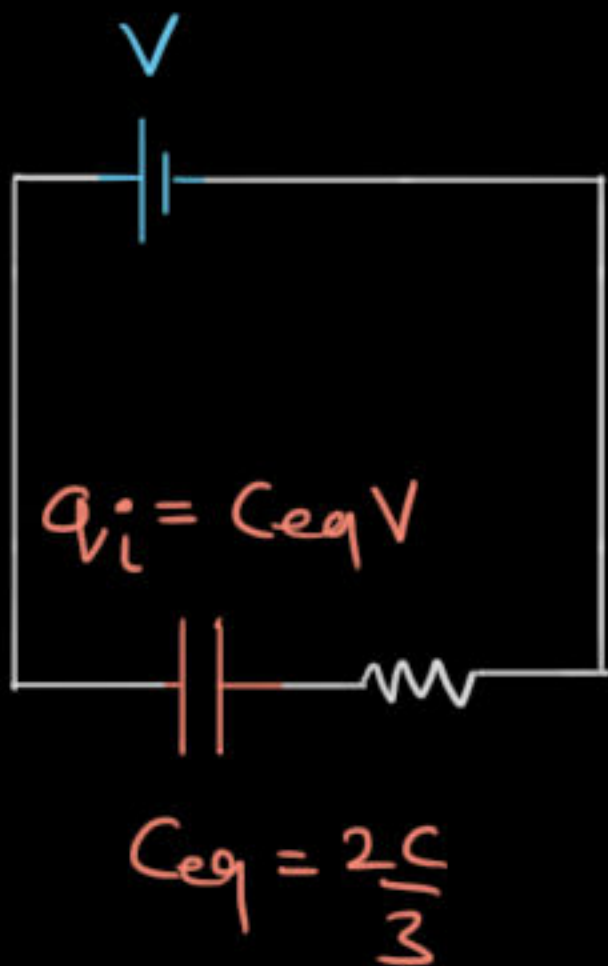
$$= V(q_f - q_i) - \frac{1}{2} V^2 (C_f - C_i)$$

$$= V \left(2CV - 2\frac{CV}{3} \right) - \frac{V^2}{2} \left(2C - \frac{2C}{3} \right)$$

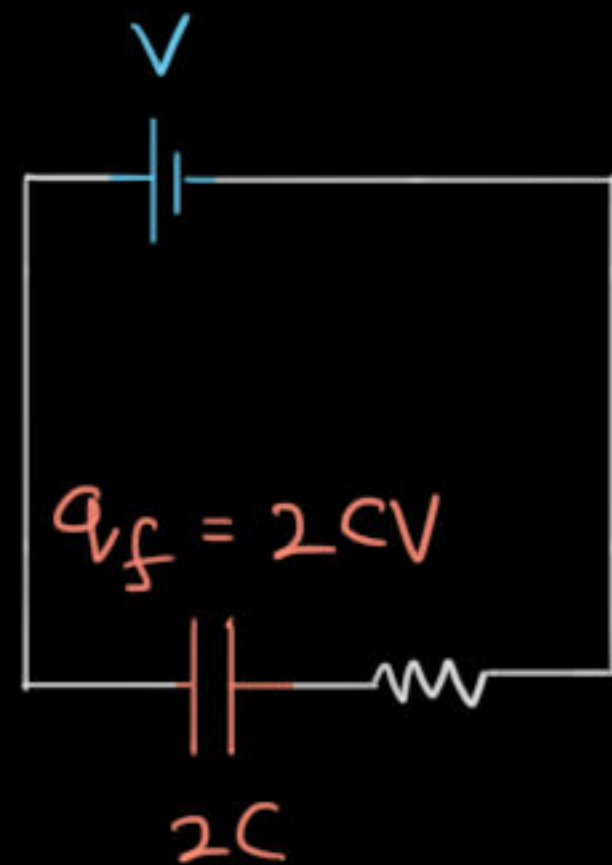
$$= \frac{4CV^2}{3} - \frac{2}{3} CV^2$$

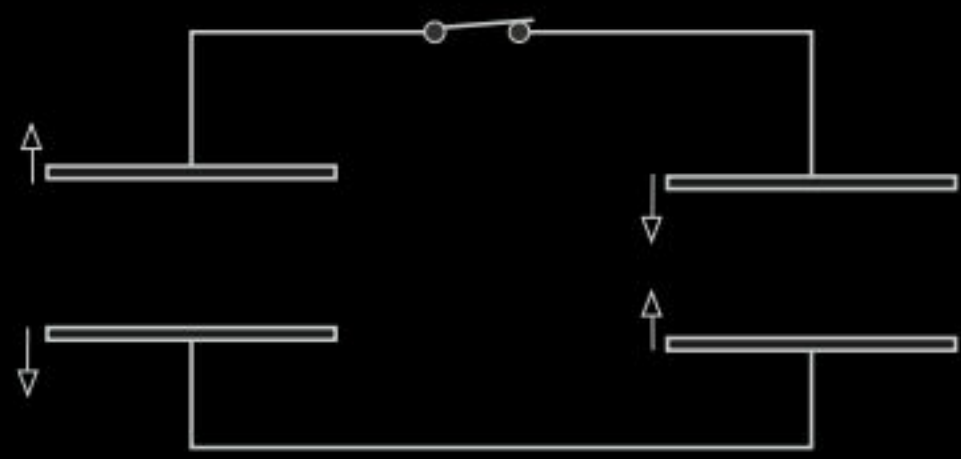
$$= \frac{2}{3} CV^2 //$$

Initial

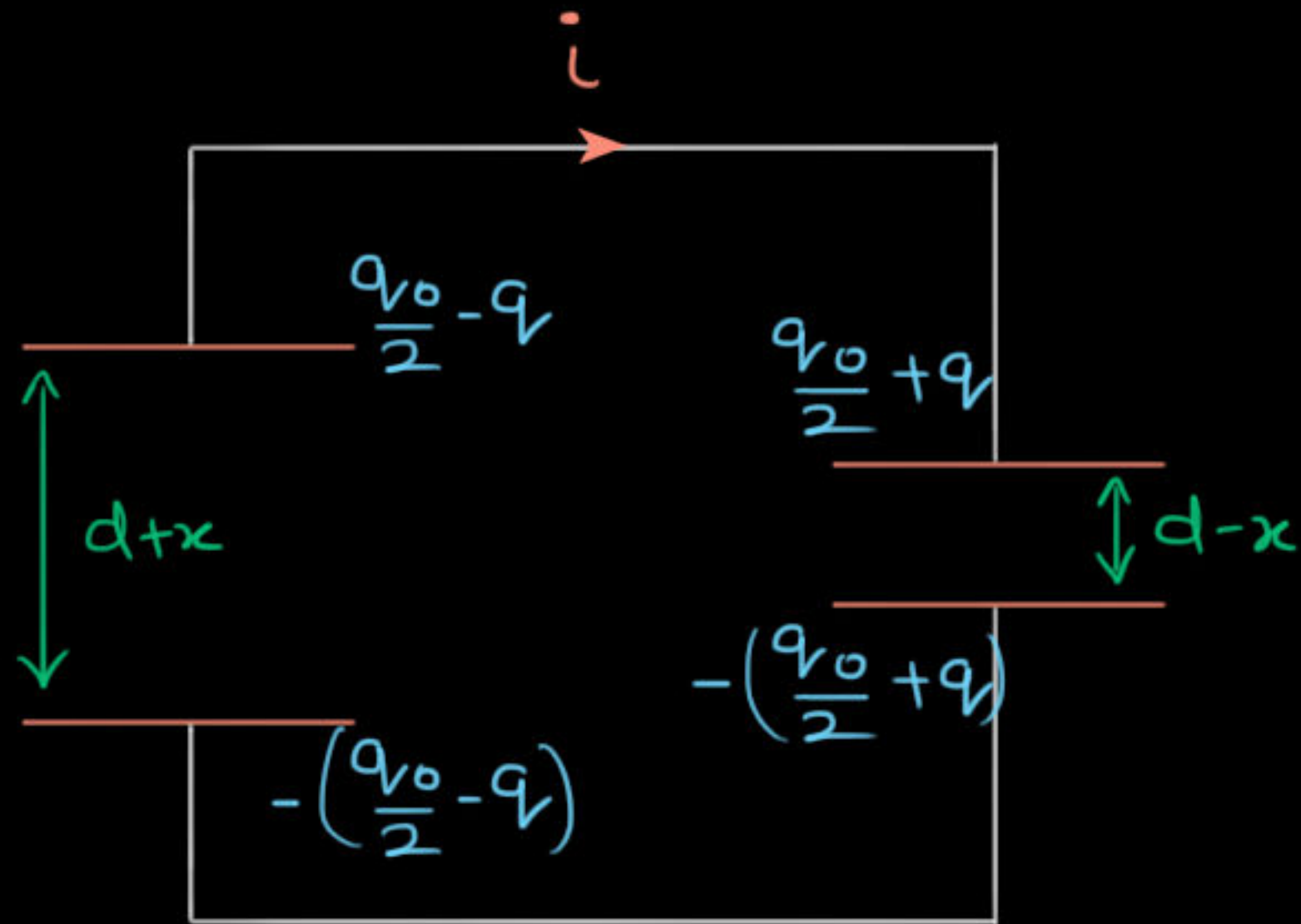


Final





42. Two identical parallel plate capacitors are connected as shown in the figure. Initially both have total charge q_0 and separation between their plates x_0 . At an instant, plates of one capacitor begin to move towards each other and plates of the other capacitor away from each other. If velocity of a plate relative to the other in both the capacitors has the same magnitude v , find current in the circuit.



$$\frac{\frac{q_{v0}}{2} - q_v}{\left(\frac{A\epsilon_0}{d+x}\right)} = \frac{\frac{q_{v0}}{2} + q_v}{\left(\frac{A\epsilon_0}{d-x}\right)}$$

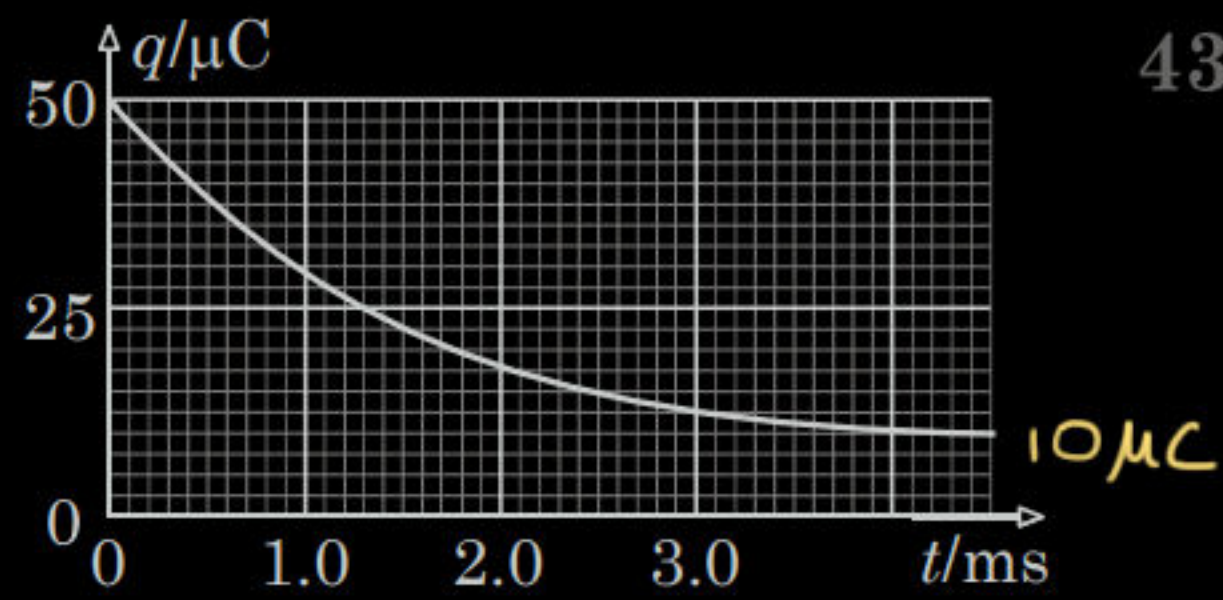
$$\Rightarrow \frac{d+x}{d-x} = \frac{\frac{q_{v0}}{2} + q_v}{\frac{q_{v0}}{2} - q_v}$$

$$\Rightarrow \frac{d}{x} = \frac{\frac{q_{v0}}{2}}{q_v}$$

$$\Rightarrow q_v = \frac{q_{v0}}{2d} x$$

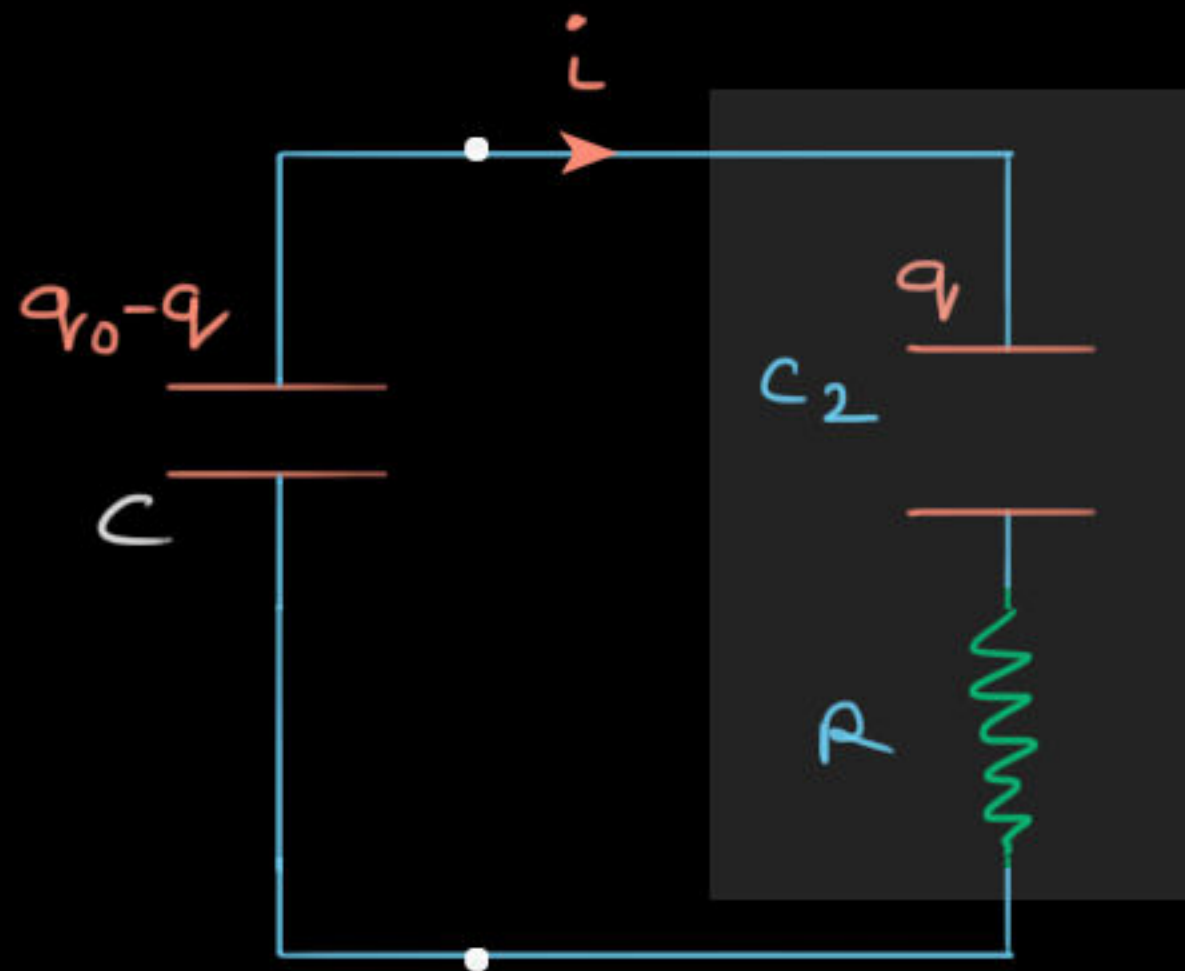
$$\therefore i = \frac{dq_v}{dt} = \frac{q_{v0}}{2d} v$$

Youtube / Arihant Senior



$$q_0 = 50$$

As the net charge on capacitor doesn't drop to zero, unknown capacitor must be in series with resistor.

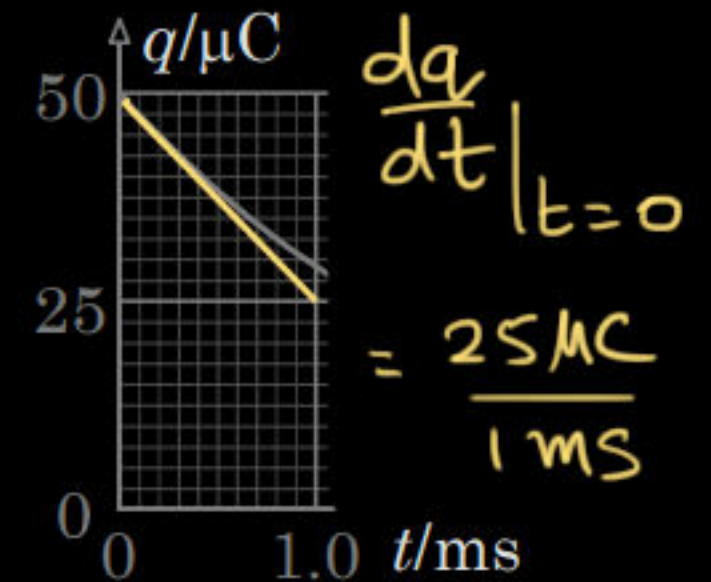


43. A circuit consisting of an uncharged capacitor and a resistance sealed inside a black-box and two terminals are taken outside. At time $t = 0$ a capacitor of capacitance $C = 1.0 \mu\text{F}$ having a charge $q_0 = 50 \mu\text{C}$ is connected across the terminals taken outside the black-box. A plot of the charge on this capacitor with time is shown in the figure. Find resistance R_{in} and capacitance C_{in} that are inside the black-box.

$$\textcircled{a} t = \infty \quad \frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{10}{C} = \frac{(50-10)}{C_2} \Rightarrow C_2 = 4C //$$

$$\textcircled{a} t = 0 \quad \frac{q_0 - q}{C} - \frac{q}{4C} - iR = 0$$

$$\Rightarrow R = \frac{q_0}{iC} = \frac{q_0}{C \frac{dq}{dt} \big|_{t=0}} = \frac{50 \cdot 10^{-6}}{(10^{-6}) \frac{25 \cdot 10^{-6}}{10^{-3}}} = 2 \text{ k}\Omega //$$



Due to fixed charge $*q_0$ at centre, a constant potential difference develops across the two plates.

Charge flows until that potential difference becomes zero.

Eventual charge distribution is as shown.

So basically its a spherical capacitor, that is being charged to q_0 .

44. A thick spherical shell of inner and outer radii r_1 and r_2 is made of a material of resistivity ρ and dielectric constant ϵ_r . Initially the shell is electrically neutral and has no charge accumulation anywhere. If at the instant $t = 0$, a point charge q_0 is somehow placed at the centre, find charge on the outer surface of the shell as a function of time t . Ignore any effects due to magnetism or radiation.

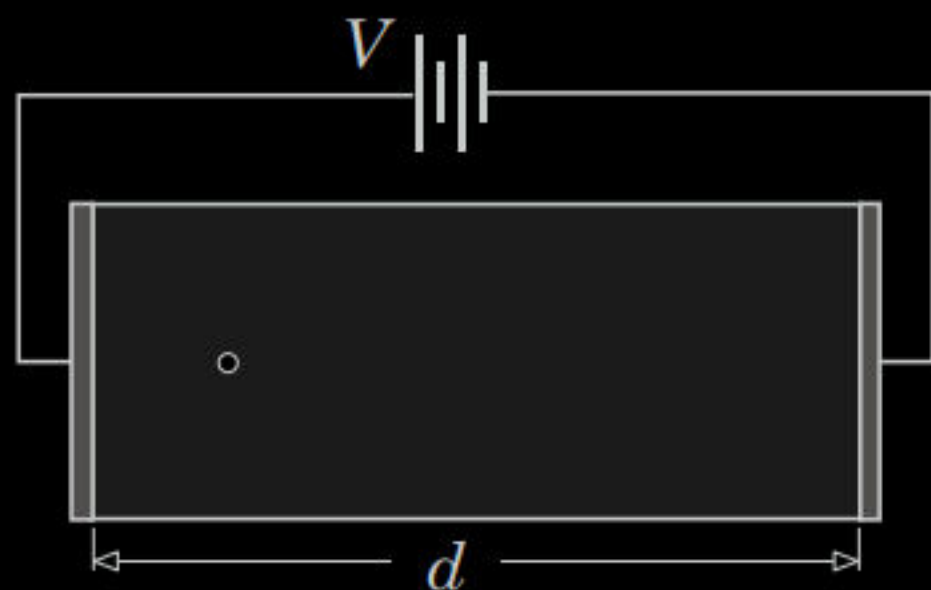


For charging of a capacitor:

$$q = q_0 (1 - e^{-t/RC})$$

$$\downarrow RC = \frac{\rho}{\epsilon_r \epsilon_0}$$

$$\Rightarrow q = q_0 (1 - e^{-t/\frac{\rho}{\epsilon_r \epsilon_0}})$$



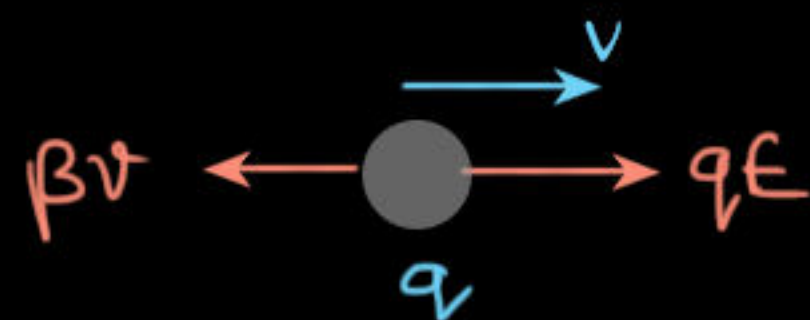
45. Space between plates of a parallel plate capacitor is filled by a weakly conducting viscous fluid of resistivity ρ and dielectric constant ϵ_r . Distance between the plates is d . The capacitor is connected across an ideal battery of terminal voltage V and a small conducting and almost massless ball having a charge q_0 is released in the fluid. Force of viscous drag on the ball in the fluid is $F = \beta v$, where v is velocity of the ball and β is a known factor. If the ball stops before striking any of the plates, find the maximum distance the ball will move. Assume no gravity.

Charge from the ball will start to leak.

When the charge approaches zero, ball will gradually halt.

Small ball acts like a spherical capacitor with other plate at infinity, filled with dielectric.

Let that capacitance be C .
So, we have a capacitor discharging from q_0 .



$$qE - \beta v = m \frac{dv}{dt}$$

$$\Rightarrow q_0 e^{-t/RC} E - \beta \frac{dx}{dt} = m \frac{dv}{dt}$$

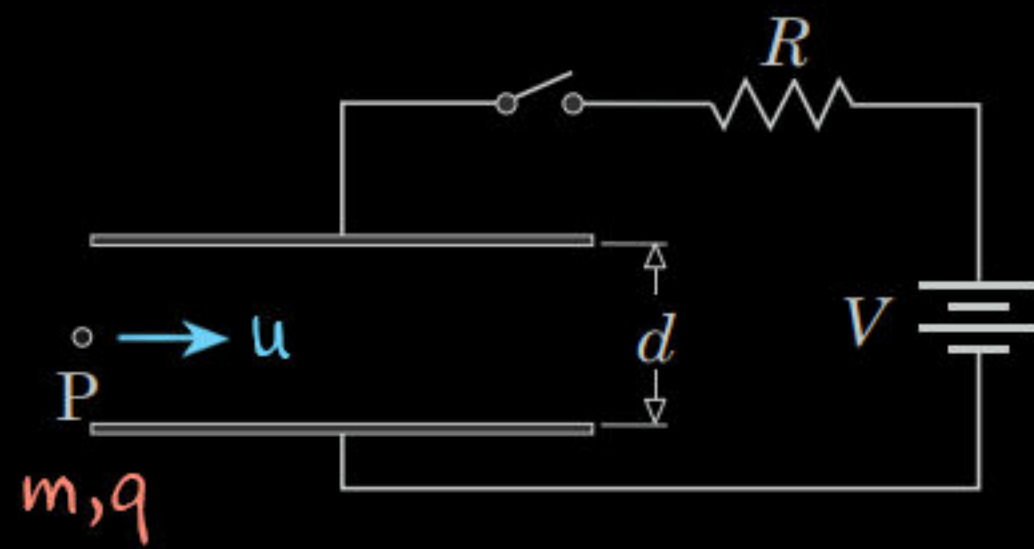
$$\Rightarrow q_0 E \int_0^{\infty} e^{-t/RC} dt - \beta \int_0^{x_0} dx = m \int_0^0 dv$$

$$\Rightarrow q_0 E RC - \beta x_0 = 0$$

$$E \rightarrow \frac{V}{d}, RC \rightarrow \frac{\rho d}{\epsilon_0 \epsilon_r} \Rightarrow x_0 = \frac{q_0 E RC}{\beta} = \frac{q_0 V \rho \epsilon_0 \epsilon_r}{\beta d}$$

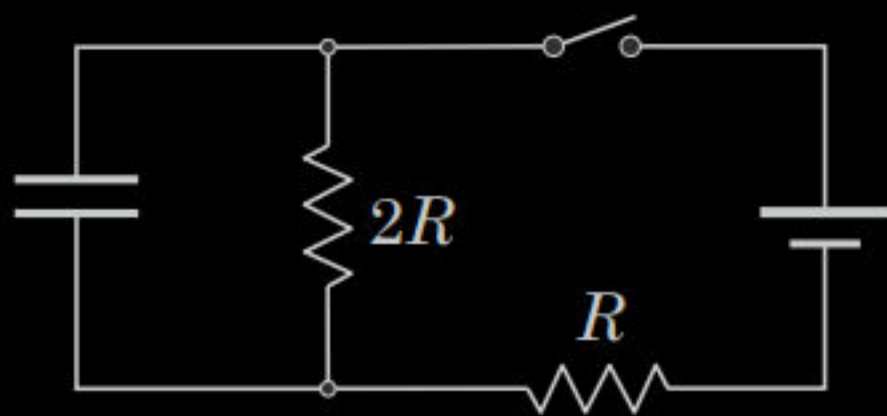
by Ankit Singhvi

Youtube / Arihant Senior

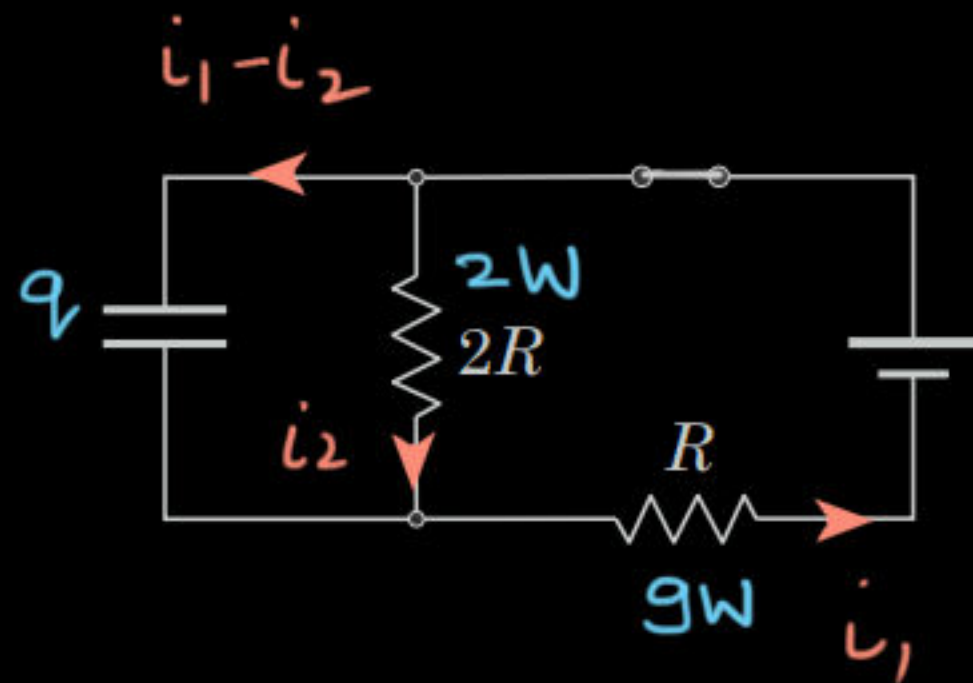


46. In the setup shown, when a particle P of charge q and mass m enters with velocity u in an uncharged parallel plate capacitor of capacitance C , the switch is closed. If time spent by the charge particle inside the capacitor is negligible as compared to the time constant RC of the circuit, find how acceleration of the particle varies with time t . Assume that the particle neither collides with the plates nor affects charge distribution on the plates. Also, neglect the edge effects and gravity.

$$\begin{aligned}
 ma &= qE \quad \left(E = \frac{V}{d} (1 - e^{-t/RC}) \right) \\
 \Rightarrow a &= \frac{q}{m} \cdot \frac{V}{d} (1 - e^{-t/RC}) \\
 \downarrow \quad 1 - e^{-x} &\approx x \text{ for small } x. \\
 \therefore a &= \frac{qVt}{mdRC} //
 \end{aligned}$$



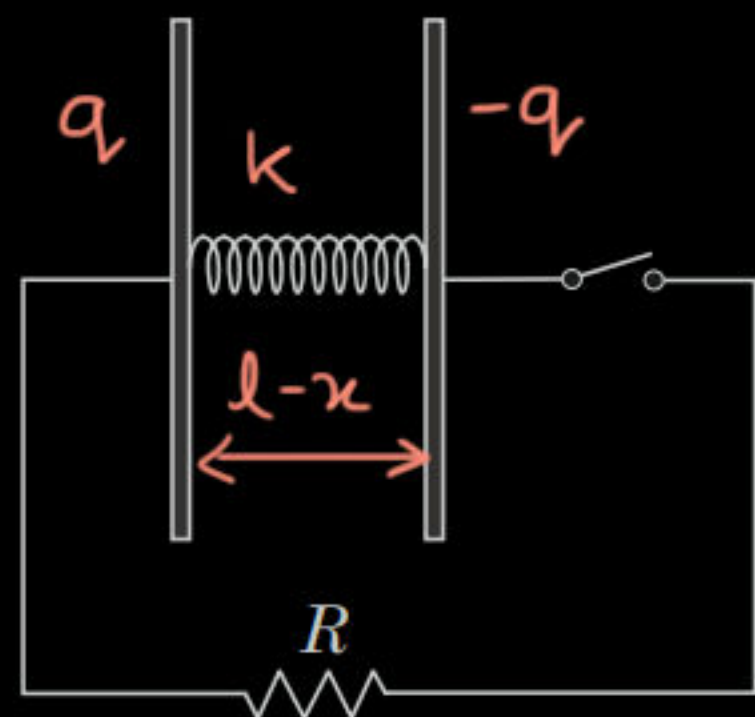
47. In the electric circuit shown, power dissipated in the resistances R and $2R$ at an instant after the switch is closed are 9 W and 2 W respectively. What is the rate of increase in the energy stored in the capacitor at this instant?



$$P_R = i_1^2 R = 9 \Rightarrow i_1 = 3/\sqrt{R}$$

$$P_{2R} = i_2^2 (2R) = 2 \Rightarrow i_2 = 1/\sqrt{R}$$

$$\begin{aligned} U_C &= \frac{q^2}{2C} \\ \Rightarrow \frac{dU_C}{dt} &= \frac{2q}{2C} \frac{dq}{dt} = i_2 (2R) (i_1 - i_2) \\ &= \left(\frac{1}{\sqrt{R}} \right) 2R \left(\frac{2}{\sqrt{R}} \right) \\ &= 4 \text{ W} \end{aligned}$$



48. Midpoints of plates of a parallel plate capacitor are connected with the help of a light spring of stiffness k , relaxed length l and made of an insulating material. Each plate has an area A and mass m . One of the plates of the capacitor is given a charge $+q$ and the other a charge $-q$ and then the capacitor is allowed to discharge through a resistance R . Find expressions for total heat developed in the resistance under the following conditions.

(a) Resistance R is very high.



A Plates will discharge slowly
So, eventually KE will be zero

(b) Resistance R is very small.



B Plates will discharge immediately
So, PE of spring will convert to KE.

Force: $kx = \frac{q^2}{2A\epsilon_0}$

Heat = $-(\Delta U_{sp} + \Delta U_c)$

Heat = $(U_i - U_f)_{sp} + (U_i - U_f)_c$

= $\frac{1}{2}kx^2 + \frac{q^2(l-x)}{2A\epsilon_0}$

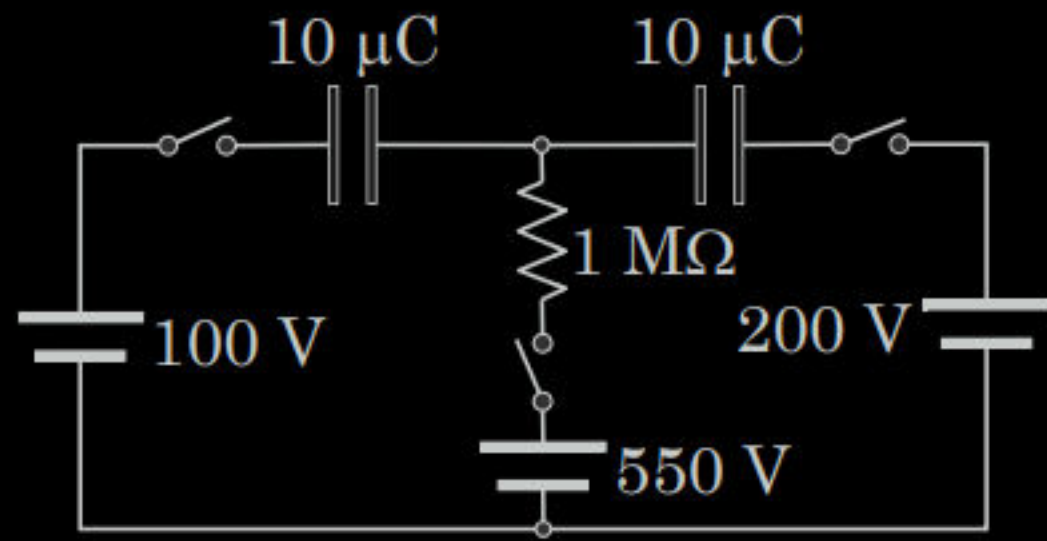
= $\frac{1}{2} \times \frac{q^4}{4A^2\epsilon_0^2k^2} + \frac{q^2l}{2A\epsilon_0} - \frac{q^4}{4A^2\epsilon_0^2k} = \frac{q^2l}{2A\epsilon_0} - \frac{q^4}{8A^2\epsilon_0^2k}$

Heat = $(U_i - U_f)_c$

= $\frac{q^2(l-x)}{2A\epsilon_0}$

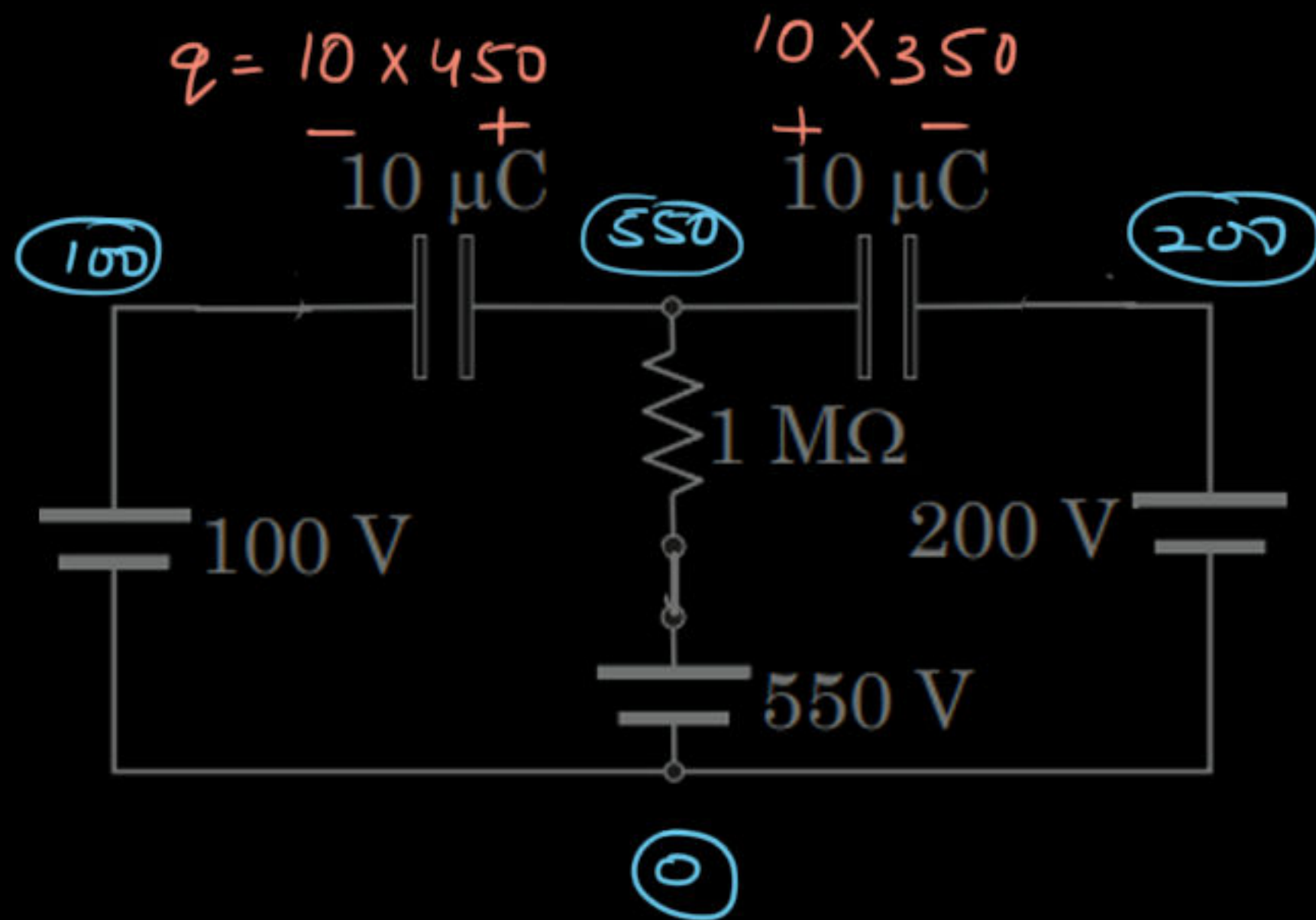
= $\frac{q^2l}{2A\epsilon_0} - \frac{q^4}{4A^2\epsilon_0^2k}$

49. The circuit shown consists of two identical capacitor $10\ \mu\text{F}$ each, a $1.0\ \text{M}\Omega$ resistor, three ideal voltage sources of terminal voltages $100\ \text{V}$, $550\ \text{V}$, $200\ \text{V}$ and three switches. Calculate amount of heat dissipated in the resistor over a long time after all the switches are closed simultaneously.



We only need to bother with initial and final situation. Not intermediate (When $550\ \text{V}$ is unable to provide charge due to high resistance.)

↓ Eventually



$$W_b = \Delta U + \text{Heat}$$

$$\therefore \text{Heat} = W_b - \Delta U$$

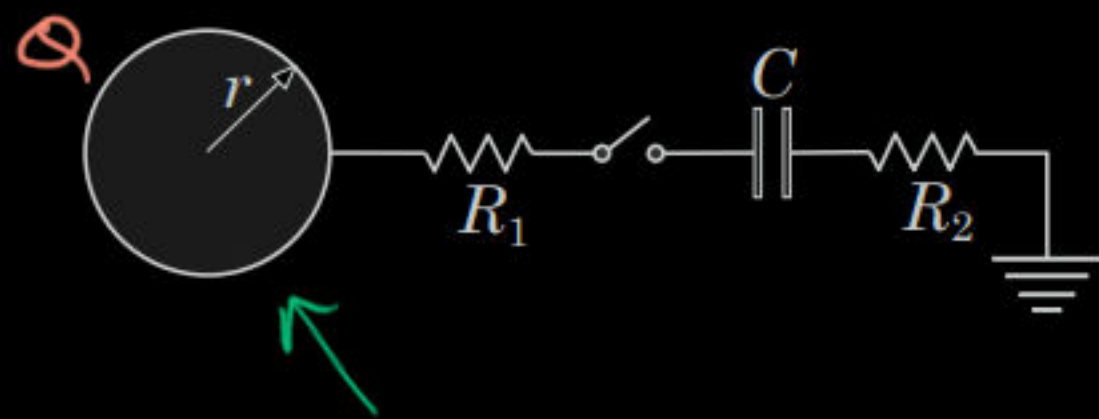
$$= [10 \times 450 + 10 \times 350] 550$$

$$- [10 \times 450] 100$$

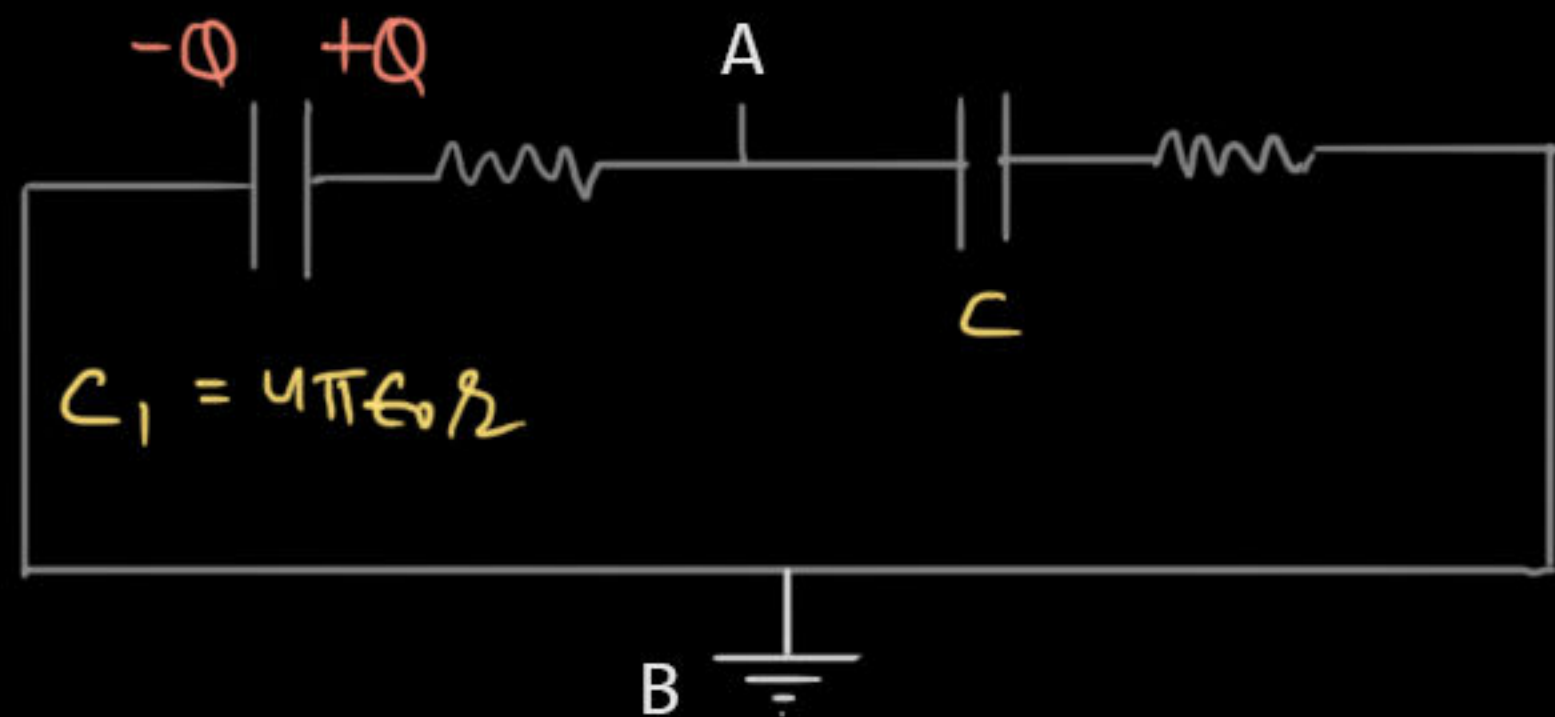
$$- [10 \times 350] 200$$

$$- \frac{1}{2} (10\ \mu\text{C}) 450^2 - \frac{1}{2} (10\ \mu\text{C}) 350^2 \} - \Delta U$$

$$= 1.625\ \text{J}$$



This single conducting sphere can be thought of as a capacitor with other plate at infinity at zero potential and $-Q$ charge.



We can see that same current flows in both resistors at any given instant.

$$\therefore P_1 = \int i^2 R_1 dt \quad P_2 = \int i^2 R_2 dt \quad \Rightarrow \quad \frac{P_1}{P_2} = \frac{R_1}{R_2} \quad (1)$$

50. One plate of a parallel plate capacitor of capacitance C is connected through a resistance R_1 and a key to a conducting ball of radius r . The other plate of the capacitor is earthed through another resistance R_2 as shown in the figure. The ball is far away from the earth as well as from the rest of the circuit. If the ball is given a charge Q and then the key is closed, what amount of heat will be dissipated in R_1 and R_2 ?

$$\text{Total heat} = -\Delta U = U_i - U_f$$

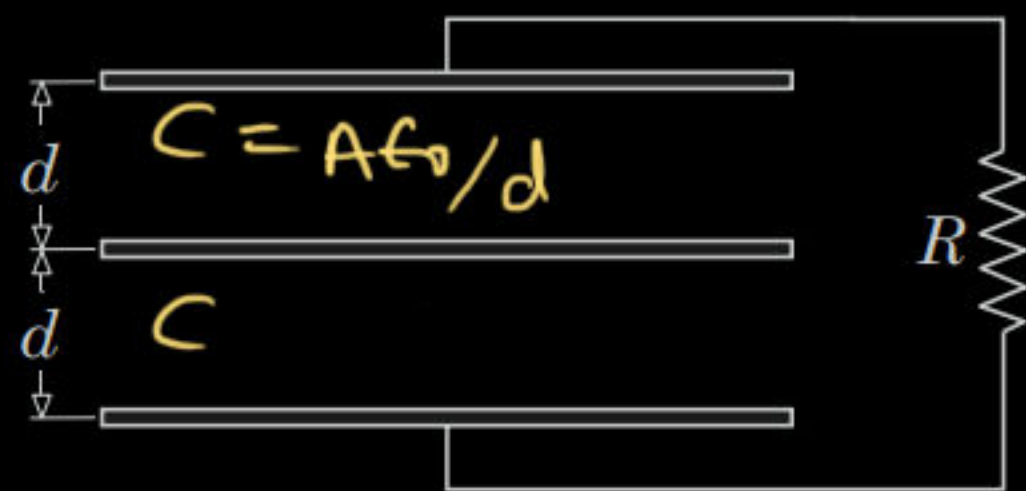
$$= \frac{1}{2} \frac{Q^2}{C_1} - \frac{1}{2} \frac{Q^2}{(C_1 + C_2)} \quad \left\{ \begin{array}{l} \text{As between A and B,} \\ \text{both capacitors are} \\ \text{in parallel.} \end{array} \right.$$

$$= \frac{Q^2}{2} \frac{C_2}{C_1(C_1 + C_2)} = \frac{Q^2 C}{8\pi\epsilon_0 r^2 (4\pi\epsilon_0 r^2 + C)} \quad (2)$$

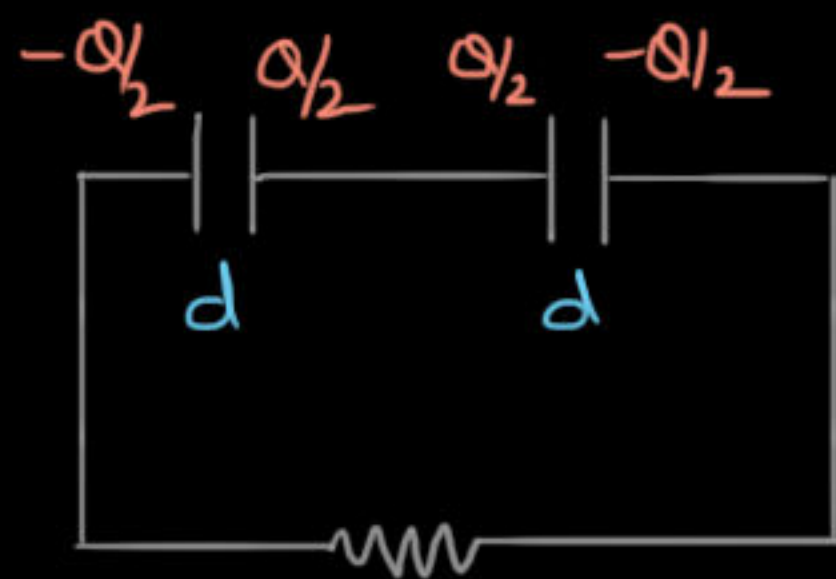
H

From 1, 2: $H_{R_1} = H \frac{R_1}{R_1 + R_2}$, $H_{R_2} = H \frac{R_2}{R_1 + R_2}$

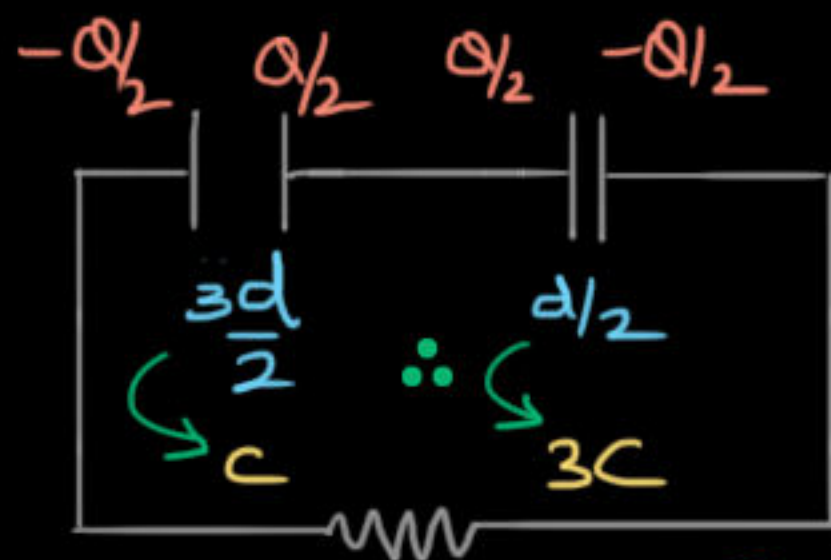
Youtube / Arihant Senior



51. Three identical thin metal plates each of area A are arranged parallel to each other with separation d between adjacent plates and the outer plates are connected through a high resistance R as shown in the figure. A charge Q is given to the middle plate and then the middle plate is quickly shifted towards one of the outer plates by a distance $d/2$. How much heat will be dissipated in the resistor after this shift? Neglect gravity and assume free space conditions.



quickly

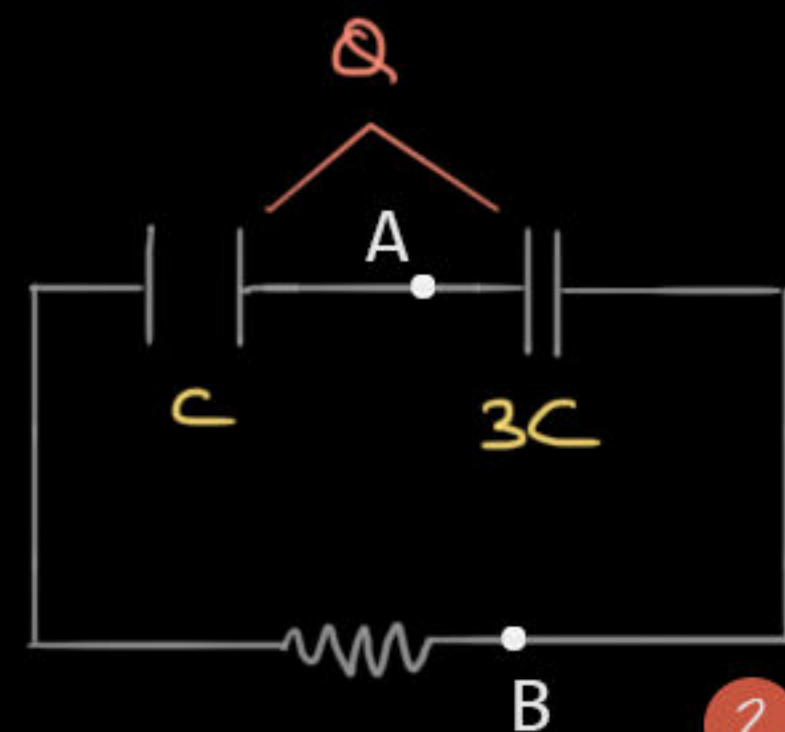


$$C = \frac{A\epsilon_0}{3d/2}$$

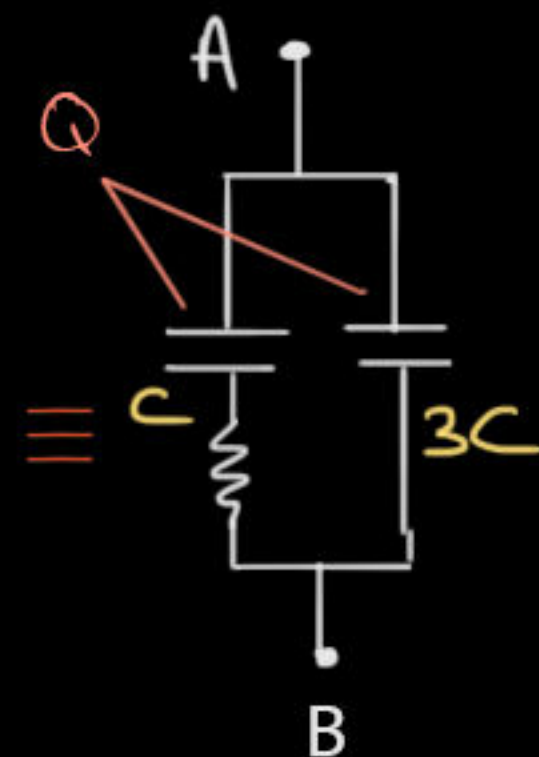
1

eventually

Heat



2

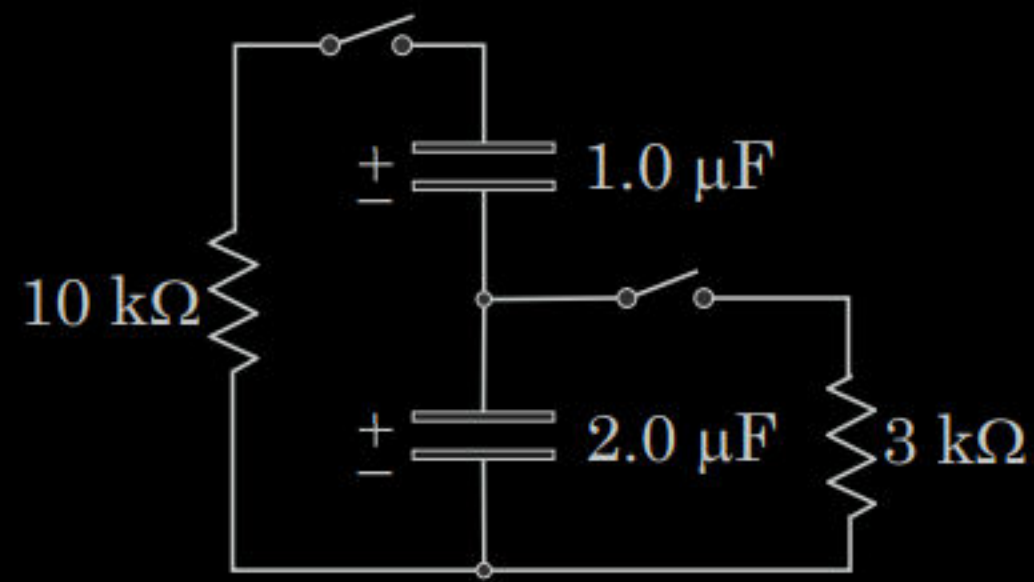


Between 1 and 2: Heat = $U_i - U_f$

$$= \left[\frac{1}{2} \frac{(Q/2)^2}{C} + \frac{1}{2} \frac{(Q/2)^2}{3C} \right] - \frac{1}{2} \frac{Q^2}{C+3C}$$

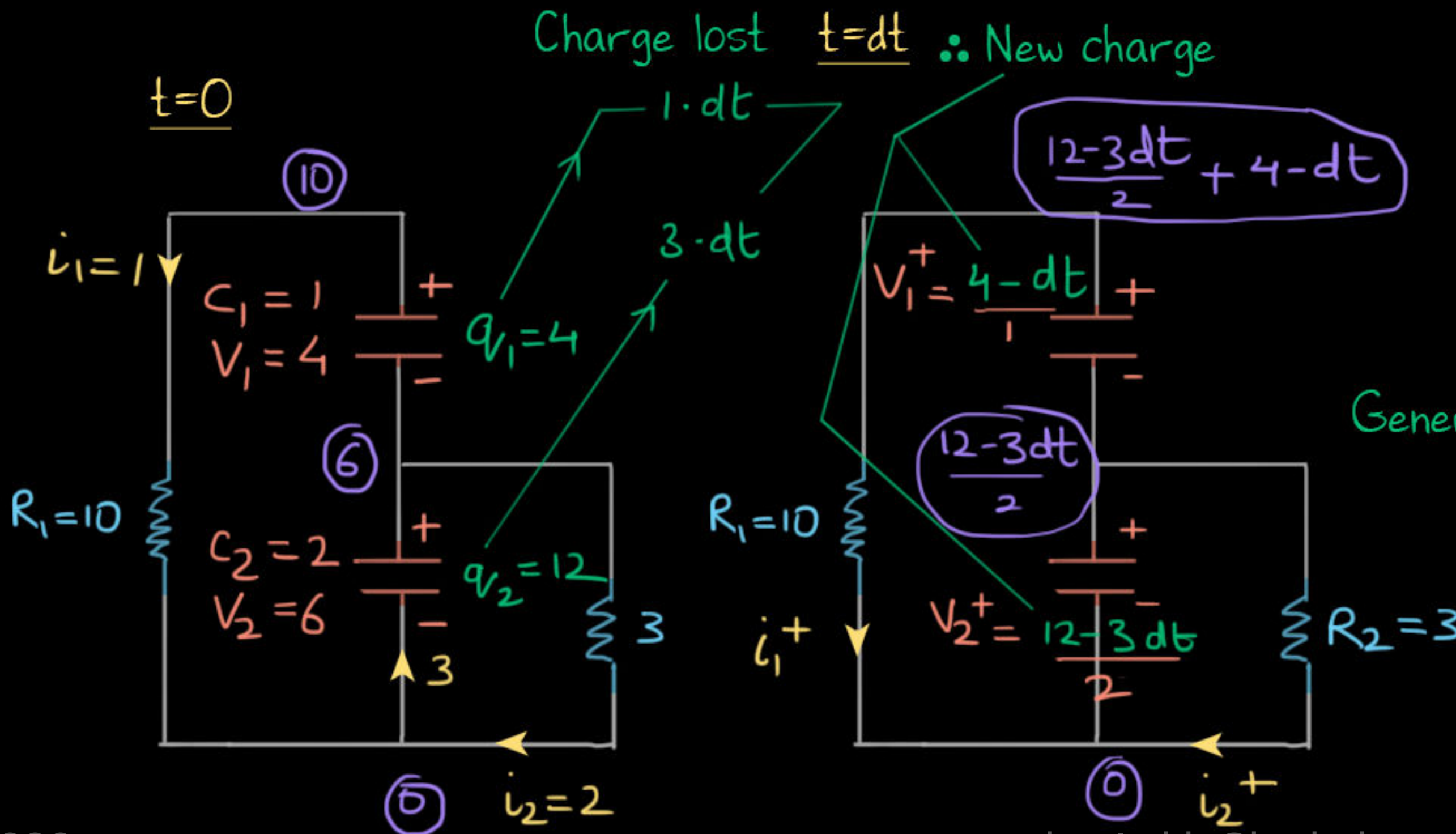
$$= \frac{1}{2} \frac{Q^2}{C} \left(\frac{1}{4} + \frac{1}{12} - \frac{1}{4} \right) = \frac{Q^2}{24C} = \frac{Q^2 3d/2}{24 \cdot A\epsilon_0} = \frac{Q^2 d}{16 A\epsilon_0}$$

Note: Disregarded $Q/2$ on both outer plates, as its unchanged during whole operation.



52. The circuit shown consists of two capacitors of capacitances $1.0 \mu\text{F}$ and $2.0 \mu\text{F}$, two resistors of resistances $10 \text{ k}\Omega$ and $3 \text{ k}\Omega$ and two switches. Initially the switches are open and the capacitors are charged to potentials 4.0 V and 6.0 V respectively according to the polarities shown. If both the switches are closed simultaneously, what are amounts of heat dissipated in the $10 \text{ k}\Omega$ and the $3 \text{ k}\Omega$ resistors over a time large enough for complete discharge of both the capacitors?

|||



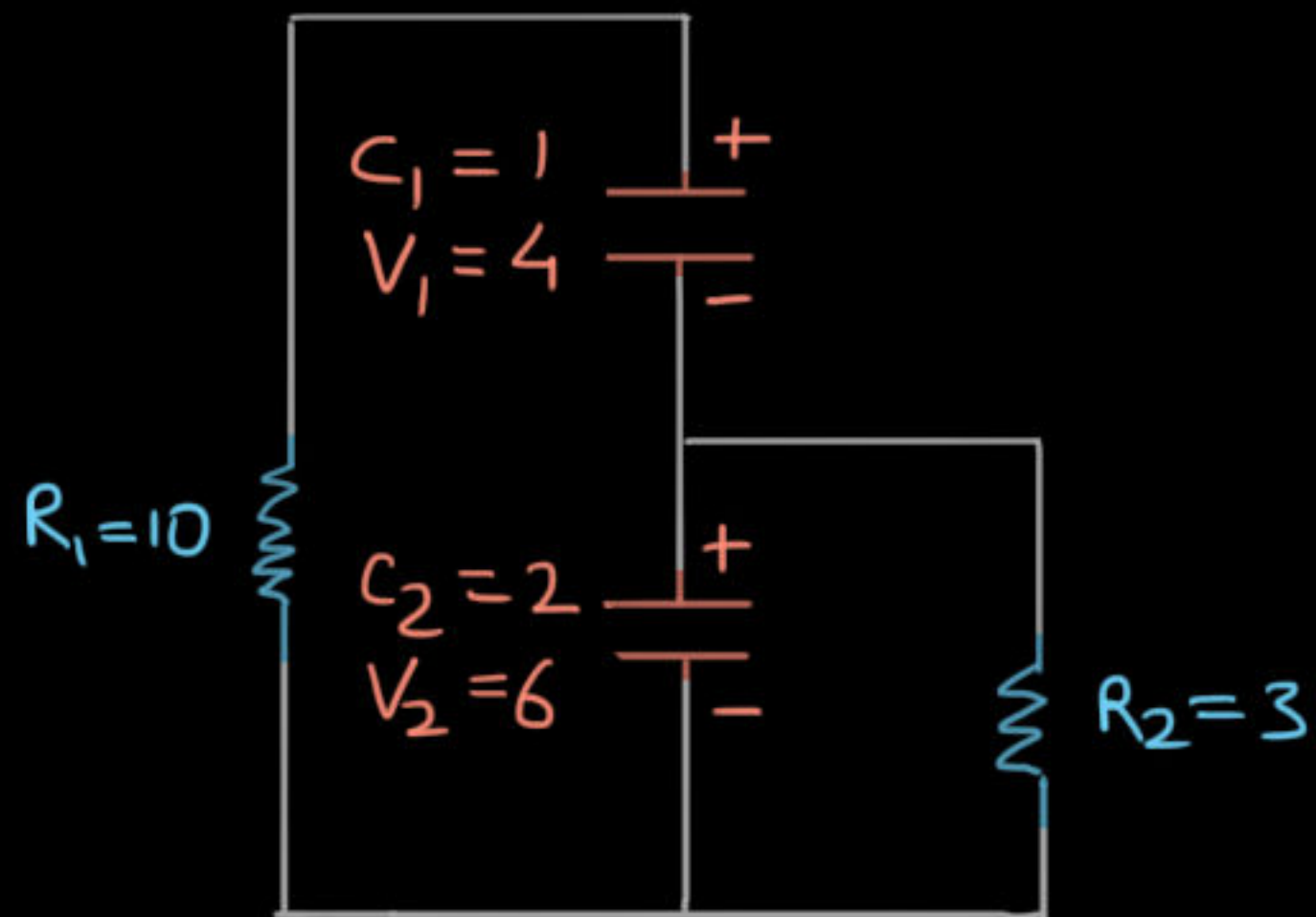
$$\frac{i_1}{i_2} = \frac{1}{2} \quad 1$$

$$\frac{i_1^+}{i_2^+} = \frac{1}{10} \left[\frac{12-3dt}{2} + 4-dt \right] = \frac{1-dt/4}{2(1-dt/4)} = \frac{1}{2}$$

Generalising: $\frac{i_1}{i_2} = \frac{i_1^+}{i_2^+} = \frac{i_1^{++}}{i_2^{++}} = \dots = \frac{1}{2}$

$$\Rightarrow \frac{P_1}{P_2} = \frac{i_1^2 R_1}{i_2^2 R_2} = \frac{1}{4} \frac{R_1}{R_2} = \frac{5}{6}$$

$$\frac{P_1}{P_2} = \frac{5}{6}$$

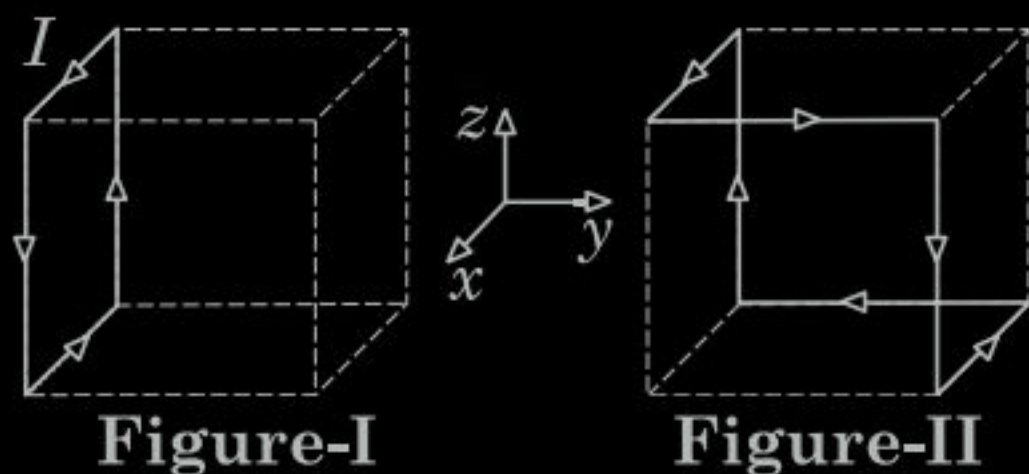


$$\begin{aligned}
 \text{Total Heat} &= U_i - \cancel{U_f}^0 \\
 &= \frac{1}{2} (1) (4)^2 + \frac{1}{2} (2) (6)^2 \\
 &= 44 \mu\text{J}
 \end{aligned}$$

In μC

$$\therefore \text{Heat in } R_1 = \left(\frac{5}{5+6} \right) 44 = 20 \mu\text{J}_{//}$$

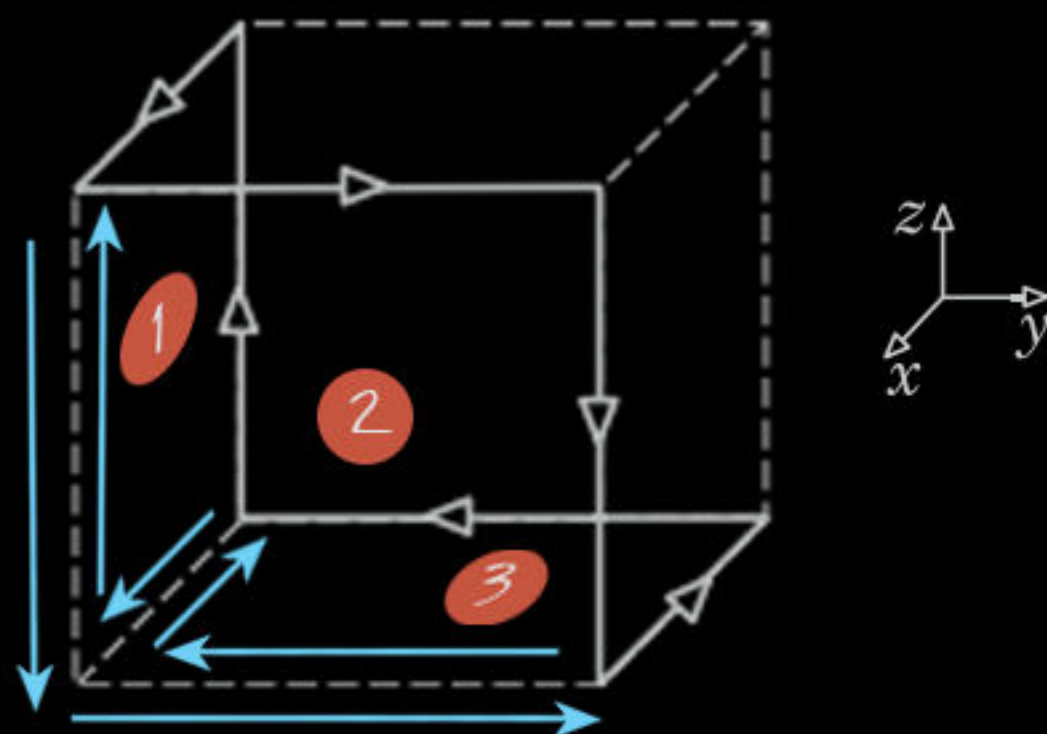
$$\text{Heat in } R_2 = 44 - 20 = 24 \mu\text{J}_{//}$$



1. Current I flowing along edges of one face of a cube as shown in the figure-I, produces magnetic field $\vec{B} = B_0 \hat{j}$ at the centre of the cube. Consider another identical cube, where the current I flows along the path shown in the figure-II. How much magnetic field exists at the centre of the second cube?

- (a) Zero
 (b) $B_0 (\hat{i} + \hat{j} + \hat{k})$
 ✓ (c) $B_0 (-\hat{i} + \hat{j} + \hat{k})$
 (d) $B_0 (-\hat{i} + \hat{j} - \hat{k})$

Adding opposite direction line current pairs, to complete three loops.



$$\begin{aligned} \therefore B \text{ due to } 1 & B_0 \hat{j} \\ B \text{ due to } 2 & B_0 (-\hat{i}) \\ B \text{ due to } 3 & B_0 \hat{k} \end{aligned}$$

We know, B due to a current carrying sheet is:

$$B = \frac{\mu_0 i}{2} \rightarrow A/m$$

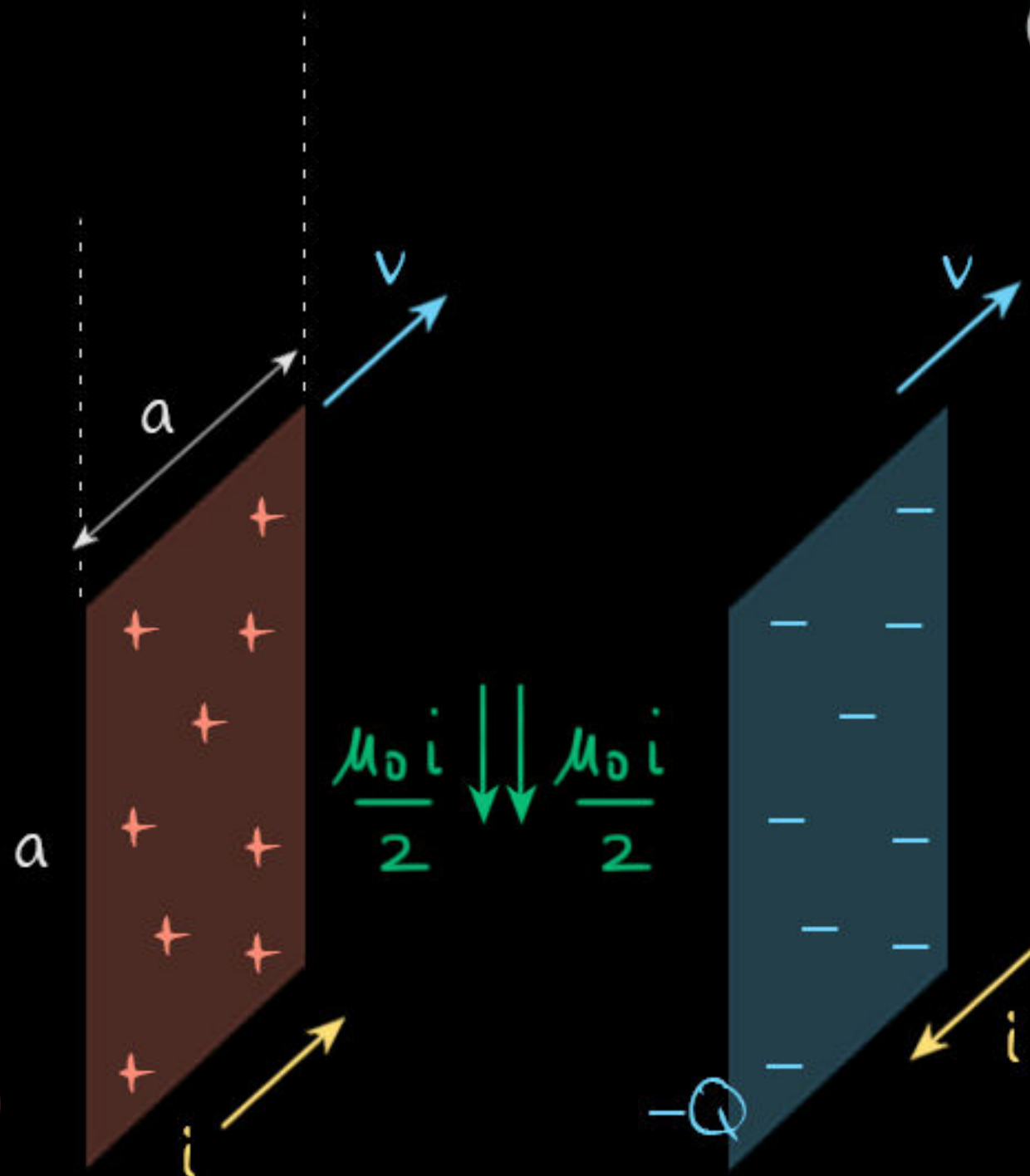
2. A parallel plate capacitor consists of square plates of edge length a separated by a distance d that is much smaller than the edge length. It is charged to a potential V and made to move with a constant velocity v directed along one of its edges. How much magnetic field exists inside the capacitor?

✓ (a) $\frac{\mu_0 \epsilon_0 v V}{d}$

(b) $\frac{\mu_0 \epsilon_0 v V}{a}$

(c) $\frac{\mu_0 \epsilon_0 d v V}{a^2}$

(d) None of these



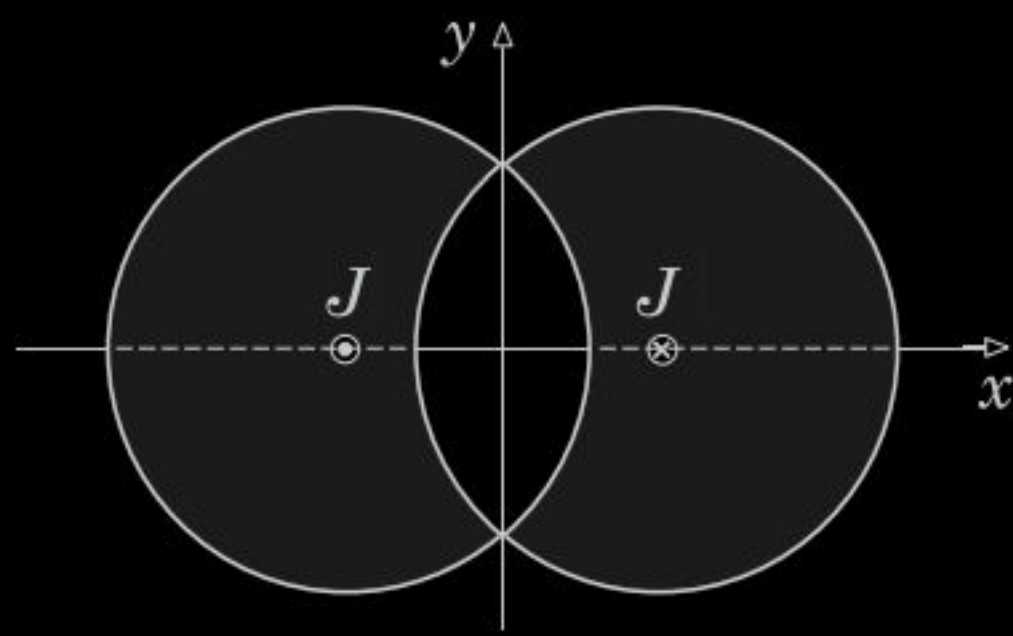
$$\therefore B = \frac{\mu_0 i}{2} + \frac{\mu_0 i'}{2} = \mu_0 i'$$

$i' = \frac{1}{a} \frac{Q}{T} = \frac{1}{a} \frac{Q}{\frac{d}{v}} = \frac{Q v}{a d}$

$$= \mu_0 \frac{1}{a} \left(\frac{d^2 \epsilon_0}{d} \cdot v \right) = \frac{\mu_0 \epsilon_0 v V}{d}$$

by Ankit Singhvi

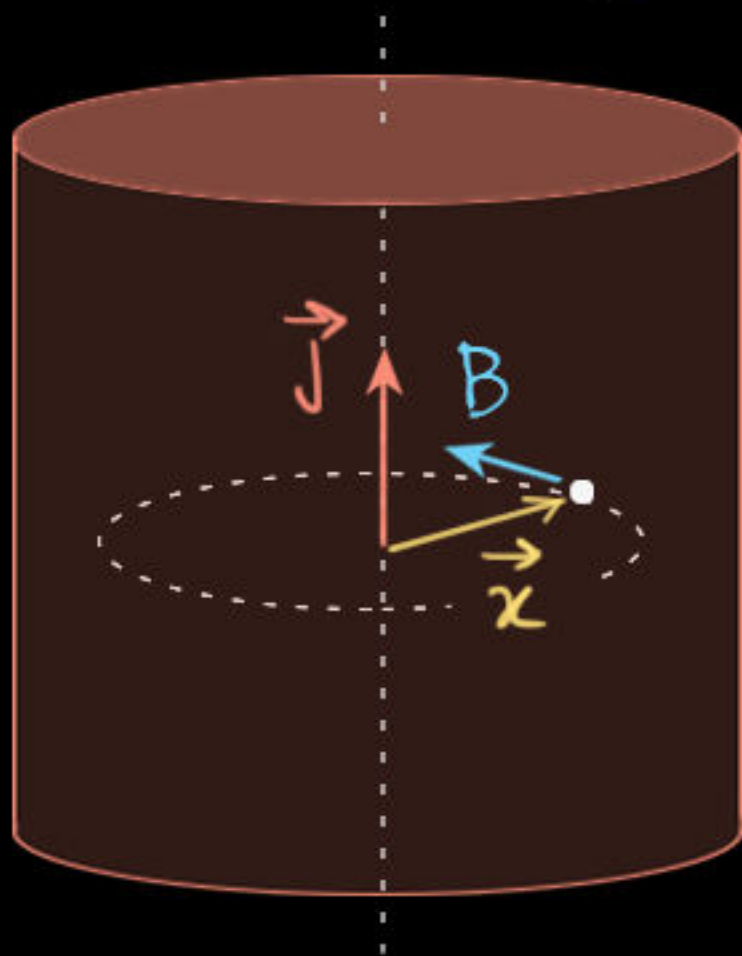
Youtube / Arihant Senior



Irodov 3.233

We know B inside a current carrying cylinder is given by:

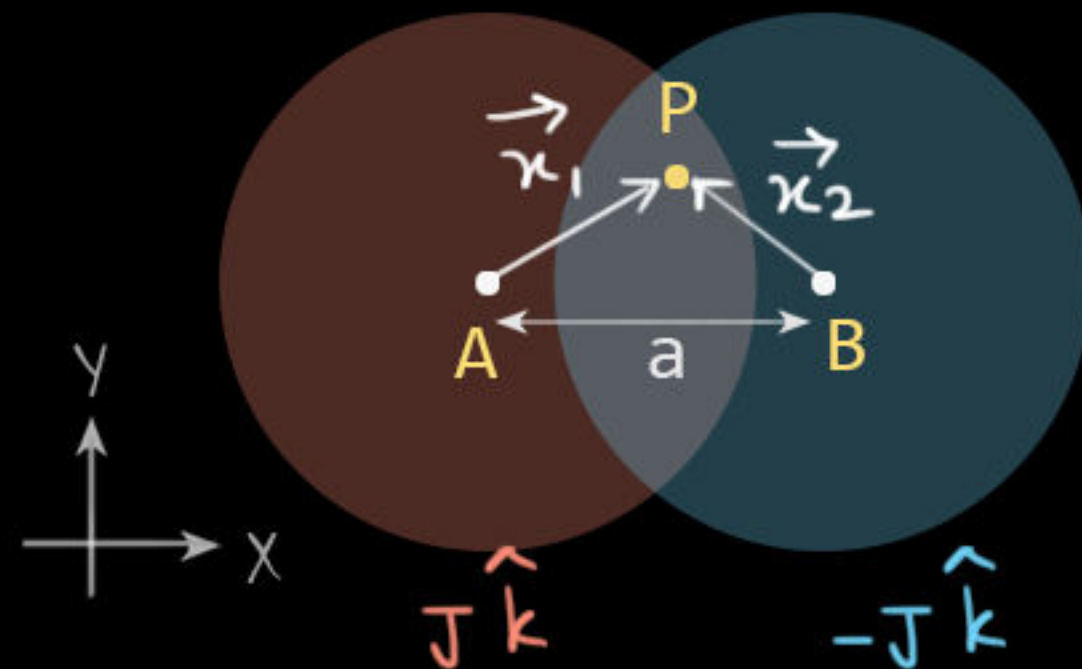
$$\vec{B} = \frac{\mu_0 (\vec{J} \times \vec{r})}{2}$$



3. A long straight conductor has uniform cross-section in the shape of two identical overlapped circles with **centre-to-centre spacing a** . Material from overlapped section has been removed from entire length of the conductor and portions to the right and to the left of the y - z plane are insulated from each other. These portions carry uniform currents of current density **J in the positive and negative z -directions** respectively. Permeability of both the conductors is the same as that of vacuum. Which of the following statements best describes **magnetic field in the empty space inside the composite conductor**.

- ✓ (c) It is uniform, points everywhere in the positive y -direction and has magnitude $\frac{1}{2} \mu_0 J a$.

Sol: Given shape is similar to one made by superposition of two cylinders carrying current in opposite directions. So in overlapping region current will be zero. Let's find B at a random point P inside.

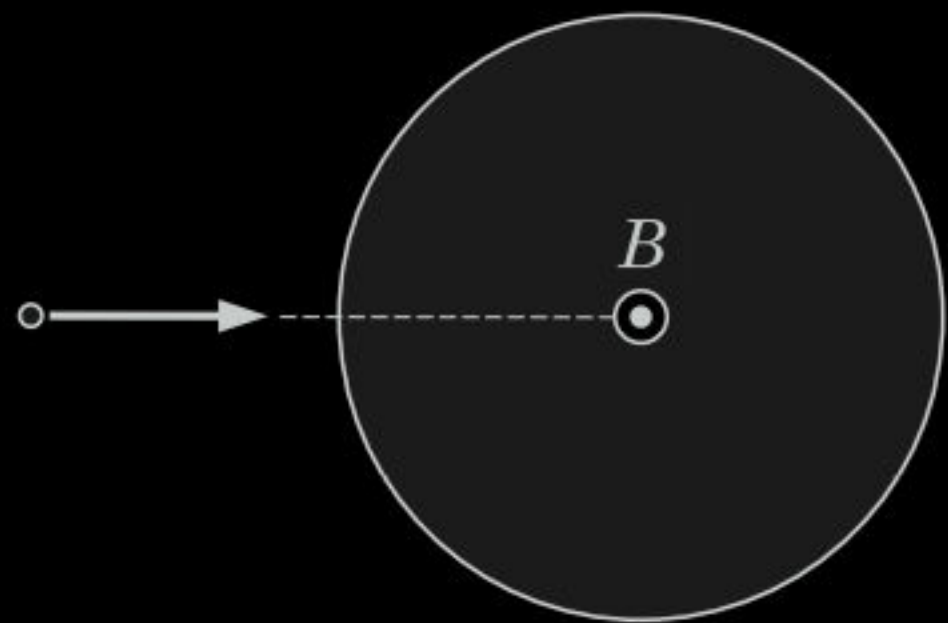


$$\begin{aligned} \therefore B_P &= \frac{\mu_0}{2} (J \hat{k} \times \vec{r}_1) + \frac{\mu_0}{2} (-J \hat{k} \times \vec{r}_2) \\ &= \frac{\mu_0}{2} [J \hat{k} \times (\vec{r}_1 - \vec{r}_2)] \\ &= \frac{\mu_0}{2} (J \hat{k} \times \vec{AB}) = \frac{\mu_0}{2} J a \hat{j} \end{aligned}$$



by Ankit Singhvi

Youtube / Arihant Senior



4. Several α -particles of different speeds enter a uniform magnetic field confined into a cylindrical region. If all the α -particles enter the field radially, what can you say about time intervals spent by them in the magnetic field?

- ✓ (a) Faster is the particle, lesser is the time.
- (b) Slower is the particle, lesser is the time.
- ✓ (c) Slower is the particle, greater is the time.
- (d) The time is same for all the particles.

Time to complete whole circle be T_0 ($\frac{2\pi m}{qB}$, same for all)

∴ Time taken to travel arc PQ: $t = \frac{\cancel{2\theta}}{\cancel{2\pi}} T_0$

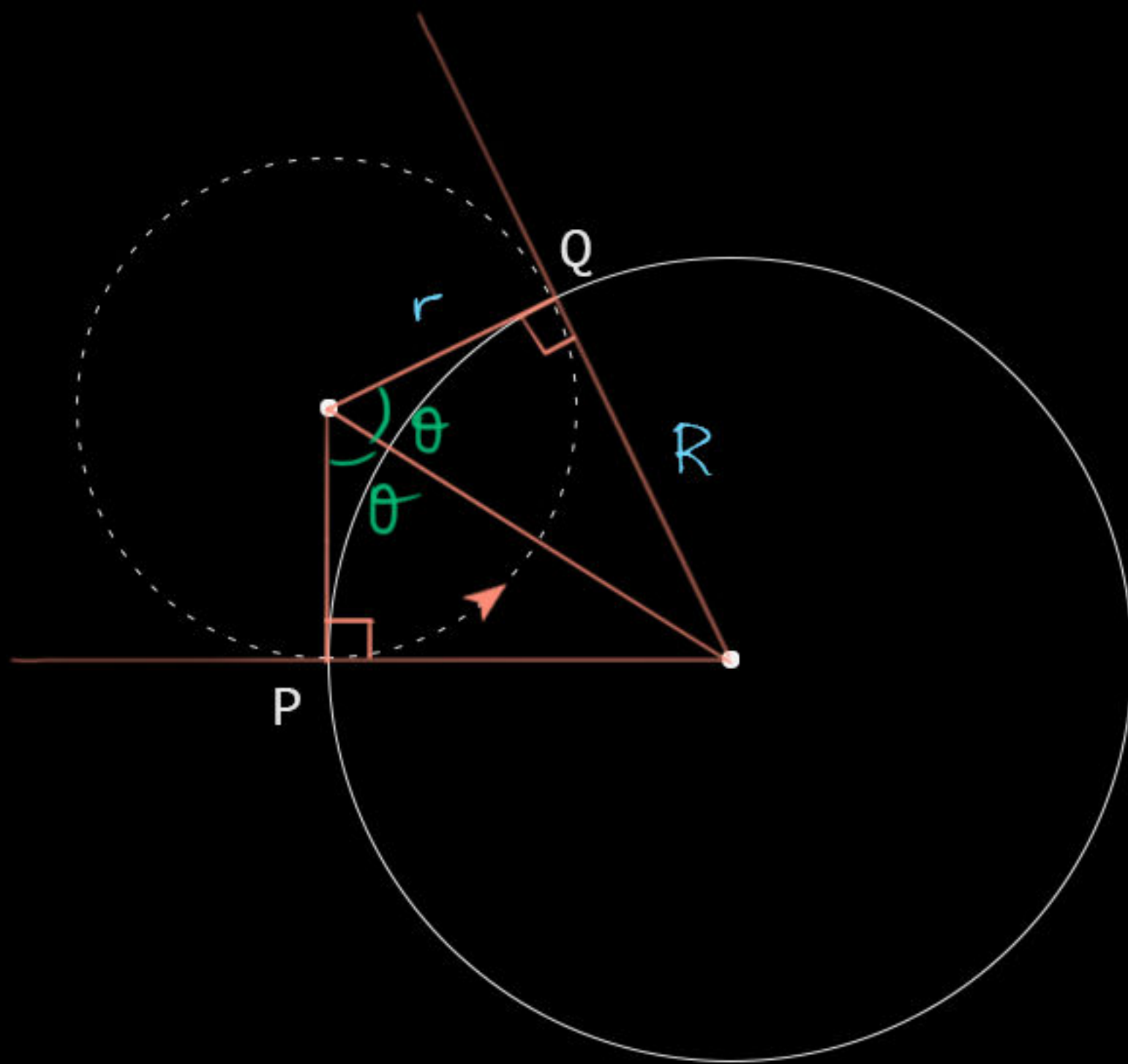
$$= \frac{T_0}{\pi} \tan^{-1} \left(\frac{R}{r} \right)$$

$\rightarrow \frac{mv}{qB}$

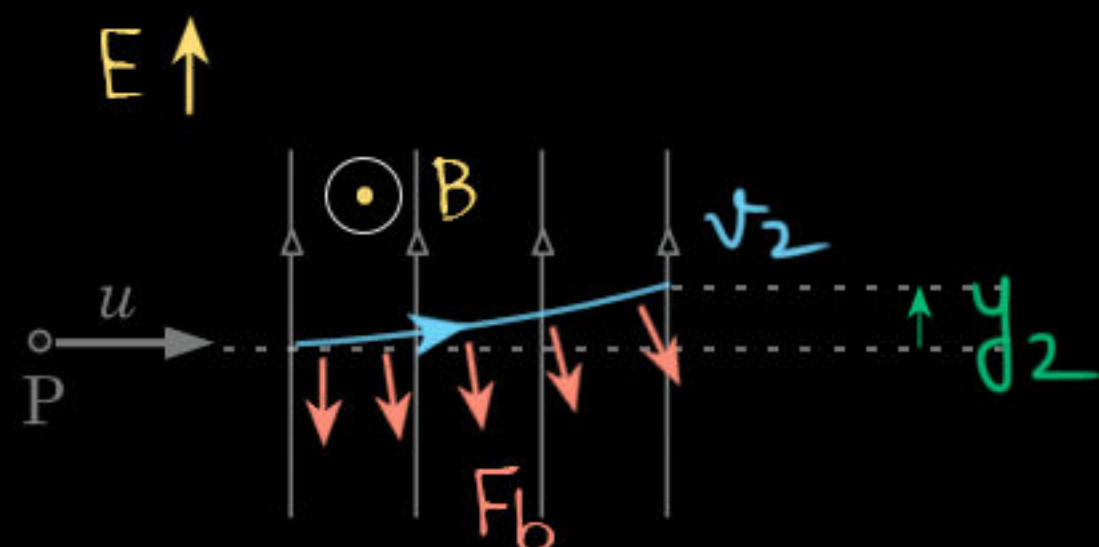
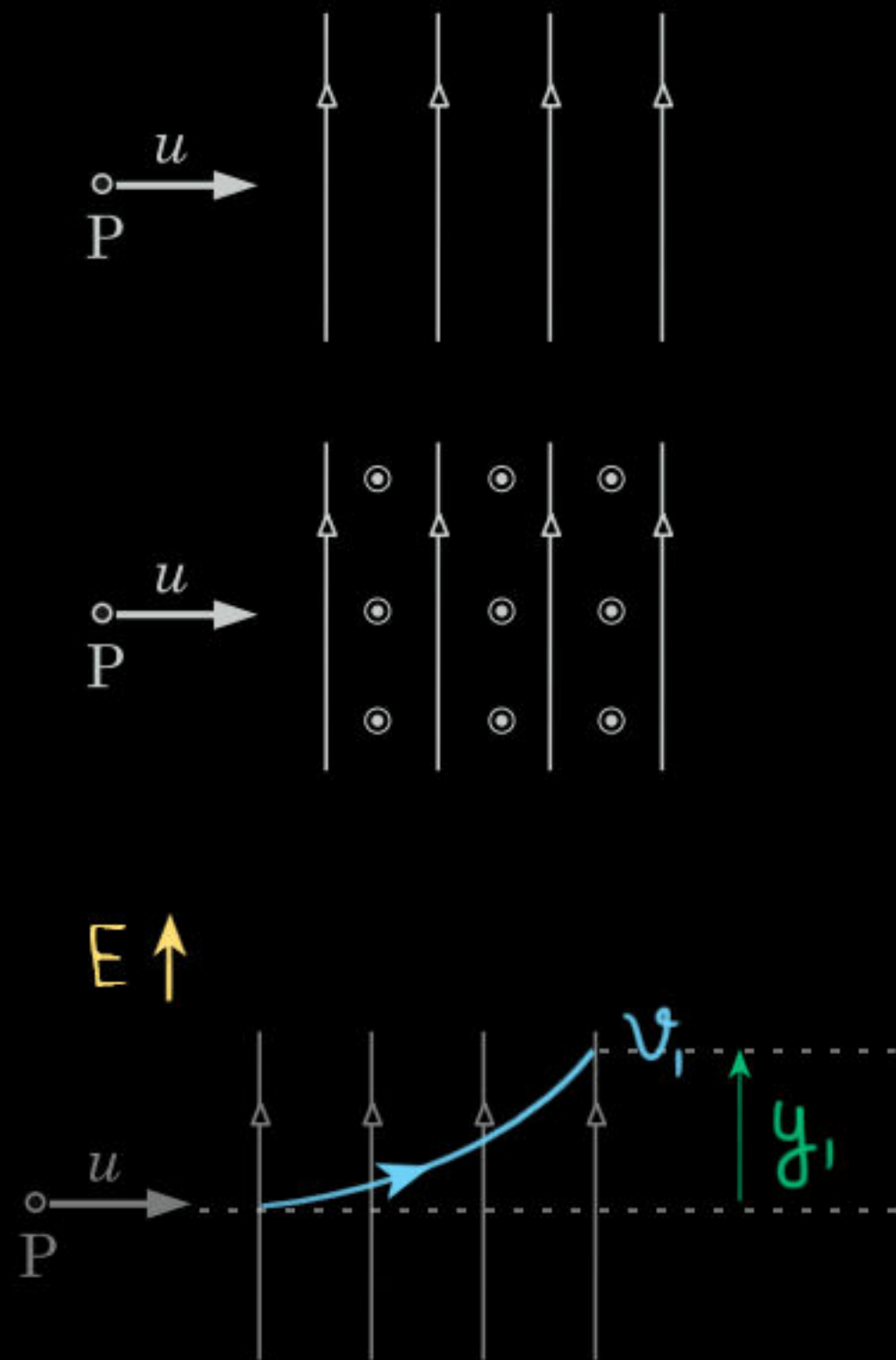
$$\Rightarrow t = \frac{T_0}{\pi} \tan^{-1} \frac{qBr}{mv}$$

- ∴ More v takes less time.
- Less v takes more time.

Youtube / Arihant Senior



by Ankit Singhvi



5. A positively charged particle P crosses a region of uniform electric field of intensity E . Velocity u of the particle at the entry on the left side of the region is perpendicular to the electric field as shown in the first figure. Speed of the particle at exit on the right side is v_1 . Now the experiment is repeated with a uniform magnetic field of induction B superimposed on the electric field as shown in the second figure, keeping entry velocity of the particle unchanged. In this experiment, speed of the particle at exit on the right side is v_2 . What can you conclude regarding the speeds v_1 and v_2 ?

✓ (a) If $u < \frac{E}{B}$, then $v_2 < v_1$

✓ (b) If $u = \frac{E}{B}$, then $v_2 < v_1$

(c) If $u = \frac{E}{B}$, then $v_2 = v_1$

✓ (d) If $u > \frac{E}{B}$, then $v_2 < v_1$

Exit velocity only depends on y as: $qEy = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ ①

F_b acts downwards.

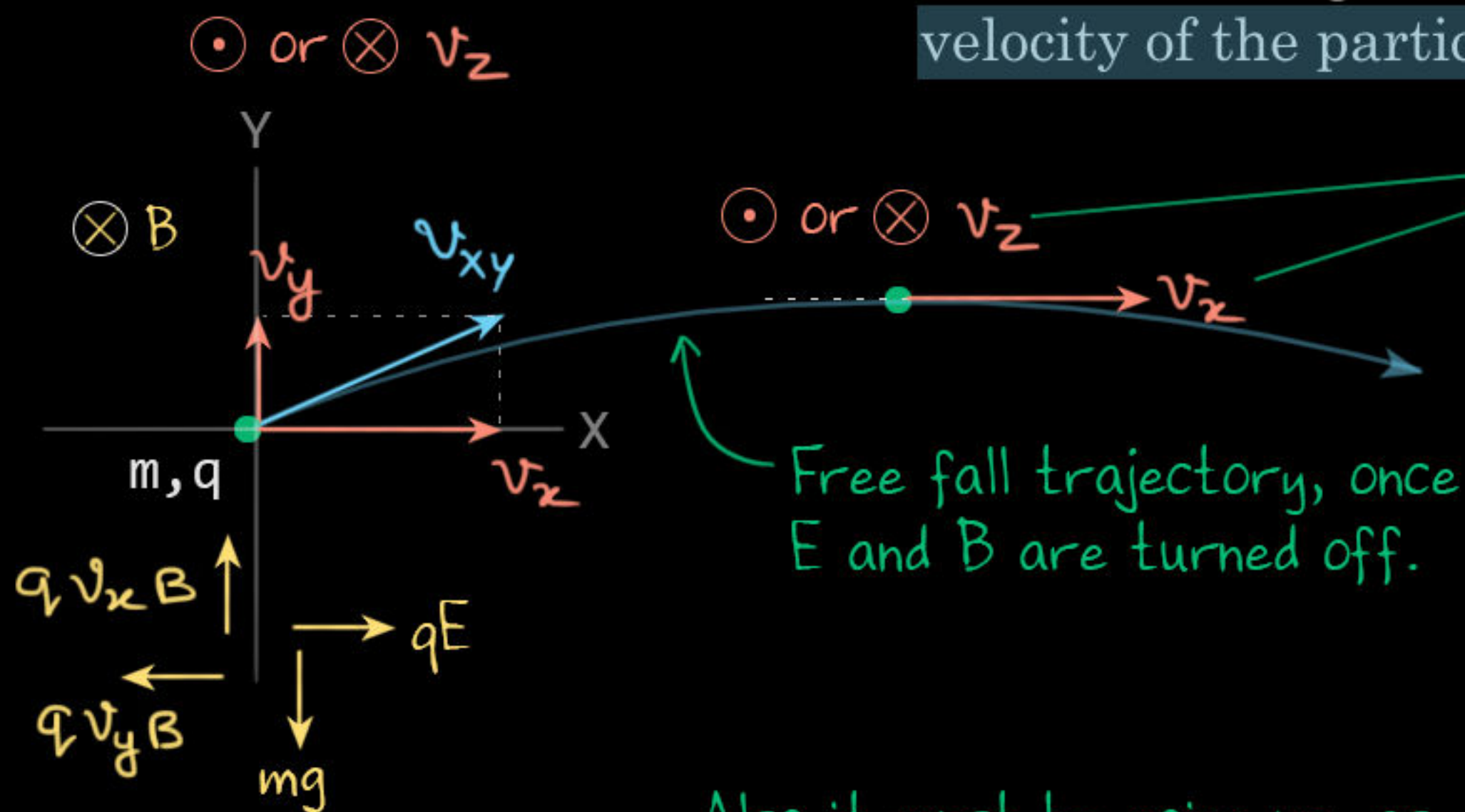
So, when the particle will exit: $y_2 < y_1$ ②

∴ From 1, 2: $v_2 < v_1$ //

Let the initial velocity of particle be v_x , v_y and v_z as shown.

Accordingly we can assume directions of E and B such that net force on particle is zero.

6. A particle of mass m and charge q is moving with a constant speed in a region of space, where a uniform and constant magnetic field $\vec{B}$ and an electric field $\vec{E}$ coexist in mutually perpendicular horizontal directions in addition to gravitational field of intensity g . After the magnetic and electric fields are switched off, the minimum kinetic energy of the particle is observed to be half of the kinetic energy when all these fields were coexisting. What can you certainly conclude for component of velocity of the particle along the direction of the magnetic field?



$$qv_x B = mg \quad (1)$$

$$qv_y B = qE \quad (2)$$

Also it must be going up, as KE is supposed to reduce to a minimum after E and B are turned off and body free falls.

During free fall, these horizontal velocity components will be unchanged. Only v_y will reduce to zero on top, where KE will be minimum.

$$\text{Given: } v_x^2 + v_z^2 = \frac{1}{2} (v_x^2 + v_y^2 + v_z^2)$$

$$\Rightarrow v_z^2 = v_y^2 - v_x^2 \quad (3)$$

$$\Rightarrow v_z^2 = \left(\frac{E}{B}\right)^2 - \left(\frac{mg}{qB}\right)^2$$

Direction either along or opposite to B .

Given: $\vec{F}_e + \vec{F}_B = 0$

$\Rightarrow -\vec{F}_e = \vec{F}_B$

For 1st electron:

$-[-e\vec{E}] = -e(v\hat{i} \times \vec{B})$

$-\frac{E}{v} = \hat{i} \times \vec{B}$ ①

For 2nd electron:

$-\frac{E}{v} = \hat{j} \times \vec{B}$ ②

7. In a region of free space, where a uniform and constant electric field of intensity $\vec{E}$ and a magnetic field of induction $\vec{B}$ coexist, an electron projected with speed v in the positive x -direction, moves undeviated without change in speed. In addition, an electron projected with velocity v in the positive y -direction in this region; also, moves undeviated without change in speed. Which of the following conclusions can you make?

✓(a) $|\vec{E}| < v|\vec{B}|$

(b) $|\vec{E}| = v|\vec{B}|$

✓(c) The electric field is perpendicular to the x - y plane.

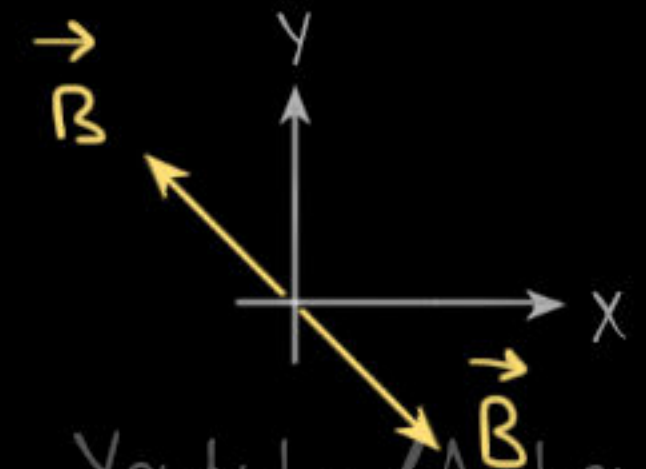
✓(d) The magnetic field is either at 45° or 135° with the positive x -direction.

Ⓐ Taking magnitudes of both sides in 1: $\frac{E}{v} = B \sin \theta \Rightarrow |\vec{E}| < v|\vec{B}|$

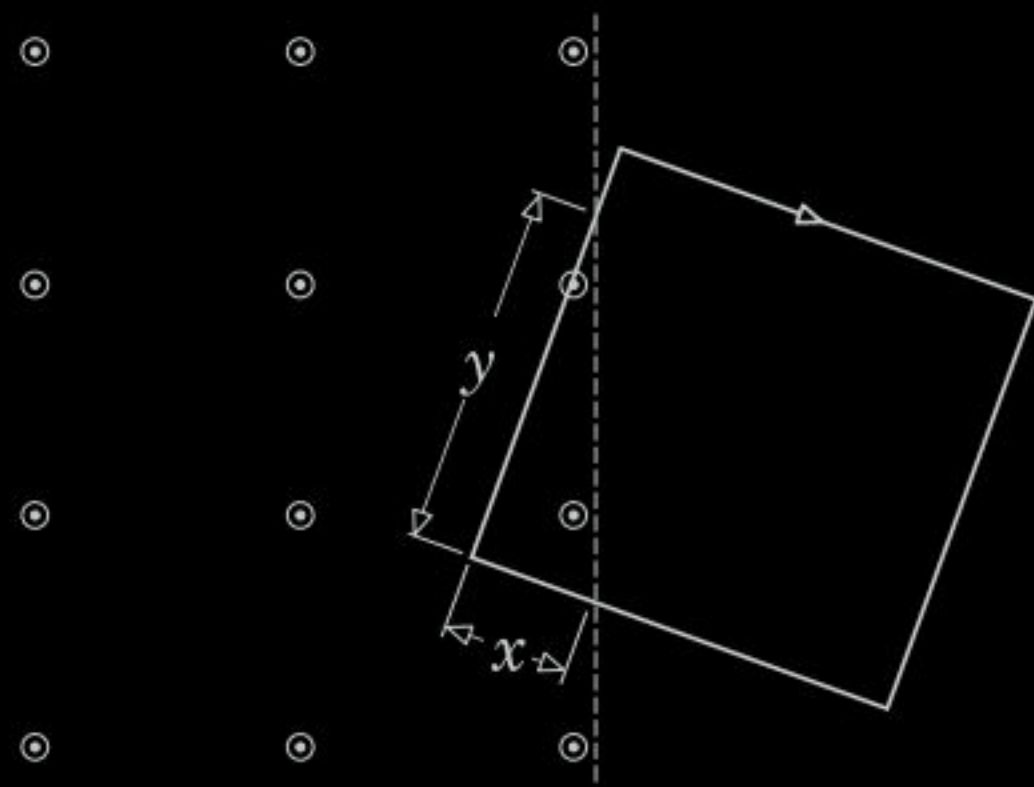
Ⓒ From 1 and 2, E is perpendicular to both X and Y axis respectively, which is possible only if E lies perpendicular to XY plane.

Ⓓ Subtracting 1 and 2 : $0 = (\hat{i} - \hat{j}) \times \vec{B}$

$\Rightarrow B$ is parallel to $\hat{i} - \hat{j}$



Youtube / Arihant Senior



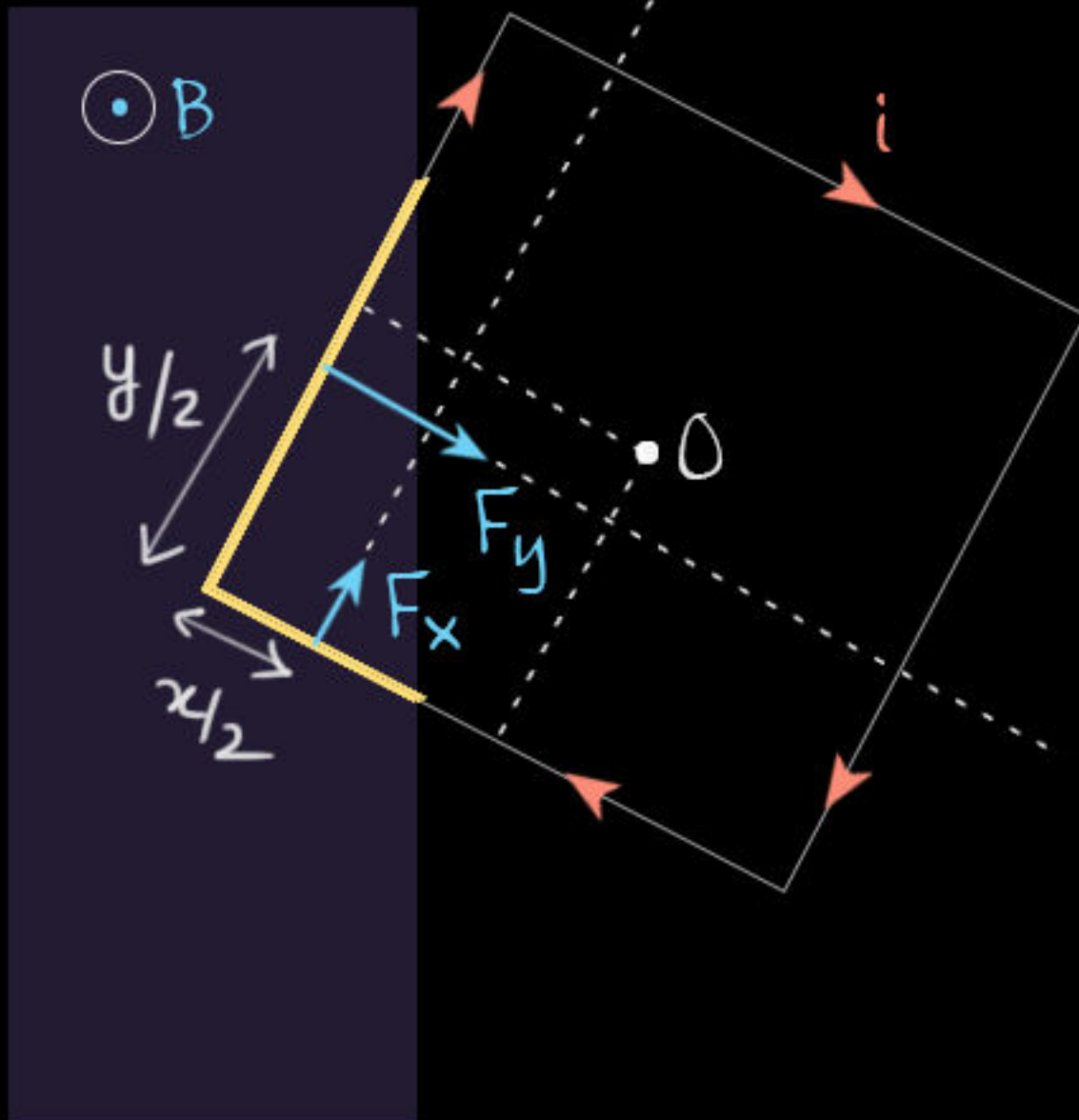
8. A rigid square loop of side length l carrying an electric current is held motionless on a frictionless horizontal tabletop. A uniform magnetic field pointing upwards is switched on everywhere to the left of the dashed line as shown in the figure and then the loop is released. Considering different lengths of the side segments x and y ($x < y$), which of the following conclusions can you make?

✓ (a) If $x < \frac{l}{2}$ and $y = \frac{l}{2}$, the loop starts rotating anticlockwise.

✓ (b) If $x < \frac{l}{2}$ and $y < \frac{l}{2}$, the loop starts rotating anticlockwise.

(c) If $x < \frac{l}{2}$ and $y > \frac{l}{2}$, the loop starts rotating clockwise.

✓ (d) If $x < \frac{l}{2}$ and $y > \frac{l}{2}$, more information is required to decide which way the loop starts rotating.



Clockwise torque about O is:

$$\begin{aligned} \tau_{CW} &= \overset{Bix}{F_x} \left(\frac{l}{2} - \frac{x}{2} \right) - \overset{Biy}{F_y} \left(\frac{l}{2} - \frac{y}{2} \right) \\ &= \frac{Bi}{2} [xl - x^2 - yl + y^2] \\ &= \frac{Bi}{2} (y-x)(x+y-l) \end{aligned}$$

↗ +ve ↘

We know, force on a coil of magnetic moment M , placed parallel to a non-uniform magnetic field is given by:

$$F = M \frac{dB}{dx}$$

9. Two coaxial circular coils of radii a and b are separated by a large distance x and carry equal currents I . If $a \gg b$, what is the force of mutual interaction between them.

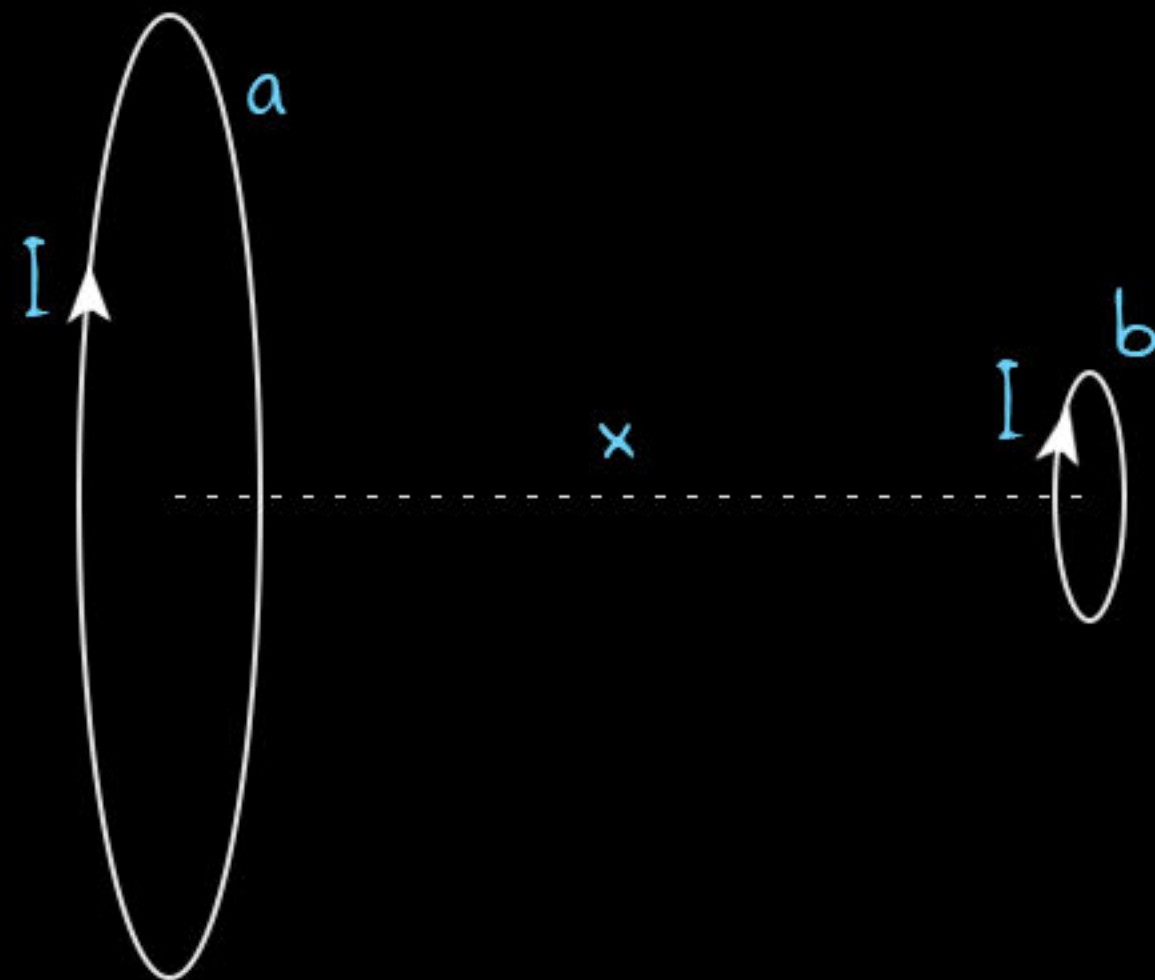
(a) $\frac{\mu_0 I^2 a^2 b}{(a^2 + x^2)^{1.5}}$

(b) $\frac{3\pi\mu_0 I^2 abx}{(a^2 + x^2)^{1.5}}$

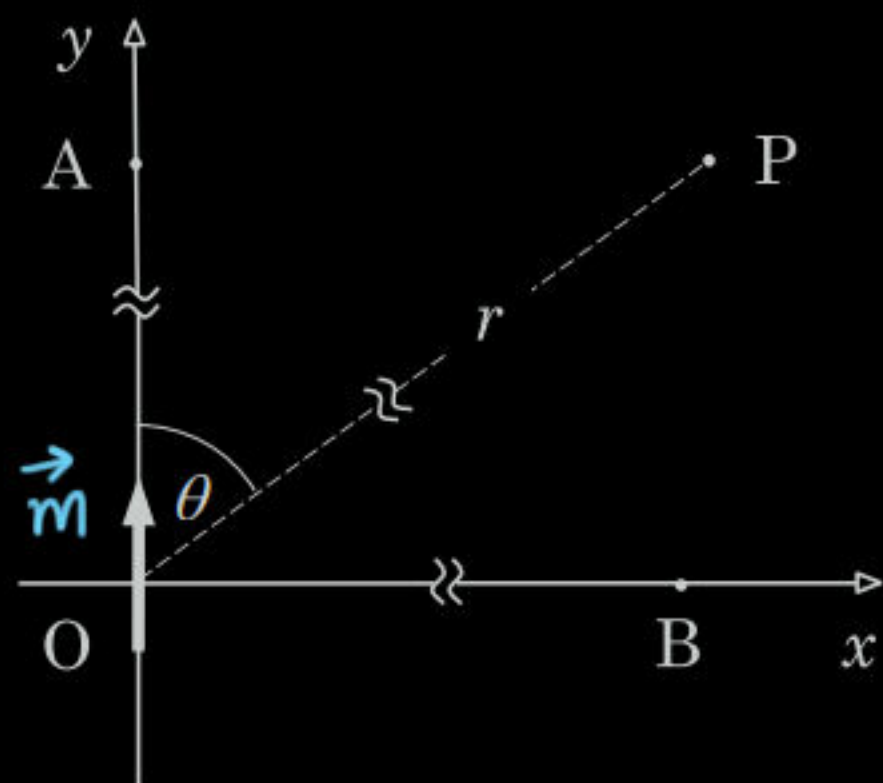
(c) $\frac{\pi\mu_0 I^2 a^3 b}{(a^2 + x^2)^2}$

✓ (d) $\frac{3\pi\mu_0 I^2 a^2 b^2 x}{2(a^2 + x^2)^{2.5}}$

Finding force on smaller coil due to non-uniform field of bigger coil:



$$\begin{aligned}
 F &= M \frac{dB}{dx} \\
 &\leftarrow I\pi b^2 \quad \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \\
 &= I\pi b^2 \frac{\mu_0 I a^2}{2} \frac{d}{dx} \left[\frac{1}{(x^2 + a^2)^{3/2}} \right] \\
 &= \frac{3}{2} \pi \mu_0 I^2 a^2 b^2 x \frac{1}{(x^2 + a^2)^{5/2}} //
 \end{aligned}$$



10. In the given figure, dipole moment $\vec{m}$ of a small magnetic dipole placed at the origin is shown by a small directed line segment. Magnetic induction vectors at distant points A (0, y) on the y-axis and B (x, 0) on the x-axis respectively are

✓ (d) $\frac{\mu_0 m}{2\pi y^3} \hat{j}$ and $-\frac{\mu_0 m}{4\pi x^3} \hat{j}$

$E_\theta = \frac{k p \sin \theta}{r^3}$
 $E_r = \frac{2k p \cos \theta}{r^3}$
 Electric dipole

Given:
 $\equiv$

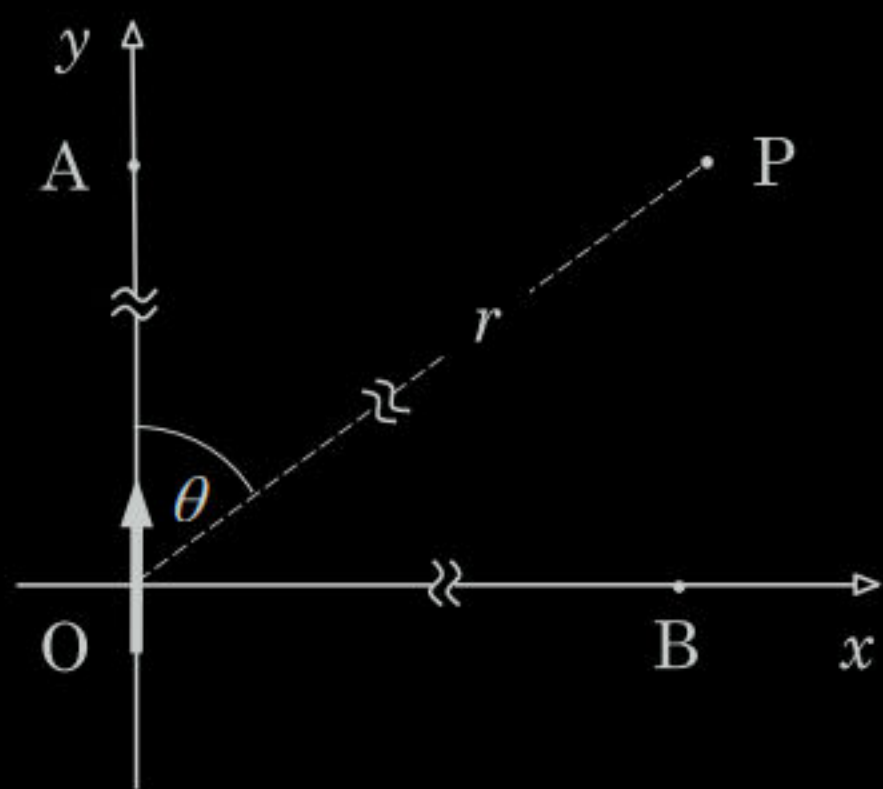
$B_\theta = \frac{\mu_0 m \sin \theta}{4\pi r^3}$
 $B_r = \frac{2\mu_0 m \cos \theta}{4\pi r^3}$
 Magnetic dipole

@A: $\theta = 0$

@B: $\theta = \pi/2$

$\therefore B_r = \frac{2\mu_0 m}{4\pi y^3} \hat{j}$

$\therefore B_\theta = \frac{\mu_0 m}{4\pi x^3} (-\hat{j})$

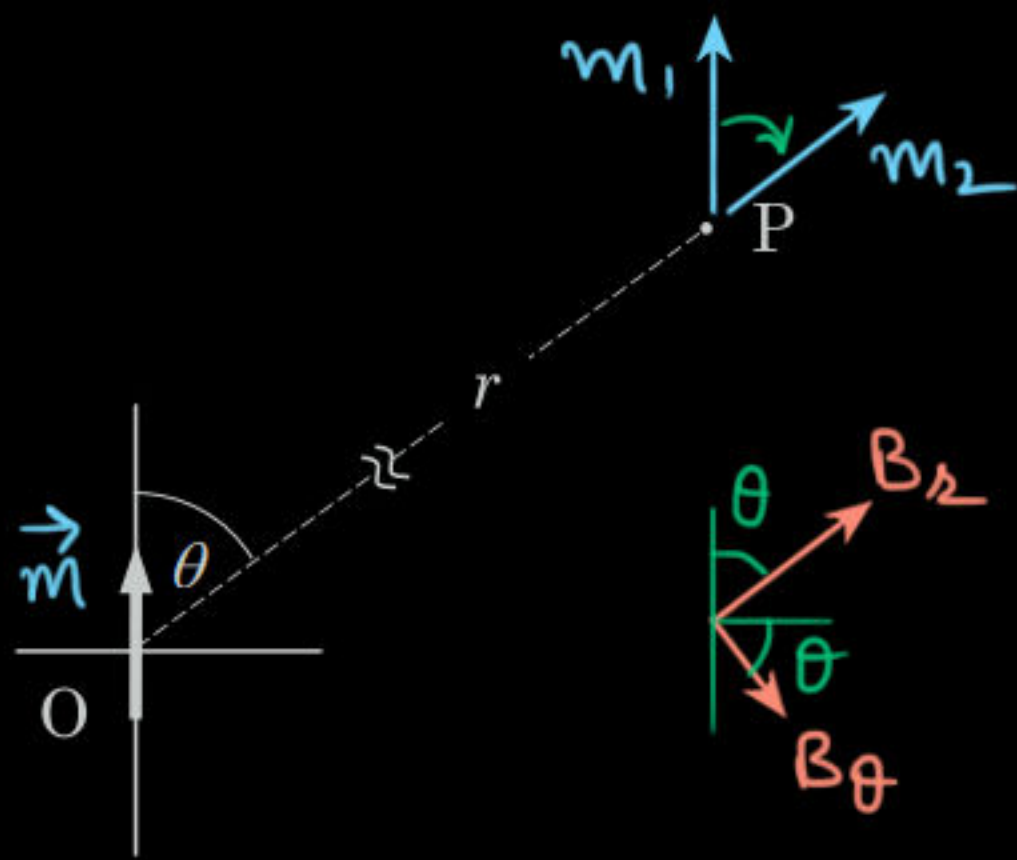


11. If unit vectors in the radial and the transverse directions are denoted by $\hat{e}_r$ and $\hat{e}_\theta$, magnetic induction vector due to the dipole at a point P shown in the figure is given by

✓ (a) $\frac{\mu_0 m}{4\pi r^3} \{ (2 \cos \theta) \hat{e}_r + (\sin \theta) \hat{e}_\theta \}$

$$B_r = 2 \frac{\mu_0}{4\pi} \frac{m \cos \theta}{r^3} \hat{e}_r$$

$$B_\theta = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^3} \hat{e}_\theta$$



12. A small magnetic dipole of magnetic moment $m_P j$ is placed at the point P. Find work done by the forces of magnetic interactions, if this new dipole is rotated to align it along the position vector of the point P.

(a) $\frac{\mu_0 m m_P}{4\pi r^3} \cos \theta (2 - \sin \theta)$

(c) $\frac{\mu_0 m m_P}{4\pi r^3} \sin \theta (2 - \cos \theta)$

✓(b) $\frac{\mu_0 m m_P}{4\pi r^3} (1 + 2 \cos \theta - 3 \cos^2 \theta)$ (d) $\frac{\mu_0 m m_P}{4\pi r^3} (1 + 2 \cos \theta - 3 \sin^2 \theta)$

$$U_E = -\vec{p} \cdot \vec{E}$$

$$U_B = -\vec{m} \cdot \vec{B}$$

$$B_r = 2 \frac{\mu_0}{4\pi} \frac{m \cos \theta}{r^3} \hat{e}_r$$

$$B_\theta = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^3} \hat{e}_\theta$$

$$W_B = -\Delta U$$

$$= U_1 - U_2$$

$$= -(\vec{m}_1 \cdot \vec{B}) - [-(\vec{m}_2 \cdot \vec{B})]$$

$$= -[m_P (B_r \cos \theta - B_\theta \sin \theta)] + m_P B_r$$

$$= m_P \frac{\mu_0}{4\pi} \frac{m}{r^3} \{-(2 \cos^2 \theta - \sin^2 \theta) + 2 \cos \theta\}$$

$$= \frac{\mu_0 m m_P}{4\pi r^3} (1 + 2 \cos \theta - 3 \cos^2 \theta) //$$

In a homogeneous, non-magnetic, highly insulating and viscous medium, a moving particle experiences a viscous drag given by equation $\vec{f} = -b\vec{v}$. Here b is a positive constant and $\vec{v}$ is velocity of the particle.

A particle having charge q projected with an unknown velocity from a point in the medium almost stops (its speed becomes practically negligible) after travelling a distance of 10 m in a straight line.

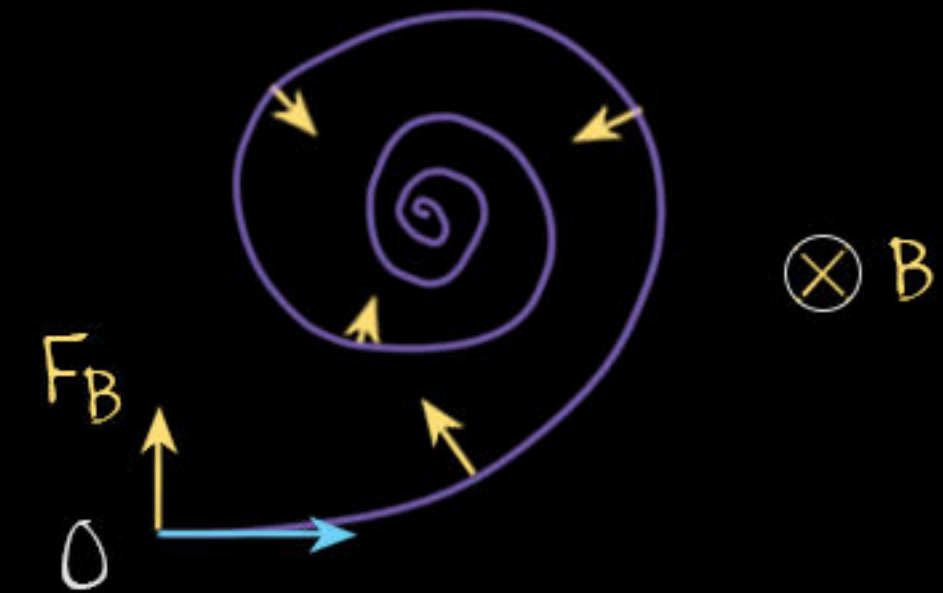
Now a uniform and constant magnetic field is established in the region and the same particle is again projected with the same velocity directed perpendicular to the magnetic field.

13. In the presence of the magnetic field, the particle moves

- (a) on a circular path.
- (b) on a helical path.
- ✓ (c) spiralling into a point.
- (d) spiralling away from a point

We know,

$$R = \frac{mv}{qB}$$

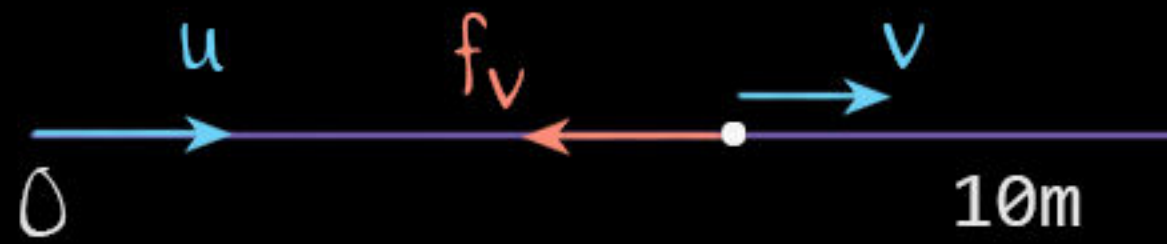


∴ As velocity reduces, radius of the circle will keep reducing too. Hence it will eventually spiral in to a point.

by Ankit Singhvi

Youtube / Arihant Senior

Without B

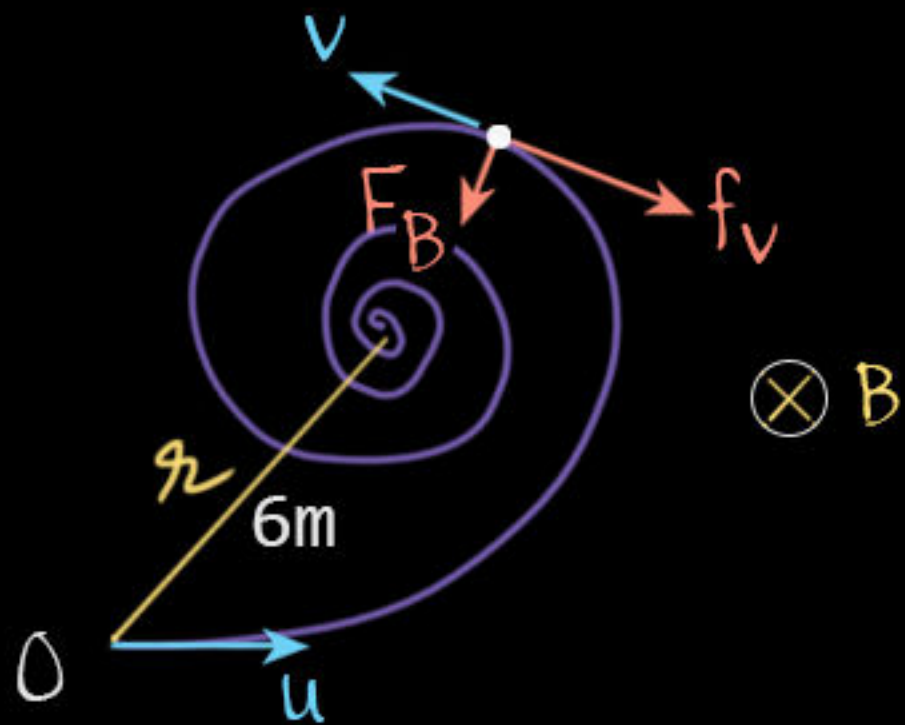


$$F = -bv$$

$$\Rightarrow m \cancel{v} \frac{dv}{dx} = -b \cancel{v}$$

$$\Rightarrow m \int_u^0 dv = -b \int_0^{10} dx$$

$$\Rightarrow u = \frac{10b}{m}$$



14. In the presence of the magnetic field, if the particle practically stops 6 m away from the point of projection, the magnetic field is equal to

With B $\vec{F} = \vec{F}_B + \vec{f}_v$

(a) $\frac{4b}{3q}$

$$\Rightarrow m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) - b\vec{v}$$

$$\Rightarrow m d\vec{v} = q(\vec{v} dt \times \vec{B}) - b(\vec{v} dt)$$

$$\Rightarrow m \int_{\vec{u}}^0 d\vec{v} = q \left[\left(\int_{\vec{u}}^0 \vec{v} dt \right) \times \vec{B} \right] - b \int_{\vec{u}}^0 \vec{v} dt$$

$$\Rightarrow -m\vec{u} = q(\vec{r} \times \vec{B}) - b\vec{r}$$

These two are orthogonal vectors.

$$\Rightarrow (m\vec{u})^2 = (q\vec{r} \times \vec{B})^2 + (b\vec{r})^2$$

$$\Rightarrow 100b^2 = (q\vec{r} \times \vec{B})^2 + (b\vec{r})^2$$

$$\vec{r} = 6 \Rightarrow B = \frac{4b}{3q}$$

15. Magnetic induction is now doubled in magnitude without altering its direction. How far from the point of projection will the particle practically stop?

(a) $\frac{\sqrt{73}}{2}$ m

(b) $\frac{8}{\sqrt{73}}$ m

(c) $\frac{20}{\sqrt{73}}$ m

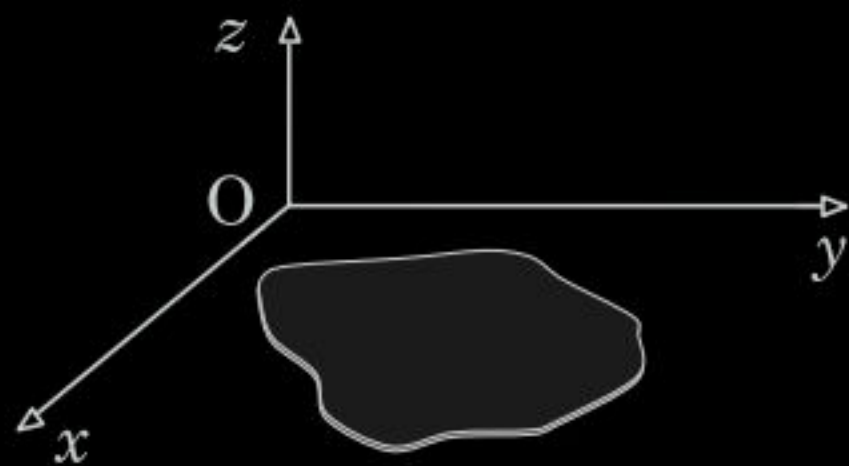
(d) $\frac{30}{\sqrt{73}}$ m

From previous slide

$$100b^2 = (q \cancel{r} B)^2 + (b \cancel{r})^2 = 2 \left(\frac{4b}{3q} \right)^2$$

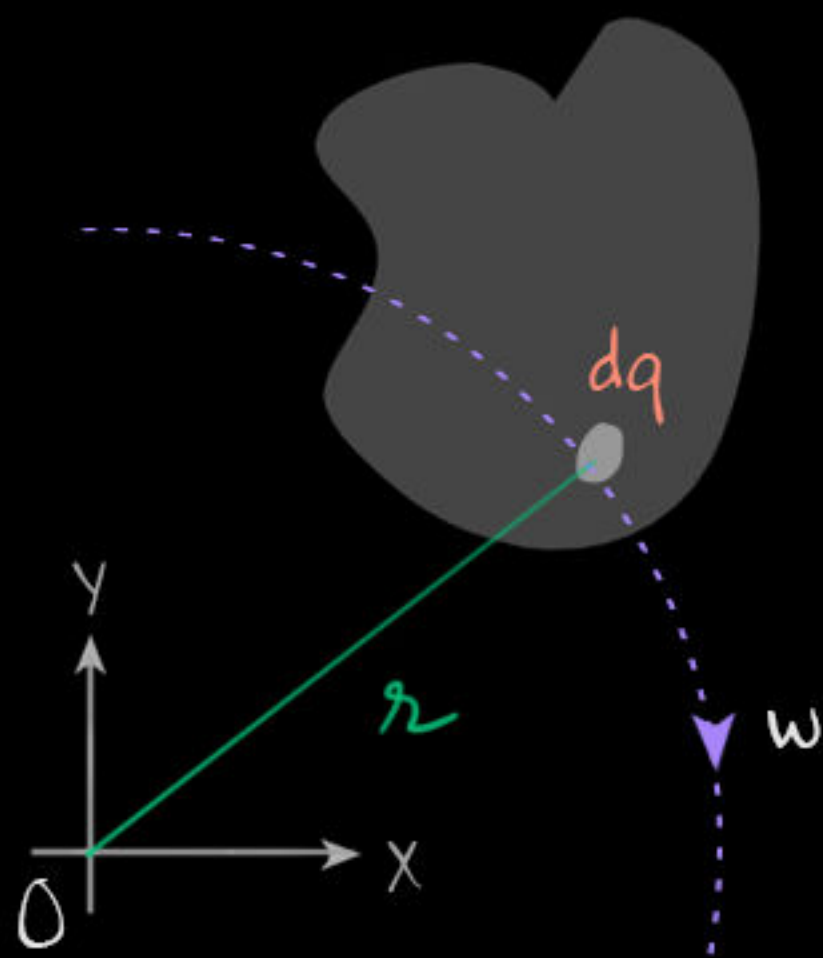
$$\Rightarrow 100 \cancel{b}^2 = \left(\frac{8}{3} \cancel{r} \cancel{b} \right)^2 + (\cancel{b} \cancel{r})^2$$

$$\Rightarrow \cancel{r} = \frac{30}{\sqrt{73}} //$$



1. A lamina made of an insulating material carrying some amount of charge is placed in the x - y plane of a coordinate system as shown in the figure. Electrostatic potential at the origin O due to the charges on the lamina is V_0 . Now the lamina is made to rotate with a constant angular velocity ω about the z -axis. Find magnetic field at the origin created by the moving charges of the lamina.

Top view:



$$V_0 = \int \frac{k dq}{r} \quad (1)$$

$$B_0 = \int \frac{\mu_0 di}{2r}$$

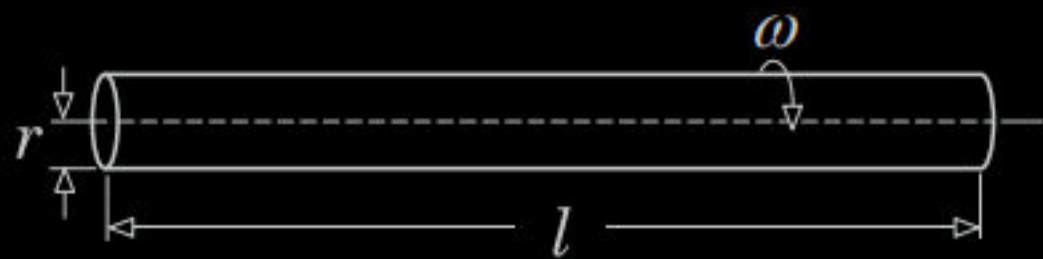
$$\frac{dq}{T} = \frac{dq}{2\pi/\omega}$$

$$= \frac{\mu_0 \omega}{2 \cdot 2\pi} \int \frac{dq}{r}$$

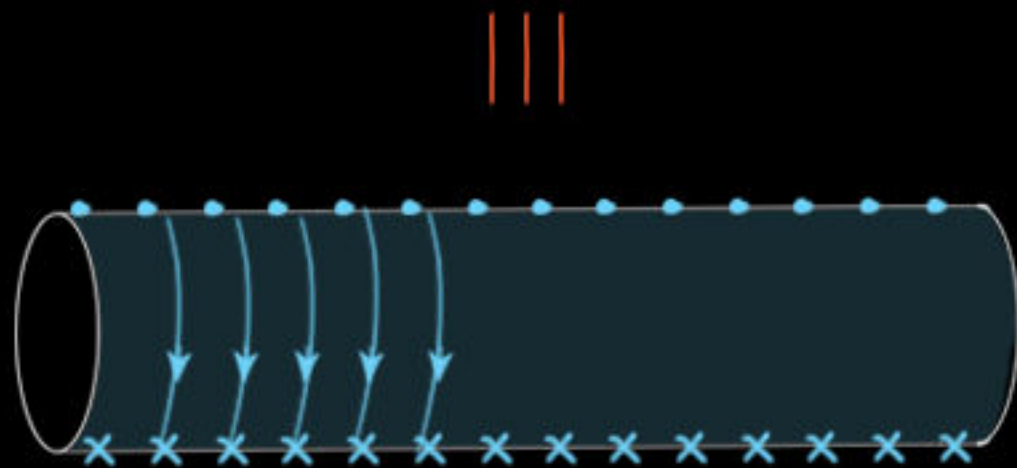
From 1

$$= \frac{\mu_0 \omega}{4\pi} \frac{V_0}{k} = \mu_0 \epsilon_0 \omega V_0 //$$

$\frac{1}{4\pi\epsilon_0}$



A rotating charged cylinder acts similar to a solenoid



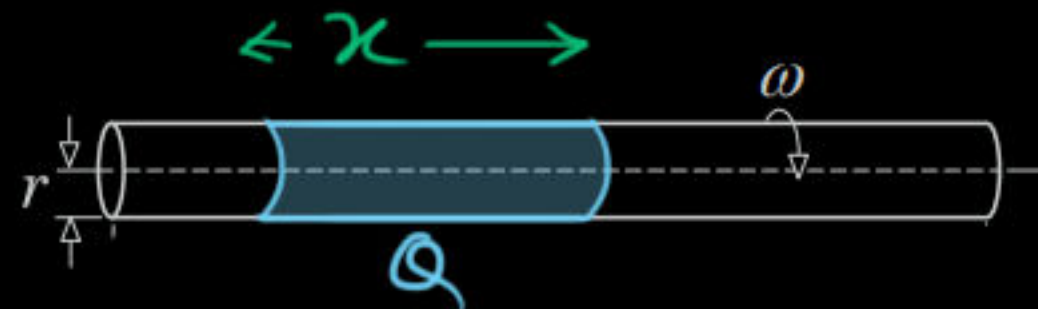
And we know, magnetic field due to solenoid is given by:

$$B_{in} = \mu_0 j \quad \text{A/m}$$

$$B_{out} = 0$$

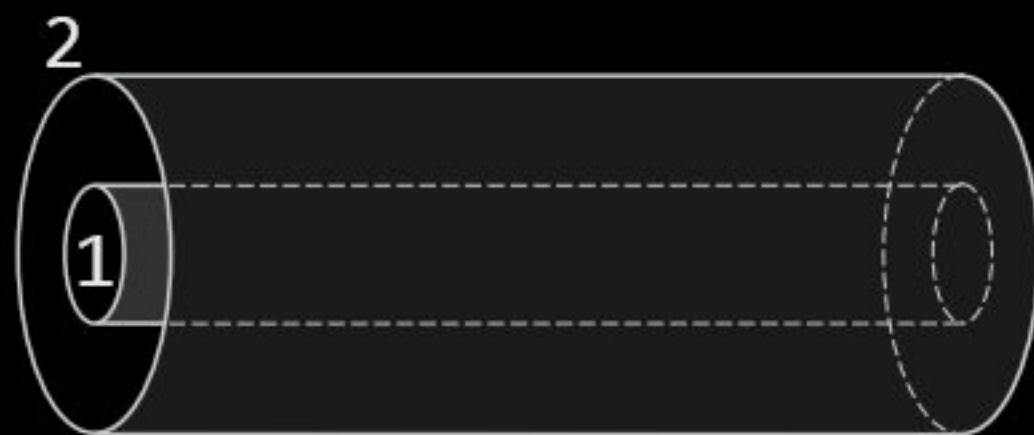
Proof: Irodov 3.238

2. A long cylindrical dielectric rod of radius r and length l ($l \gg r$) carrying uniform charge of surface charge density σ on its curved surface is rotating with a constant angular velocity ω about its own axis. Find magnetic induction at the centre of the rod. Relative magnetic permeability of the material of the rod is μ_r .



$$\text{Here, } j = \frac{i}{x} = \frac{Q}{T x} = \frac{\sigma \cancel{2\pi r} \cancel{x}}{\cancel{2\pi / \omega} x} = \sigma \omega r$$

$$\therefore B = \mu_0 \mu_r j = \mu_0 \mu_r \sigma \omega r //$$



For rotating cylinder, that is similar to solenoid:

$$\vec{j} = \sigma \omega \vec{r}$$

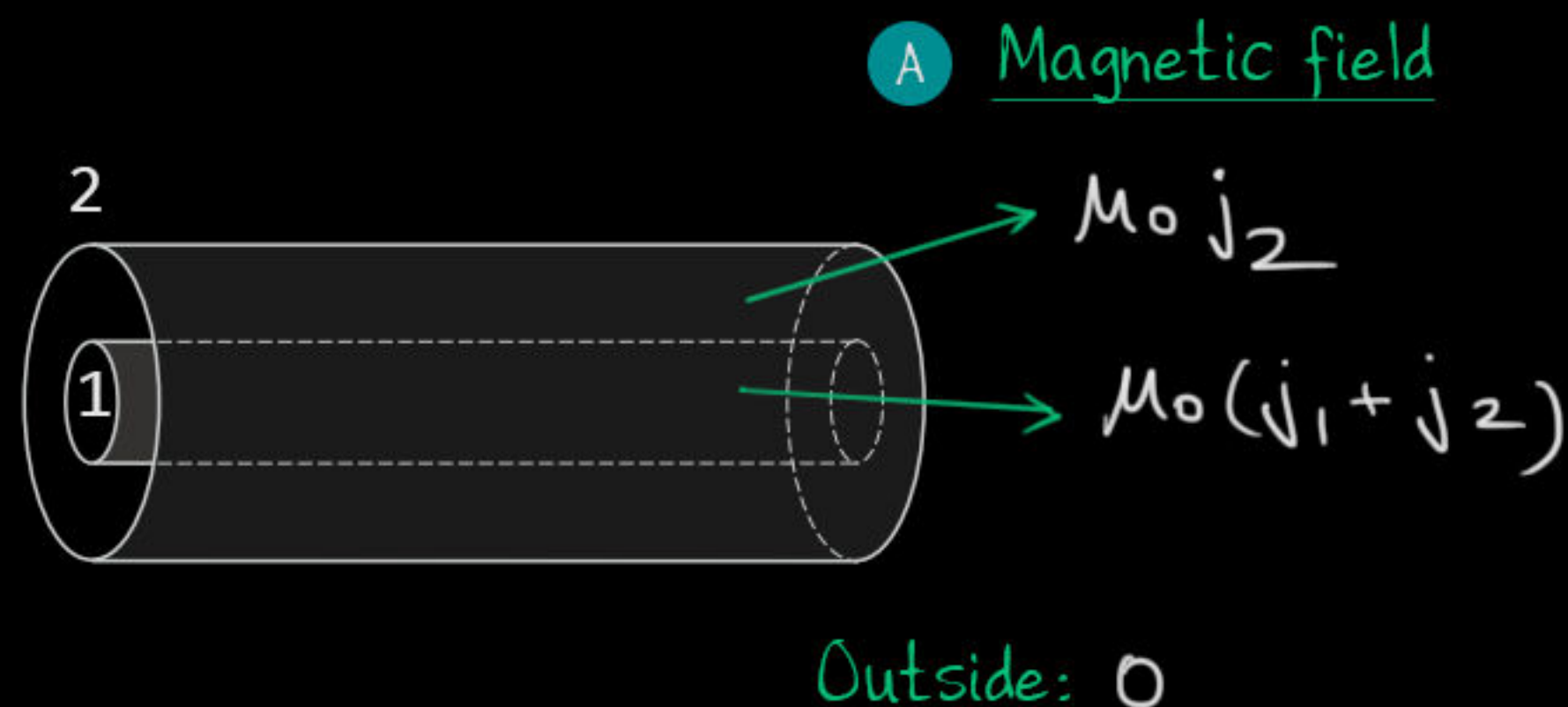
(Proven in previous problem)

$$\therefore j_1 = \sigma_1 \omega_1 r_1$$

$$j_2 = \sigma_2 \omega_2 r_2$$

3. Consider two infinitely long coaxial dielectric cylindrical shells. The inner and the outer shells are of radius R_1 and R_2 and carry uniform charges of surface charge densities σ_1 and σ_2 respectively. These shells begin to rotate in the same sense with angular velocities ω_1 and ω_2 .

(a) Find magnetic induction B at a distance r from the common axis.



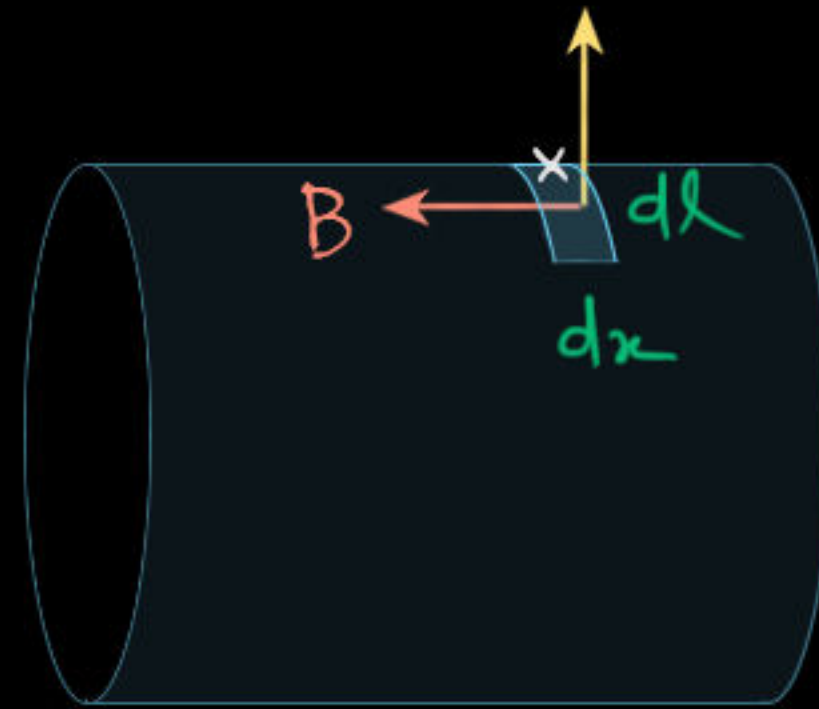
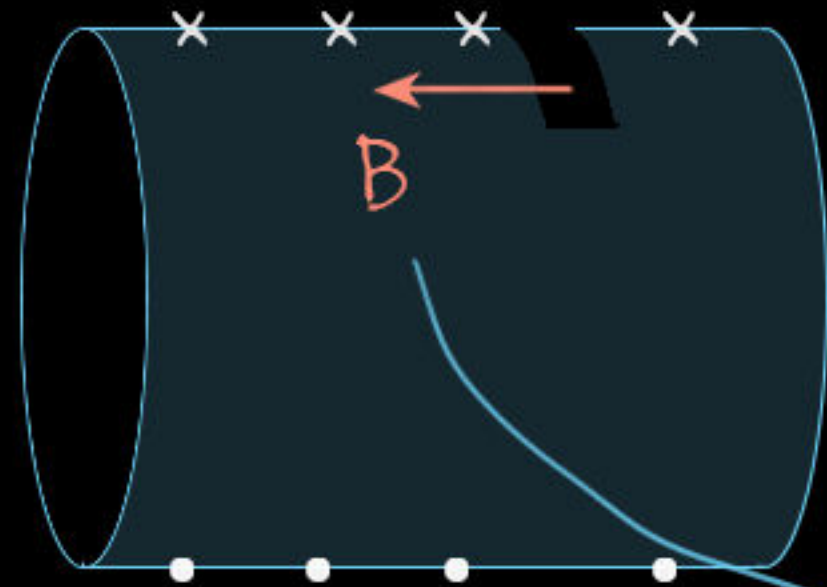
(b) Find pressures p_1 and p_2 on the surfaces the inner and the outer cylinders due to magnetic forces.

Let field at the location of element, without the element itself be B .

$$j_1 = \sigma_1 \omega_1 r_1$$

$$j_2 = \sigma_2 \omega_2 r_2$$

$$F = B i dl = B j dx dl$$



$$\therefore P = \frac{F}{A} = \frac{B j dx dl}{dx dl} = B j$$

Sol:

Proof: Irodov 3.263

$$B = \frac{\mu_0 j_2}{2}$$

$$\Rightarrow P_2 = B j_2 = \frac{\mu_0 j_2^2}{2} //$$

$$B = \mu_0 j_2 + \frac{\mu_0 j_1}{2} \Rightarrow P_1 = B j_1 = \mu_0 \left(j_2 + \frac{j_1}{2} \right) j_1 //$$



A From Irodov 3.247

$$B = \frac{2}{3} \mu_0 \sigma \omega R$$

$$= \frac{2}{3} \mu_0 \frac{Q}{4\pi R^2} \omega R \frac{\epsilon_0}{\epsilon_0}$$

$$= \frac{2}{3} \mu_0 \epsilon_0 \omega V //$$

4. An isolated conducting sphere of radius R maintained at a potential V is being rotated at a constant angular velocity ω about one of its diameter. Find magnetic dipole moment $\vec{m}$ of the sphere and magnetic induction $\vec{B}$ at its centre.

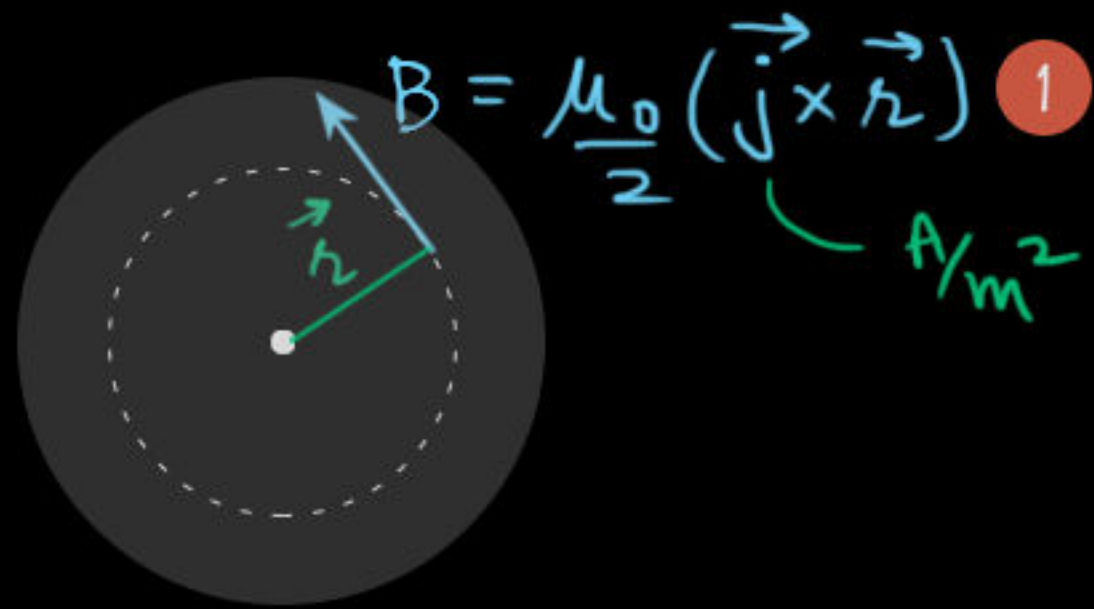
B We know, $\frac{\mu}{L} = \frac{q}{2m}$ If charge and mass distribution of a rotating charged body is similar.

Lets replace the given conducting sphere with shell, as then above condition will be satisfied and magnetic moment will remain the same.

$$\frac{\mu}{L} = \frac{q}{2m}$$
$$L = I\omega = \frac{2}{3} m R^2 \omega$$

$$\Rightarrow \mu = \frac{1}{3} q R^2 \omega \cdot \frac{4\pi\epsilon_0}{4\pi\epsilon_0} \cdot \frac{R}{R} = \frac{4\pi\epsilon_0}{3} \omega R^3 V //$$

Inside a cylindrical, uniform current carrying wire:



Proof: Irodov 3.233

Also, energy densities:

$$u_E = \frac{\epsilon_0 E^2}{2} \quad (2)$$

$$u_B = \frac{B^2}{2\mu_0} \quad (3)$$

5. A uniform electric field is established inside a long cylindrical straight rod along its axis. Energy density of this electric field is uniform and is equal to $u_E = 2 \times 10^{-17} \text{ J/m}^3$. Inside the rod, energy density of magnetic field is nonuniform and assumes a value $u_B = 0.4 \text{ J/m}^3$ at a distance $r = 2 \text{ cm}$ from the axis of the rod. Find electrical resistivity of the material of the rod.

Sol:

$$\Rightarrow \sigma = \frac{J}{E}$$

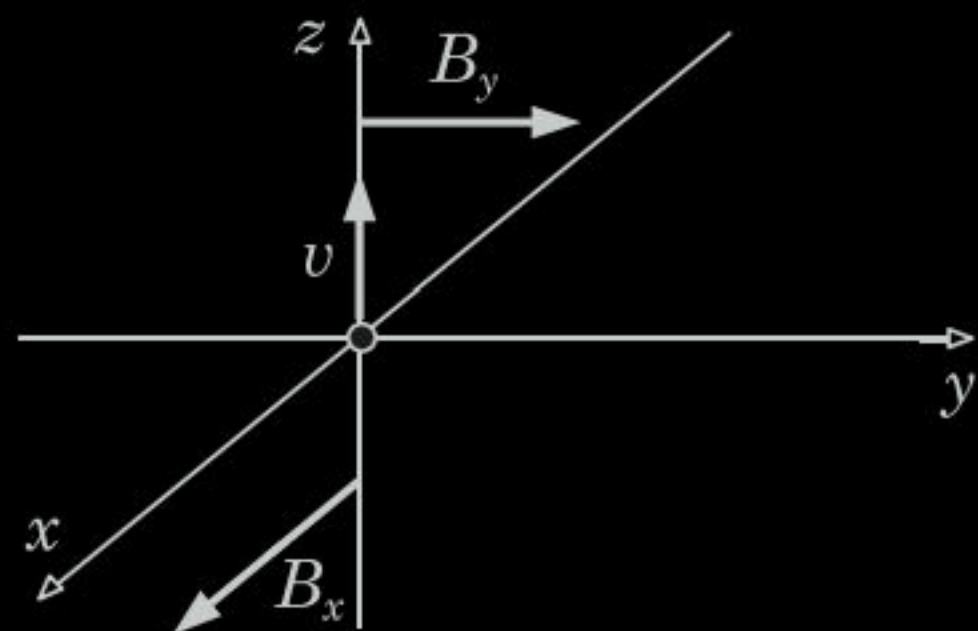
1 $\rightarrow \frac{2B}{\mu_0 r}$

2 $\rightarrow \sqrt{\frac{2u_E}{\epsilon_0}}$

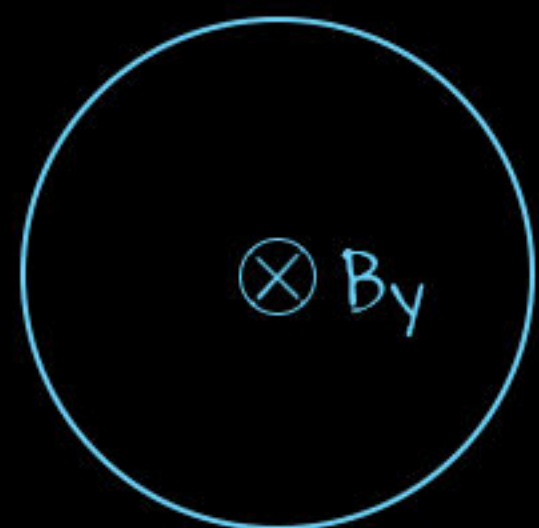
3 $\rightarrow \sqrt{2\mu_0 u_B}$

$$\Rightarrow \sigma = \frac{2}{r} \sqrt{\frac{\epsilon_0 u_B}{\mu_0 u_E}}$$

$$\therefore \rho = \frac{1}{\sigma} = \frac{r}{2} \sqrt{\frac{\mu_0 u_E}{\epsilon_0 u_B}}$$



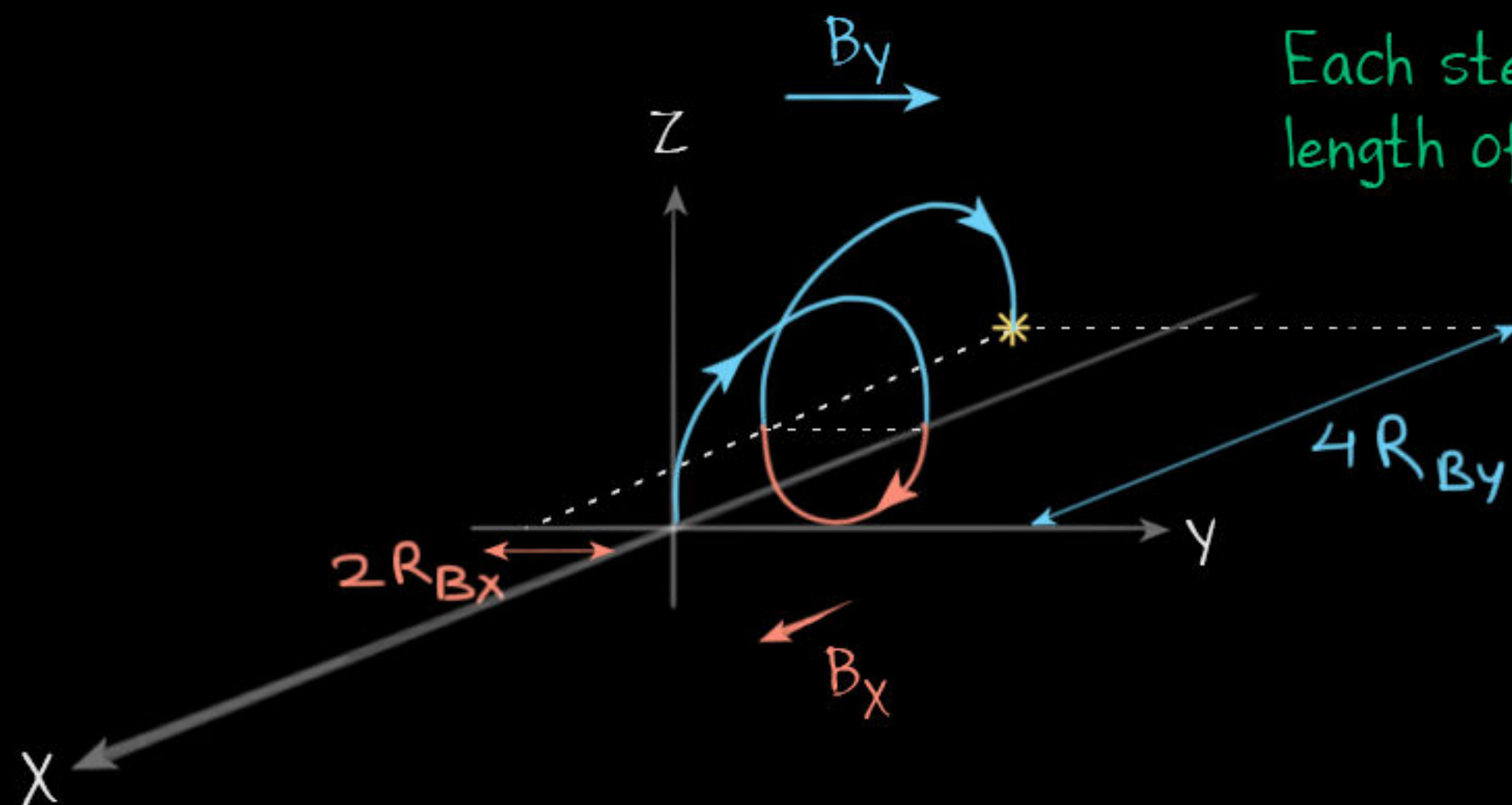
6. A particle of mass m and charge q is projected from the origin with a velocity v in the positive z -direction. Above the plane $z = 0$, a uniform and constant magnetic field of induction B_y exists in the positive y -direction and below the plane $z = 0$, a uniform and constant magnetic field of induction B_x exists in the positive x -direction. Determine coordinates of the point where the particle crosses the plane $z = 0$ third time.



$$R_{By} = \frac{mv}{qB_y}$$



$$R_{Bx} = \frac{mv}{qB_x}$$



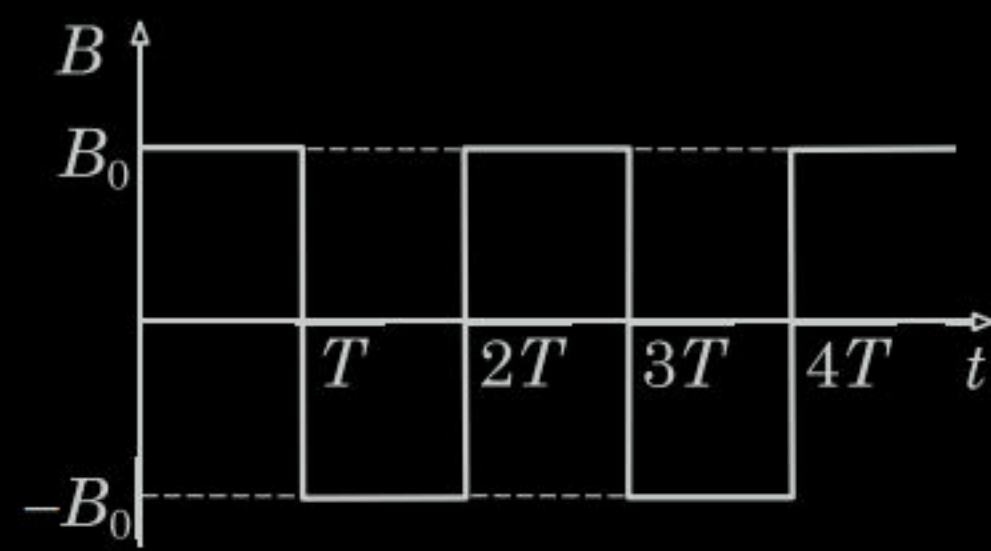
Each step covers the length of diameter

$$\therefore \text{Location on third crossing} = (-4R_{By}, -2R_{Bx})$$

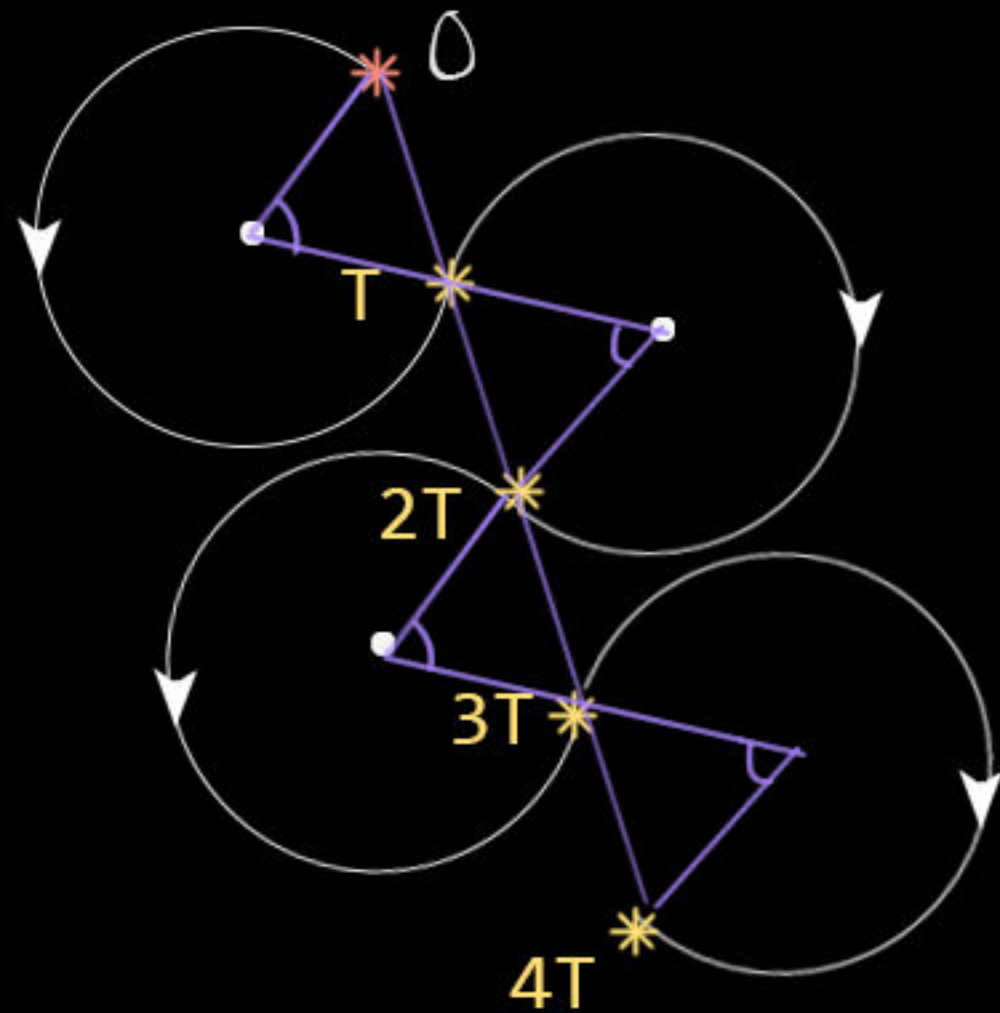
$$= \left(-4 \frac{mv}{qB_y}, -2 \frac{mv}{qB_x} \right)$$

by Ankit Singhvi

Youtube / Arihant Senior



7. In a region of free space, a uniform magnetic field of induction B varies with time t as shown in the graph. At the instant $t = 0$, a particle of charge q and mass m enters the region with a velocity v perpendicular to the direction of the magnetic field. Neglect the influence of the induced electric field and explore motion of the particle. For what values of the charge q , will the particle follow a closed path?



If particle is at any other point than O at end of T , it will start to move away.

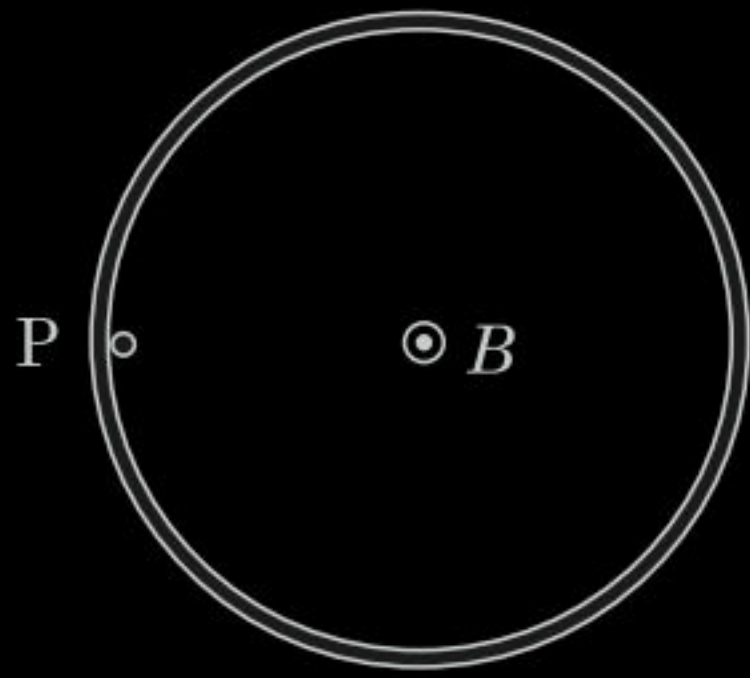
It shall move in equidistant steps in a straight line.



Only continuous loops are 8 shaped. That is, particle must be back at starting point O after each T .

$$\therefore T = n \left(\frac{2\pi m}{qB} \right)$$

$$\Rightarrow q = \frac{n 2\pi m}{BT}, \quad n = 1, 2, 3, \dots$$



8. In a hollow rigid cylinder of radius R gravity is absent and a uniform magnetic field of induction B is established everywhere parallel to the axis of the cylinder. The cylinder is made of an insulating material and is fixed by some external means. A very small ball of mass m and charge q is projected normally from a point P on the inner surface of the cylinder. If the ball returns to the point P after a certain number of elastic collisions with the cylinder, find range of velocity v of projection of the ball.

Between any two strikes, θ is constant.

- ∴ In order for particle to come strike back at P :

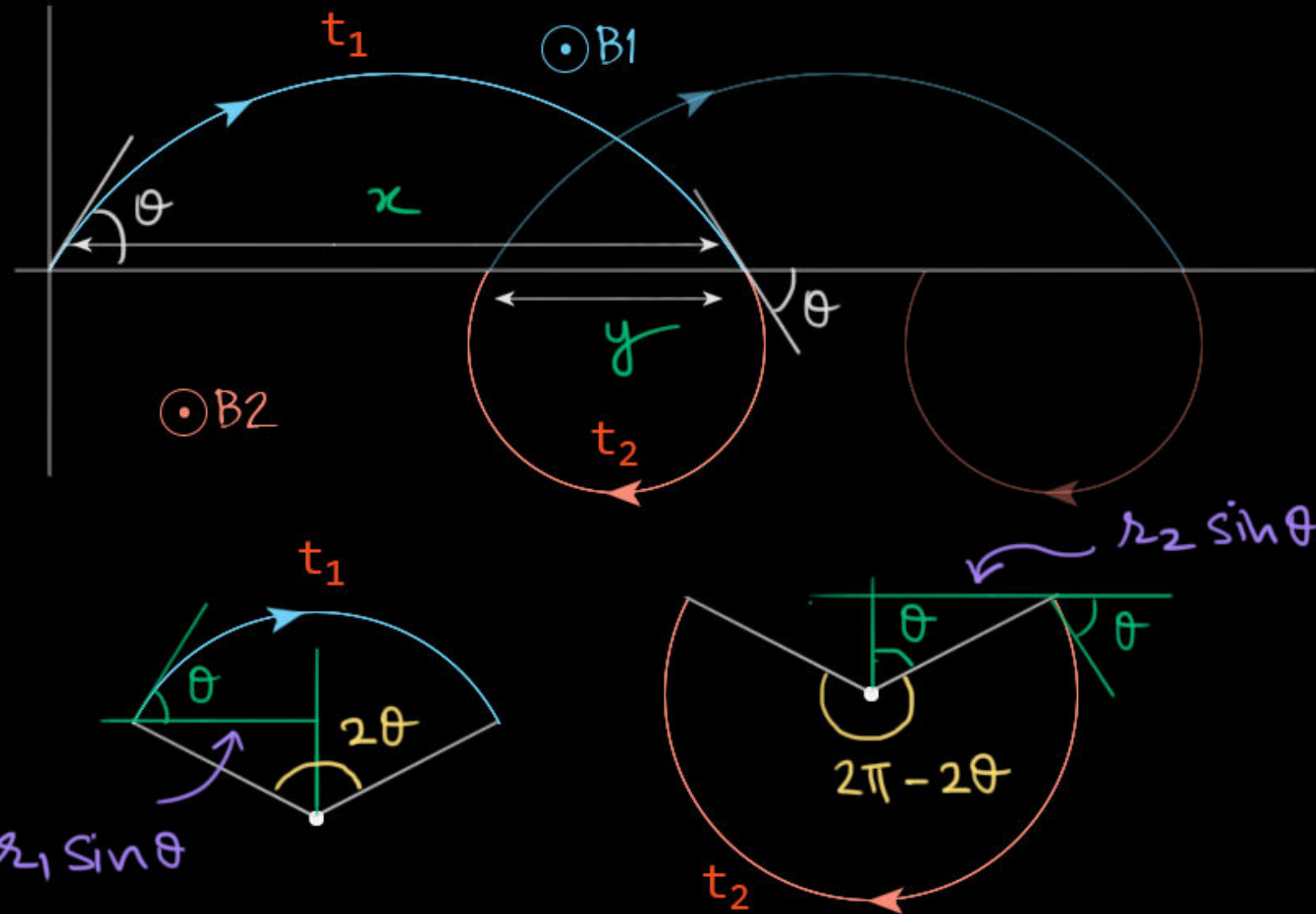
Number of 'steps' particle will take to come back at P .

$$n_1 \theta = n_2 2\pi, \quad n_1 \text{ and } n_2 \text{ can be any natural numbers}$$

$$\Rightarrow \frac{\theta}{2\pi} = \frac{n_2}{n_1}$$

- ∴ It will always come back to P .
As every value on LHS can be represented by ratio of some pair of natural numbers.

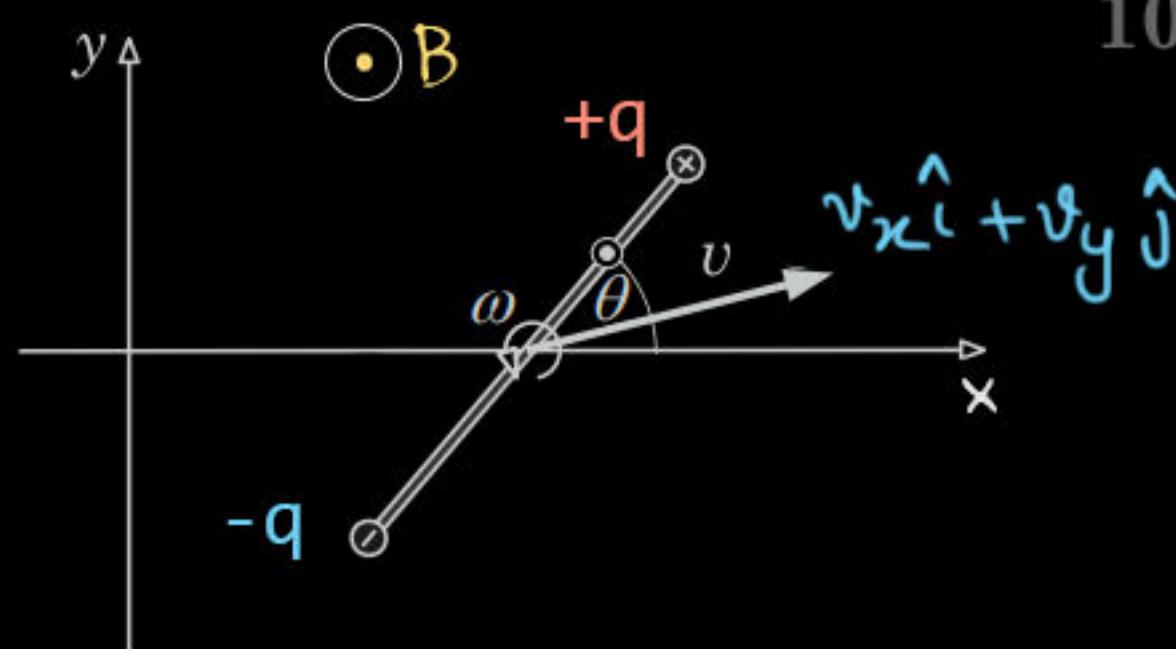
Note: It's given that ball enters normally. But, it doesn't matter.


$$v_{av} = \frac{x - y}{t_1 + t_2}$$

$$= \frac{2r_1 \sin \theta - 2r_2 \sin \theta}{\frac{2\theta}{2\pi} \cdot \frac{2\pi m}{qB_1} + \frac{(2\pi - 2\theta)}{2\pi} \cdot \frac{2\pi m}{qB_2}}$$

$$= \frac{v \sin \theta \left(\frac{1}{B_1} - \frac{1}{B_2} \right)}{\frac{\theta}{B_1} + \frac{(\pi - \theta)}{B_2}} = \frac{v \sin \theta (B_2 - B_1)}{(B_2 - B_1)\theta + B_1\pi}$$

Youtube / Arihant Senior



Only velocity components along the rod can cause torque.

So let's break translational velocity ($\vec{v}_x + \vec{v}_y$) into components along and perpendicular to rod ($\vec{v}_1 + \vec{v}_2$).

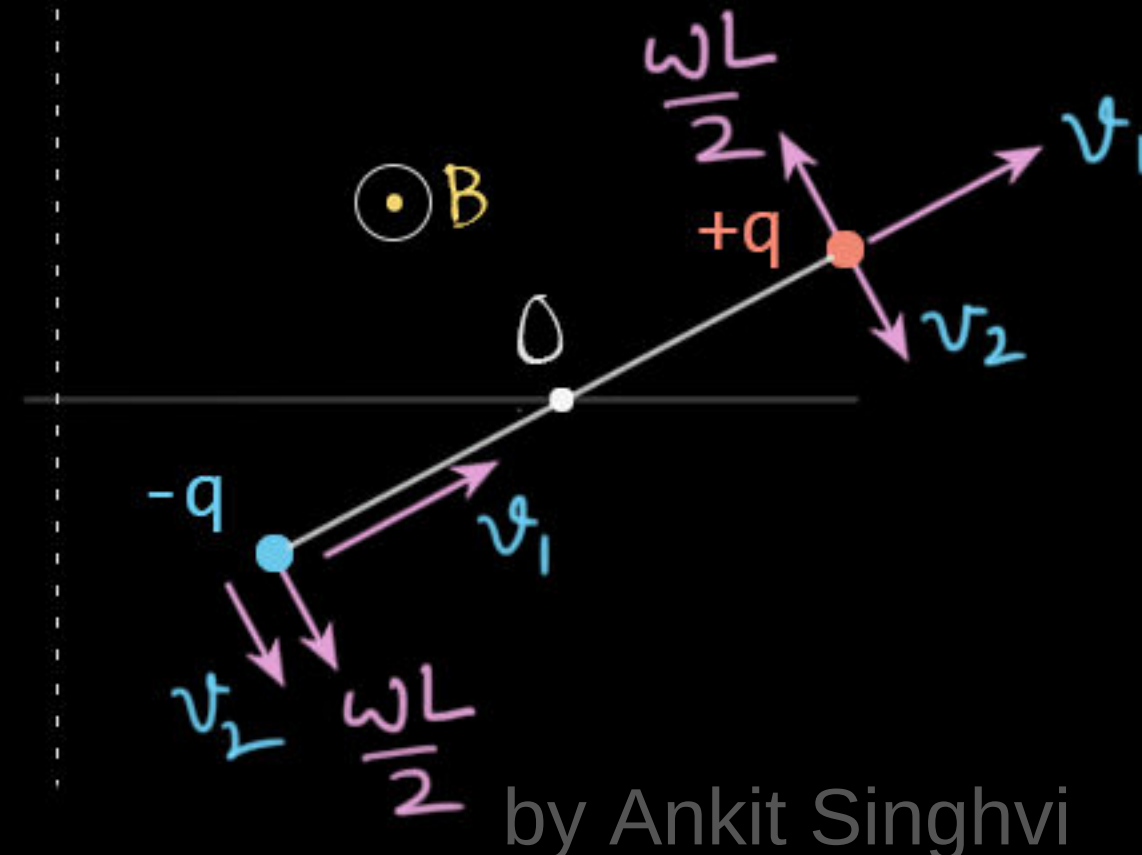
$$v_1 = v_x \cos \theta + v_y \sin \theta$$

$$v_2 = v_x \sin \theta - v_y \cos \theta$$

10. A dumbbell is made by fixing two particles at the ends of a thin rigid rod of length l . The particles carry unlike charges of equal modulus q . The dumbbell is moving in a region of space, where a uniform magnetic field of induction B pointing everywhere in the positive z -direction is present. At a particular instant when the rod of the dumbbell makes an angle θ with positive x -direction, it is rotating with angular velocity $\omega \hat{k}$ and its centre is moving with velocity $\vec{v}_c = v_x \hat{i} + v_y \hat{j}$. Find net force $\vec{F}$ on the dumbbell and net torque $\vec{\tau}$ about the centre of the dumbbell at this instant.

$$\vec{v}_x + \vec{v}_y = \vec{v}_1 + \vec{v}_2$$

∴ Final velocity of each charge w.r.t ground is as shown:



Force: is only due to $\frac{\omega L}{2}$

$$\therefore F = 2 \times (q v B)$$

$$= 2 \cdot q \cdot \frac{\omega L}{2} B$$

$$= q \omega L B \parallel \theta$$

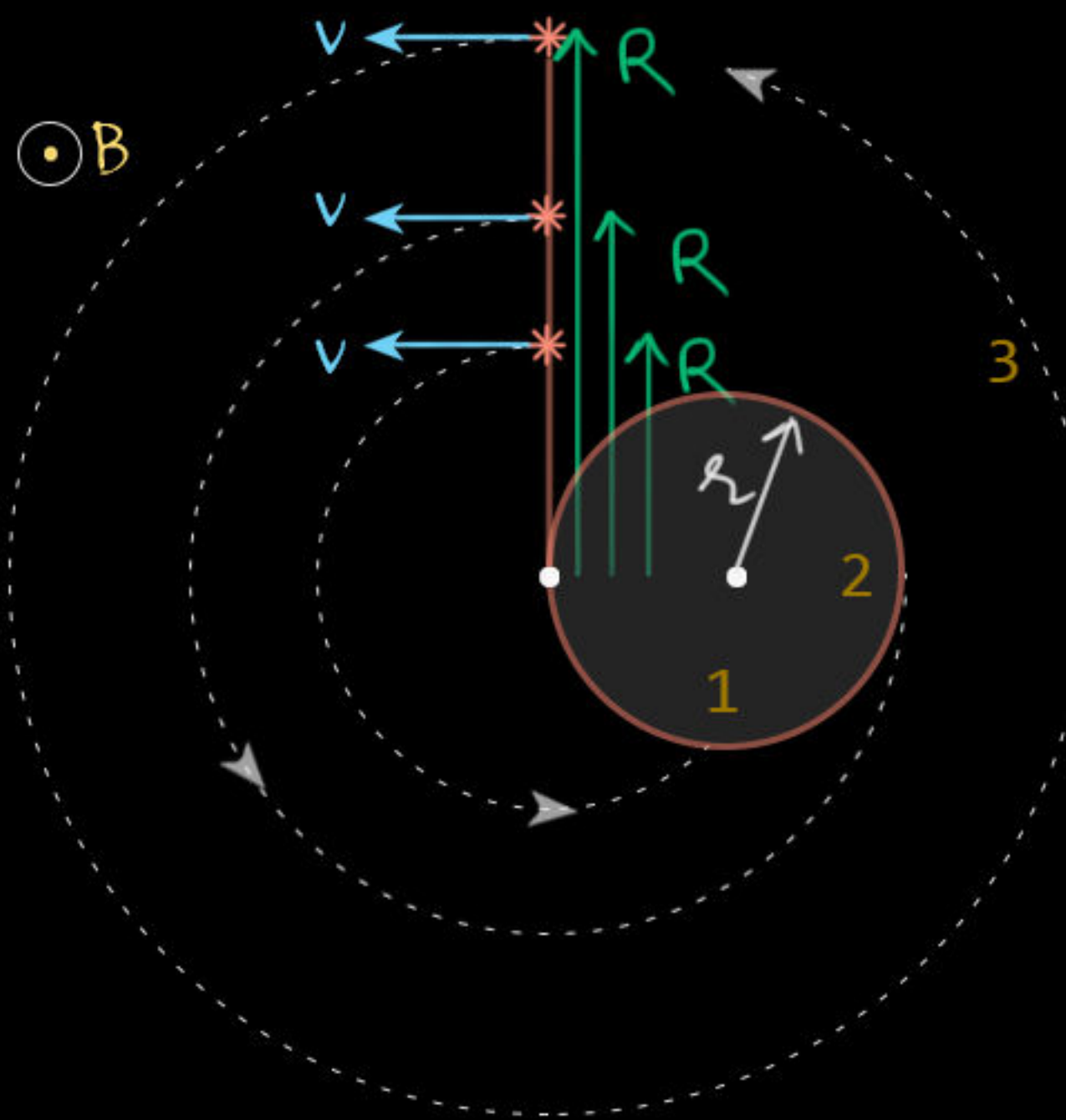
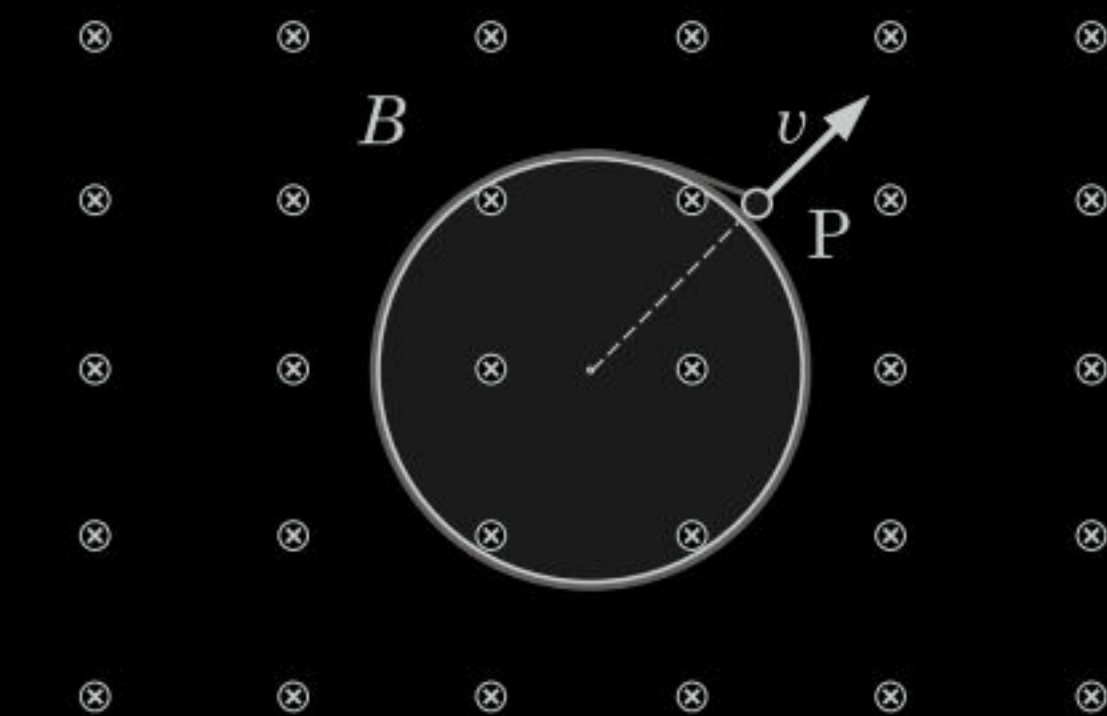
Torque: is only due to v_1

$$\therefore \tau = 2 \times (q v B) \frac{l}{2}$$

$$= q B l v_1 \perp \otimes$$

Youtube / Arihant Senior

by Ankit Singhvi



11. A long thin inextensible light thread is wound on a fixed cylinder of radius r and a positively charged bead P of mass m is attached to the free end of the thread. A uniform magnetic field of induction B directed along the axis of the cylinder exists in the region surrounding the cylinder as shown in the figure. Initially the thread is fully wound on the cylinder and the bead touches it. If the bead is projected with a velocity v radially away from the cylinder, it will eventually touch the cylinder tangentially. What should be the magnitude of charge on the bead to fulfil the above-mentioned conditions?

Speed v is constant, as thread is always normal to v .

Without B , it unwinds in a spiral, never touching the cylinder again.

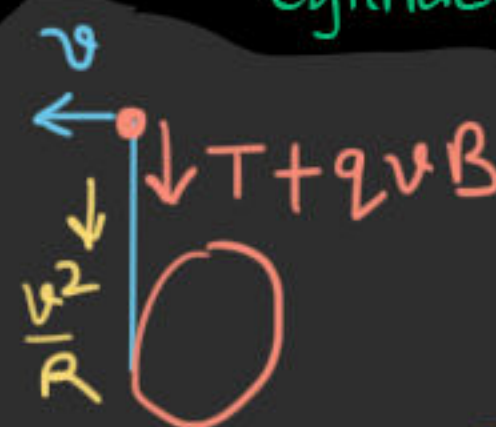
But due to B , after thread length is $R = mv/qB$, Tension vanishes and particle will move in circle of radius R .

It may or may not hit the cylinder as seen in three different cases.

$$R < 2r$$

✗ 1 Particle will hit the cylinder

✓ 2 Particle will just brush the cylinder as tangent.



$$T + qvB = \frac{mv^2}{R}$$

$$@ R = \frac{mv}{qB}, T = 0$$

$$R = 2r$$

$$\Rightarrow \frac{mv}{qB} = 2r$$

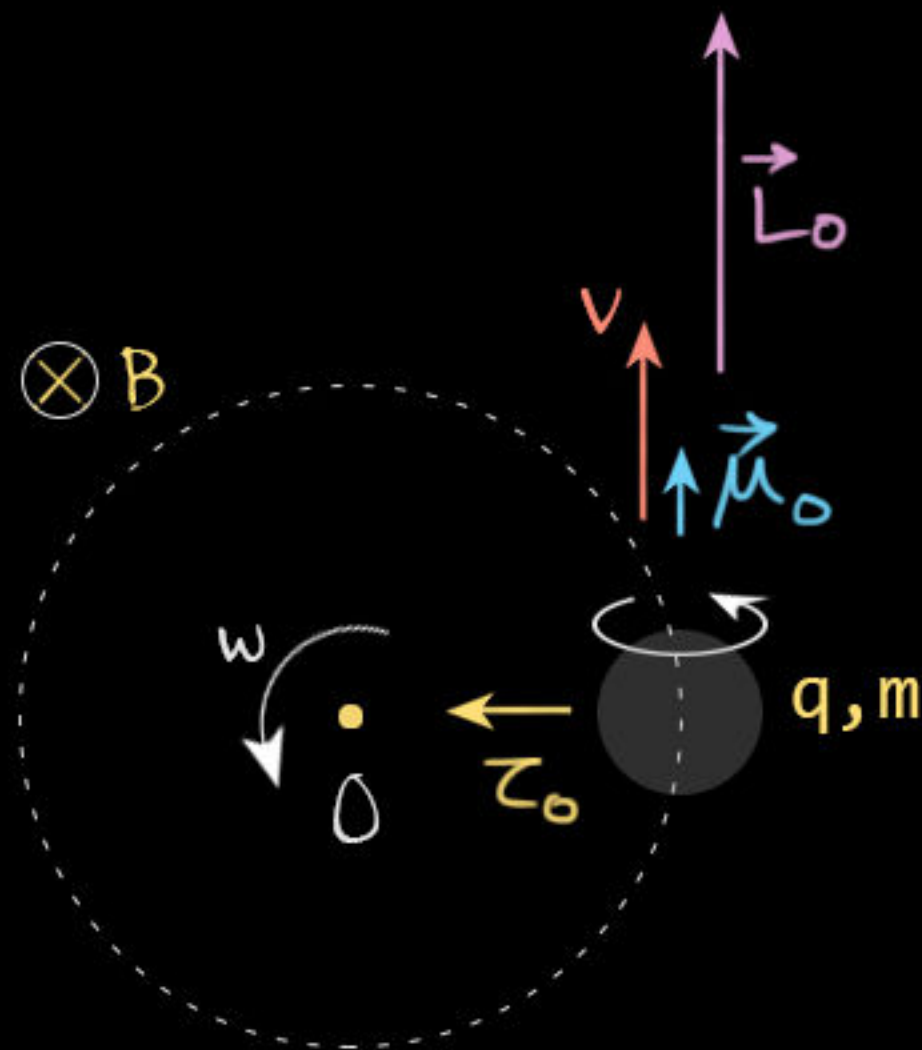
$$\Rightarrow q = \frac{mv}{2Br} //$$

✗ 3 Particle will circle around the cylinder. $R > 2r$

Since similarity in charge and mass distribution is not given, we can NOT use

$$\vec{\mu} = \frac{q}{2m} \vec{L}$$

Let $\vec{\mu}_0$ and $\vec{L}_0$ be magnetic moment and angular momentum due to spin alone.



12. An insulating bead of mass M and total charge q spinning with angular momentum $\vec{L}_0$ enters a region of uniform magnetic field with a velocity that is in the direction of its angular momentum vector. If the velocity vector and the angular momentum vector always remain parallel to each other and the bead moves on a circular path, find magnetic dipole moment vector of the bead associated with its spin. $\rightarrow$ i.e μ_0

$$= \vec{\mu}_0 + \frac{q}{2\pi/\omega} \pi r^2 \hat{k} \text{ parallel to } \vec{B} = \frac{\vec{L}_0}{\omega} + \frac{mvr\omega \hat{k}}{\omega}$$

$\vec{\tau}_0 = \vec{\mu} \times \vec{B} = \frac{d\vec{L}}{dt}$ Constant magnitude but changing direction. Constant

$$\Rightarrow \mu_0 B + 0 = \frac{dL_0}{dt} + 0 \text{ (Both directions towards center)}$$

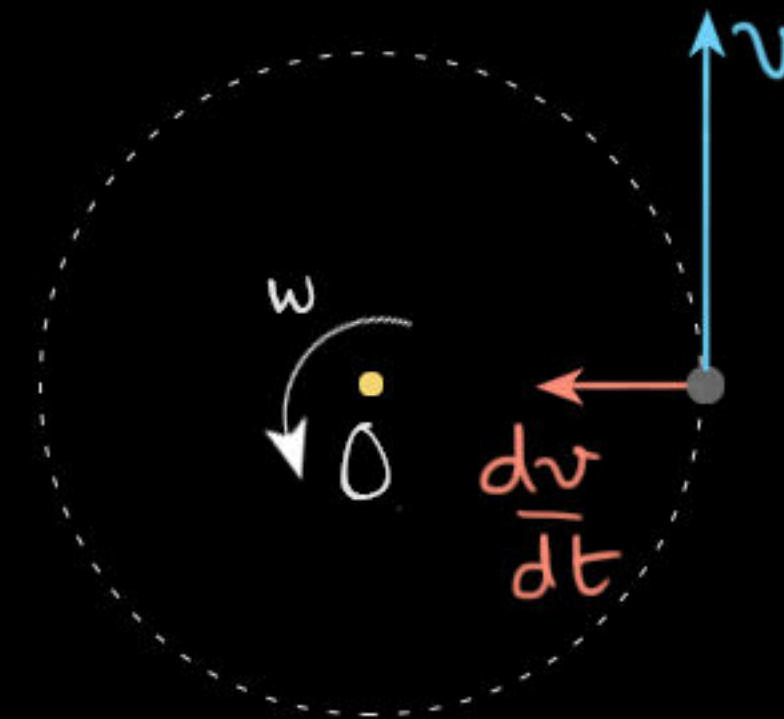
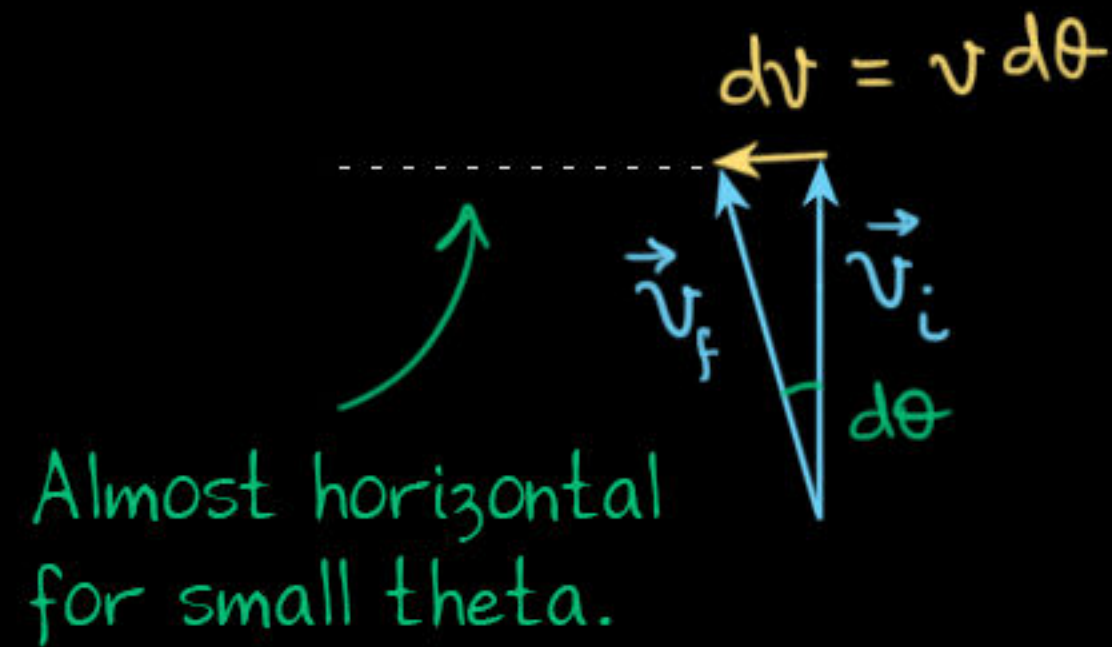
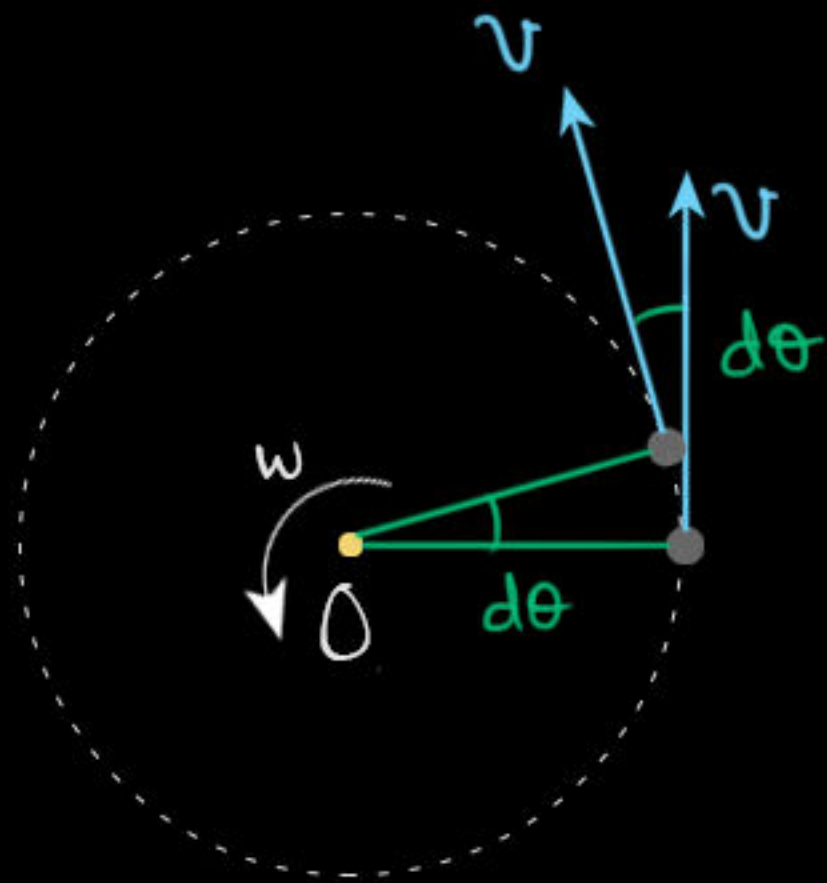
↓ Explained in next two slides

$$\Rightarrow \mu_0 B = L_0 \omega \quad \frac{qB}{m} \text{ (Magnitudes)}$$

$$\Rightarrow \vec{\mu}_0 = \frac{q}{m} \vec{L}_0$$

Proof:

$d\vec{v}/dt$ for uniform circular motion (constant v) = $v\omega$, towards centre

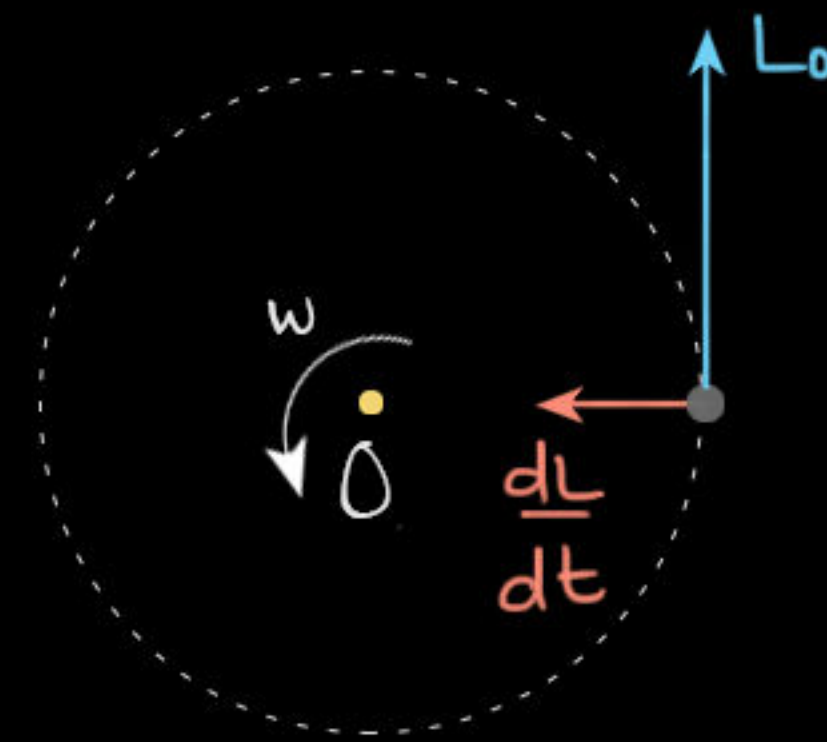
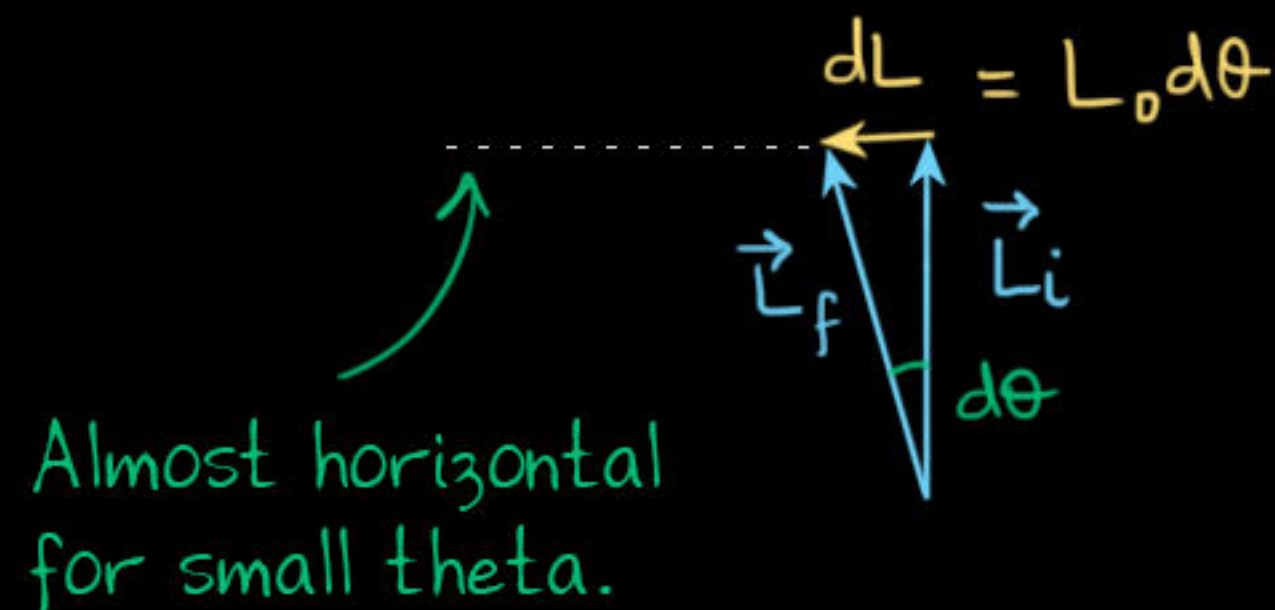
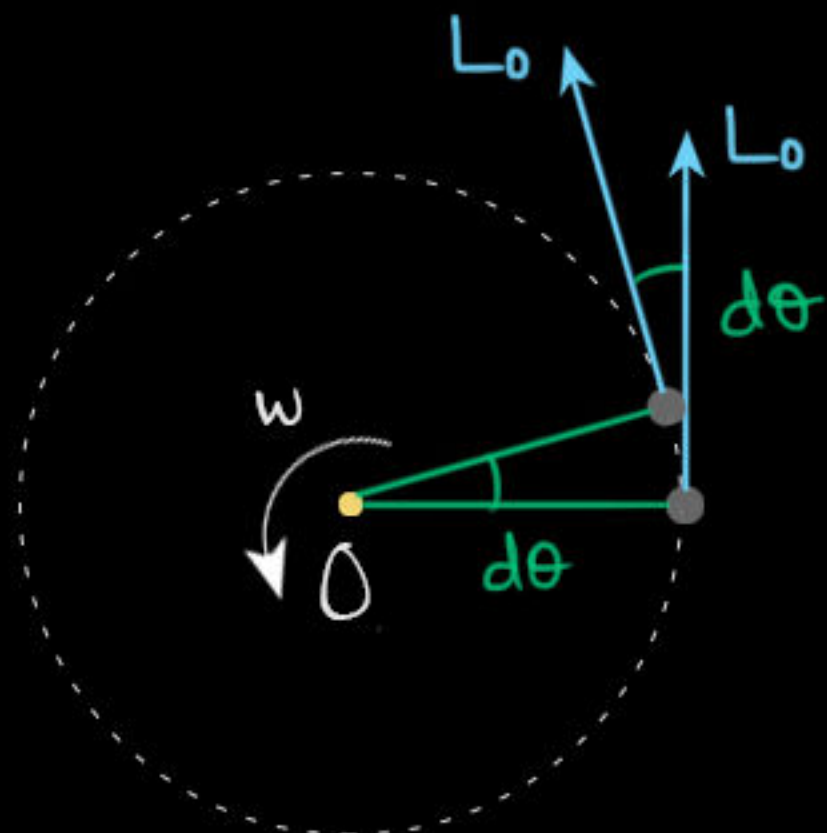


$$\therefore \frac{dv}{dt} = v \frac{d\theta}{dt} = v\omega //$$

Towards centre O.

Proof:

Similarly, $d\vec{L}/dt$ for uniform circular motion (constant v) = $L_0\omega$, towards centre



$$\frac{dL}{dt} = L_0 \frac{d\theta}{dt} = L_0\omega //$$

Towards centre O.

Different effects of same torque, due to same field B .

Lets compare effect of $\vec{B}$ on:

① Given insulated, spinning and rotating bead of magnetic dipole moment $\vec{\mu}_0$, and

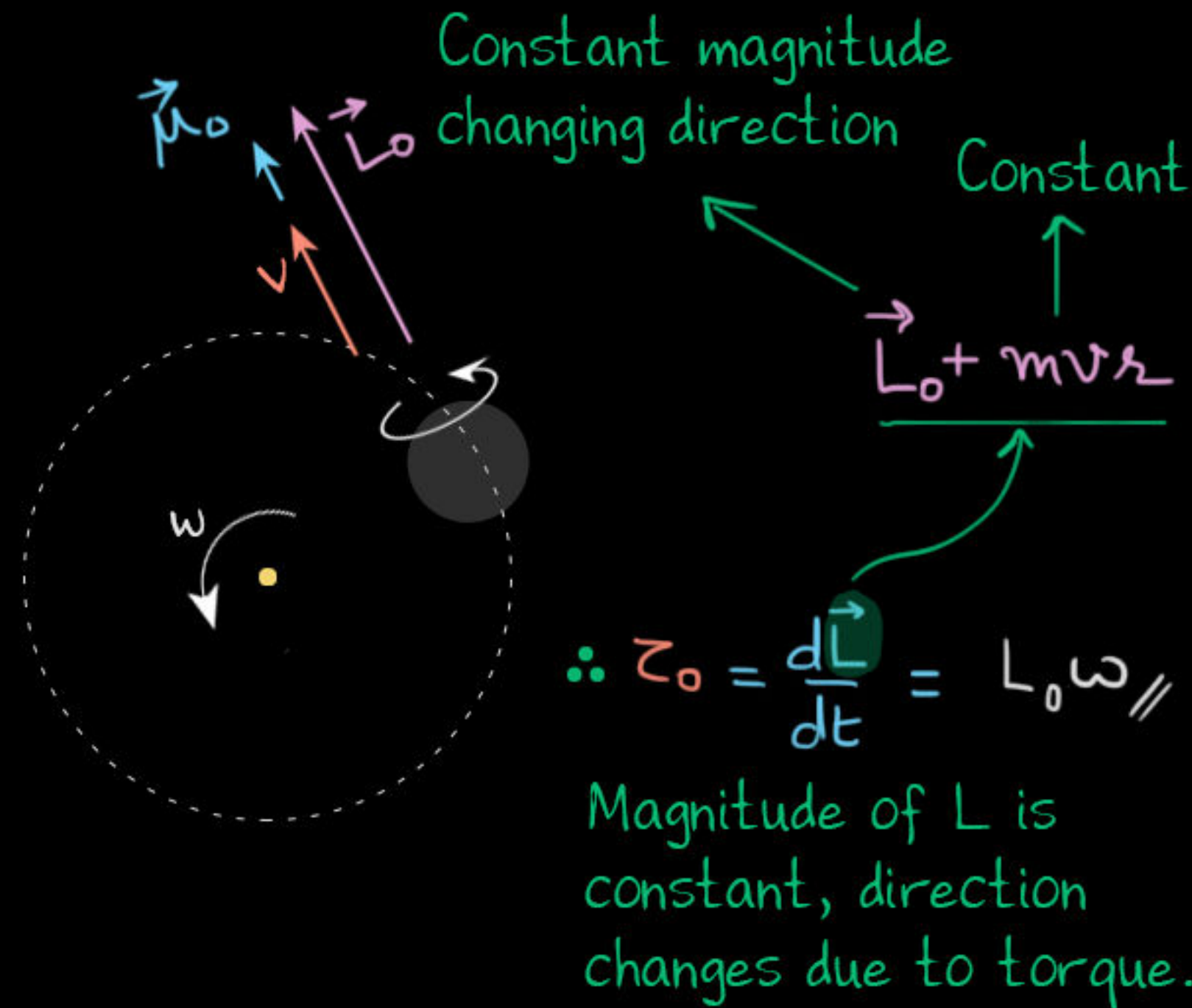
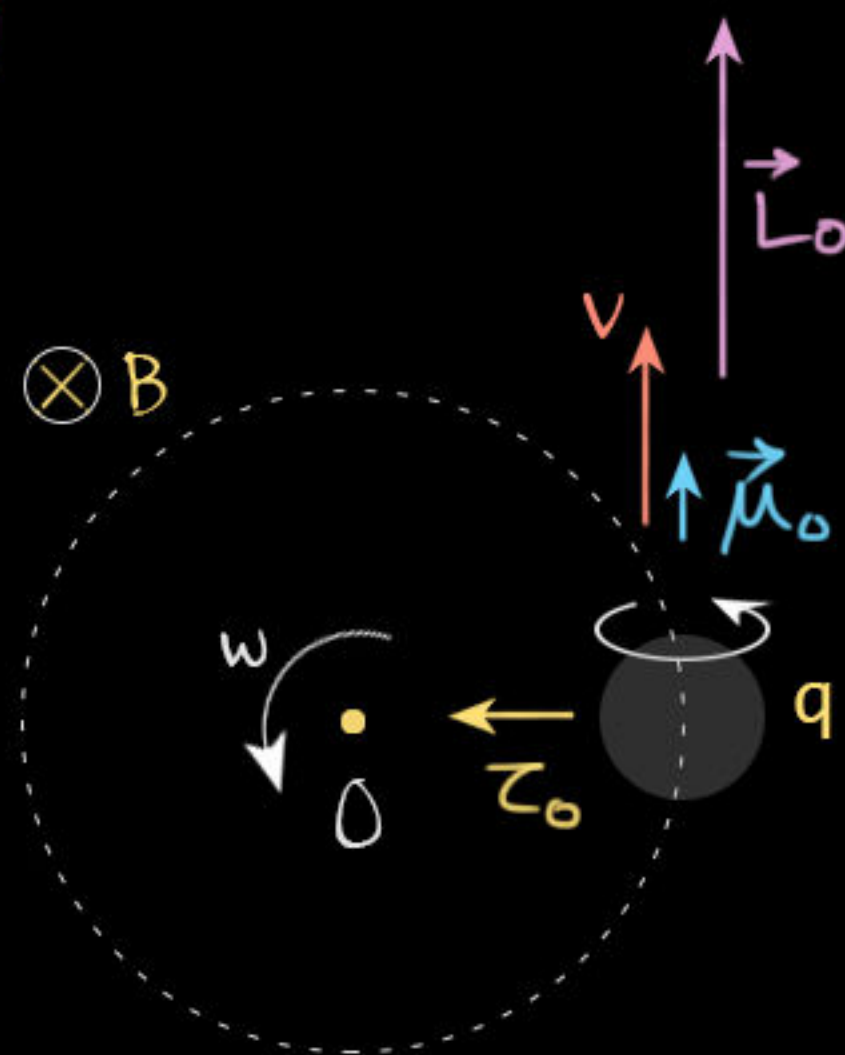
② A conducting, immobile, current carrying wire loop of same magnetic moment ($\vec{\mu}_0$).

$$\vec{\tau}_0 = \vec{\mu} \times \vec{B} = \vec{\mu}_0 \times \vec{B}$$

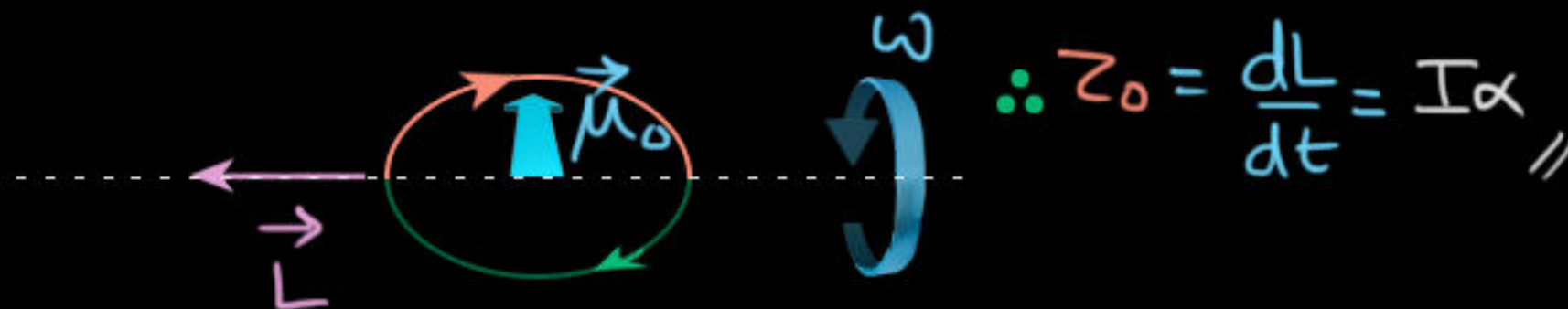
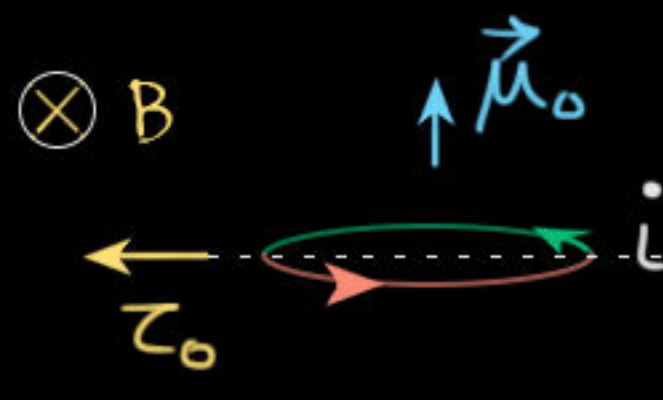
is same in both

But effect of that torque is different. In ①, torque causes bead to change its axis of spin. whereas in ②, torque causes whole loop to rotate about X axis.

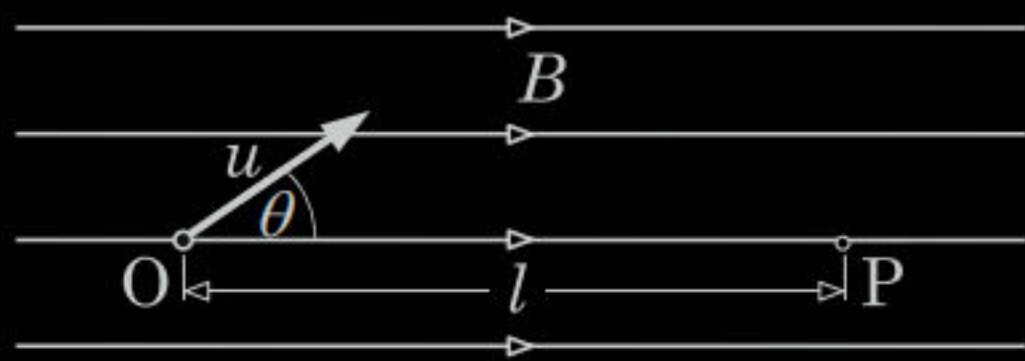
①



②



Direction of L is constant, magnitude changes due to torque.

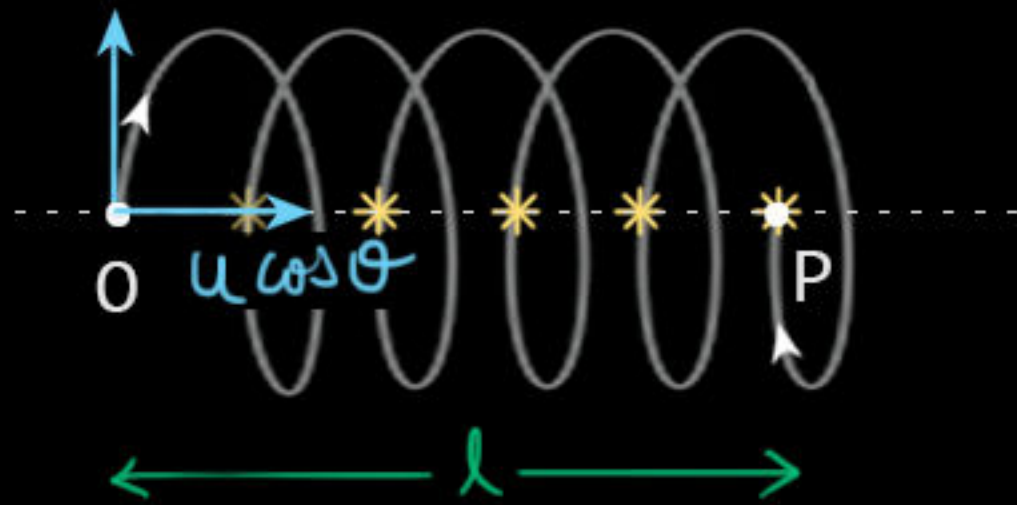


13. In a uniform magnetic field from a point O, a particle of charge q and mass m is projected with velocity u at an angle θ with the magnetic field. For what value of magnetic induction will the particle pass through the point P that is a distance l apart from the point of projection as shown in the figure?

Particle will move on a helical path parallel to field.

As OP is parallel to field, helix will pass through OP (*) everytime particle completes the circle.

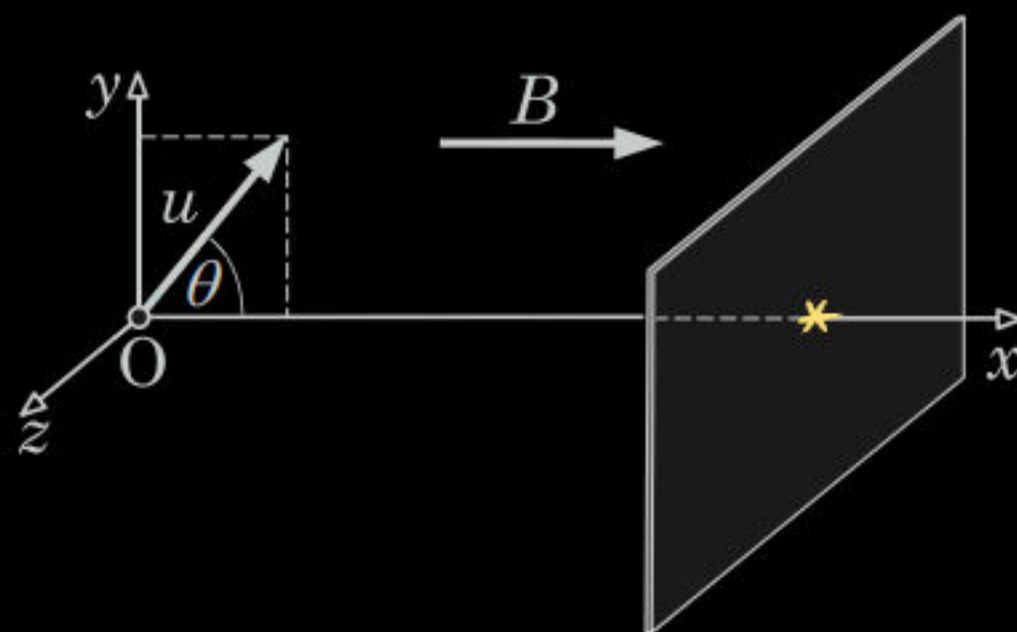
$u \sin \theta$



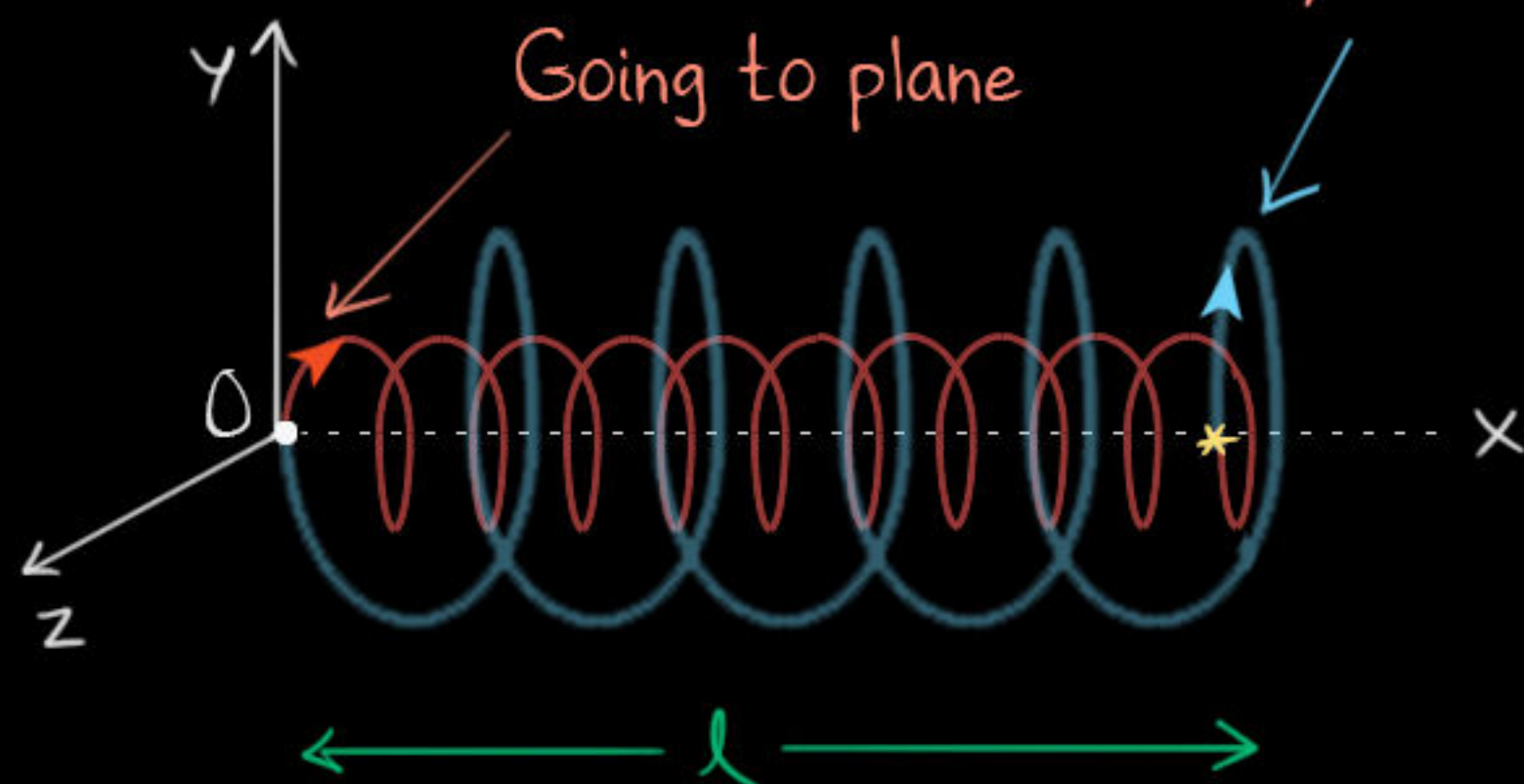
- To pass P, total time it takes to reach P ($l/u \cos \theta$) must be an integral multiple of time period of circle completion ($2\pi m/qB$)

$$\text{i.e: } \frac{l}{u \cos \theta} = n \cdot \frac{2\pi m}{qB}$$

$$\Rightarrow B = n \frac{2\pi m u \cos \theta}{ql}, \quad n = 1, 2, 3, \dots$$



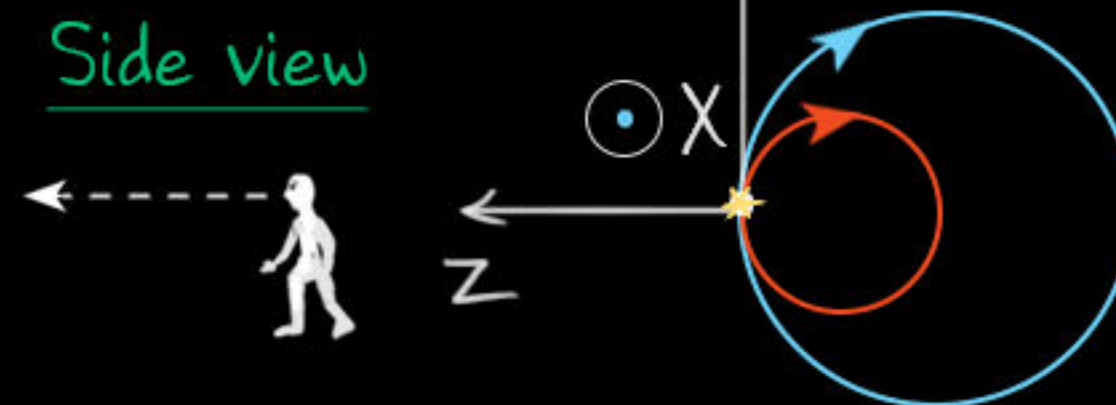
As it returns to O, so particle must have hit the plane on X axis (*)



Radius, time period (and therefore pitch) on return journey is doubled.
Velocity remains the same.

14. In a region of space, a uniform magnetic field of induction B is established everywhere pointing in the positive x -direction. A rigid plane is fixed perpendicular to the x -axis at a certain distance from the origin. A particle of mass m and charge q is projected from the origin with its velocity vector $\vec{u}$ in the x - y plane making an angle θ with the x -axis as shown in the figure. After an elastic collision with the plane, the particle loses half of its charge and then returns to the point of projection crossing the x -axis n times. Find distance of the plane from the origin.

On its way back, after striking the plane



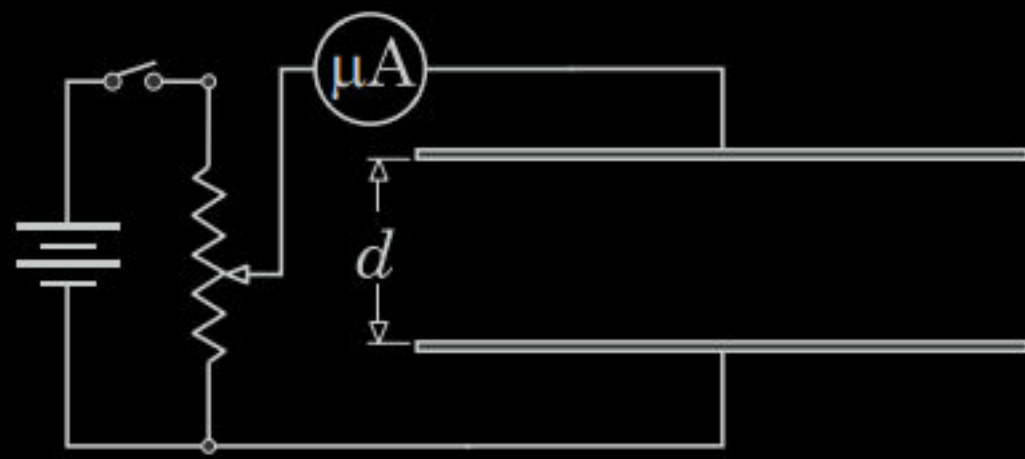
Time period after strike:

$$T = \frac{2\pi m}{q_{1/2} B} \quad (1)$$

To return to O:

$$l = (u \cos \theta) (nT) \quad (2)$$

$$= u \cos \theta \frac{n(4\pi m)}{qB}$$



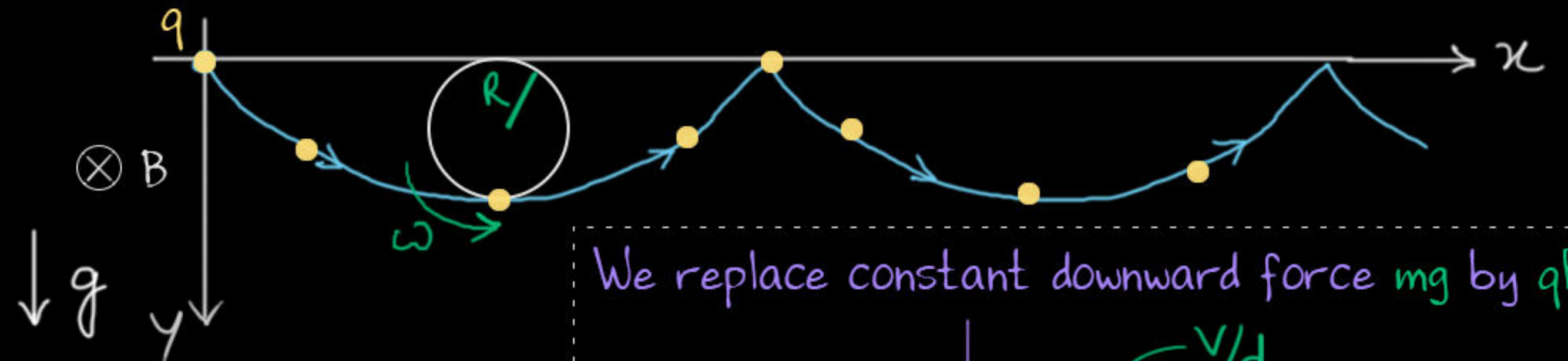
15. Consider the arrangement shown, where distance between the plates of a parallel plate capacitor is d . When a potential difference V is applied between the plates, electrons are emitted from the negative plate with negligible initial velocity. These electrons are attracted towards the positive plate creating a small electric current, which is registered in a micro-ammeter. What minimum magnetic field must be applied perpendicular to the electric field in the capacitor so that the micro-ammeter registers no current? Denote charge and mass of an electron by symbols e and m respectively and neglect effects of gravity.

Irodov 3.392: If we drop a particle in gravitational field where horizontal magnetic field is turned on, it follows a cycloid path.

Corresponding positions of that particle are given by following equations.

$$x = \underbrace{\frac{g}{\omega^2}}_{\text{Radius } R} (\omega t - \sin \omega t)$$

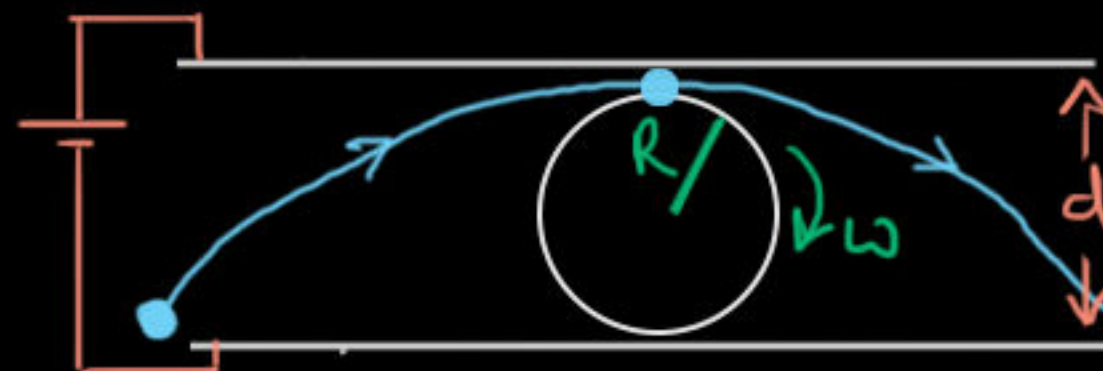
$$y = \frac{g}{\omega^2} (1 - \cos \omega t)$$



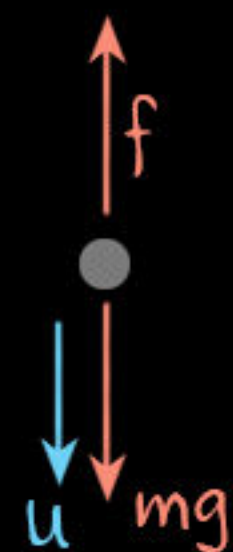
We replace constant downward force mg by qE .

Sol: For electron to miss the top: $d > 2R$

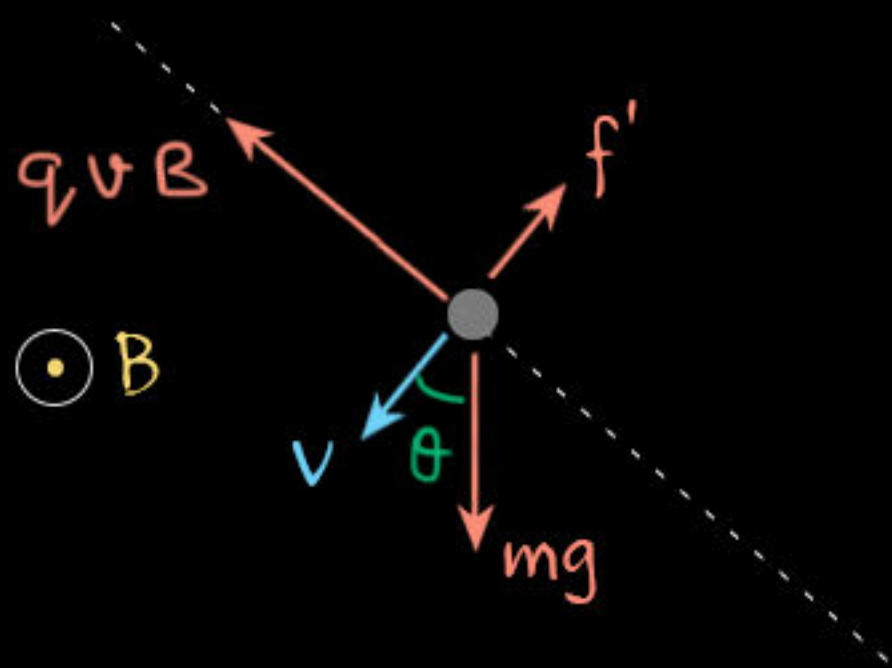
$$\Rightarrow d > \frac{2g}{\omega^2}$$



$$\Rightarrow d > \frac{2m^2 q V}{q^2 B^2 m d} \Rightarrow B > \frac{1}{d} \sqrt{\frac{2mV}{q}}$$



$$P_1 = mgu$$



$$P_2 = mgv \cos \theta$$

Wasted

16. A small ball of mass $m = 1.0 \text{ g}$ and charge $q = 2.0 \text{ } \mu\text{C}$ is falling vertically with terminal velocity $u = 0.1 \text{ m/s}$ in a viscous fluid. After a uniform and constant horizontal magnetic field of induction B is switched on, it is observed that the ball acquires another terminal velocity and power dissipated in the viscous fluid becomes $\eta = 0.5$ times of the power dissipated prior to switching on the magnetic field. At what maximum value of magnetic induction is this possible? Dependence of the viscous forces on the velocity is unknown.

$$\text{Given: } P_2 = \eta P_1$$

$$\Rightarrow \cancel{mg} v \cos \theta = \eta \cancel{mg} u \quad (1)$$

$$\text{Force balance: } mg \sin \theta = qvB \quad (2)$$

$$\Rightarrow mg \sin \theta = q \frac{\eta u}{\cos \theta} B$$

$$\Rightarrow B = \frac{mg \sin \theta \cos \theta}{\eta qu}$$

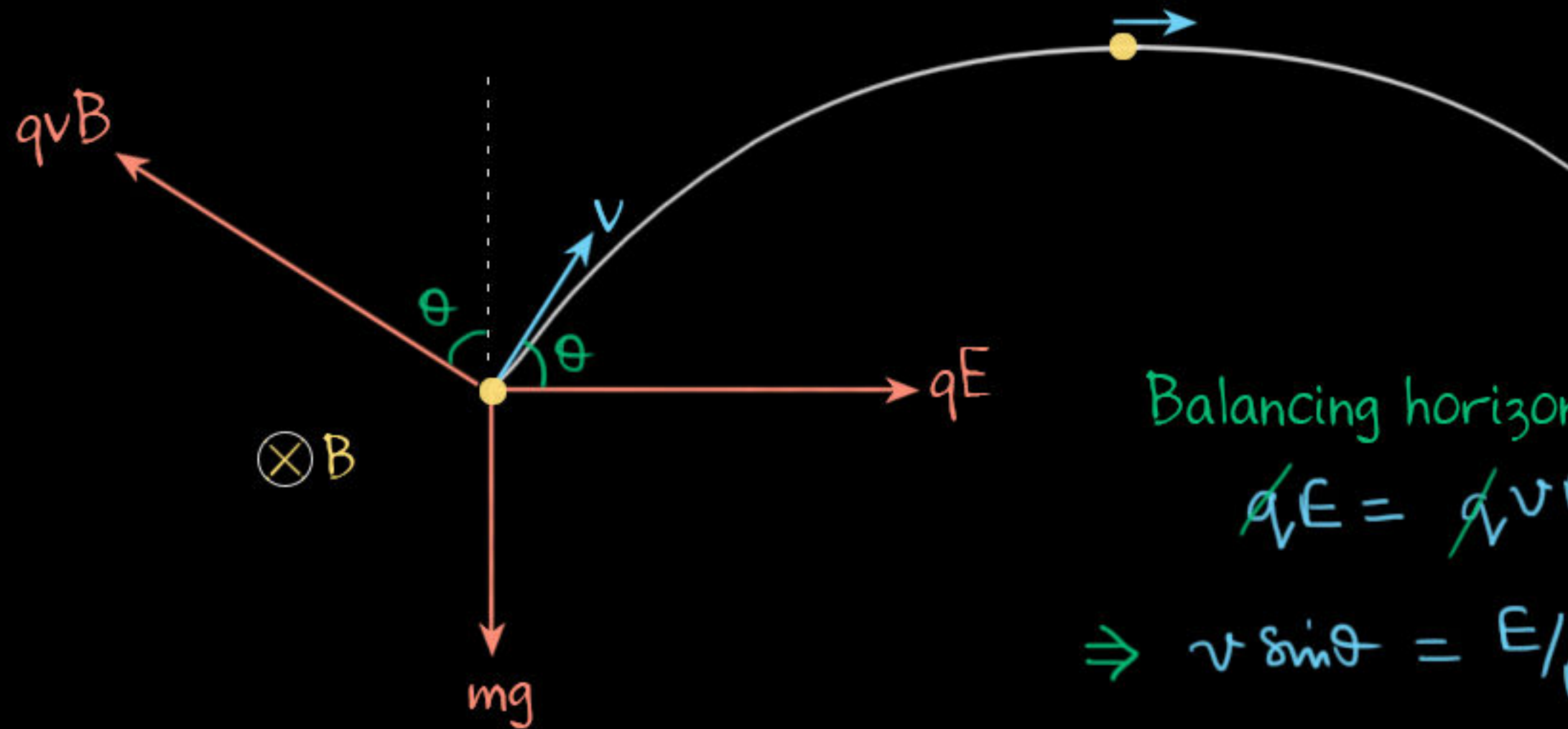
$$\therefore B_{\max} = \frac{1}{2} \frac{mg}{\eta qu} \quad @ \theta = \pi/4$$

As E and B suddenly vanish, particle will free fall as a projectile with minimum velocity at top.

$$T = \frac{v \sin \theta}{g} \quad (1)$$

17. In a region of space, a uniform electric field of strength E , a uniform magnetic field of induction B and a uniform gravitational field of strength g coexist in mutually perpendicular directions. A charge particle is moving with a uniform velocity in this region. Once the particle leaves this region and begins moving into another region where there is only gravitational field of strength g , its speed begins to decrease. How long after departure from the first region, will the velocity of the particle become minimum?

∴ Particle must be going up initially.



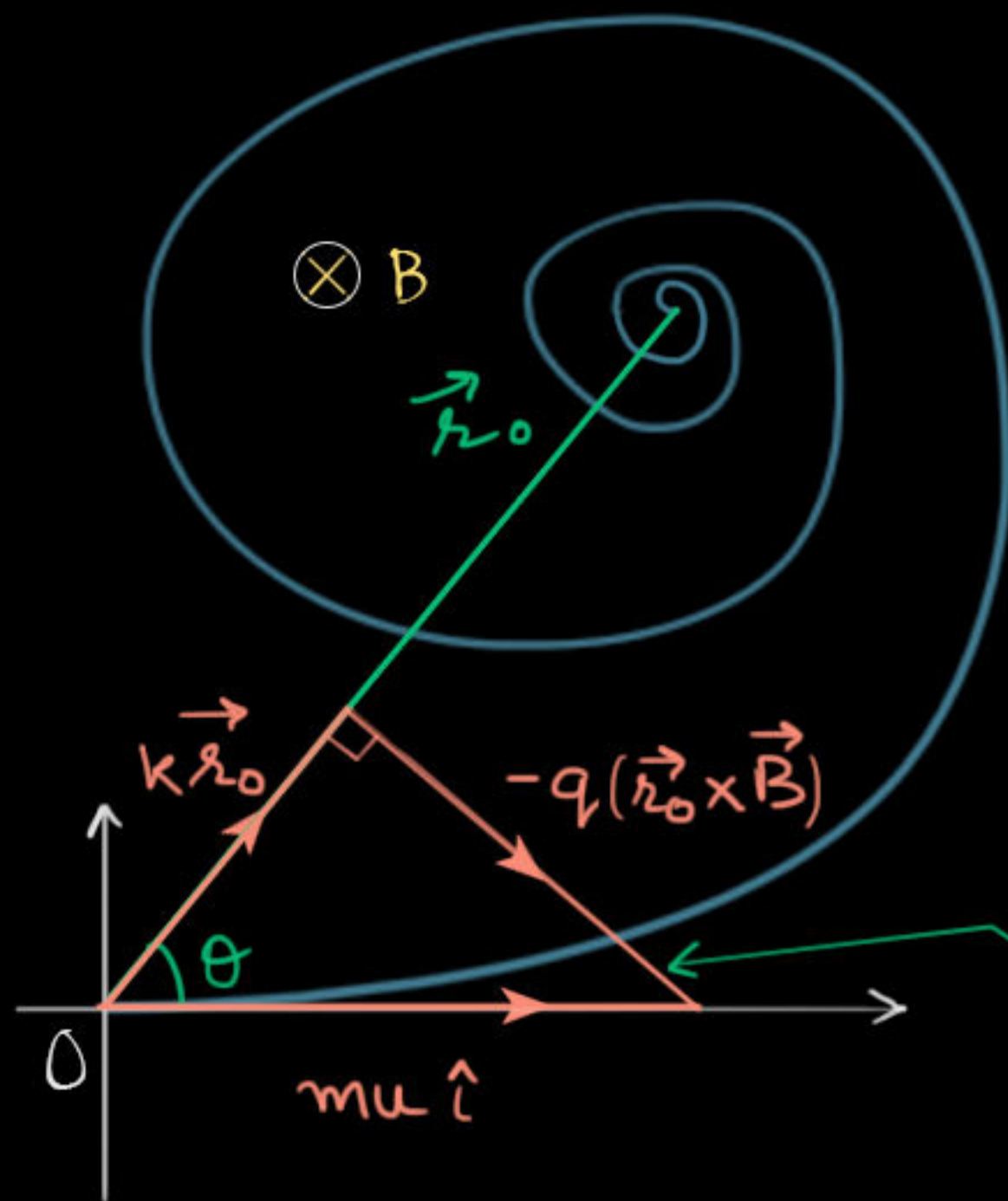
Balancing horizontal forces:

$$qE = qvB \sin \theta$$

$$\Rightarrow v \sin \theta = E/B \quad (2)$$

From 1,2:

$$T = \frac{E}{Bg}$$



$$\tan \theta = \frac{qBr_0}{k r_0} = \frac{qB}{k}$$

⇓

1

18. In a region of space, a uniform magnetic field of induction $\vec{B} = -B\hat{k}$ is established. A particle moving with velocity $\vec{v}$ in this region experiences a viscous drag $\vec{f} = -k\vec{v}$, where k is a positive constant. A particle of mass m and charge q is projected with velocity $\vec{u} = u\hat{i}$ in this region. Find its displacement when it practically stops.

Reference: MCQ 13,14,15

$$m \frac{d\vec{v}}{dt} = -k\vec{v} + q(\vec{v} \times \vec{B})$$

$$\Rightarrow m \int_{u\hat{i}}^0 d\vec{v} = -k \int_0^{\vec{r}_0} \vec{v} dt + q \left[\left(\int_0^{\vec{r}_0} \vec{v} dt \right) \times \vec{B} \right]$$

$$\Rightarrow -mu\hat{i} = -k\vec{r}_0 + q(\vec{r}_0 \times \vec{B})$$

$$mu\hat{i} = k\vec{r}_0 - q(\vec{r}_0 \times \vec{B})$$

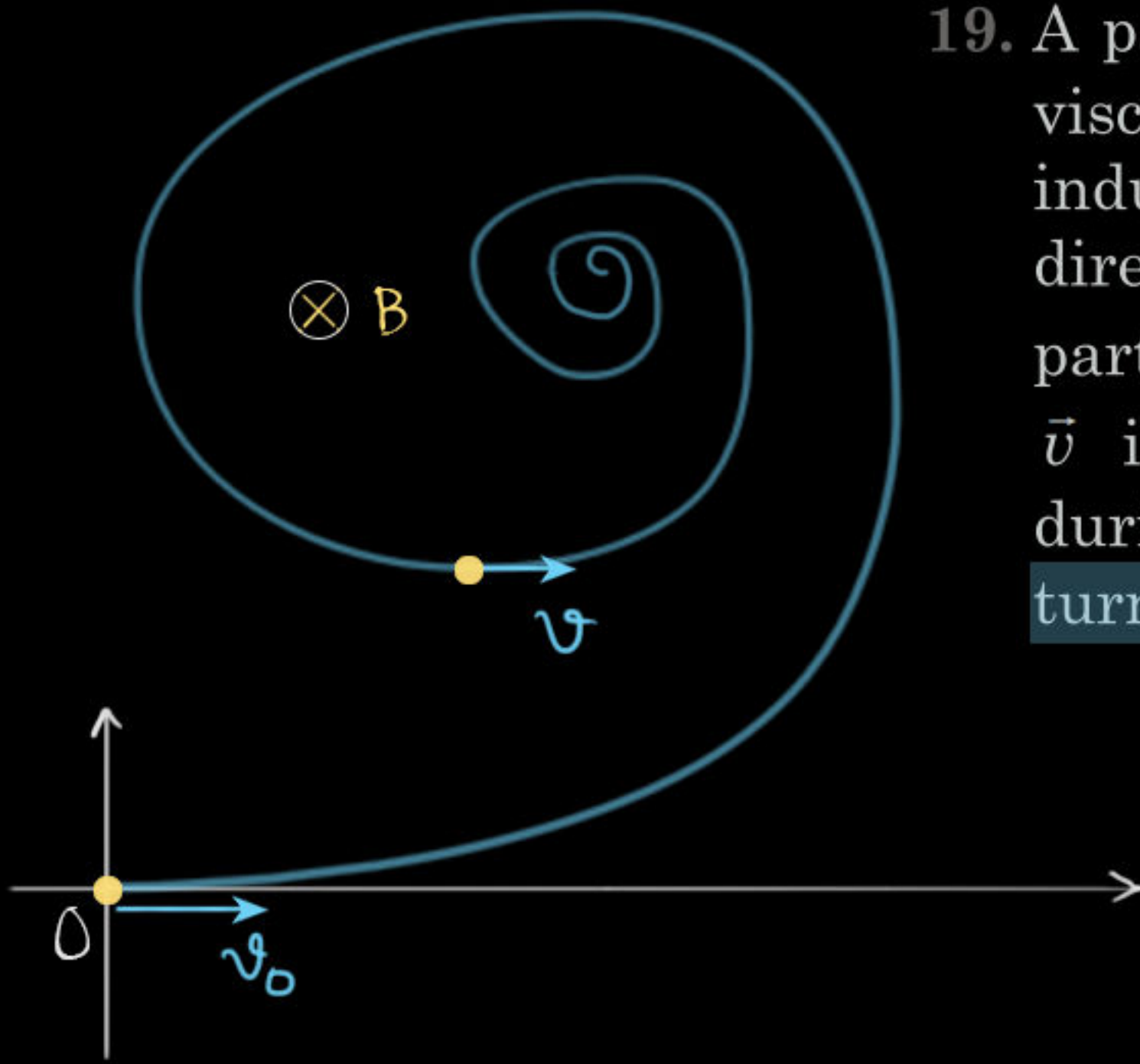
Orthogonal

Magnitudes: $(mu)^2 = (kr_0)^2 + (qBr_0)^2 \Rightarrow r_0 = \frac{mu}{\sqrt{k^2 + q^2 B^2}}$ 2

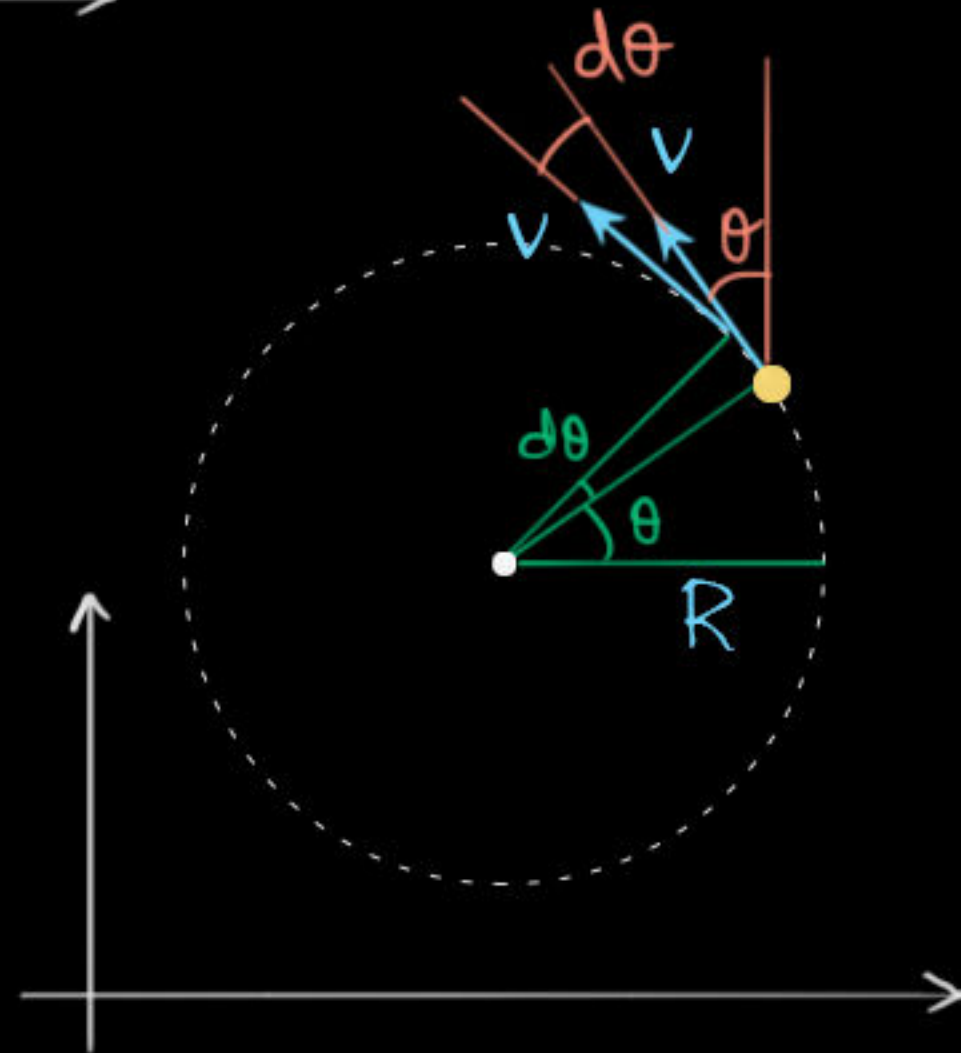
$$\therefore \vec{r}_0 = r_0 \cos \theta \hat{i} + r_0 \sin \theta \hat{j} \xrightarrow{1,2} \frac{mu k}{k^2 + q^2 B^2} \hat{i} + \frac{mu q B}{k^2 + q^2 B^2} \hat{j}$$

by Ankit Singhvi

Youtube / Arihant Senior



19. A particle of mass m and charge q is projected with a velocity v_0 in a viscous medium, where a uniform and constant magnetic field of induction B exists everywhere in a direction perpendicular to the direction of projection of the particle. The force of viscous drag on the particle is given by the law $\vec{f} = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is velocity of the particle. Find distance travelled by the particle during a time interval from the instant of projection until velocity vector turns by 2π radians.



In its spiral path, at a particular moment, let's say its velocity is v , RoC R , and it makes θ with the horizontal.

We know, $\frac{d\theta}{dt} = \omega = \frac{qB}{m}$ (a constant)

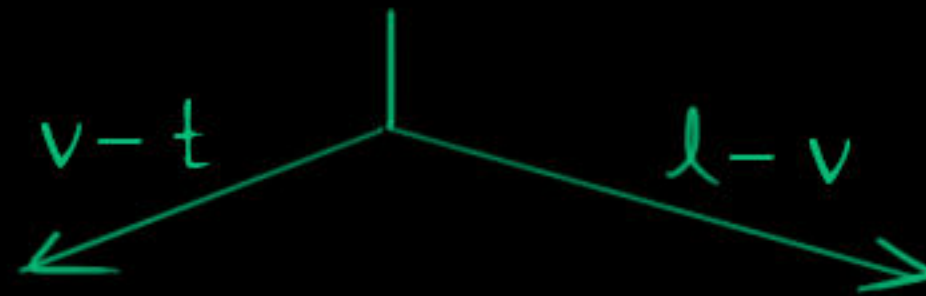
$\therefore \vec{v}$ rotates with constant ω , even though RoC varies continually.

Sol: To turn 2π , time taken,

$$t = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} \quad (1)$$

Relating tangential speed and acceleration (along the path)

$$ma = -bv$$



$$m \frac{dv}{dt} = -bv$$

$$\Rightarrow \int_{v_0}^v \frac{dv}{v} = -\frac{b}{m} \int_0^t dt$$

$$\Rightarrow v = v_0 e^{-bt/m} \quad (2)$$

$$m \cancel{v} \frac{dv}{dx} = -b \cancel{v}$$

$$\Rightarrow m \int_{v_0}^v dv = -b \int_0^{l_0} dx$$

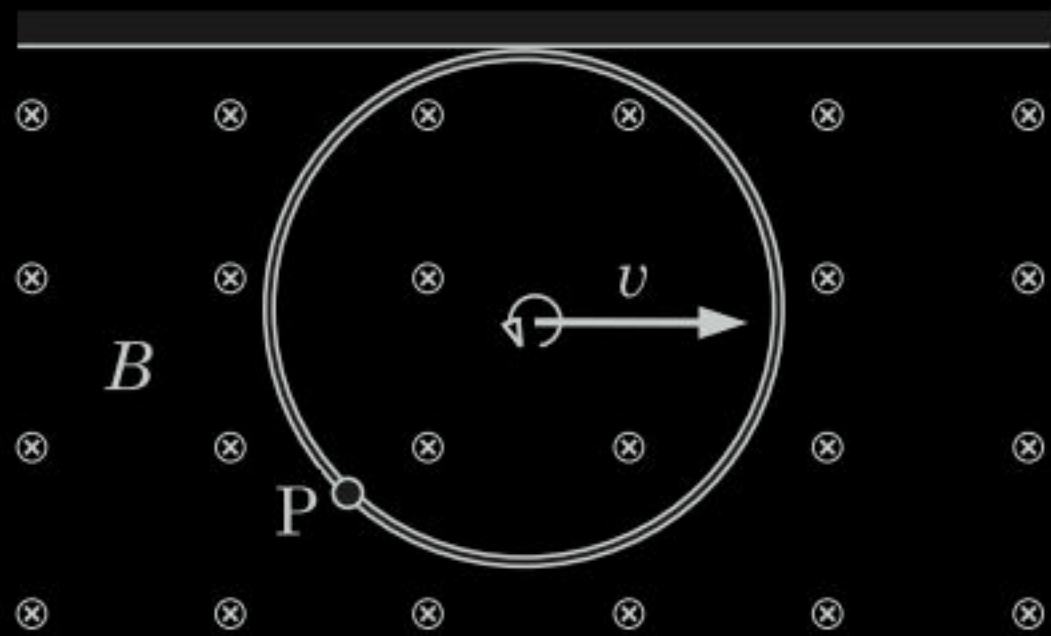
$$\Rightarrow l_0 = \frac{m(v_0 - v)}{b}$$

$\downarrow 1,2$

$$= \frac{m}{b} (v_0 - v_0 e^{-\frac{b}{m} \frac{2\pi m}{qB}})$$

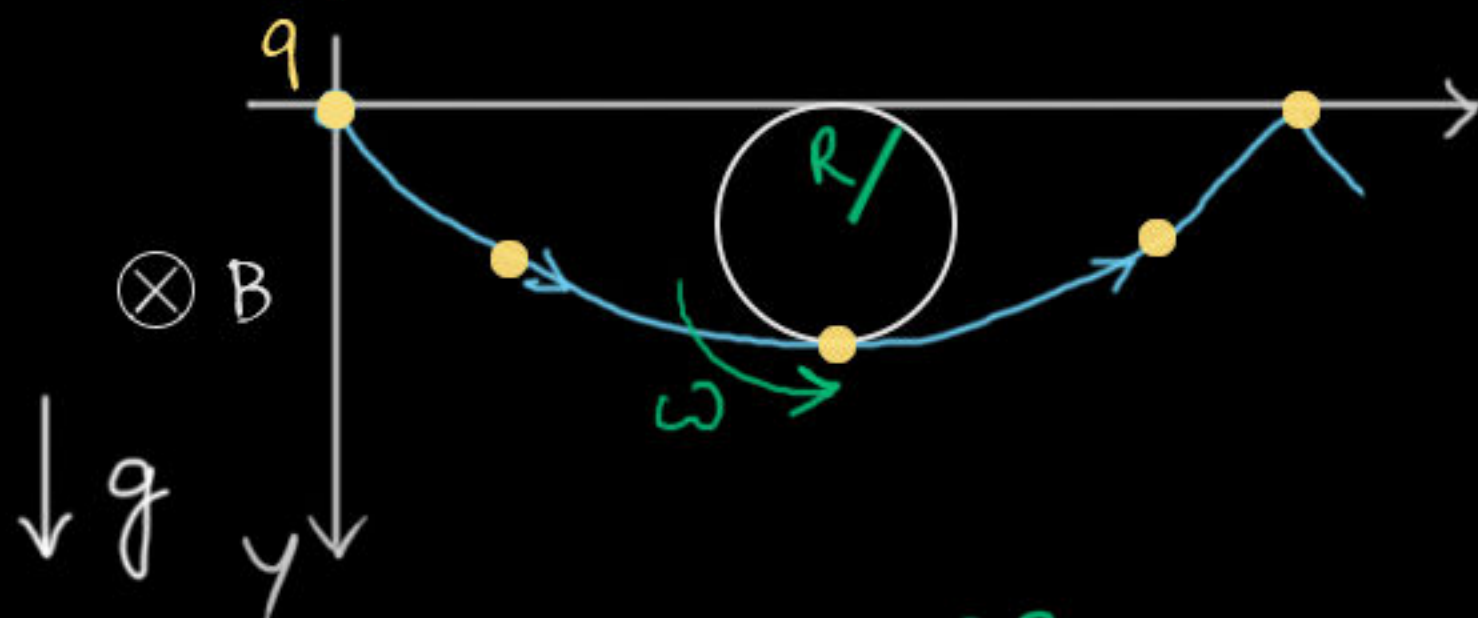
$$= \frac{mv_0}{b} (1 - e^{-2\pi b/qB}) //$$

Youtube / Arihant Senior



20. A rigid ring is made to roll along the ceiling of a room, where a horizontal uniform and static magnetic field of induction B exists perpendicular to the plane of the ring. Velocity of the centre of the ring is constant and its modulus is v . A charge particle P of mass m is fixed on the ring. What should the charge q on the particle and radius r of the ring be so that there is no force of interaction between the ring and the particle?

Prerequisite: Irodov 3.392, BYU 15



$$x = \frac{g}{\omega^2} (\omega t - \sin \omega t)$$

Radius R

$$y = \frac{g}{\omega^2} (1 - \cos \omega t)$$

Sol: If P follows a natural cycloid path (of given R and ω) with Electrostatic and magnetic forces alone, then reaction force from ring will be zero.

$$R = \frac{g}{\omega^2} \quad \frac{v}{R}$$

$$\Rightarrow R = \frac{g R^2}{v^2}$$

$$\Rightarrow R = v^2 / g$$

$$\omega = \frac{qB}{m} = \frac{v}{R}$$

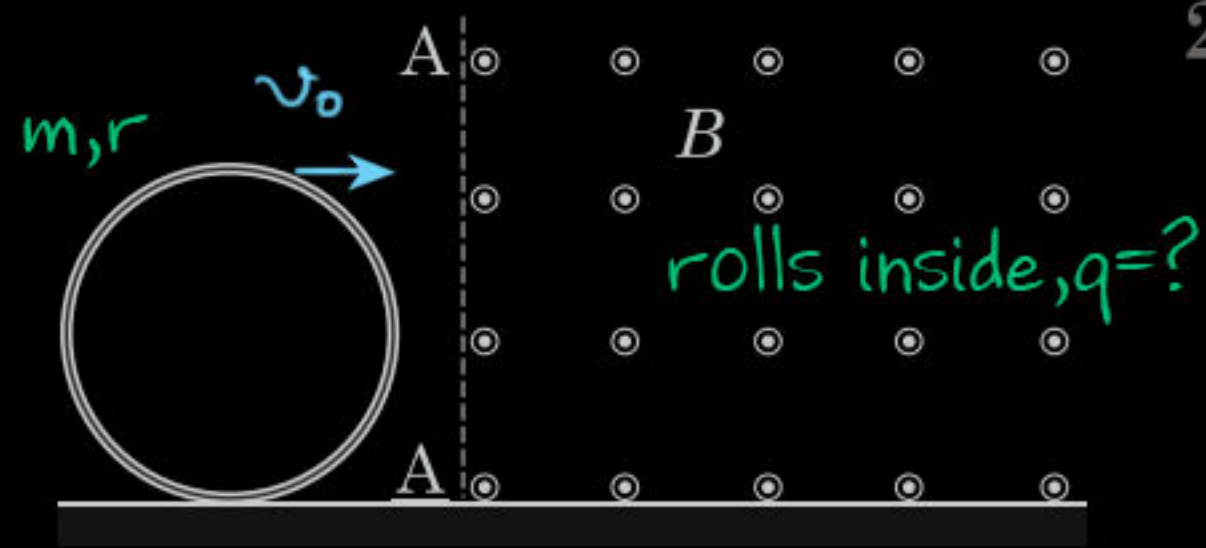
$$\Rightarrow q = \frac{mv}{BR}$$

$$= \frac{mv}{B \frac{v^2}{g}}$$

$$= \frac{mg}{Bv}$$

by Ankit Singhvi

Youtube / Arihant Senior



Hint: Irodov 3.311

Sol:

Energy conservation:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

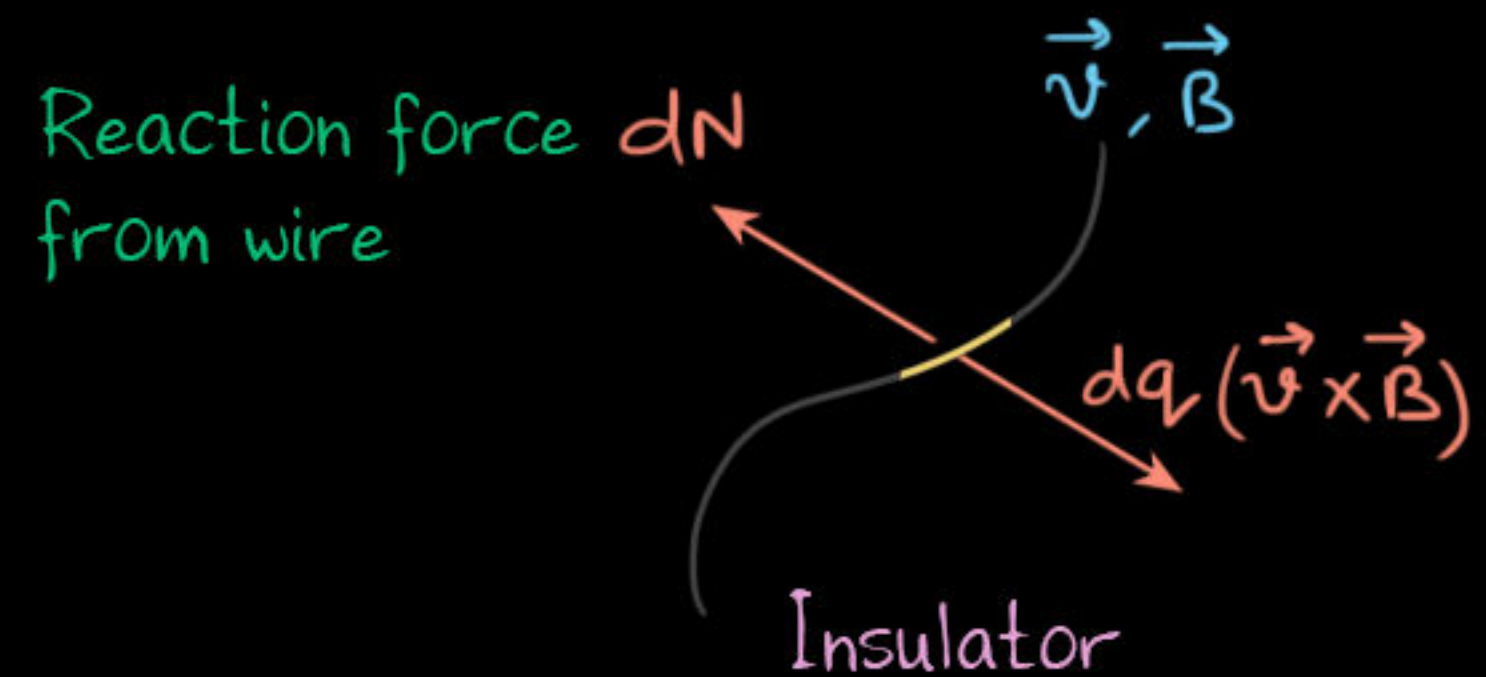
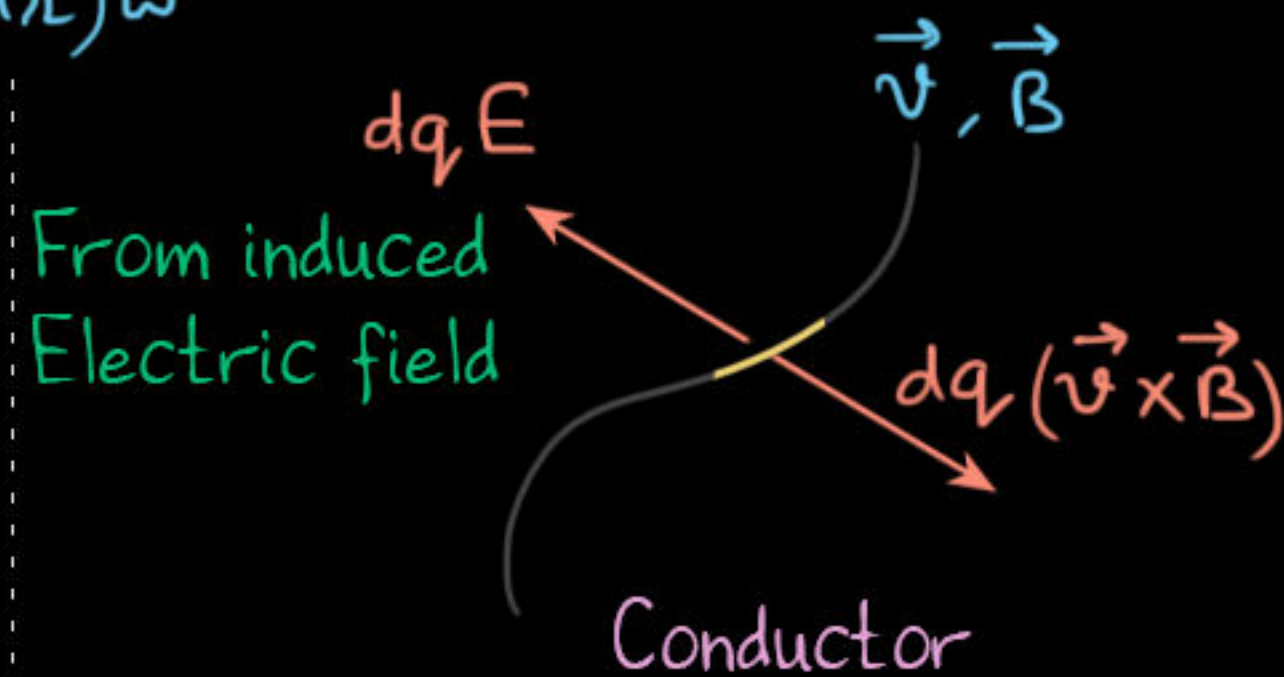
$$\frac{1}{2} m v_0^2 = \frac{1}{2} m (v_0)^2 + \frac{1}{2} (m r^2) \omega^2$$

$$\Rightarrow \omega = v_0 / \sqrt{2} r \quad (1)$$

21. A ring of radius r made of an insulating material has mass m and carries uniform charge. Initially it rests on a frictionless horizontal tabletop with its plane vertical. A uniform horizontal magnetic field of induction B pointing everywhere parallel to the axis of the ring is established in the region to the right of a vertical plane AA as shown in the figure. The ring is pushed forward to acquire a velocity v_0 without any rotation. What should the charge on the ring be so that it starts rolling on the tabletop after completely entering the region of the magnetic field?

Concept: Lets take two similarly shaped wires. One is conductor, another is insulator with distributed charge.

For a small charge dq on both at similar locations, lets write forces as they are moving with v in an uniform B .



As F_{net} on dq is zero in both cases: $dN = dqE$

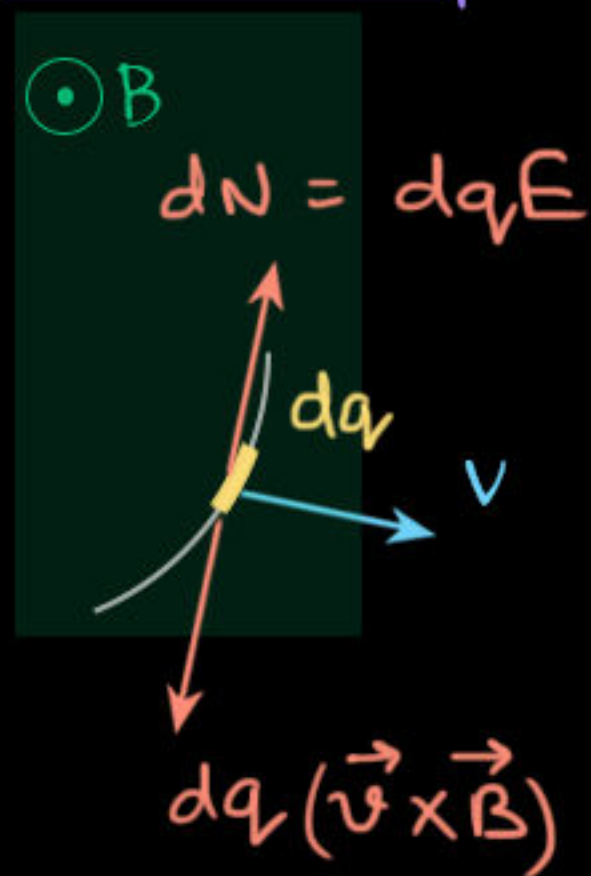
by Ankit Singhvi

Youtube / Arihant Senior

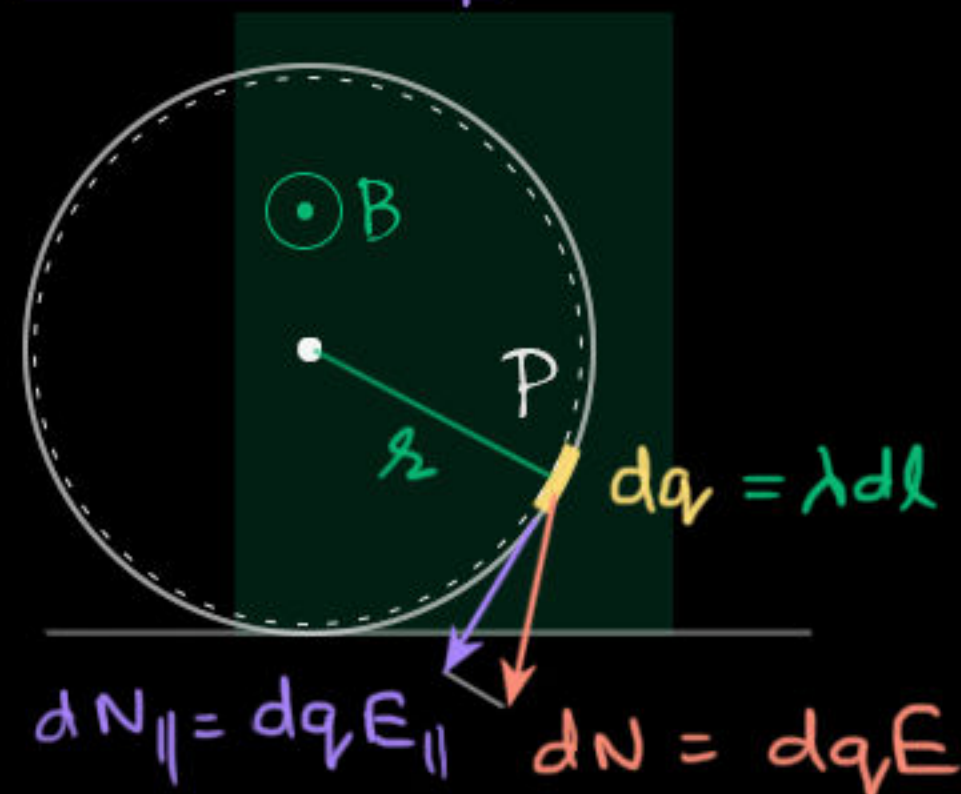
Sol: At any random moment, let's consider a circular path overlapping the loop, and a small element dq on it.

Mathematically, we can now replace dN with dqE in our problem.

Forces on dq



Force on loop



Writing torque on the loop:

$$d\tau = r dN_{\parallel} = r dq E_{\parallel} = r \lambda dl E_{\parallel} = r \lambda \vec{E} \cdot d\vec{l}$$

$$\Rightarrow \tau = r \lambda \oint \vec{E} \cdot d\vec{l} = r \lambda \frac{d\phi}{dt} \text{ From (2)}$$

$$\Rightarrow \int \tau dt = r \lambda \int d\phi$$

$$\underbrace{I \omega}_{m r^2 \cdot \frac{v_0}{\sqrt{2} r}} = r \underbrace{\frac{Q}{2\pi r}}_{\frac{Q B \pi r^2}{2\pi r}}$$

From (1)

$$\Rightarrow \frac{m r^2 v_0}{\sqrt{2} r} = \frac{r Q B \pi r^2}{2\pi r} \Rightarrow Q = \frac{\sqrt{2} m v_0}{B r}$$

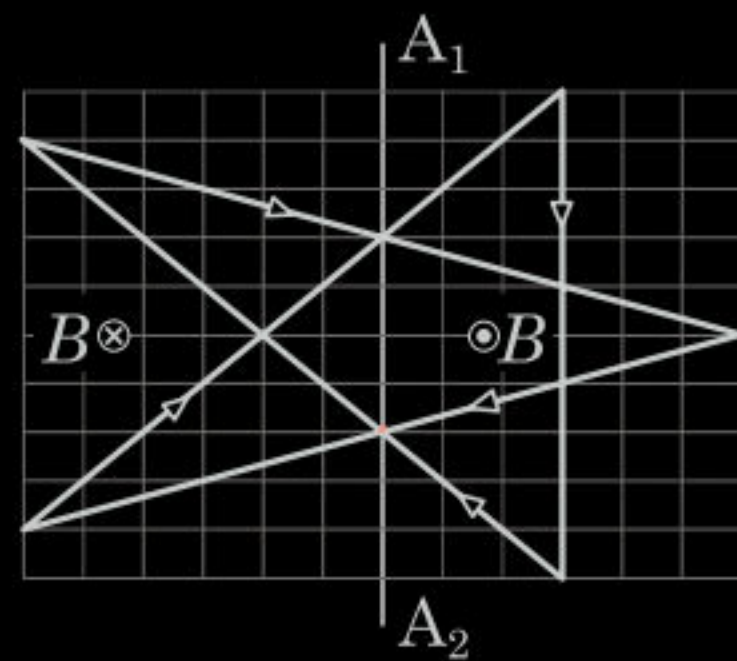
Note: Actual electric field does NOT exist or get induced. We have solved this by mathematically making the mechanical force (dN) analogous to electrostatic force (dqE).

Again, mathematically we know: Traveling along a closed path, induced emf is:

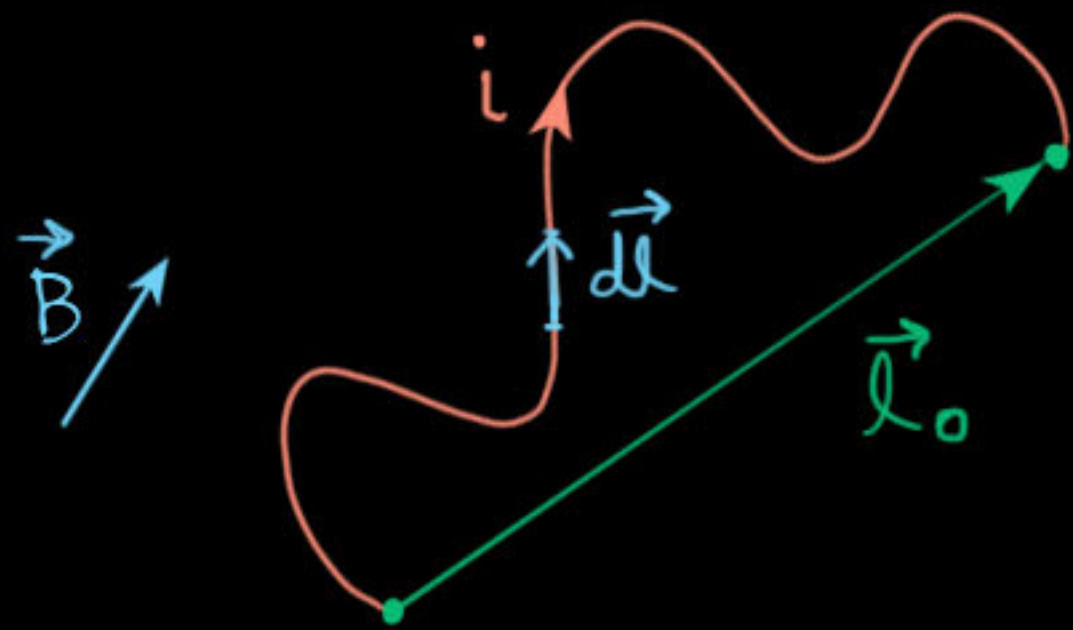
$$\oint \vec{E} \cdot d\vec{l} = \frac{d\phi}{dt} \text{ (2)}$$

by Ankit Singhvi

Youtube / Arihant Senior



22. An insulated conducting rod in shape of a five-point star like planar structure carries a current I . In the regions to the left and to the right of a line A_1A_2 , uniform magnetic fields each of induction B exist perpendicularly into and out of the plane of the star as shown in the figure. If length of a side of a unit cell of the grid shown is l , find force of interaction between the current and the magnetic field.

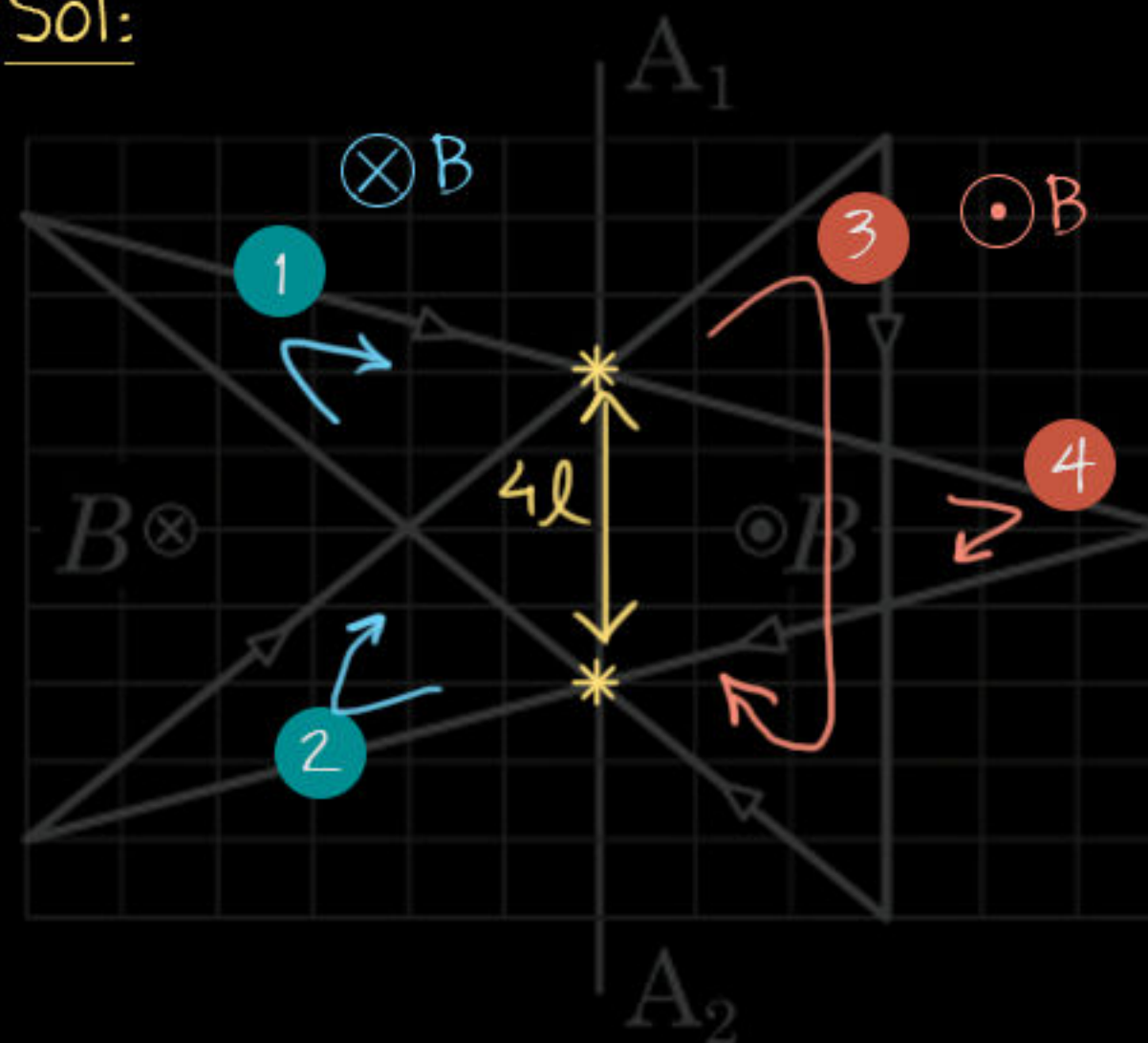


If we place a randomly shaped current carrying wire in a uniform magnetic field,

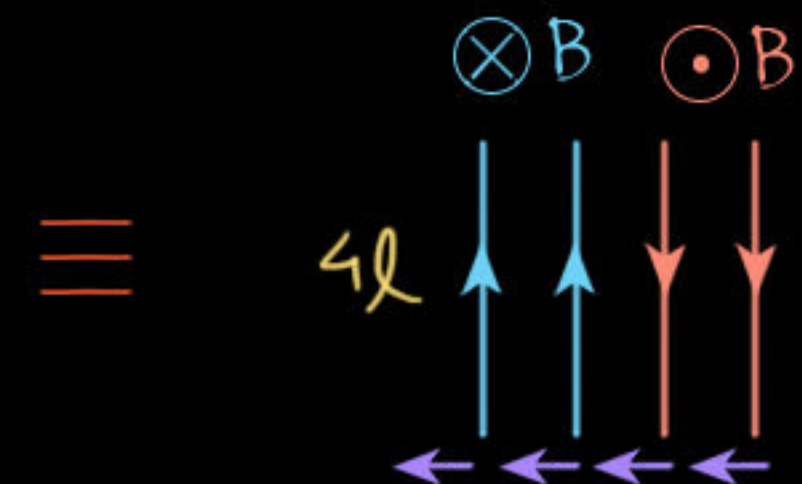
Force on that piece is given as:

$$\vec{F} = i \int (d\vec{l} \times \vec{B}) = i \left[\left(\int d\vec{l} \right) \times \vec{B} \right] = i (\vec{l}_0 \times \vec{B})$$

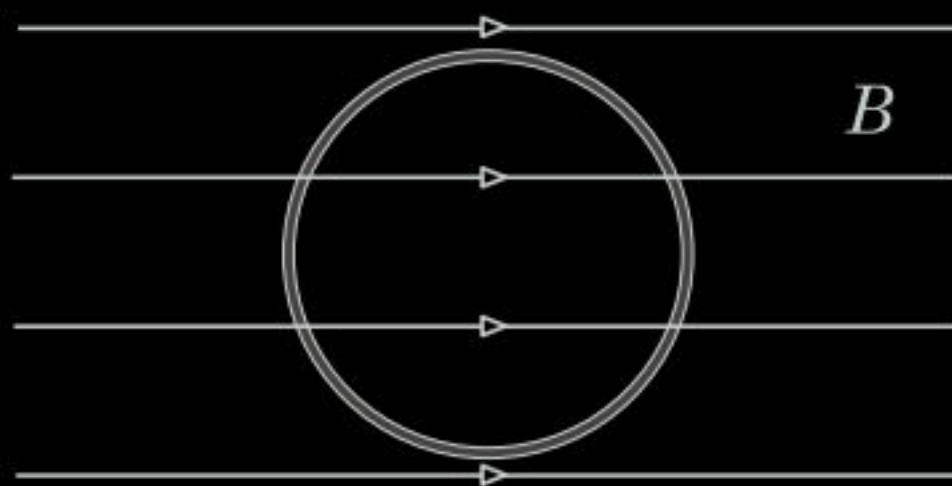
Sol:



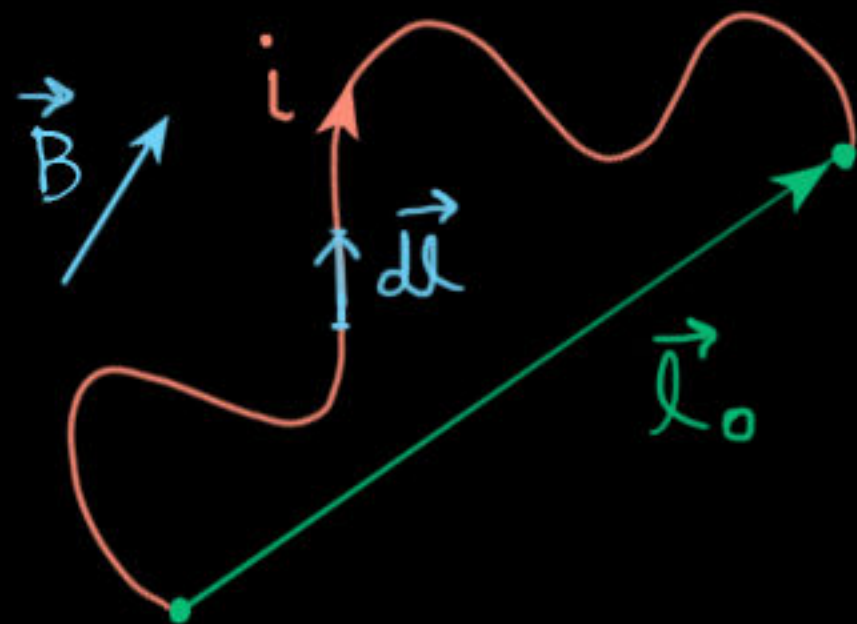
Force on each branch is:
 $Bi(4l)$ Towards left



$$\therefore F_{\text{net}} = 4 \times (Bi4l) = 16 Bil_{//}$$



23. A uniform rod of resistance R is bent into a ring of radius r . The ring is placed in a uniform magnetic field of induction B directed parallel to its plane. You have an ideal battery of electromotive force $\mathcal{E}$. How will you connect this battery between two points on the ring to obtain maximum force on the ring and how much is this maximum force $F_{\max}$?



Force on a random wire in uniform field B is given by:

$$\vec{F} = i(\vec{l}_0 \times \vec{B})$$

∴ For maximum force $\vec{l}_0 \perp \vec{B}$

In our case it means that battery terminals should be perpendicular to field as shown.

Lets say terminals subtend 2θ at center.
Battery emf is e .

$$F = Bi_1(2r \sin\theta) + Bi_2(2r \sin\theta)$$

$$\propto \sin\theta (i_1 + i_2)$$

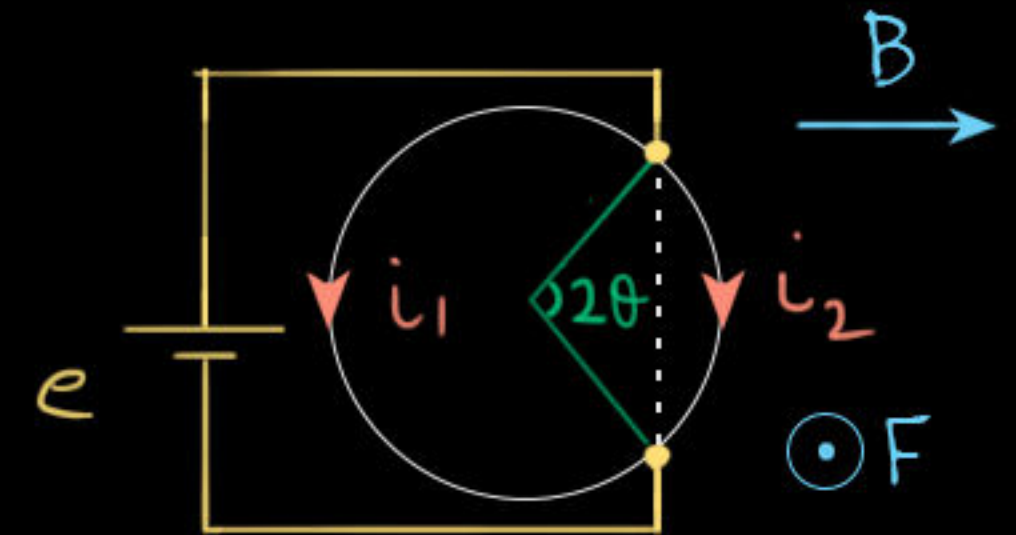
$$\propto \sin\theta \left(\frac{e}{R_1} + \frac{e}{R_2} \right)$$

$$\propto \sin\theta \left(\frac{1}{\pi - \theta} + \frac{1}{\theta} \right)$$

$$R_1 = \frac{2\pi - 2\theta}{2\pi} R$$

$$R_2 = \frac{2\theta}{2\pi} R$$

by Ankit Singhvi



$$\Rightarrow F \propto \frac{\sin\theta}{\theta(\pi - \theta)}$$

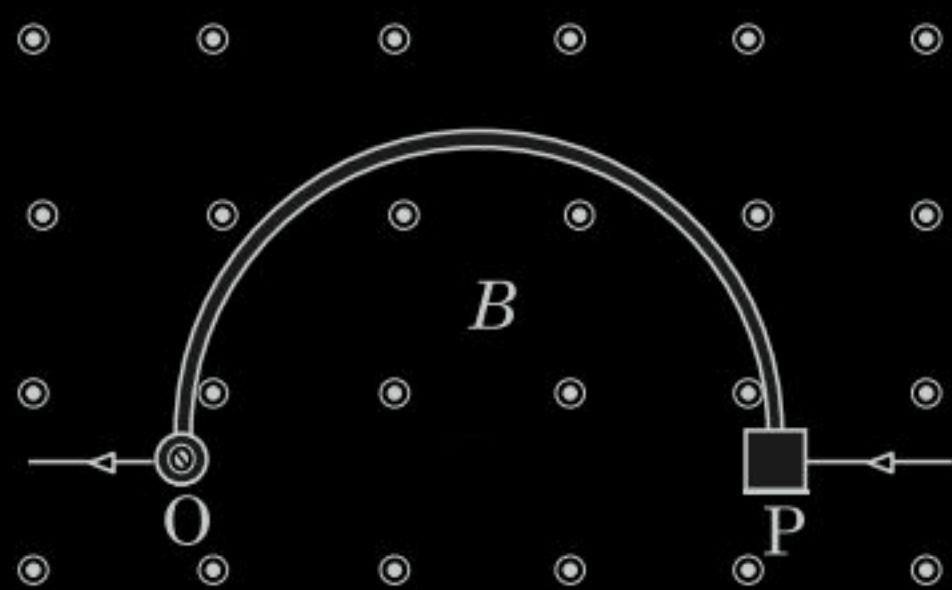
Out of scope maths

$$F_{\max} \text{ @ } \theta = \pi/2$$

$$= 2 \times \left[B \frac{e}{R_{1/2}} 2r \right]$$

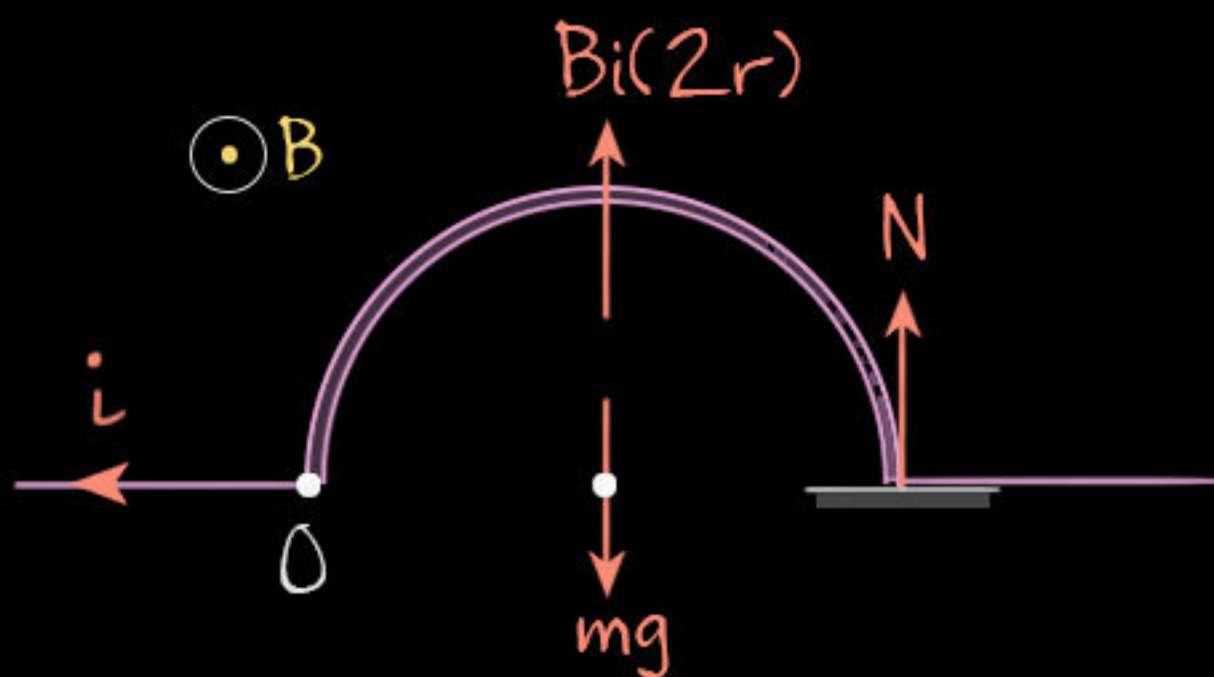
$$= \frac{8Be r}{R}$$

Youtube / Arihant Senior



24. A schematic model of a **current limiter** consists of a conducting half ring of radius r and mass m , one end O of which is hinged while the **other rests on a fixed metallic support P**. The ring can rotate in a vertical plane without friction in presence of a uniform horizontal magnetic field of induction B pointing perpendicular to the plane of the ring. The ring is connected in series in a branch of a circuit through the hinge O and the metallic contact P . **Find the minimum current, at which the limiter opens the circuit.** The acceleration of free fall is g .

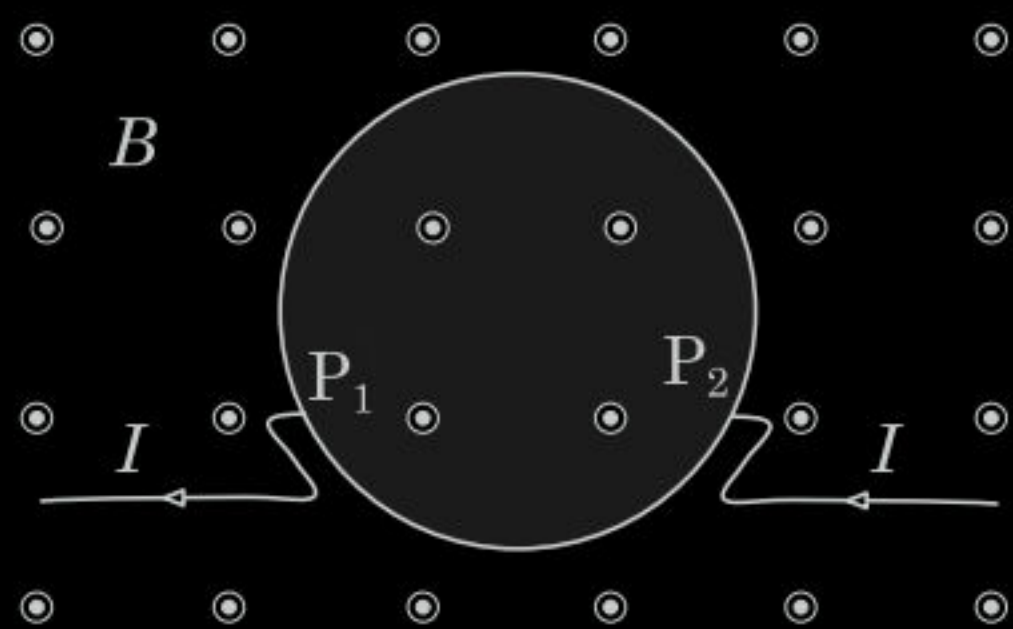
When circuit is open:



$$\odot \text{CCW } \tau_O = [Bi(2r) - mg]r + \cancel{N(2r)} > 0$$

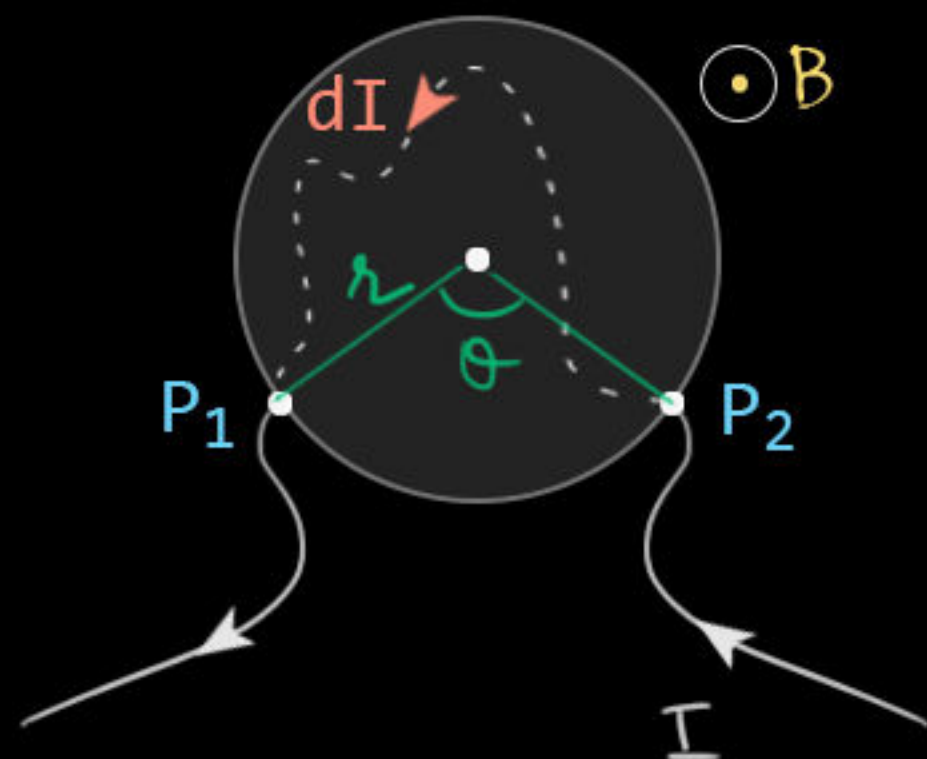
$$\Rightarrow Bi(2r) > mg$$

$$\Rightarrow i > \frac{mg}{2Br} //$$



25. A thin uniform conducting disc of radius r and mass m rests on a horizontal tabletop where a uniform magnetic field of induction B pointing vertically upwards exists. Coefficient of friction between the disc and the tabletop is μ . Two points P_1 and P_2 on the rim of the disc are connected with two flexible wires, which facilitate a current to flow through the disc. The contact points P_1 and P_2 subtend an angle θ at the centre of the disc. How much minimum current must flow through the disc so that it begins to slide? Acceleration of free fall is g .

Inside the conductor, let's say dI current flows via a random path from P_2 to P_1 due to some dq .

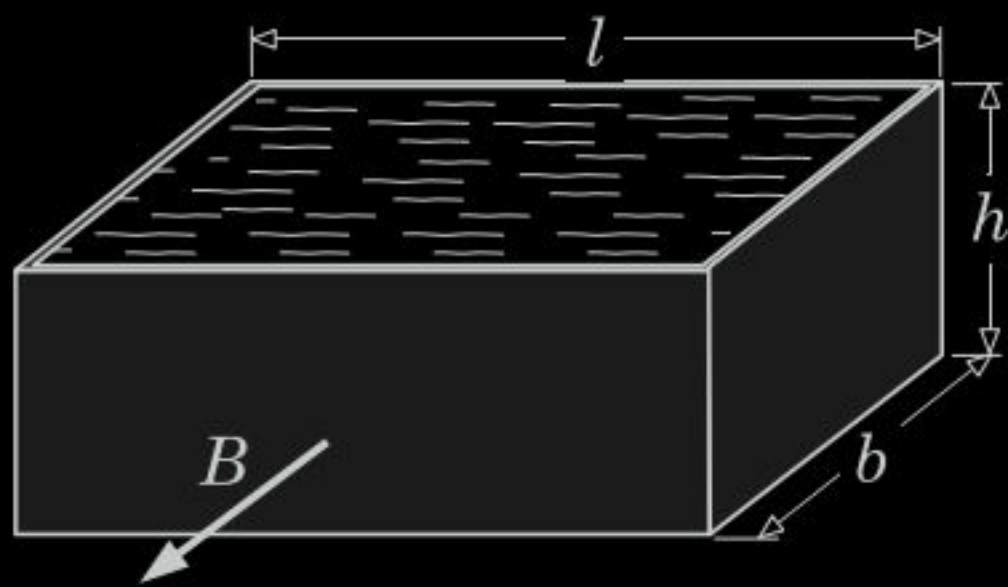


$$\therefore dF = B dI (2r \sin \theta/2)$$

$$\Rightarrow F = 2Br \sin \theta/2 \int dI$$

$$\Rightarrow F = 2Br \sin \theta/2 I > \mu mg$$

$$\Rightarrow I > \frac{\mu mg}{2Br \sin \theta/2} //$$



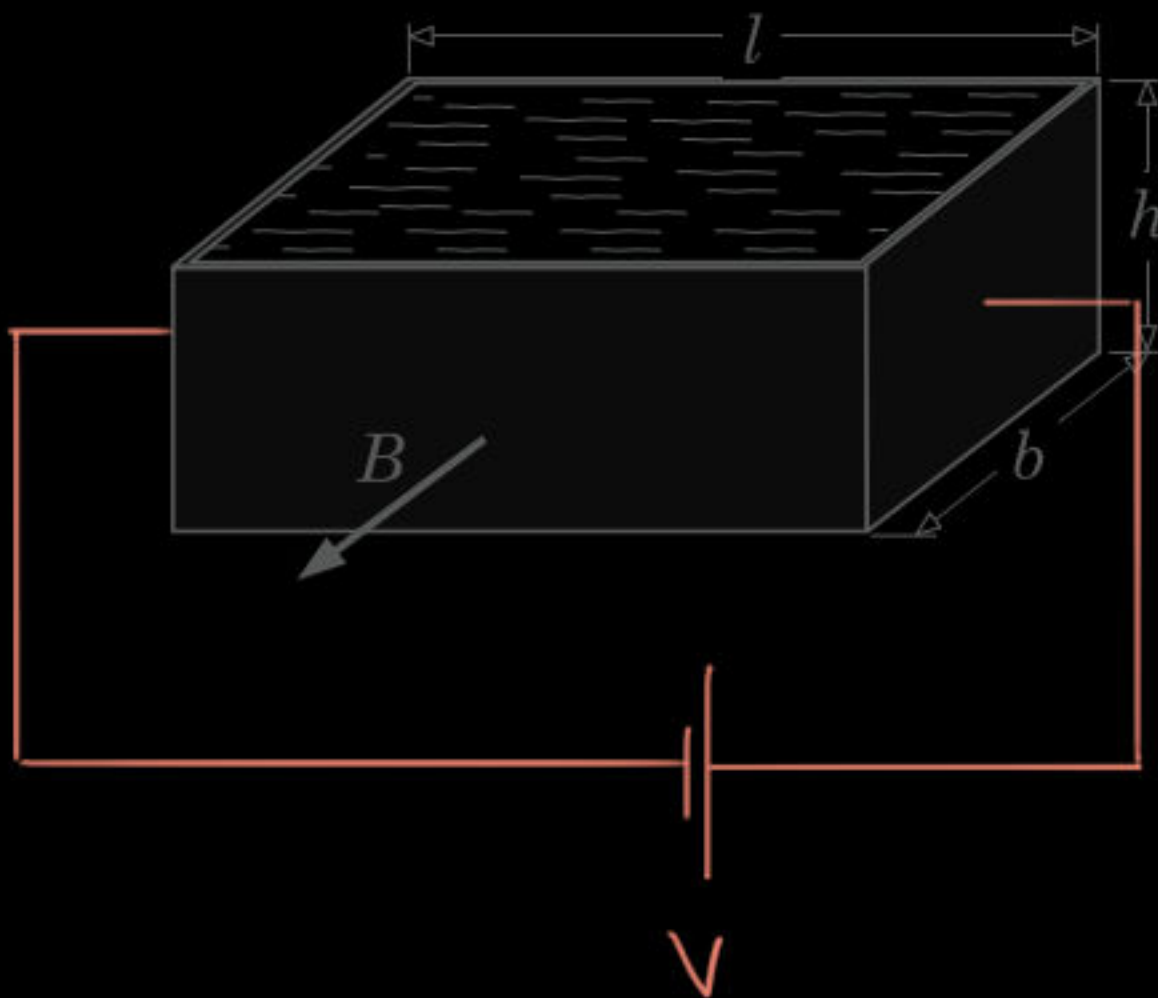
26. A conducting fluid of mass density ρ_m and electrical resistivity ρ_e is kept in an insulating vessel of dimensions $l \times b \times h$. The vessel is placed on a horizontal floor where a uniform horizontal magnetic field of induction B is established perpendicular to the face $l \times h$ as shown in the figure. How much potential difference must be applied across the fluid between the side faces designated by dimensions $b \times h$ so that the fluid pressure at the bottom of the vessel vanishes? Acceleration of free fall is g .

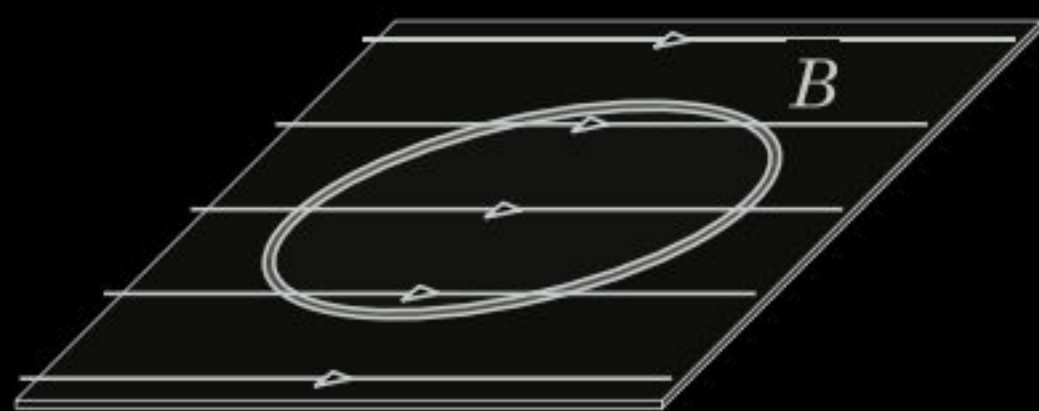
Conducting fluid is similar to conducting rod.

$$\therefore Bil = mg$$

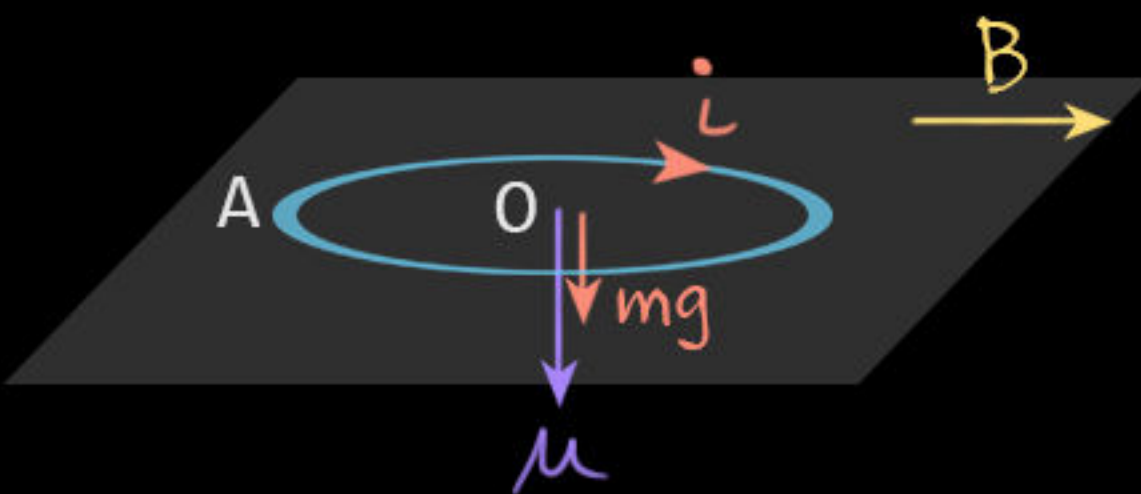
$$\Rightarrow B \cdot \frac{V}{\left(\frac{\rho_e l}{bh}\right)} \cdot l = \rho_m (l \cancel{b} \cancel{h}) g$$

$$\Rightarrow V = \frac{\rho_m \rho_e l g}{B} //$$





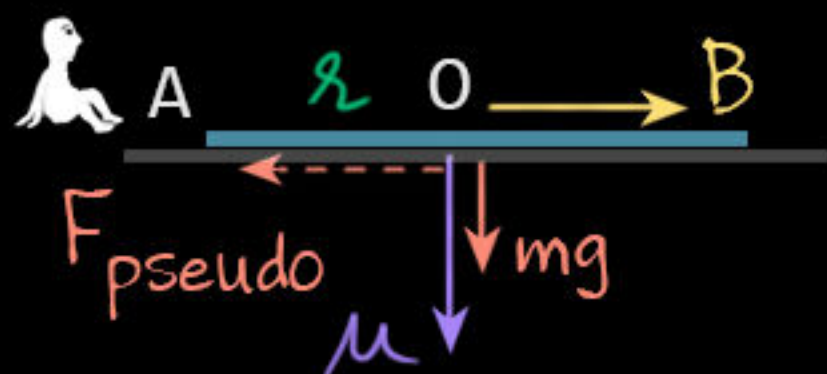
27. A uniform rigid ring of radius r and unknown mass is placed on a nonconducting frictionless horizontal tabletop, where exists a uniform horizontal magnetic field of induction B as shown in the figure. A current I that is double of the minimum current required to flip over the ring is made to circulate in the coil. Find force of normal reaction from the tabletop on the ring immediately after the current is switched on.



∴ From our observations, if ring manages to lift initially, later pseudo force will help in further tilt.
So, for ring to just flip initial torque overcome is sufficient. i.e: $\mu_0 B = mgR$ 2

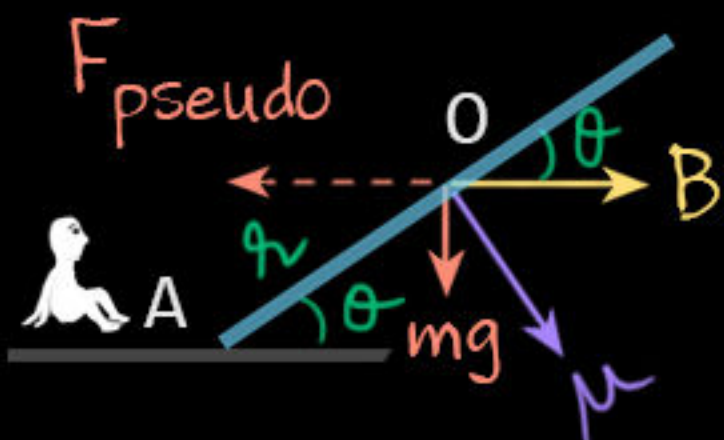
$$i_0 (\pi r^2)$$

w.r.t A



$$\begin{aligned} \vec{\mu} \times \vec{B} &= \mu B \odot \\ \vec{r} \times m\vec{g} &= mgR \otimes \end{aligned}$$

Couple torque
Axis immaterial



$$\begin{aligned} \vec{\mu} \times \vec{B} &= \mu B \sin\left(\frac{\pi}{2} - \theta\right) = \mu B \cos\theta \odot \\ \vec{r} \times m\vec{g} &= mgR \cos\theta \otimes \\ \vec{r} \times \vec{F}_{pseudo} &= F_{pseudo} \cdot R \sin\theta \odot \end{aligned}$$

When i_0 is doubled, μ doubles.

∴ Initial torque on ring will now be:

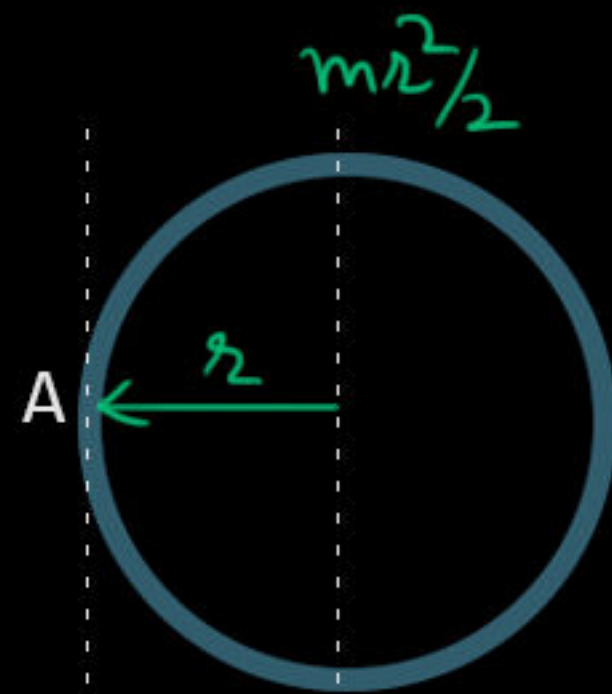
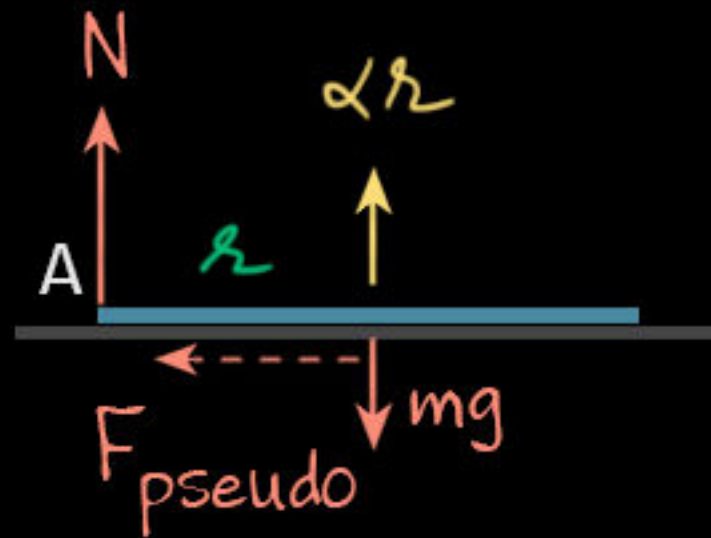
$$\begin{aligned} \tau_A &= (2\mu_0)B - mgR \odot \\ &= 2mgr - mgr \odot \\ &= mgr \odot \end{aligned}$$

Youtube / Arihant Senior

by Ankit Singhvi

w.r.t A

$$\tau_A = mgr \odot$$



$$I_A = \frac{mr^2}{2} + mr^2 = \frac{3mr^2}{2}$$

$$\tau_A = I_A \alpha$$

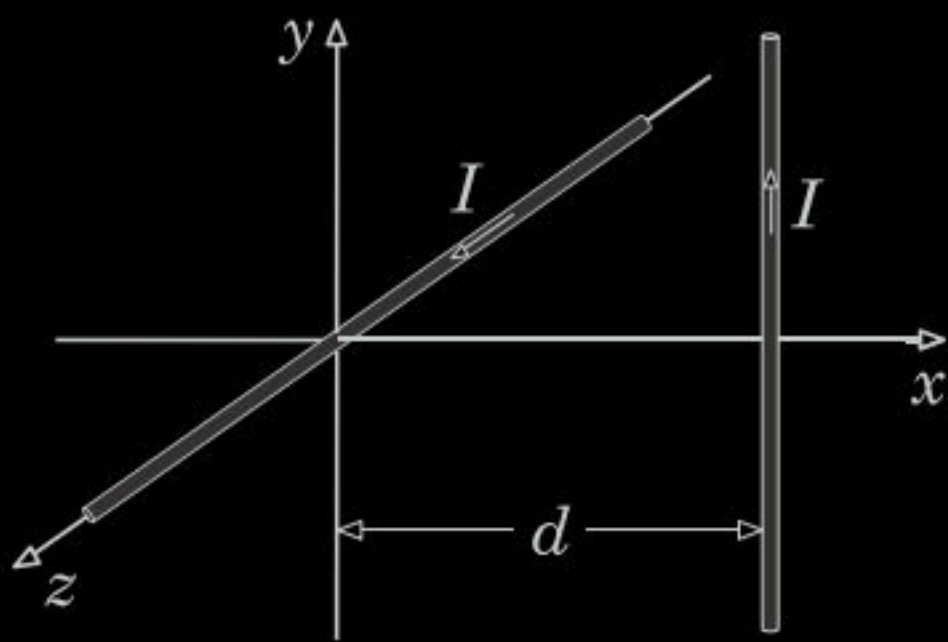
$$\Rightarrow mgr = \frac{3mr^2}{2} \alpha$$

$$\Rightarrow \alpha = \frac{2g}{3r}$$

$$N - mg = m(\alpha r)$$

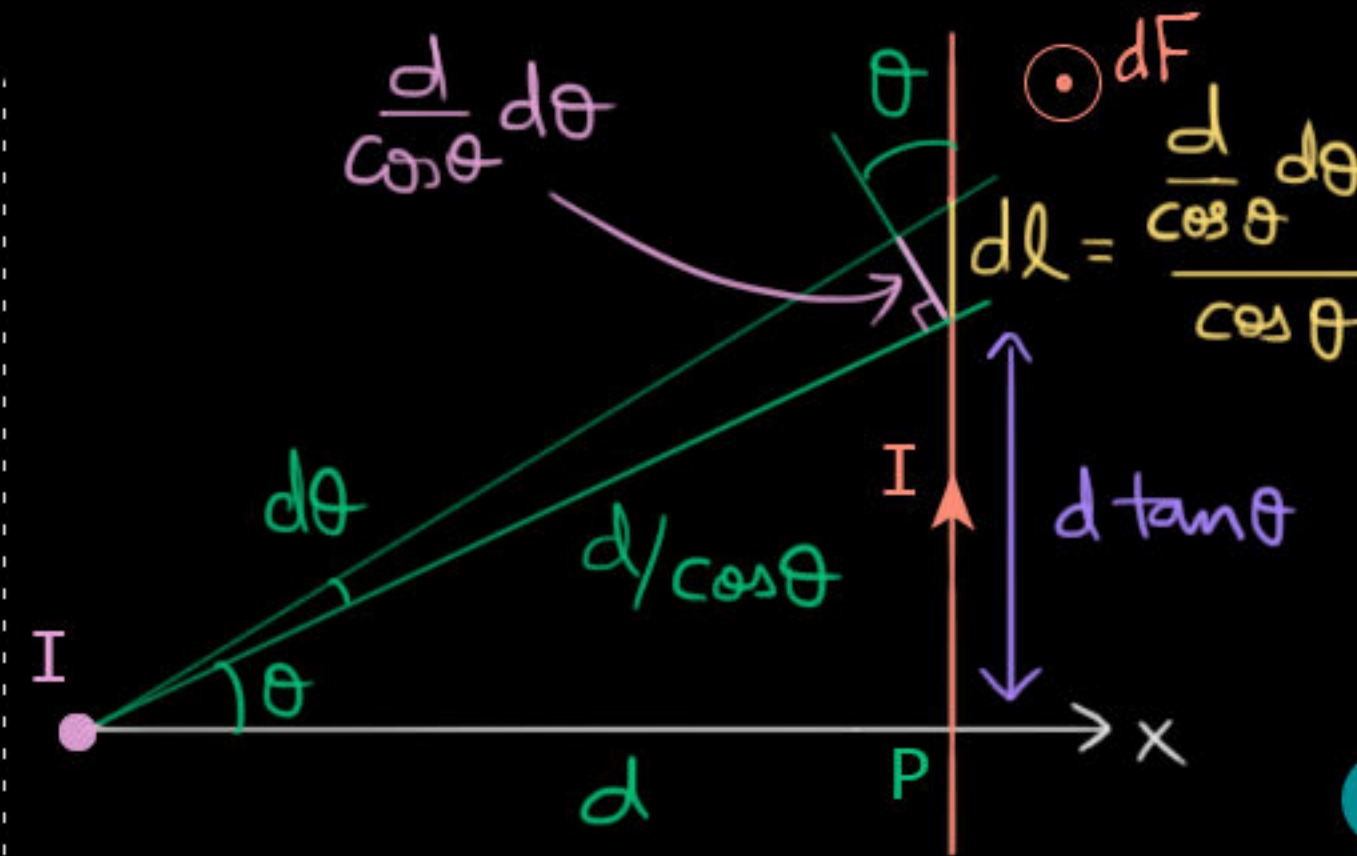
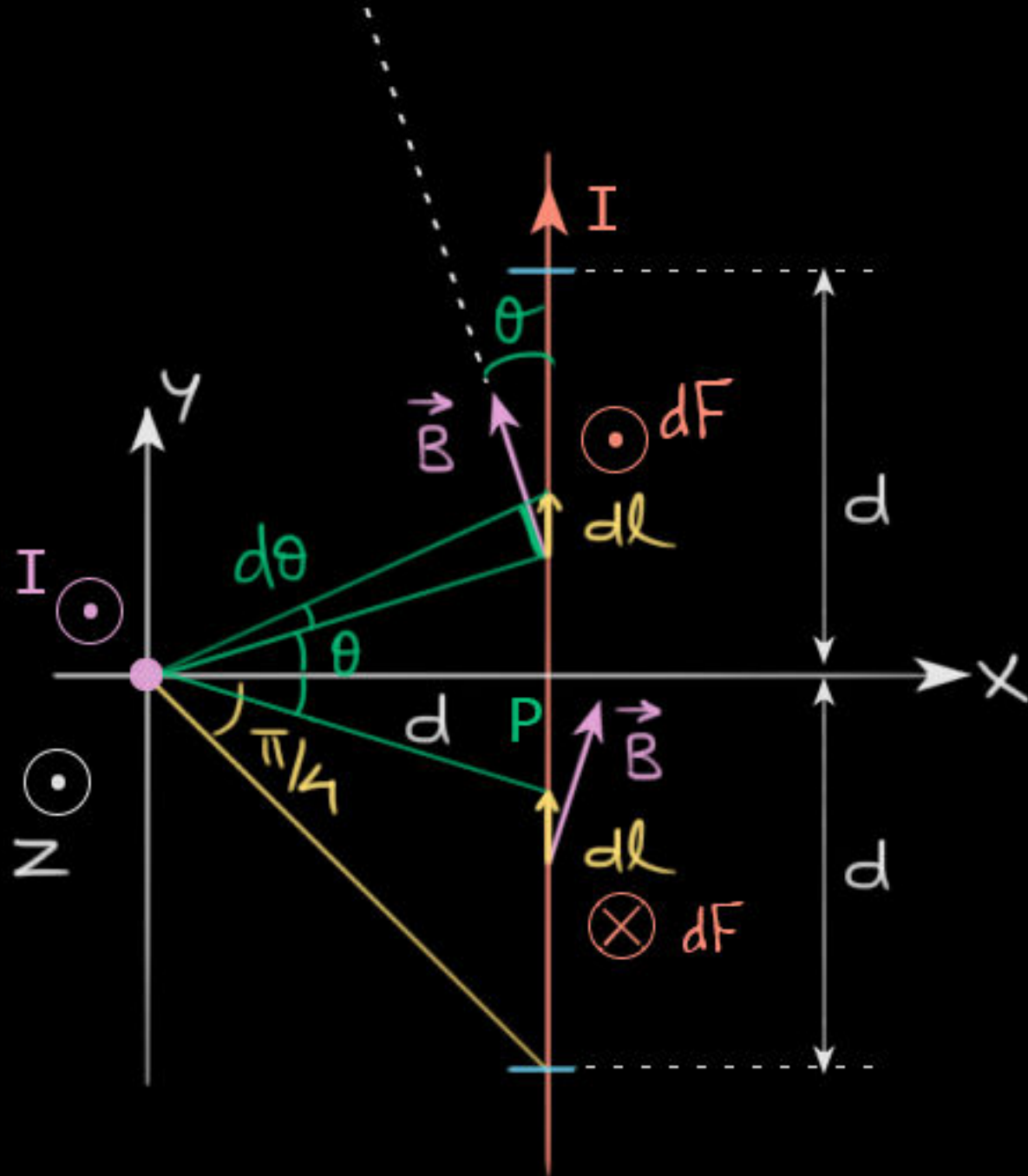
$$\Rightarrow N = mg + m\alpha r$$
$$= mg + m \frac{2g}{3r} \cdot r$$

$$= \frac{5mg}{3} //$$



28. Two mutually perpendicular infinitely long straight rods each carrying current I are fixed at a distance d apart. One rod is along the z -axis and the other is parallel to the y -axis in x - y plane as shown in the figure.

- (a) Find force experienced by a rod due to magnetic interaction.
 (b) Find torque experienced by a symmetrically located segment of length $2d$ of a rod.



$$dF = I (d\vec{l} \times \vec{B}) \cdot \hat{n}$$

$$dF = \frac{\cancel{d} d\theta}{\cos\theta} \cdot \frac{\mu_0 I}{2\pi \cancel{d} / \cos\theta} \sin\theta$$

$$\Rightarrow dF = \frac{\mu_0 I^2}{2\pi} \tan\theta d\theta$$

A For symmetrical elements about X , dF will cancel out. $\therefore F_{\text{net}} = 0$

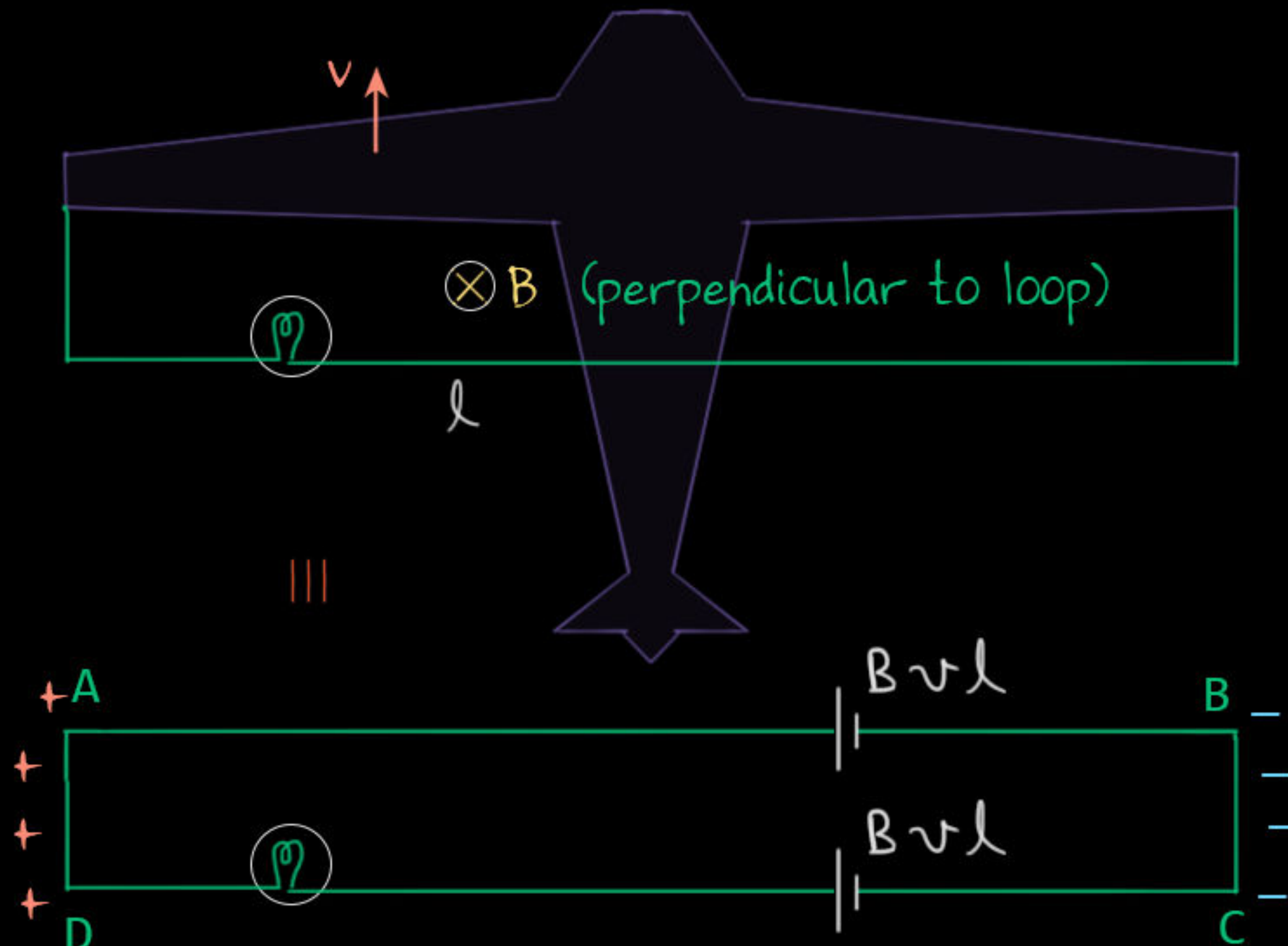
B $d\tau_p = dF \cdot d \tan\theta$

$$\Rightarrow \tau_p = \frac{\mu_0 I^2 d}{2\pi} \int_{-\pi/4}^{+\pi/4} \tan^2\theta d\theta = \left(\frac{4-\pi}{4\pi} \right) \mu_0 I^2 d$$

1. An aircraft is flying at a level height in presence the magnetic field of the Earth. If an electric bulb is connected between the two extreme ends of the wings,

✓(c) a voltage will induce across the wings but the bulb will not glow.

Method 1



Method 2

We know,

$$\otimes B \blacktriangle$$

$$\text{emf induced in loop} = \frac{d\phi}{dt}$$

In given closed loop:

$$\frac{d\phi}{dt} = 0 \Rightarrow i = 0 //$$

$$\cancel{Bvl} - iR - \cancel{Bvl} = 0$$

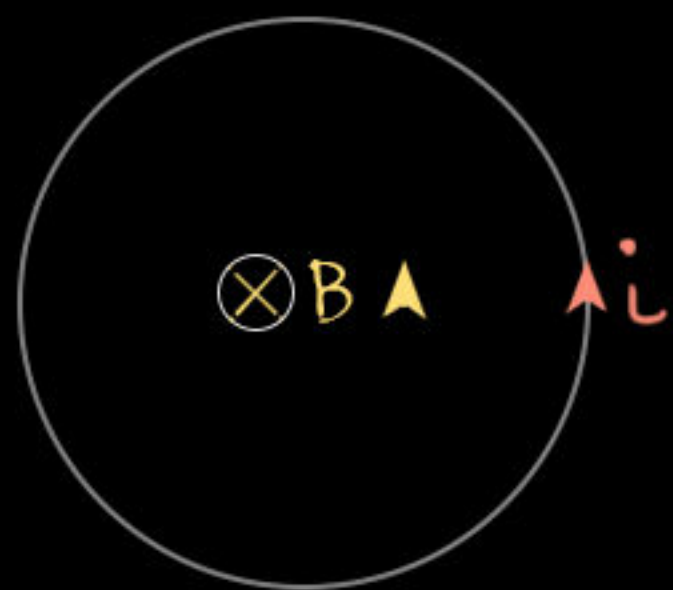
$$\Rightarrow i = 0 //$$

Shape must be circular. As for same circumference, circular shape provides maximum area.

$$A \uparrow \Rightarrow \phi \uparrow \Rightarrow \frac{d\phi}{dt} \uparrow \Rightarrow i \uparrow$$

Orientation must be along $\vec{B}$.

Dimension: Let the radius of loop be r .



2. A wire of uniform cross-section is drawn from a piece of mass m of a metal of density d and resistivity ρ . The ends of the wire are joined to make a planar closed loop and the loop is placed in a uniform magnetic field that varies with time t according to equation $B = \beta t$, where β is a positive constant. What maximum current can you obtain in the loop by properly selecting shape and dimensions of the loop?

(a) $\beta m / (\pi \rho d)$

(b) $\beta m / (2\pi \rho d)$

✓ (c) $\beta m / (4\pi \rho d)$

(d) $\beta m / (8\pi \rho d)$

$$i = \frac{d\phi/dt}{R} = \frac{\frac{d}{dt}(\beta t \cdot \pi r^2)}{\rho \frac{m}{d} / 2\pi r} = \frac{\cancel{\pi} r^2 \beta m}{8 \cdot 4 \cancel{\pi}^2 \cancel{r} d} = \frac{\beta m}{4\pi \rho d} //$$

$\left(\frac{m}{d} / 2\pi r \right) \rightarrow$ Cross sectional area of wire (Volume/length).

Just make the loop circular. Nothing else matters for maximum current.

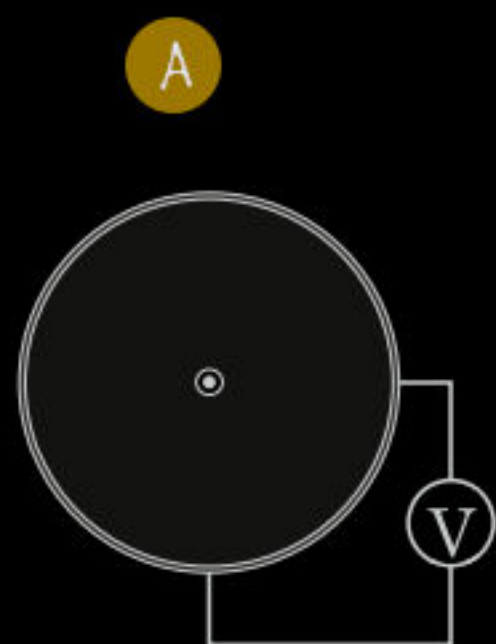


Figure-I

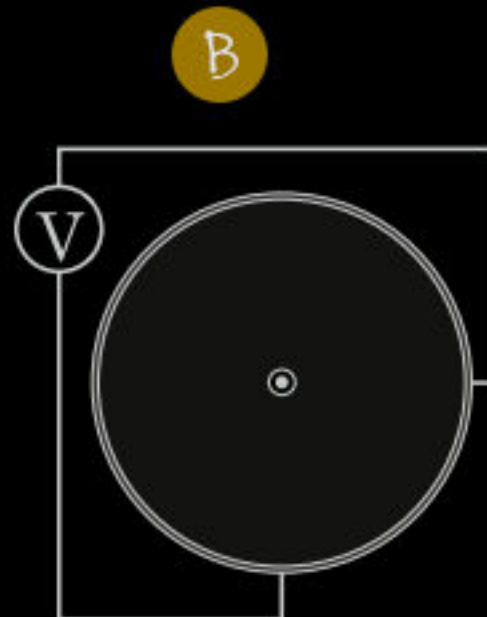


Figure-II

3. In a cylindrical region that is encircled by a uniform metallic ring, a uniform magnetic field perpendicular to the plane of the ring varies with time in such a manner that a constant electromotive force of 12 V is induced in the ring. A voltmeter of very high resistance is used to measure potential difference between two points on the ring separated by one quarter of ring. For the purpose, the voltmeter is connected in two different manners as shown in figures. In both the arrangements, the ring and connecting wires are in the same plane.

Which of the following conclusion can you draw regarding readings V_I and V_{II} of the voltmeter in the first and the second arrangement respectively?

(a) $V_I = V_{II} = 3 \text{ V}$

(b) $V_I = V_{II} = 9 \text{ V}$

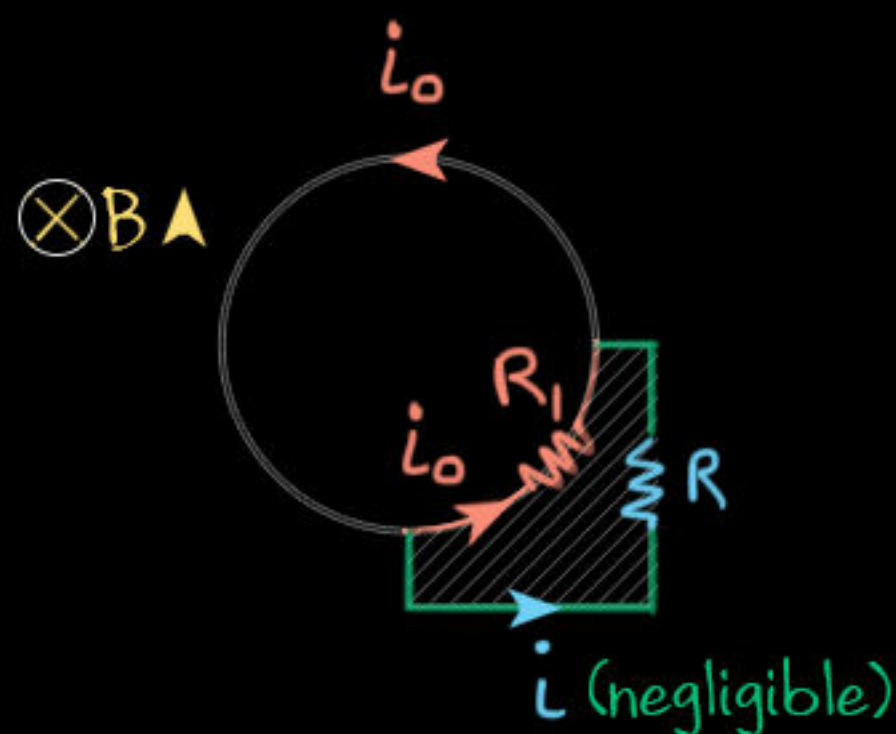
✓ (c) $V_I = 3 \text{ V}$ and $V_{II} = 9 \text{ V}$

(d) $V_I = 9 \text{ V}$ and $V_{II} = 3 \text{ V}$

Method 1

A

Voltmeter resistance be R .



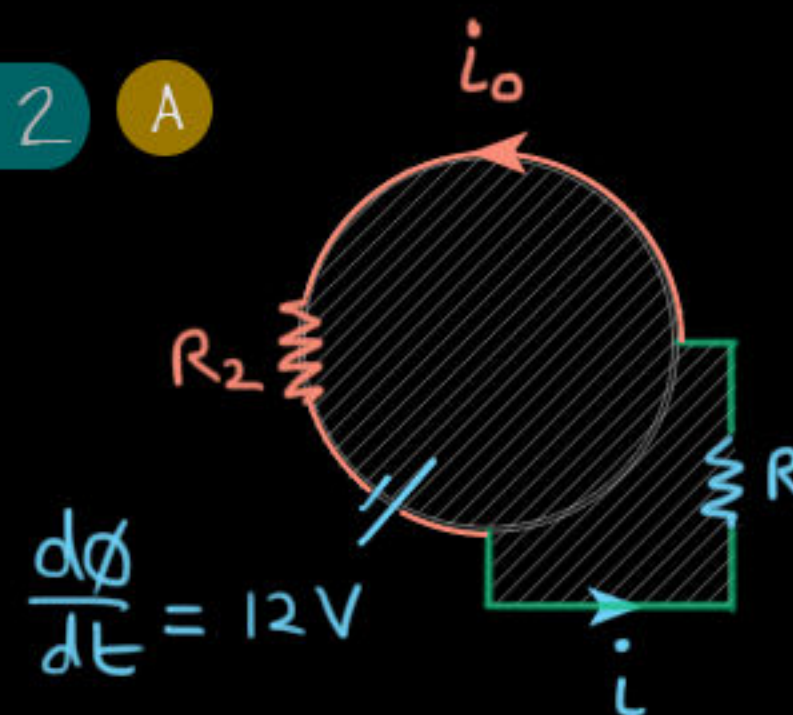
In shaded loop:

$$-i_0 R_1 + i R = 0$$

$$12/4 = 3 \quad \text{Reading}$$

Method 2

A



In shaded loop:

$$12 - i R - i_0 R_2 = 0$$

$$\frac{3}{4}(12) = 9$$

$$\Rightarrow \text{Reading} = 3 //$$

Youtube / Arihant Senior

by Ankit Singhvi

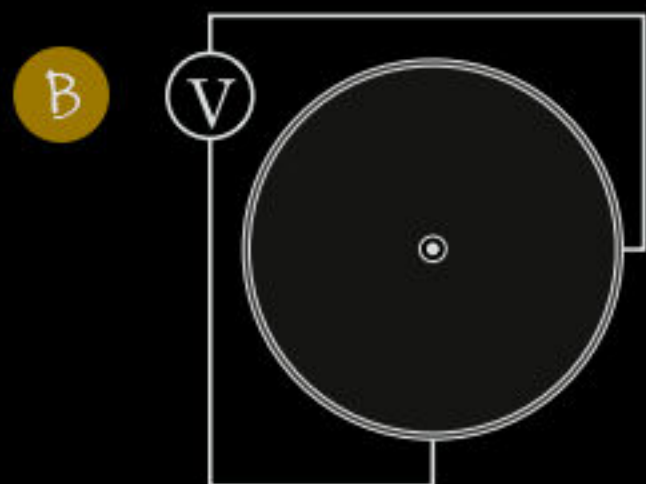
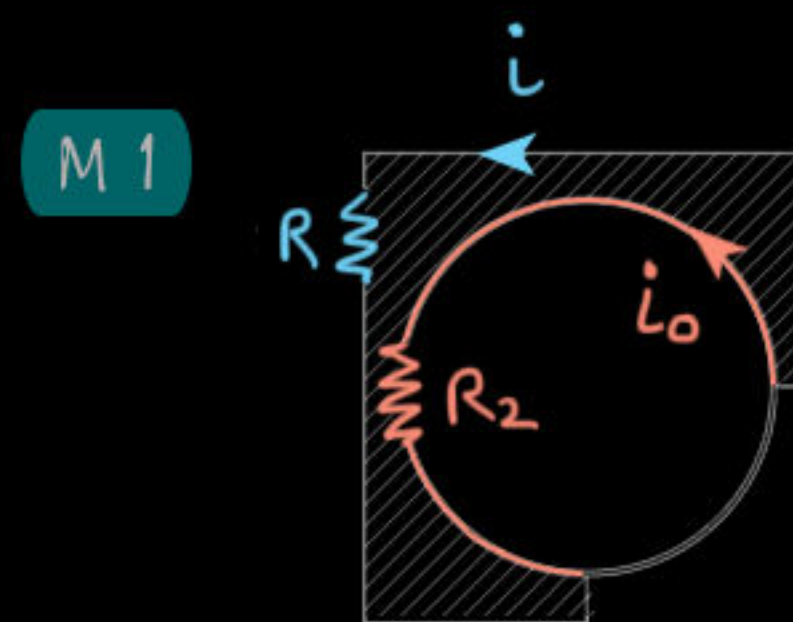


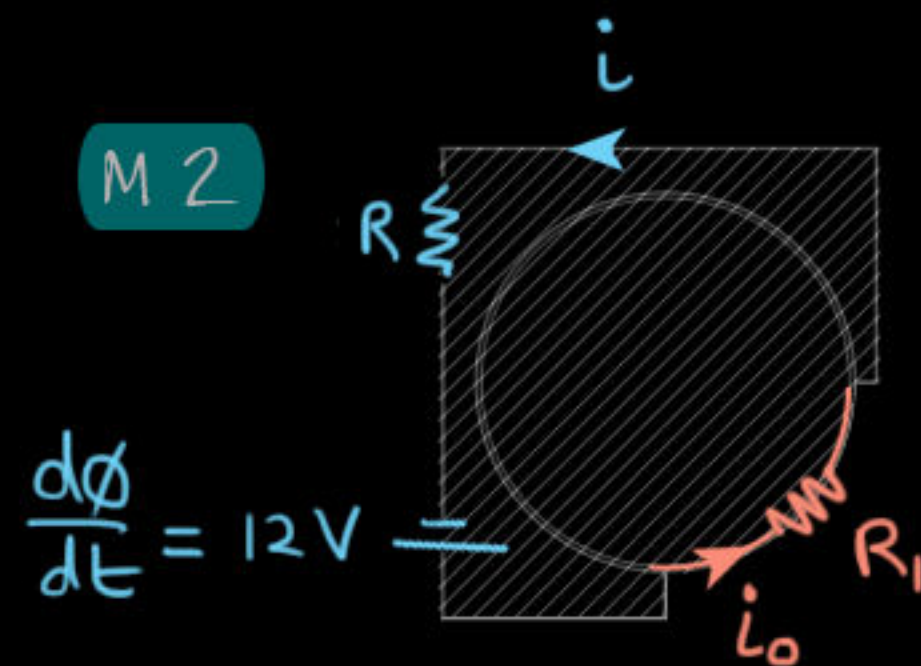
Figure-II



In shaded loop:

$$-i_0 R_2 + i R = 0$$

$$\frac{3}{4}(12) = 9 \quad \text{Reading}$$



In shaded loop:

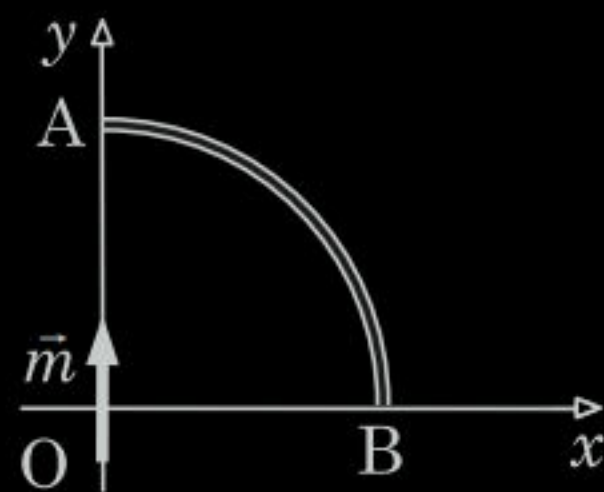
$$12 - iR - i_0 R_1 = 0$$

$$12/4 = 3$$

$$\Rightarrow \text{Reading} = 9$$

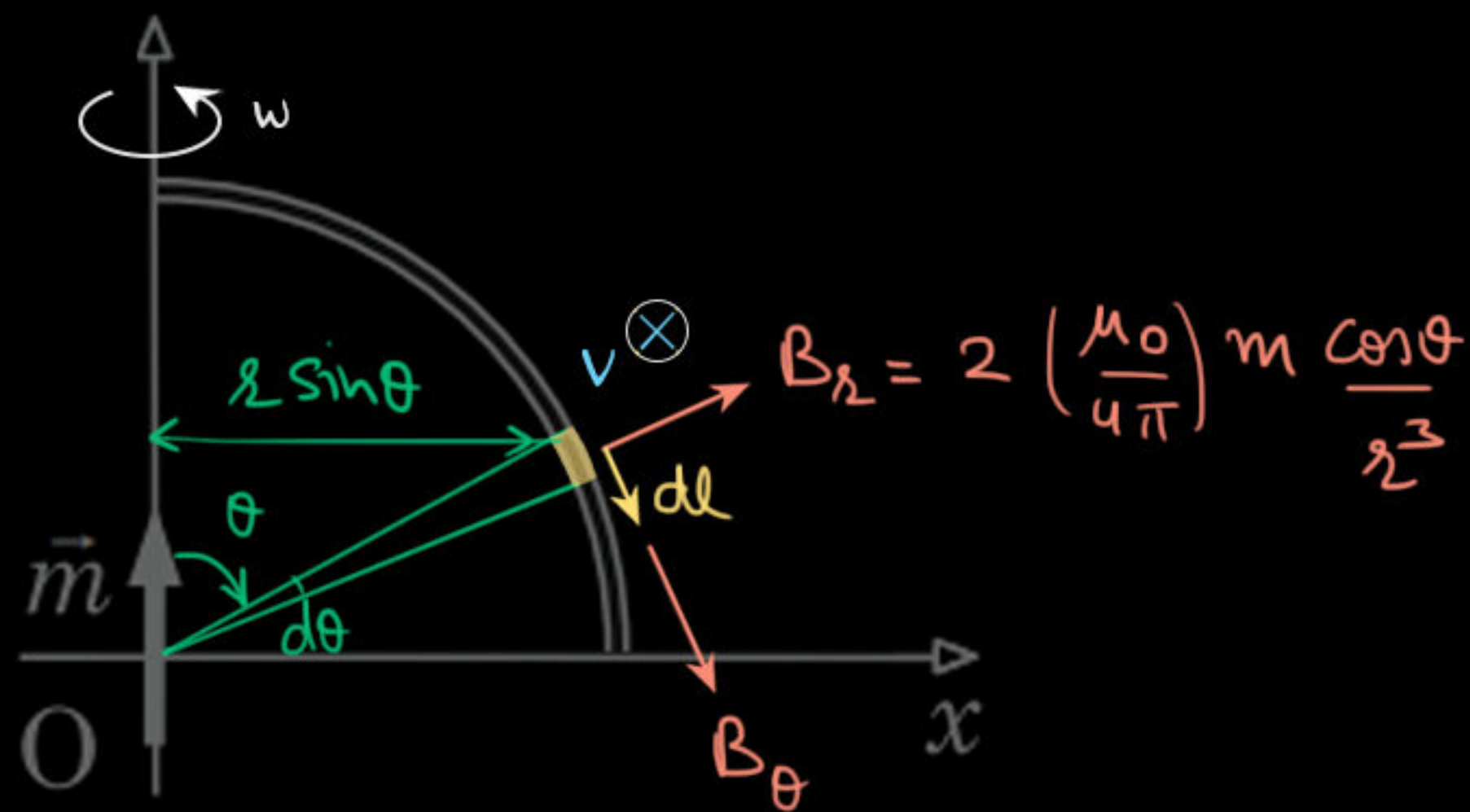
Note: Equivalent induced emf
battery location doesn't matter

Youtube / Arihant Senior



4. Consider a quarter circular conducting ring of large radius r with its centre at the origin, where a magnetic dipole of moment $\vec{m}$ is placed as shown in the figure. If the ring rotates at a constant angular velocity ω about the y -axis, electromotive force induced between its ends is

- (a) Zero
 (b) $\mu_0 m \omega / (2\pi r)$
 ✓ (c) $\mu_0 m \omega / (4\pi r)$
 (d) $\mu_0 m \omega / (8\pi r)$



Ignore B_θ , as $d\vec{l} \cdot (\vec{v} \times \vec{B}_\theta) = 0$

$$\begin{aligned} \therefore d\epsilon &= B_r v dl \quad \text{where } v = \omega r \sin\theta \text{ and } dl = r d\theta \\ \Rightarrow \epsilon &= 2 \frac{\mu_0}{4\pi} \frac{m}{r^3} \omega r \int_0^{\pi/2} \cos\theta \sin\theta d\theta \\ &= \frac{\mu_0}{4\pi} \frac{m \omega}{r} \int_0^{\pi/2} \sin 2\theta d\theta \\ &= \frac{\mu_0 m \omega}{4\pi r} // \end{aligned}$$



5. A long thin wooden cylindrical pipe of radius R carrying a uniform surface charge density σ , is rotating about its axis with an angular velocity ω that increases slowly with time t as $\omega = kt$, where k is a positive constant. Which of the following statements is/are correct for the region inside the pipe?

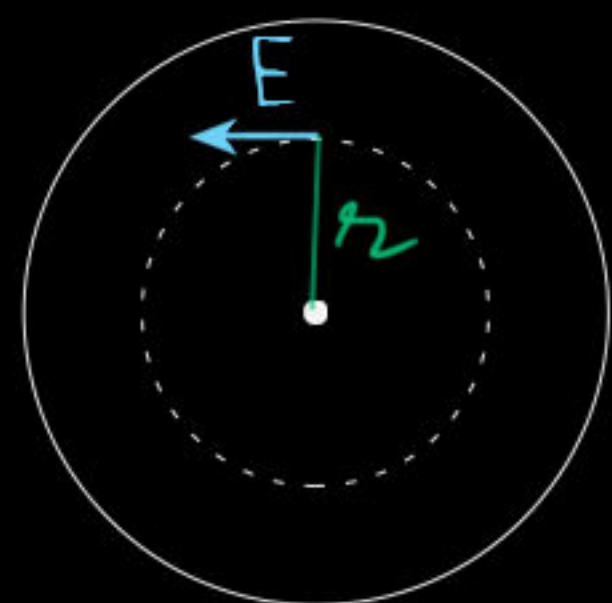
- (a) Both the magnetic and the electric fields are uniform and constant.
- ✓ (b) The magnetic field is uniform but not constant and the electric field is constant but not uniform.
- (c) The magnetic field is constant but not uniform and the electric field is uniform but not constant.
- ✓ (d) Total energy density u due to both the fields varies with time as $u = a + bt^2$, where a and b are positive constants.

Refer: Ch-14, BYU 2

This is similar to a solenoid, with uniform magnetic field inside proportional to ω .

$$B = \mu_0 \omega = \mu_0 kt \quad \text{①}$$

∴ Uniform but not constant.



E is due to induced emf.

At distance r from center its:

$$E = \frac{r}{2} \frac{dB}{dt} \xrightarrow{\text{From 1}} \frac{r}{2} \mu_0 k \quad \text{②}$$

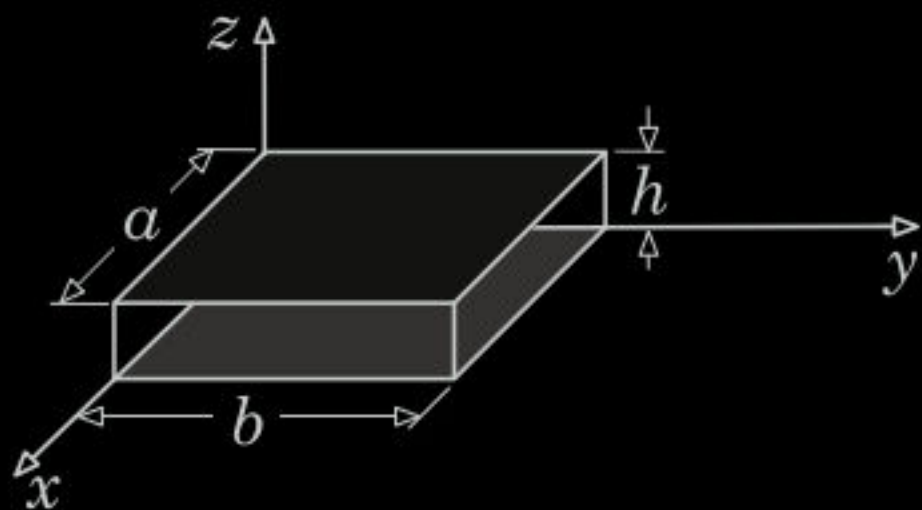
∴ Constant, but non-uniform (increases with r)

In a region:

$$u = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$

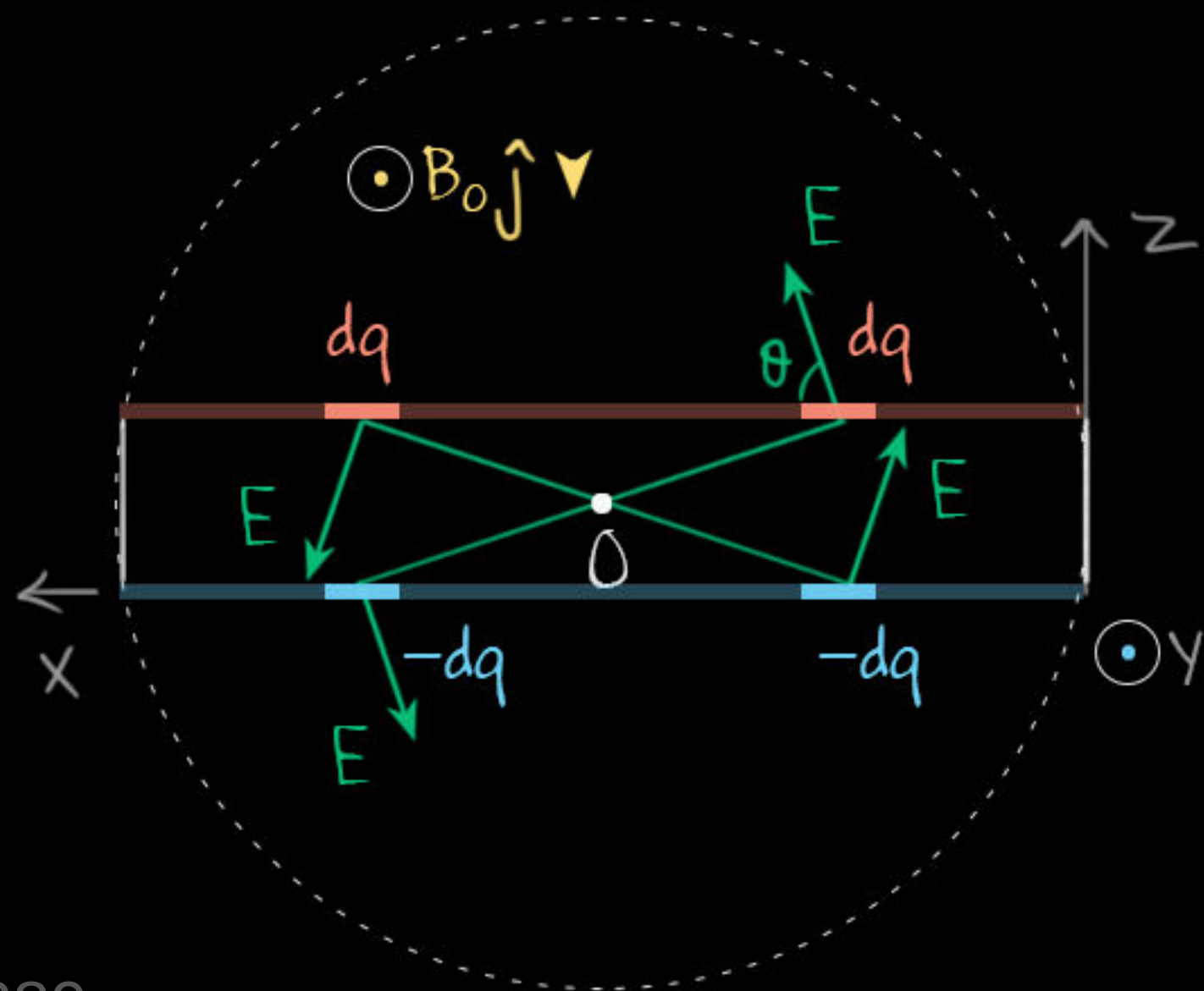
constant ② $B \propto t$ ①

$$= c_1 + c_2 t^2$$



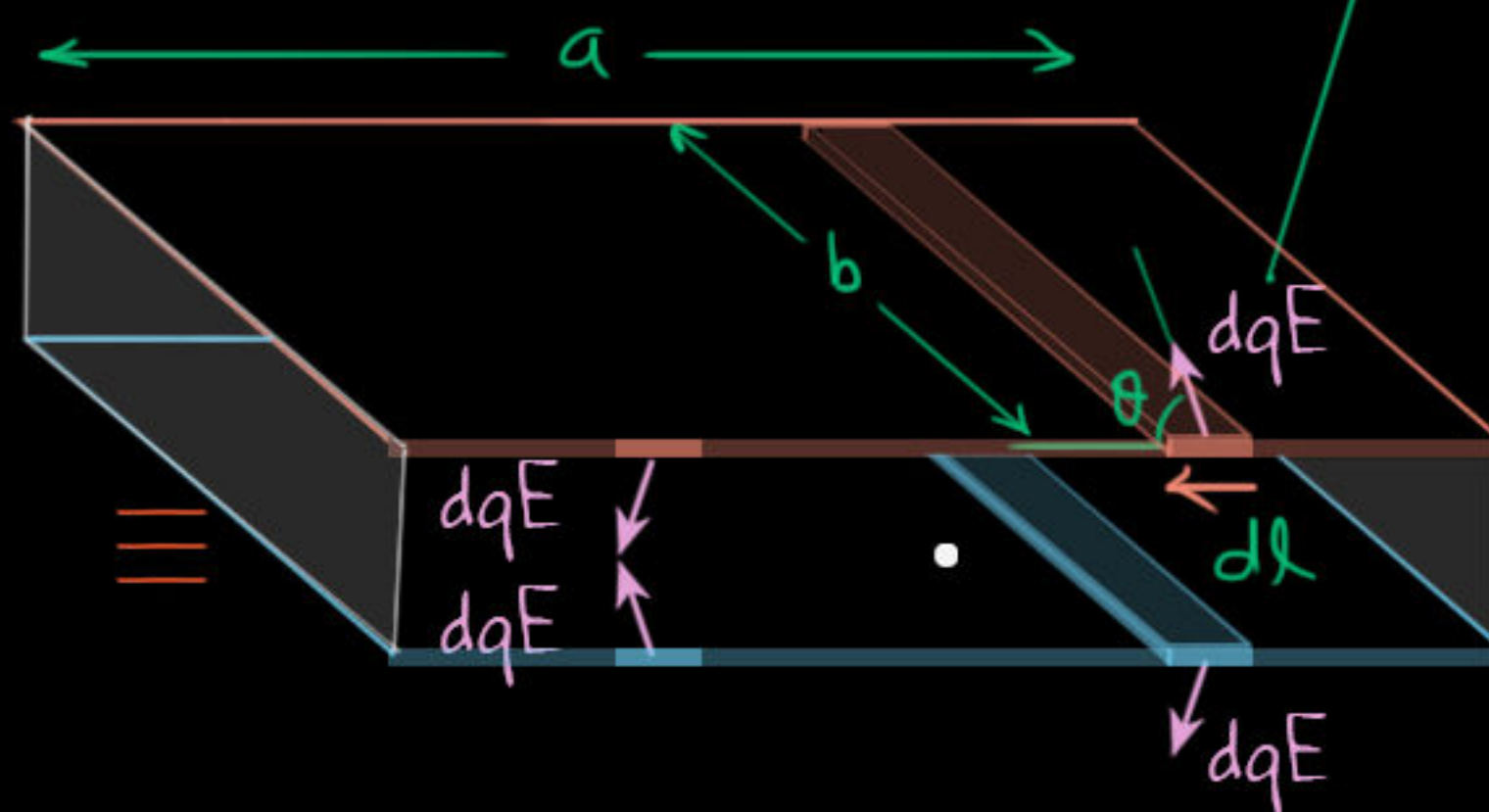
6. The top and bottom faces of a rigid insulating box of dimensions $a \times b \times h$ and mass m are charged with uniform surface charge densities σ and $-\sigma$ respectively and the box is placed in a uniform horizontal magnetic field $\vec{B} = B_0 \hat{j}$. Assuming free space conditions and h negligibly small as compared to the dimensions a and b , find how much velocity will the box acquire after the magnetic field is switched off?

As its symmetric about O , induced electric field will look like as shown:



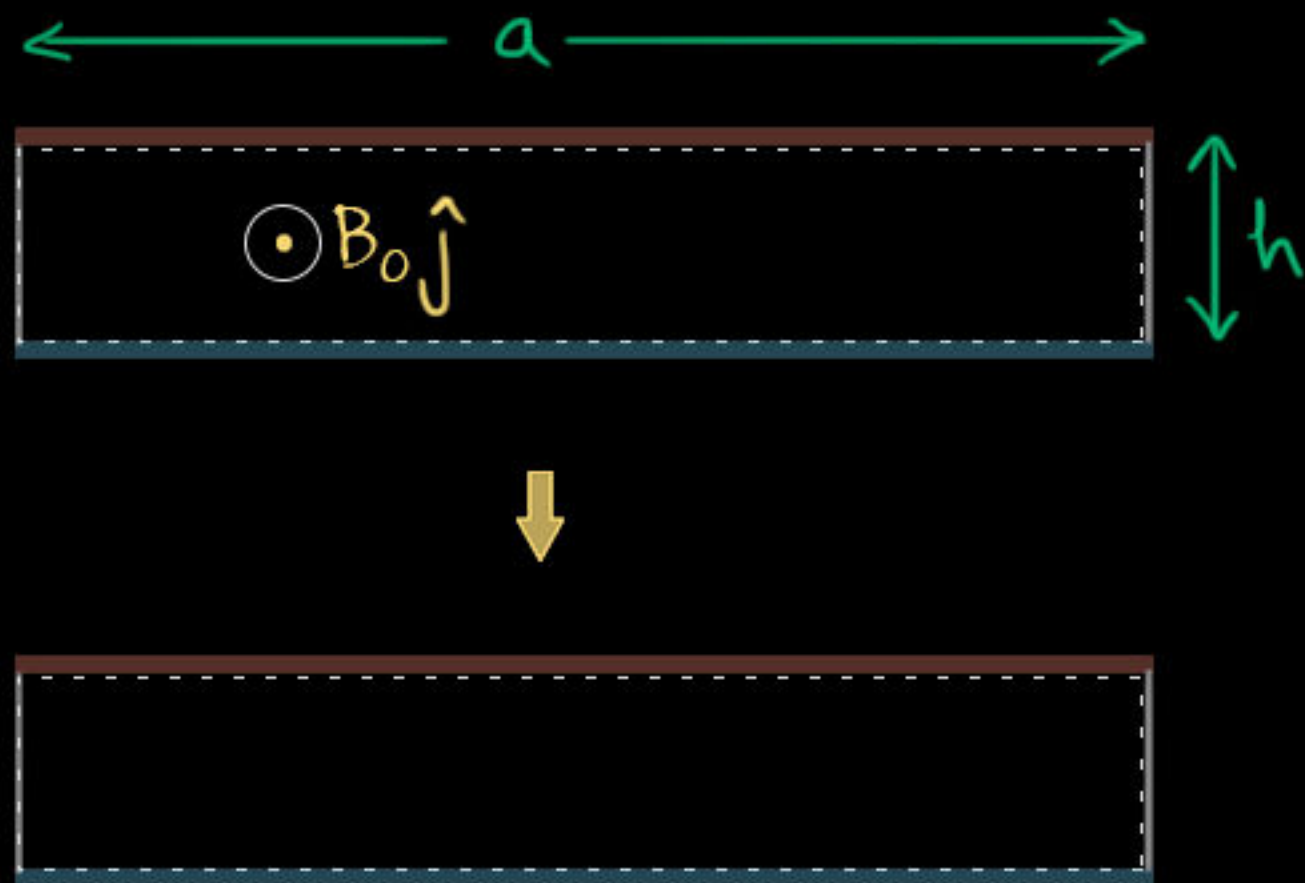
✓ (c) $\frac{\sigma abh B_0}{m} \hat{i}$

Clearly, net force will be in X direction.



$$\begin{aligned} \therefore d\vec{F}_{net} &= dq E \cos\theta \hat{i} \\ &= \sigma b dl E \cos\theta \hat{i} \\ &= \sigma b \vec{E} \cdot d\vec{l} \hat{i} \\ \Rightarrow \vec{F}_{net} &= \sigma b \oint \vec{E} \cdot d\vec{l} \hat{i} \\ &= \sigma b \frac{\Delta\phi}{\Delta t} \hat{i} \end{aligned}$$

$$\Rightarrow F_{net} \Delta t = \sigma b \Delta\phi \hat{i}$$



$$\therefore \Delta\phi = B_0 a h$$

$$f_{\text{net}} \Delta t = \sigma b \Delta\phi \hat{i}$$

$$\Rightarrow \Delta p = m v = \sigma b (B_0 a h) \hat{i}$$

$$\Rightarrow v = \frac{\sigma B_0 a b h}{m} \hat{i} //$$

Concept: In a superconducting ring, flux **never ever changes**. As any change would imply emf, and emf in zero resistance ring implies infinite current.

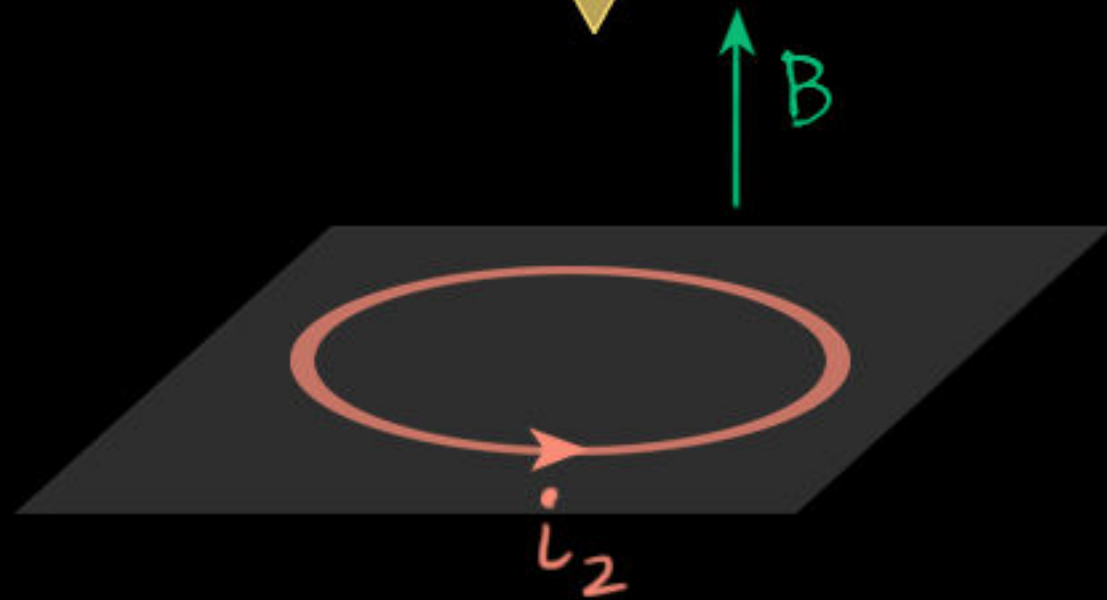
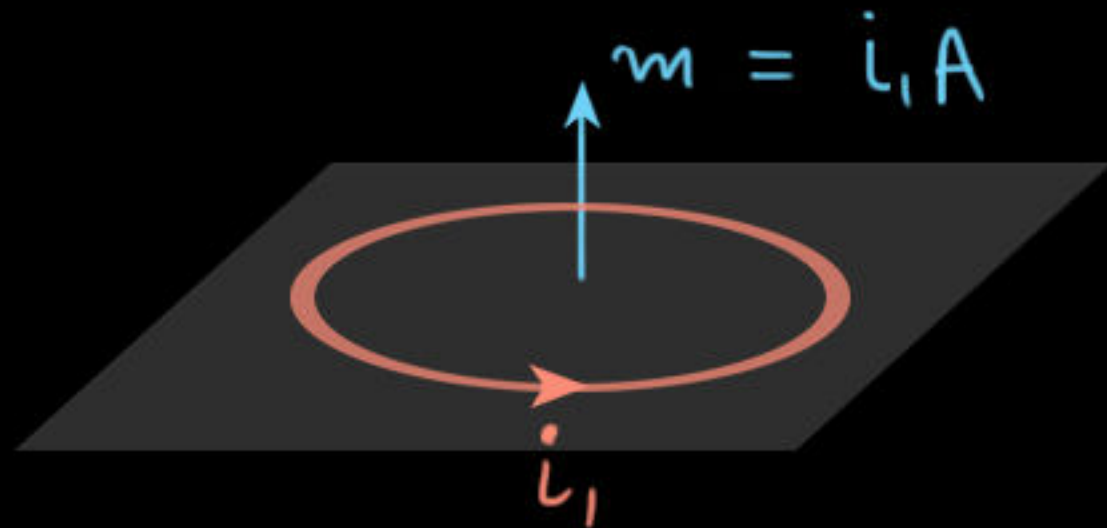
7. A superconducting rigid planar loop of area A and self-inductance L carrying a current is held motionless in a region of free space. Now a uniform magnetic field of induction B pointing everywhere parallel to the magnetic moment $\vec{m}$ of the loop is switched on. Current in the loop after the magnetic field is switched on is given by

(a) $\frac{AB}{L}$

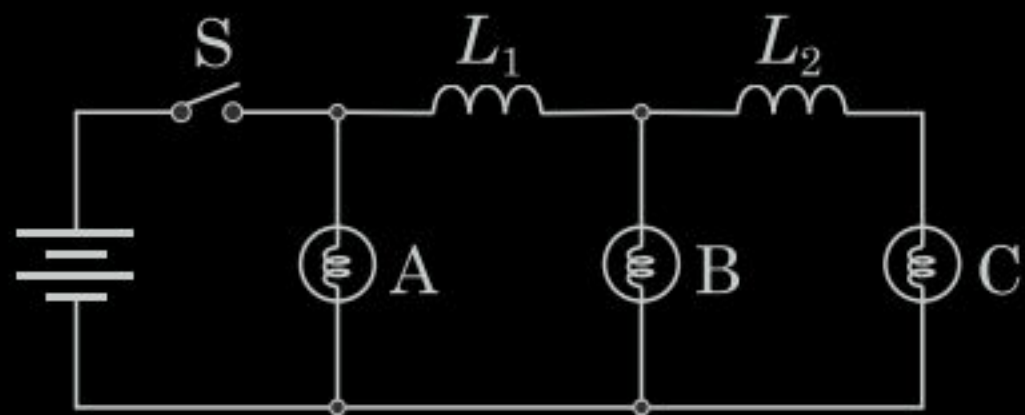
(b) $\frac{m}{A}$

✓(c) $\frac{m}{A} - \frac{AB}{L}$

(d) $\frac{m}{A} + \frac{AB}{L}$



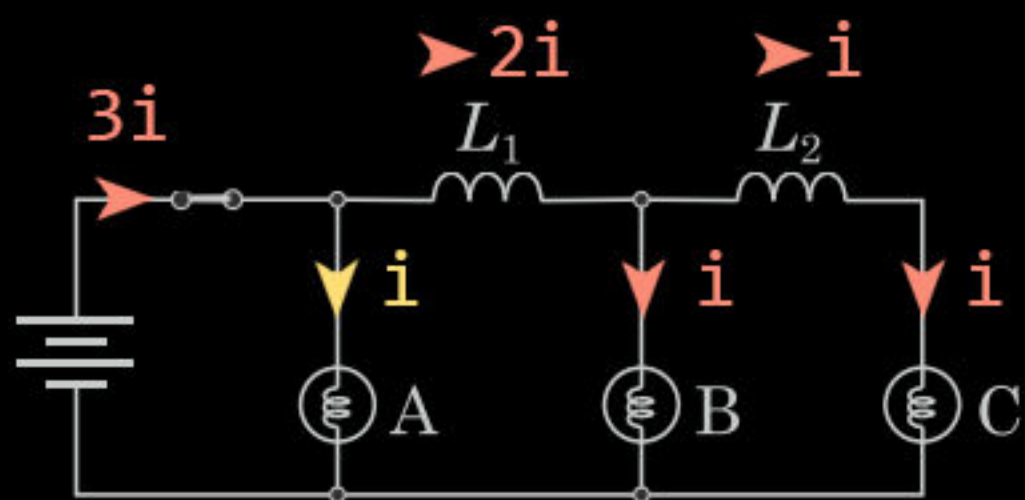
$$\begin{aligned} \phi_1 &= \phi_2 \\ \Rightarrow \frac{m}{A} &= Li_2 + BA \\ \Rightarrow i_2 &= \frac{m}{A} - \frac{BA}{L} // \end{aligned}$$



Concept: In an inductor flux doesn't change immediately. As finite emf change in infinitesimally small time will generate infinite emf.

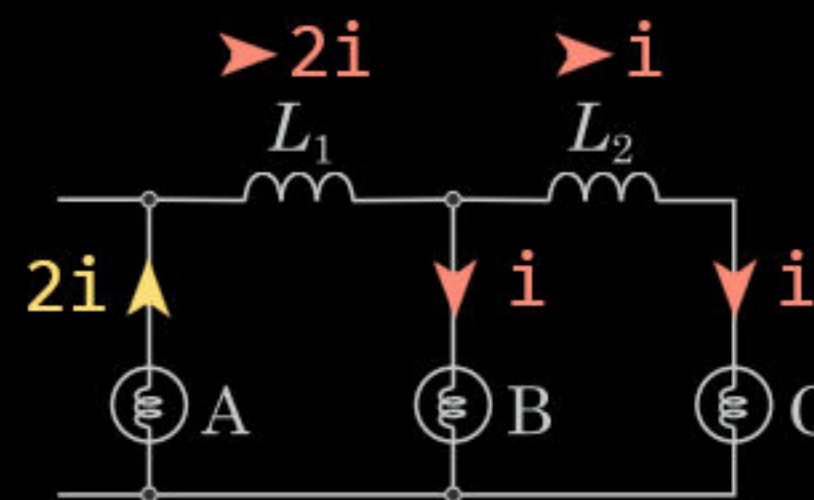
8. Three identical lamps A, B and C and two identical inductive coils L_1 and L_2 are connected to a DC power supply through a switch S as shown in the figure. Initially the switch is closed for a long time and steady state is reached. Now the switch S is opened. Which of the following statements correctly describes order of brightness of the bulbs immediately after the switch is opened?

- ✓(a) Brightness of A suddenly increases but that of B and C remains unchanged.
- (b) Brightness of B suddenly increases but that of A and C remains unchanged.
- (c) Brightness of C suddenly increases but that of A and B remains unchanged.
- (d) Brightness of all the bulbs suddenly increases equally.

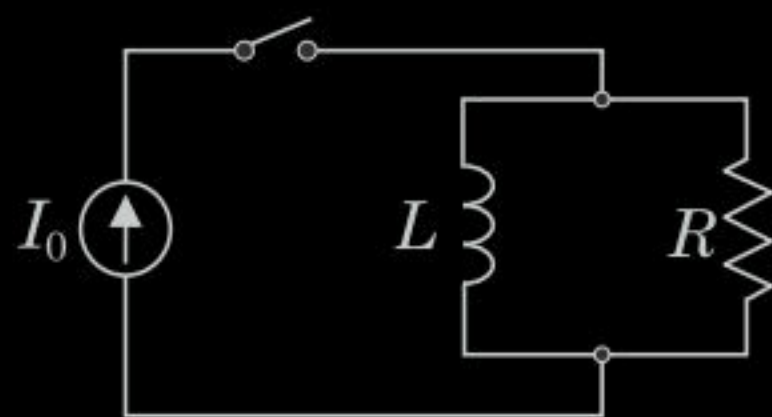


Initially

Current through both inductors will not change immediately on opening the switch.



Immediately after switch is open.

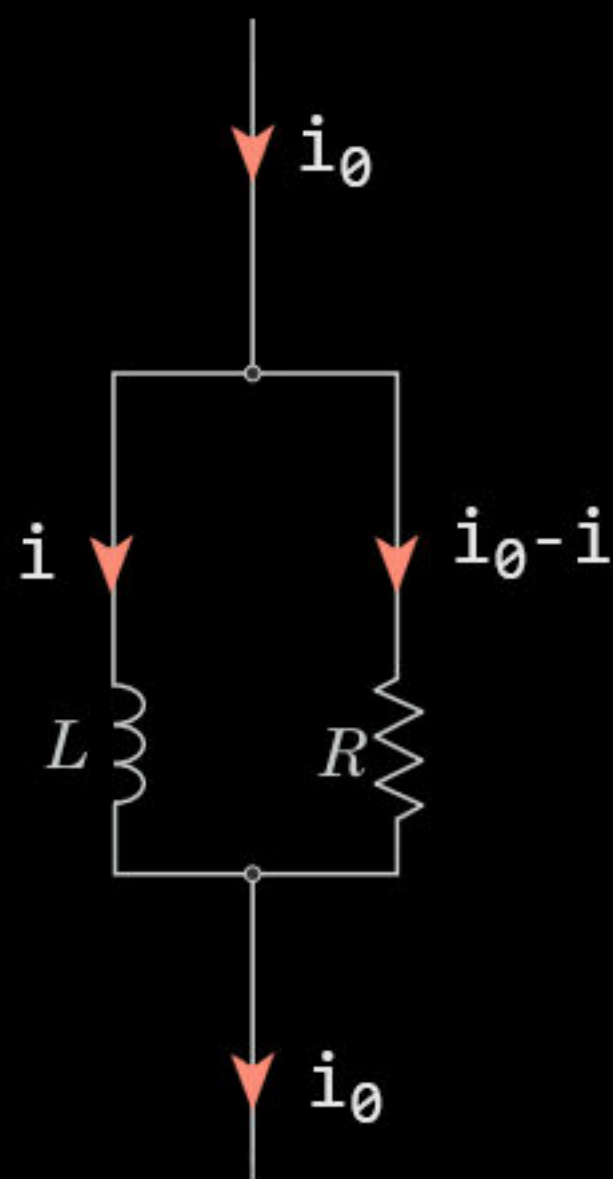


9. A circuit shown consists of a coil of inductance L , an unknown resistance R , a switch and an ideal current source of current I_0 . Find total heat dissipated in the resistance after the switch is closed.

- (a) Zero
 ✓ (c) $\frac{1}{2}LI_0^2$

(b) $\frac{1}{4}LI_0^2$

(d) LI_0^2



Let current in inductor be i at any random time.

$$L \frac{di}{dt} = (i_0 - i)R \quad (1)$$

Method 1

Integrate to find i

Integrate again as $\int (i_0 - i)^2 R dt$

Method 2 One simple integration

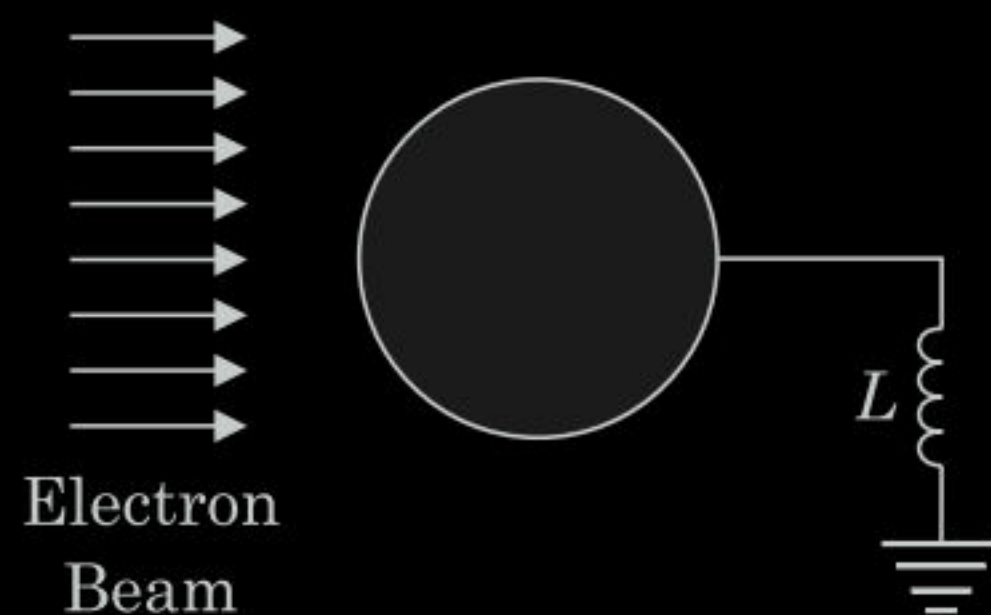
Heat dissipated in dt time:

$$dH = (i_0 - i)^2 R dt$$

$$= (i_0 - i) (i_0 - i) R dt \quad (2)$$

$$= (i_0 - i) L di$$

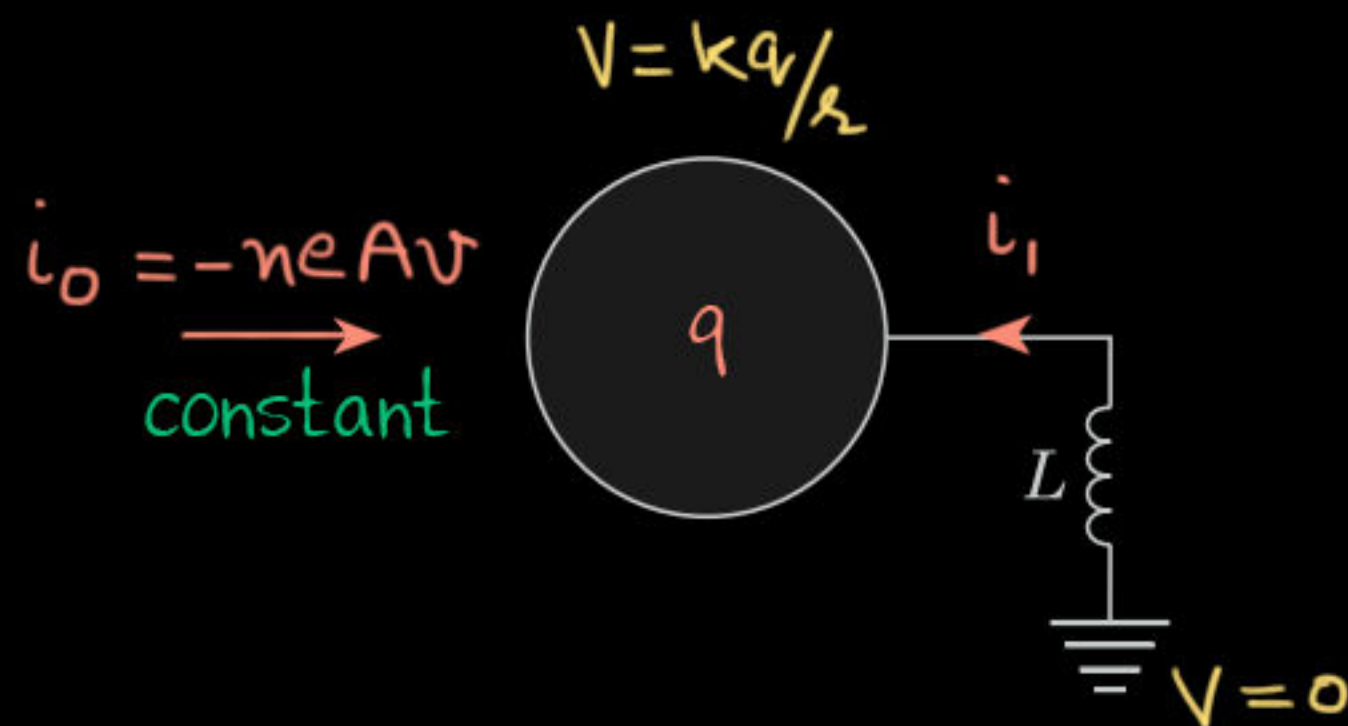
$$\Rightarrow H = L \int_0^{i_0} (i_0 - i) di = \frac{Li_0^2}{2} //$$



10. An uncharged conducting ball of radius r is earthed through an inductor L as shown in the figure. A horizontal electron beam swoops on the ball from a great distance. The density of electrons in the incident beam is n and all these electrons are moving with velocity v that is much smaller than the speed of light so the relativistic effects can be neglected. Which of the following statements is/are correct?

(a) Maximum charge on the ball is equal to $2ne\pi r^2 v \sqrt{\pi \epsilon_0 r L}$

(c) Maximum current through the inductor is equal to $2ne\pi r^2 v$



$$0 - L \frac{di_1}{dt} = \frac{kq}{r} \quad (1)$$

$$i_0 + i_1 = \frac{dq}{dt} \quad (2)$$

Eliminating q from 1,2:

$$i_0 + i_1 = -\frac{Lr}{k} \frac{d^2 i_1}{dt^2}$$

$-neAv$

$$\Rightarrow \frac{d^2 i_1}{dt^2} + \frac{k}{Lr} (i_1 - neAv) = 0$$

ω^2 its SHM

$$\therefore i_1 - neAv = i_{10} \sin(\omega t + \phi)$$

$$\Rightarrow \frac{di_1}{dt} = \omega i_{10} \cos(\omega t + \phi)$$

$$\downarrow t=0, q=0 \Rightarrow \frac{di_1}{dt} = 0 \Rightarrow \phi = \frac{\pi}{2}$$

$$i_1 - neAv = i_{10} \cos \omega t$$

$$\downarrow t=0, q=0 \Rightarrow i_1 = 0 \Rightarrow i_{10} = -neAv$$

$$i_1 = neAv (1 - \cos \omega t)$$

$$q = -\frac{Lr}{k} \frac{di_1}{dt} \Rightarrow q_{\max} = -\frac{Lr}{k} (\omega \cdot neAv) = \frac{Lr}{k} \sqrt{\frac{k}{Lr}} neAv$$

From 1:

by Ankit Singhvi

Youtube / Arihant Senior

Hint: Irodov 4.104

Lets find the energy stored in system in terms of current amplitude i .

o @ i_{max}

$$-iR - L \frac{di}{dt} + \frac{q}{C} = 0$$

$$\Rightarrow \frac{q}{C} = iR$$

$$\therefore U = \frac{q^2}{2C} + \frac{1}{2} Li^2$$

$$= \left(\frac{C}{2} R^2 + \frac{L}{2} \right) i^2$$

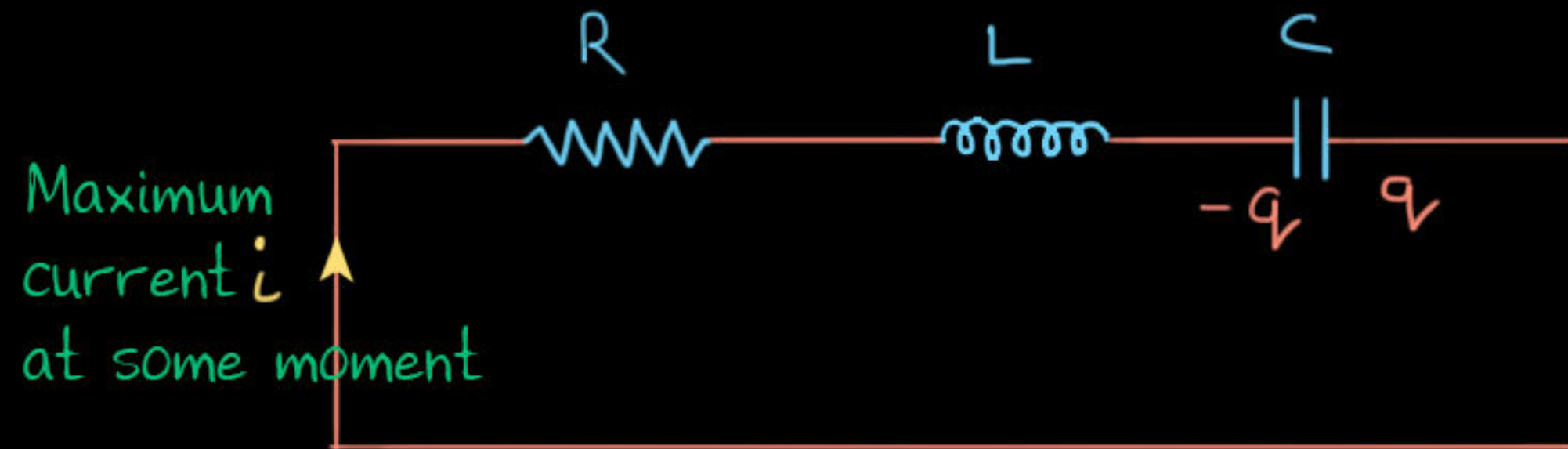
11. Consider a series combination of a resistance R , an inductance L , and a capacitance C undergoing electromagnetic oscillations. During a certain period, amplitude of current through the resistance drops from I to $0.5I$. What is the amount of heat dissipated by the resistance in this period?

(a) $0.375LI^2$

(b) $0.375I^2CR^2$

✓(c) $0.375I^2(L + CR^2)$

(d) $0.375LI^2 + 0.75I^2CR^2$

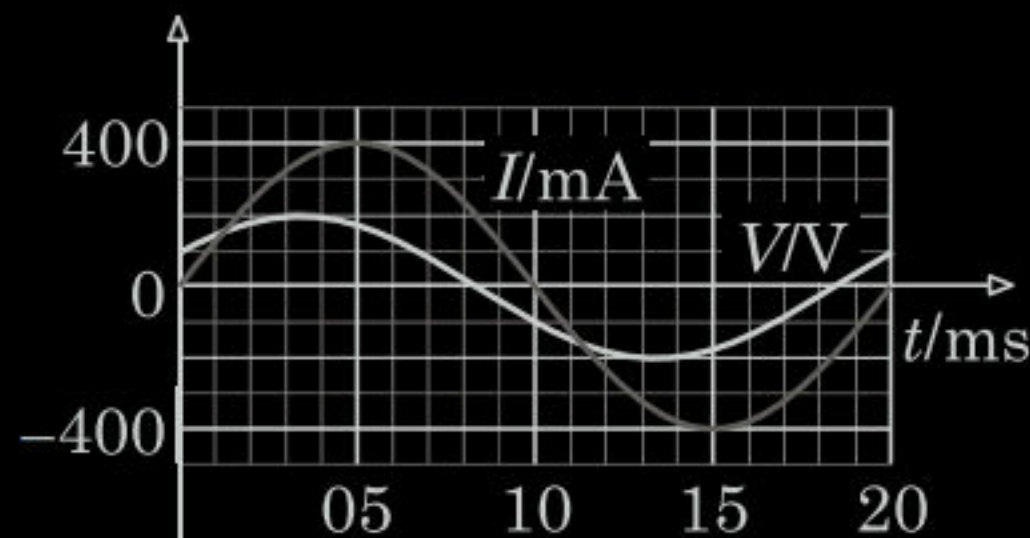


$$\text{Heat} = U_i - U_f$$

$$= \left(\frac{C}{2} R^2 + \frac{L}{2} \right) [i^2 - (0.5i)^2]$$

$$= 0.375(L + CR^2) i^2 //$$

So if we want to find the energy stored in a LCR circuit. Finding it when current is at its peak is a nice way to find it.



12. The given graph shows variation with time in the source voltage V and steady state current I drawn by a series RLC circuit. Which of the following statements is/are correct?

- ✓ (a) Current lags the voltage.
- ✓ (b) Resistance in the circuit is $250\sqrt{3} \Omega$.
- (c) If capacitive reactance is 74Ω , inductance in the circuit is approximately 560 mH .
- ✓ (d) Average power dissipation in the circuit is $20\sqrt{3} \text{ W}$.

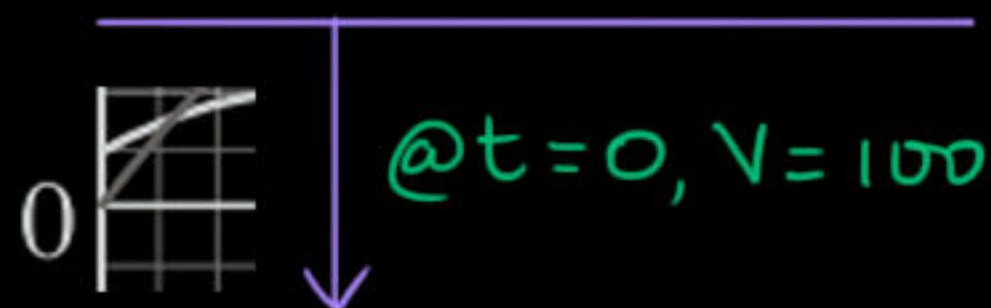
A By graph:
Current's maxima comes a bit later, so it lags.

$$i = i_0 \sin \omega t$$

$\nearrow 0.4 \quad \nearrow 2\pi/20 \cdot 10^{-3}$

$$V = V_0 \sin(\omega t + \phi)$$

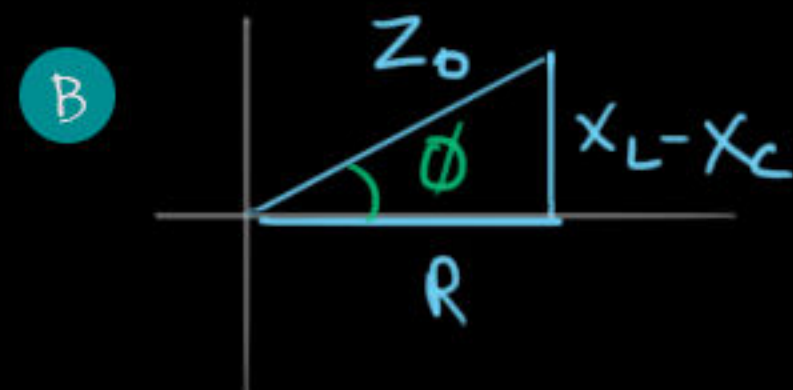
$\nearrow 200$



$$\Rightarrow 100 = 200 \sin \phi$$

$$\Rightarrow \phi = \pi/6$$

By maths: We know, V leads current by ϕ .
As, $\phi = \pi/6$ is +ve, V leads the current.



$$\begin{aligned} R &= Z_0 \cos \phi \\ &= \frac{V_0}{i_0} \cos \pi/6 = \frac{200}{0.4} \cdot \frac{\sqrt{3}}{2} \\ &= 250\sqrt{3} \end{aligned}$$

C

$$\begin{aligned} X_L - X_C &= Z_0 \sin \phi \\ \Rightarrow \omega L - 74 &= \frac{200}{0.4} \cdot \frac{1}{2} \\ \omega &= \frac{2\pi}{20 \cdot 10^{-3}} \end{aligned}$$

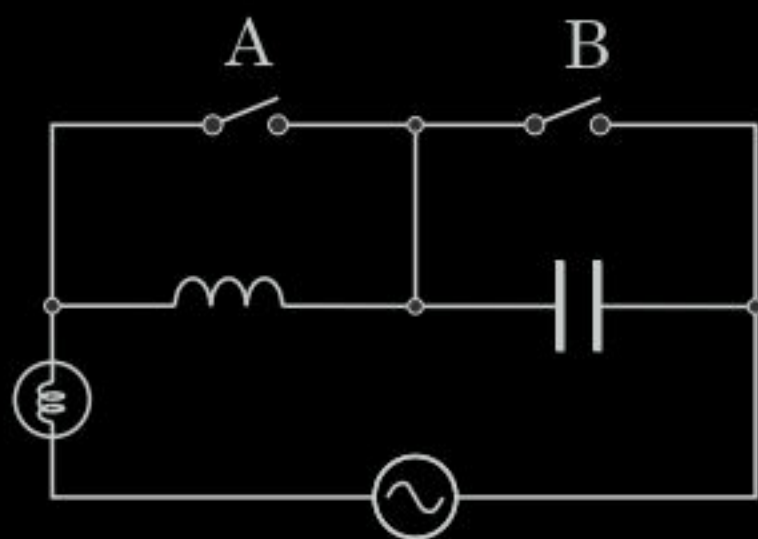
$$\Rightarrow L = 1030 \text{ mH}$$

D M1

$$\begin{aligned} P_{av} &= i_{rms}^2 R \\ &= \frac{i_0^2}{2} R \\ &= \frac{(0.4)^2}{2} \cdot 250\sqrt{3} \\ &= 20\sqrt{3} \end{aligned}$$

M2

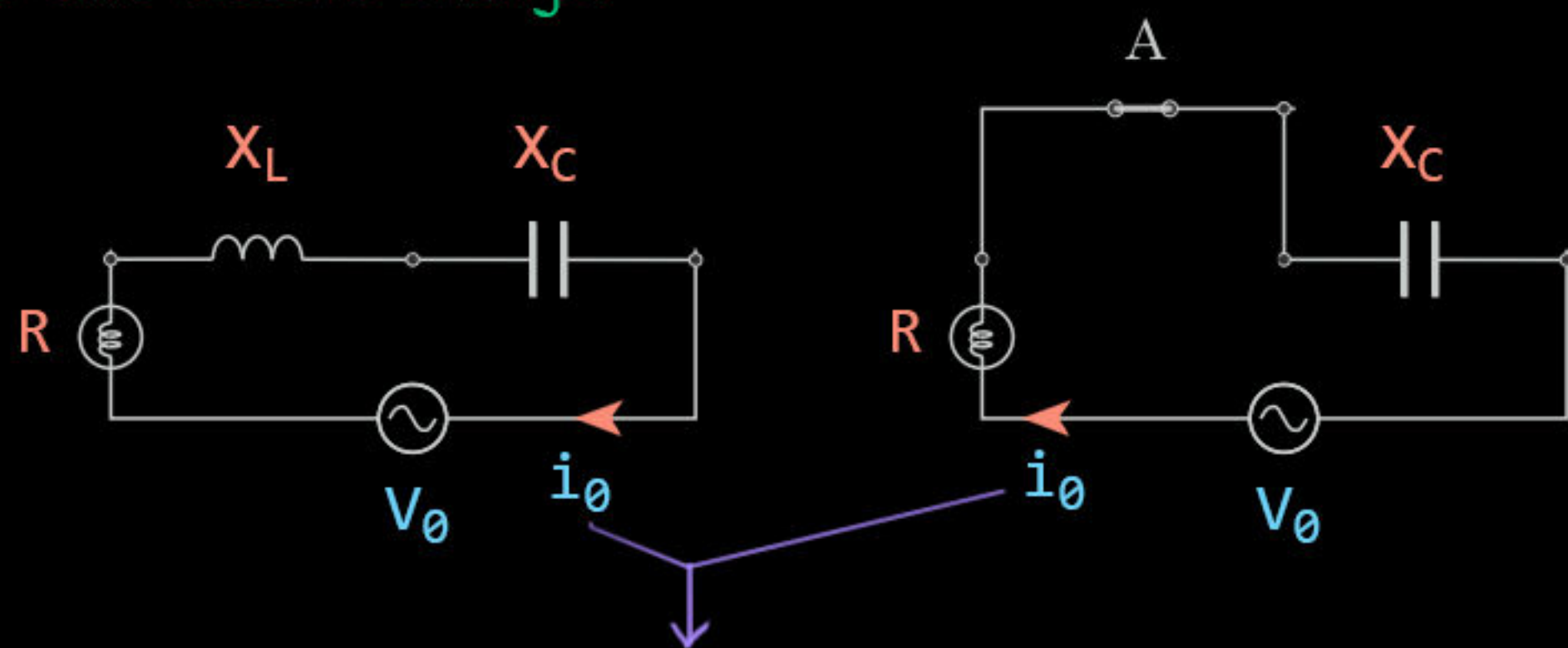
$$\begin{aligned} P_{av} &= V_{rms} i_{rms} \cos \phi \\ &= \frac{V_0}{2} \cdot \frac{i_0}{2} \cdot \frac{\sqrt{3}}{2} \\ &= \frac{200(0.4) \cdot \sqrt{3}}{2} = 20\sqrt{3} \end{aligned}$$



13. A series combination of an incandescent lamp, an inductor and a capacitor is connected across an alternating voltage source. Two switches A and B are provided to short circuit the inductor or the capacitor or both. Initially both the switches are open and the lamp glows with a certain brightness. If the switch A is closed keeping B open, brightness of the lamp remains unchanged. How does the brightness of the lamp change, if the switch B were closed keeping A open?

Concept: Same brightness of bulb means current in the circuit doesn't change.

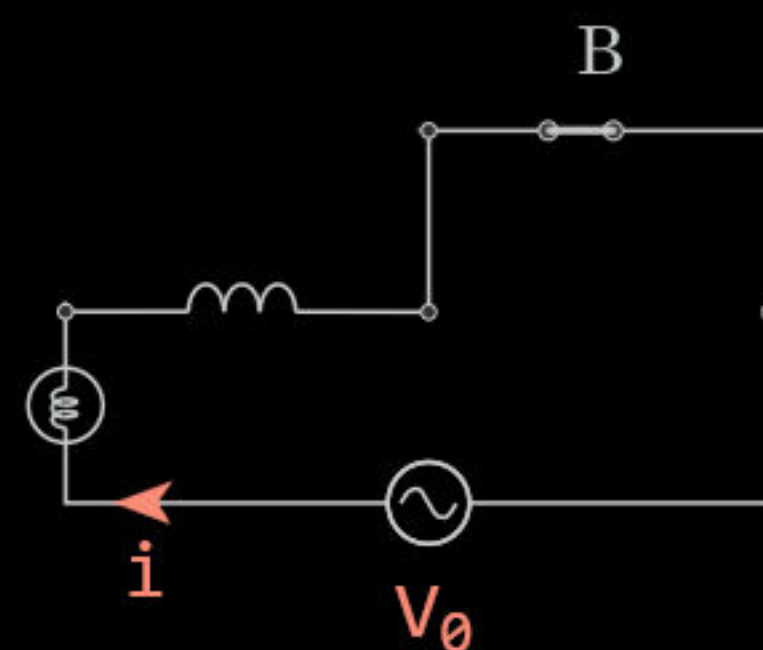
- ✓(a) Decreases
- (c) Remain unchanged
- (b) Increases
- (d) Insufficient information to decide.



Same source, same current $\Rightarrow$ Same impedance

$$\Rightarrow \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_C^2} \quad (1)$$

$$\Rightarrow X_L = 2X_C$$



$$\text{Impedance} = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{R^2 + 4X_C^2} \quad (2)$$

$2 > 1 \Rightarrow$ Bulb brightness will decrease.

M 1

Concept: Adding any one additional component in parallel (be it a resistor or capacitor or inductor) reduces the total impedance.

While adding any component in series increases the total impedance.

Just treat capacitor as another resistor and we shall get:

$$V' < V \text{ and } A' > A$$

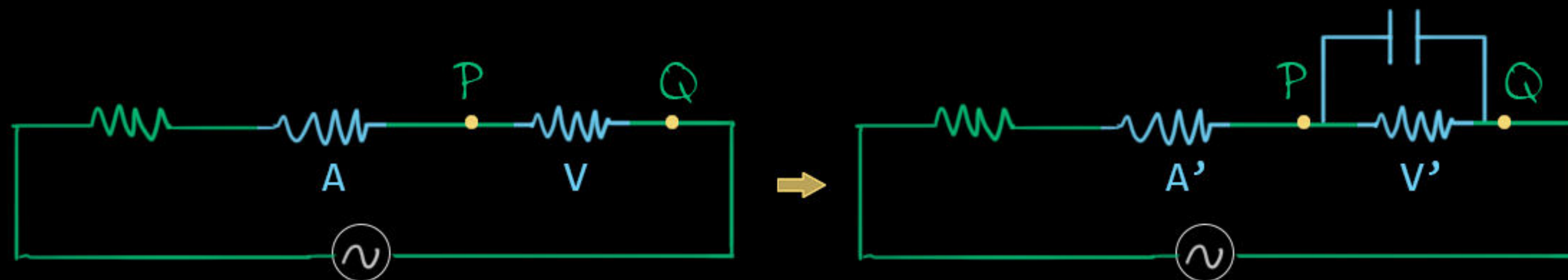
14. If an AC ammeter, an AC voltmeter and a resistor are connected in series and the combination is connected across an AC voltage source, the instruments read A and V respectively. The instruments are purely resistive in nature. If a capacitor is now connected across the voltmeter

(a) both A and V will increase

(b) both A and V will decrease

(c) A will decrease, V will increase

✓(d) A will increase, V will decrease

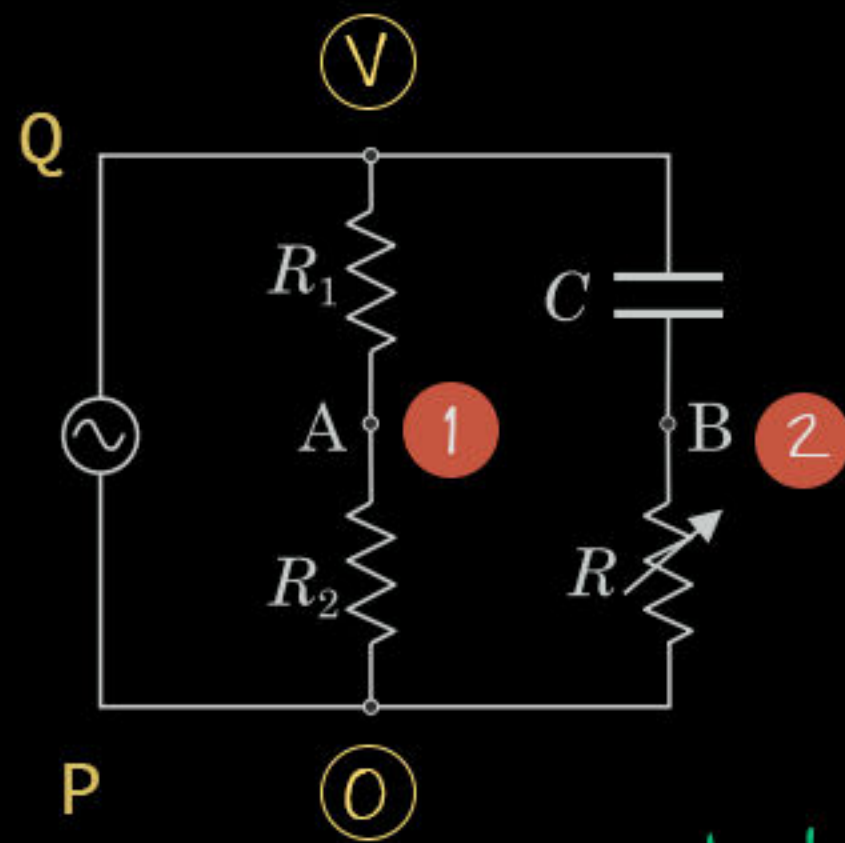


M 2

Think extreme.

If the frequency is pretty high, capacitor will act shorted and V' will be zero.

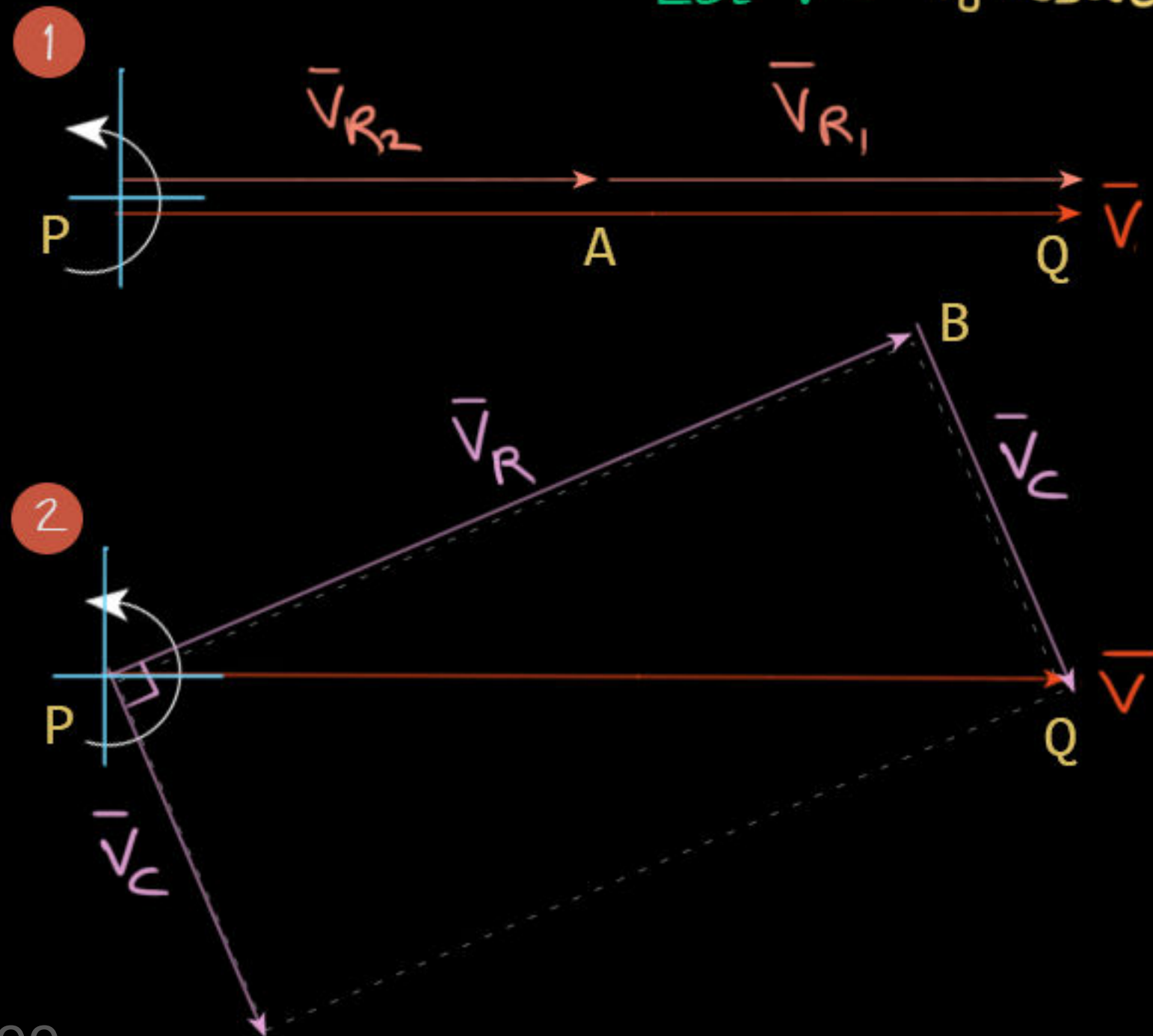
$$\therefore V' < V \text{ and } A' > A$$



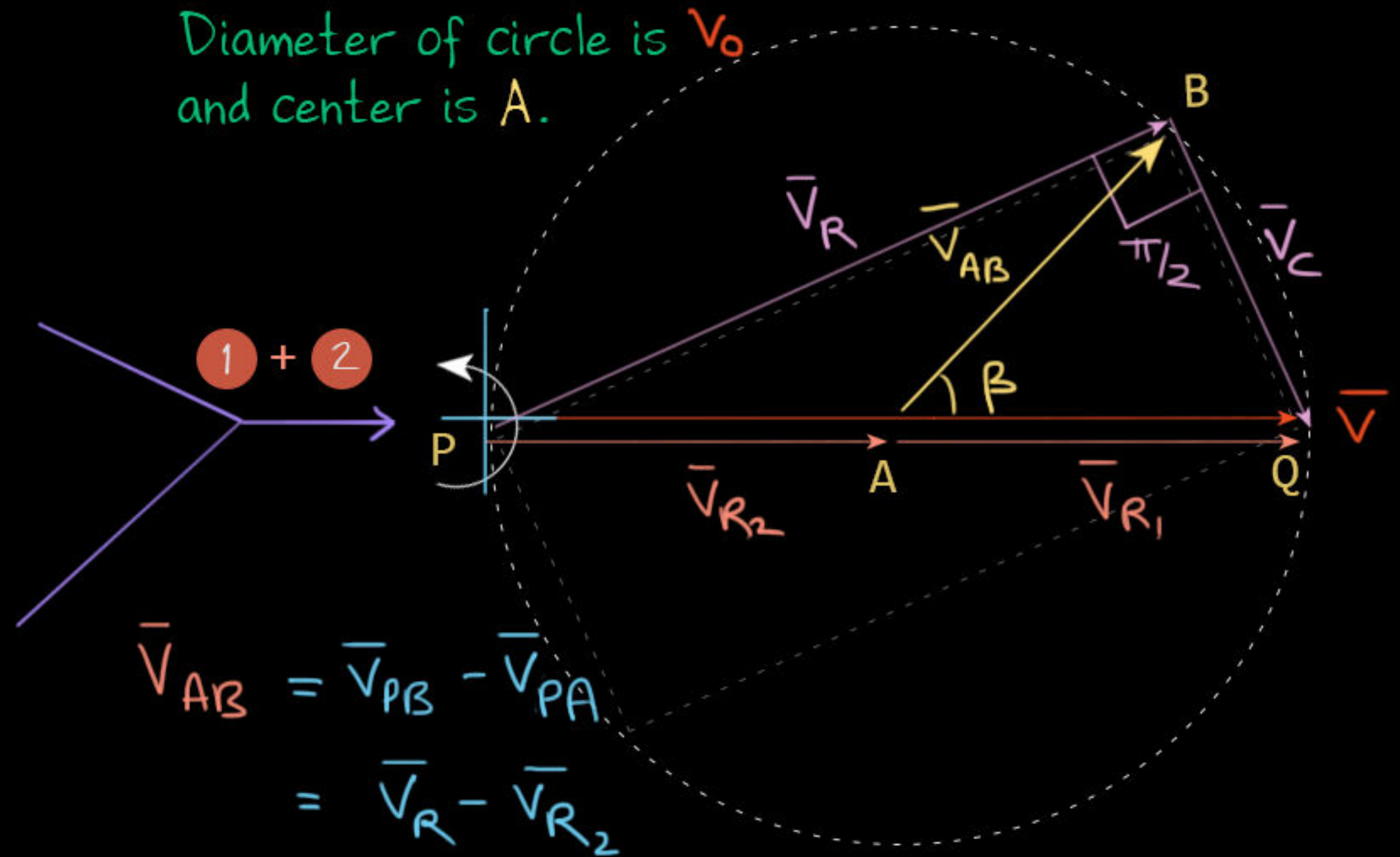
15. A circuit shown consists of two resistances R_1 and R_2 of equal and fixed values, a variable resistance R , a capacitor of capacitance C and an alternating voltage source. If the variable resistance is made to vary over a wide range, how does the peak voltage between the terminals A and B change?

- (a) First decreases then increases. (b) First increases then decreases.
(c) Decreases to a constant value. ✓(d) Remains unchanged.

Let $V = V_0 \cos \omega t$

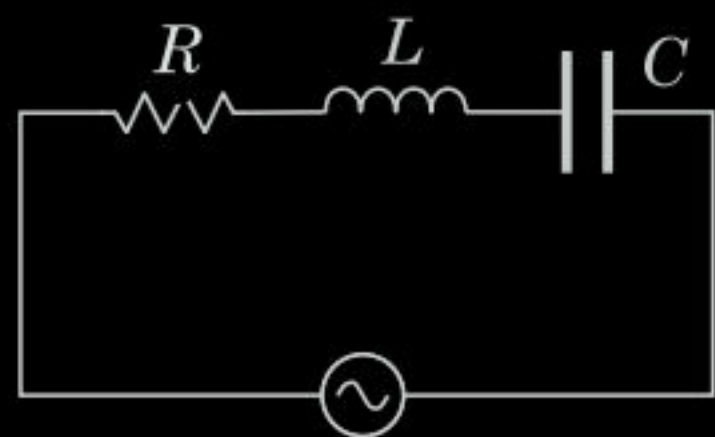


Diameter of circle is V_0
and center is A.



$$\begin{aligned}\bar{V}_{AB} &= \bar{V}_{PB} - \bar{V}_{PA} \\ &= \bar{V}_R - \bar{V}_{R_2} \\ &= \frac{V_0}{2} \cos(\omega t + \beta)\end{aligned}$$

∴ Constant peak voltage $V_0/2$.
Youtube / Arihant Senior



16. Consider the following series **RLC** circuit. Here the symbols R , L and C denote resistance, inductance and capacitance respectively. If frequency of the supply voltage is increased from a very low value to a very high value, the maximum voltages across L and C are measured at frequencies f_L and f_C respectively. Which of the following statements is correct?

✓ (d) $f_C < \frac{1}{2\pi\sqrt{LC}}$ and $f_L > \frac{1}{2\pi\sqrt{LC}}$

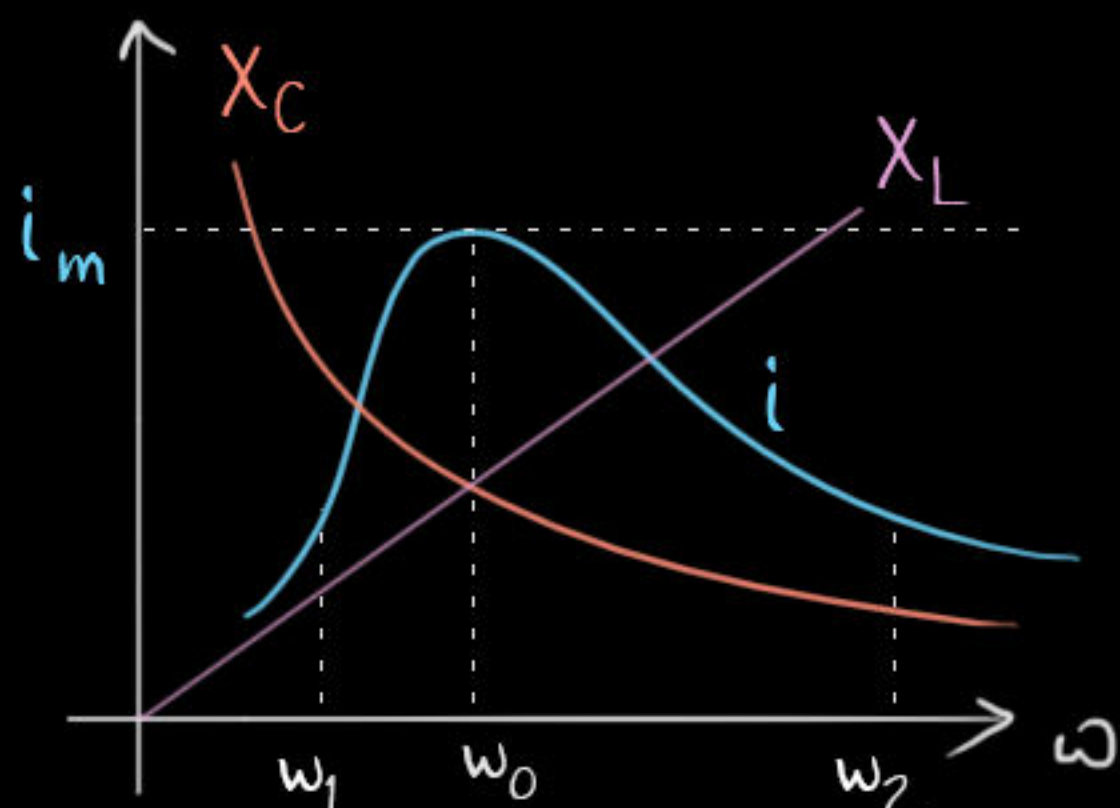
At resonant frequency ω_0 , $X_C = X_L$, both capacitor and inductor register equal voltage: $iX_C = iX_L$.

At lower frequency ω_1 , X_C is really high and therefore maximum voltage across it is found there (i.e V_0).

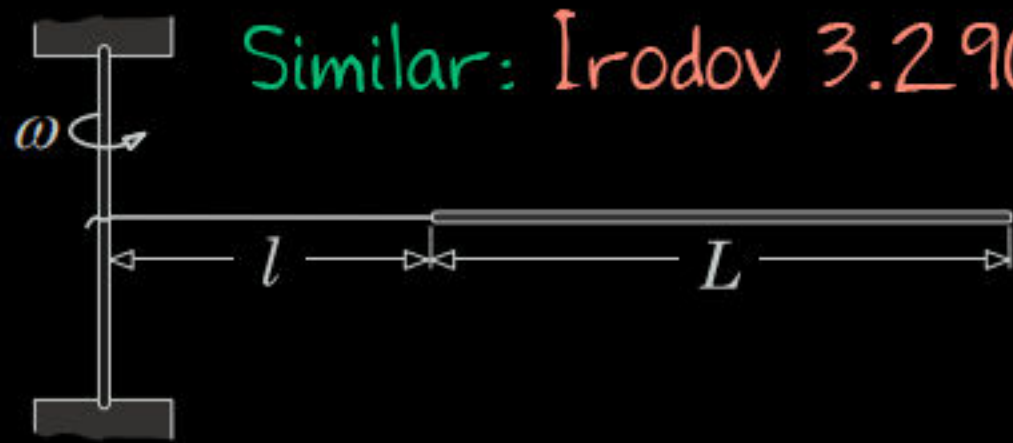
$$\therefore f_C < f_0 = \frac{1}{2\pi\sqrt{LC}}$$

At higher frequency ω_2 , X_L is really high and therefore maximum voltage across it is found there (i.e V_0).

$$\therefore f_L > f_0 = \frac{1}{2\pi\sqrt{LC}}$$



↓
Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

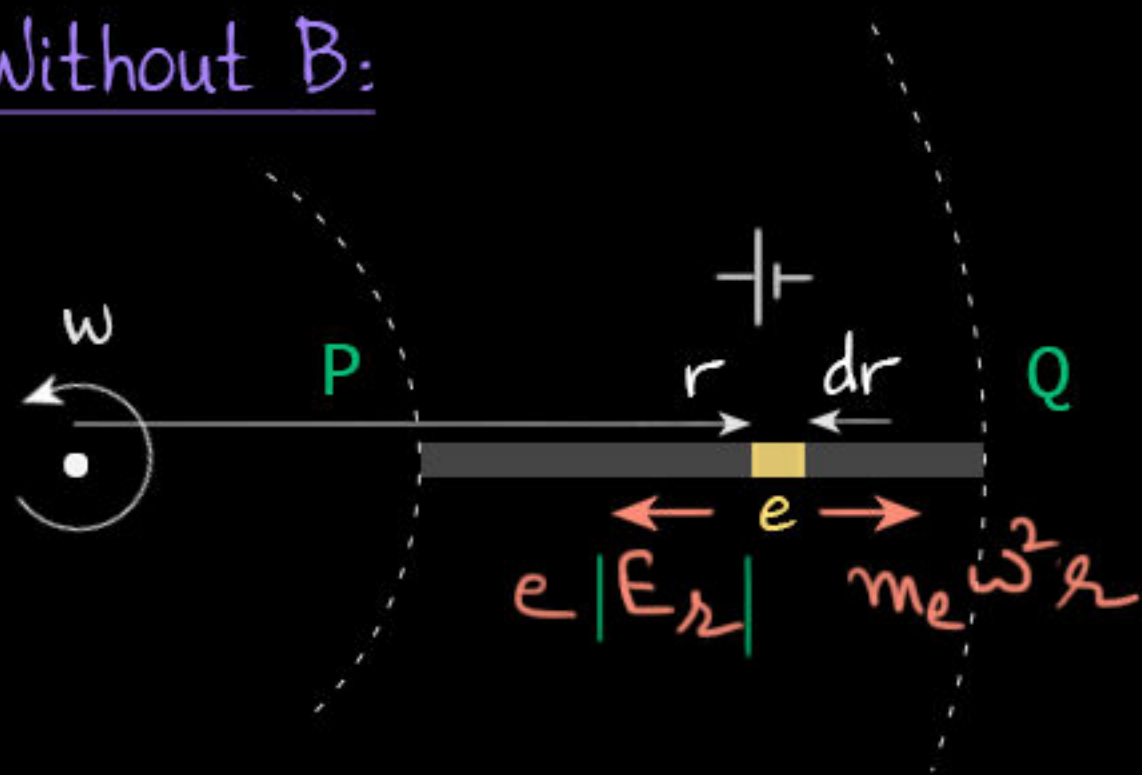


Similar: Irodov 3.290

1. A metal rod of length L , attached to a vertical shaft by a non-conducting thread of length l is rotating in a horizontal plane with a constant angular velocity ω . Find potential difference between the ends of the rod. Now a uniform magnetic field in vertically ~~upward~~ direction is switched on. What should the magnitude of the flux density of the magnetic field be so that the potential difference between the ends of the rod becomes double? Neglect gravity and denote the mass and the charge on an electron by m and e respectively.

→ downward (to match the answer)

Without B:

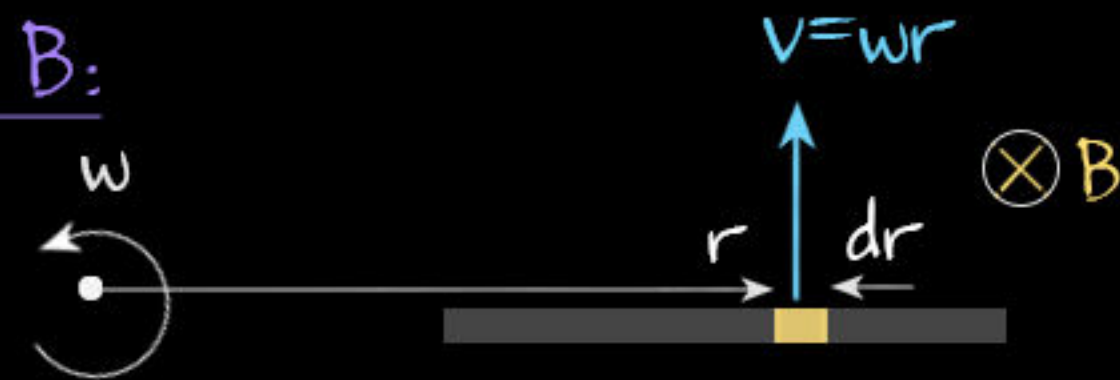


Electrons will drift towards the rim due to centrifugal force until electric field thus generated cancels it. That is: $e E_r = m_e \omega^2 r$

$$\therefore dV_\omega = E_r dr = \frac{m_e \omega^2}{e} r dr \quad \text{--- 1}$$

$$\Rightarrow V_\omega = \int_l^{l+L} \frac{m_e \omega^2}{e} r dr = \frac{m_e \omega^2}{2e} (L^2 + 2lL)$$

With B:



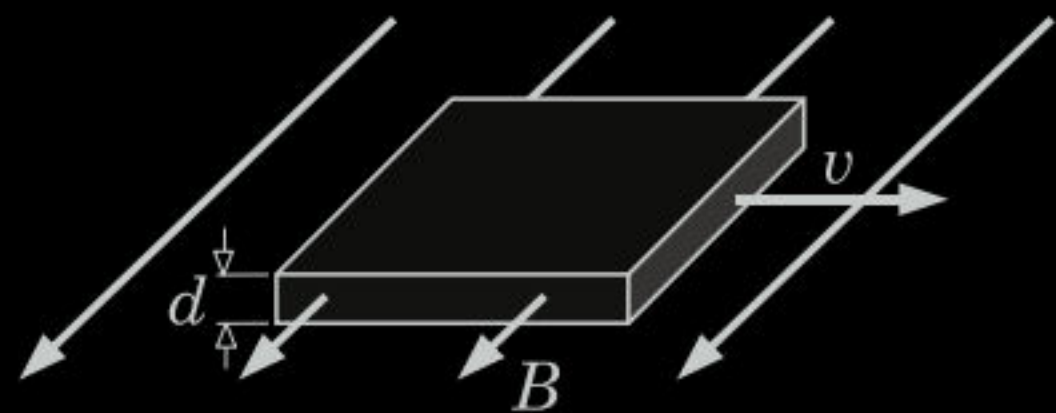
For same element, due to field B alone:

$$dV_B = B (\omega r) dr \quad \text{--- 2}$$

Total potential difference is doubled,
∴ From 1 and 2 (reusing same $\int r dr$):

$$\frac{m_e \omega^2}{e} = B \omega \Rightarrow B = \frac{m_e \omega}{e}$$

Note: If B was upwards, answer would be: $\frac{3m_e \omega}{e}$
With Q at higher potential.



We know that Electric field is defined as the force on a unit charge particle at rest.

Now we are in a train (v) with Electric (E) and magnetic field (B) all around (directions shown).

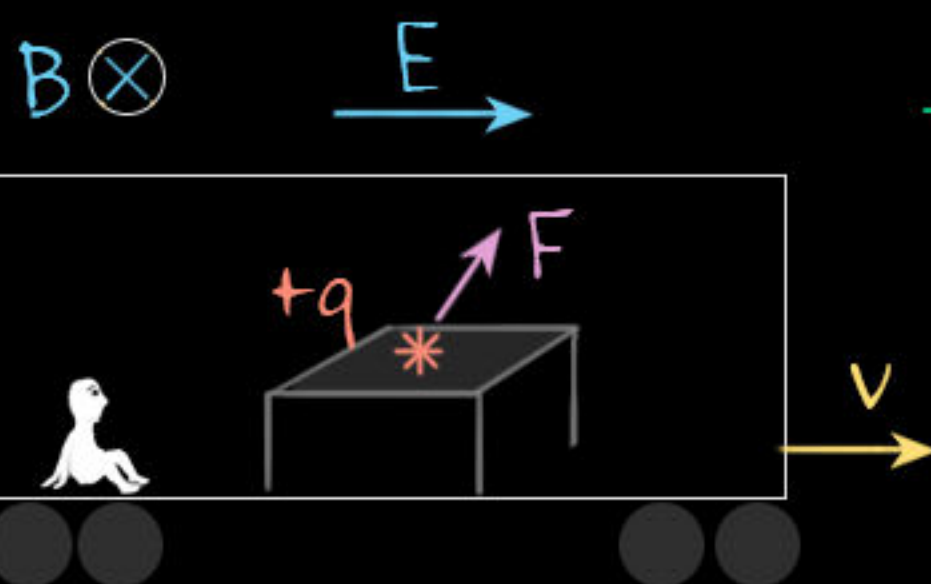
To us train is at rest.

We observe $+q$ kept on a table. It experiences a force. So we assume an electric field $\vec{F}/q$.

Lets call it E'

2. A bar of thickness d is made to move with a velocity v perpendicular to a uniform magnetic field of induction B . Top and bottom faces of the bar are parallel to the lines of magnetic field as shown in the figure.

- Find surface charge densities on the upper and the bottom faces of the bar, assuming material of the bar to be a perfect conductor.
- Find the surface charge densities on the upper and the bottom faces of the bar, assuming material of the bar to be a dielectric of dielectric constant ϵ_r .



To observer on train: $\vec{F} = q\vec{E}'$ ①

To observer on ground: $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$ ②

From 1,2: $\vec{E}' = \vec{E} + (\vec{v} \times \vec{B})$

This is the fundamental of field transformation, where effects of magnetic field are seen as effects of electric field due to moving frame.

Sol:  $\Rightarrow E' = vB \downarrow$
To a moving observer (v), an external field E' is present, and bar is at rest.

by Ankit Singhvi

A conductor

$$\frac{\sigma}{\epsilon_0} = E' = vB$$

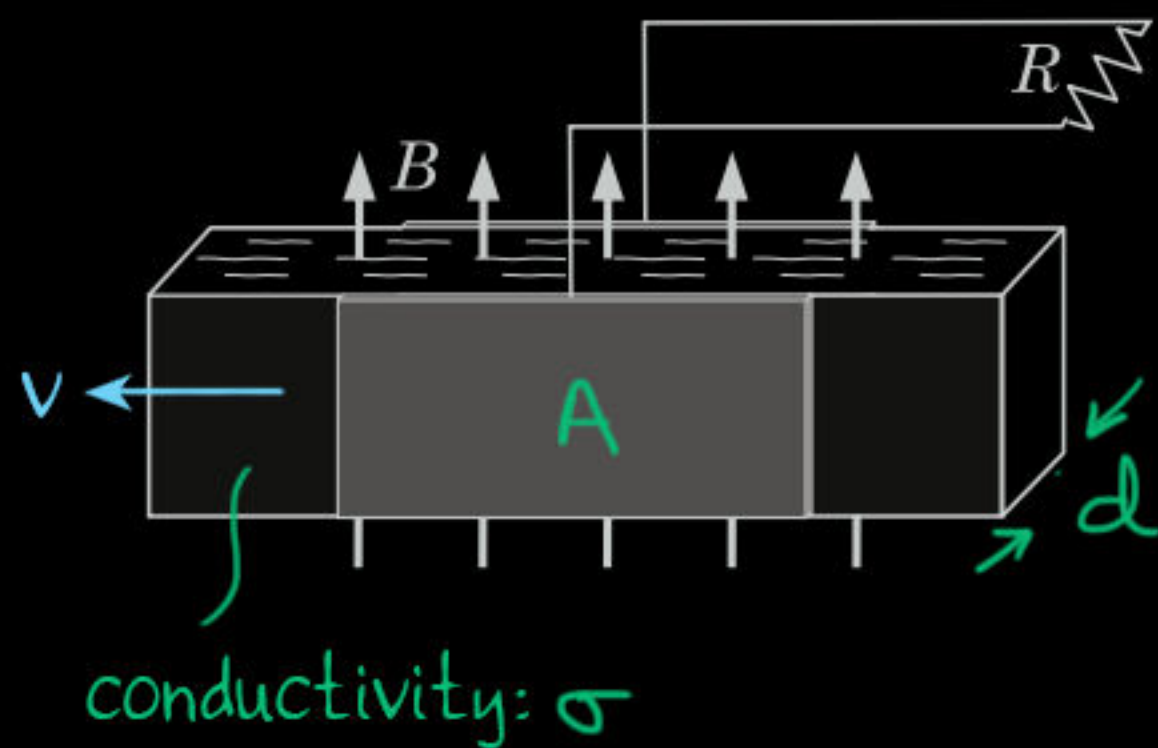
$$\Rightarrow \sigma = Bv\epsilon_0 //$$

B dielectric

$$E' - \frac{\sigma}{\epsilon_0} = \frac{E'}{\epsilon_r}$$

$$\Rightarrow \sigma = vB\epsilon_0 \left(1 - \frac{1}{\epsilon_r}\right) //$$

Youtube / Arihant Senior



Resistance r , of fluid between plates is:

$$r = \frac{\rho l}{A} = \frac{1}{\sigma} \frac{d}{A}$$

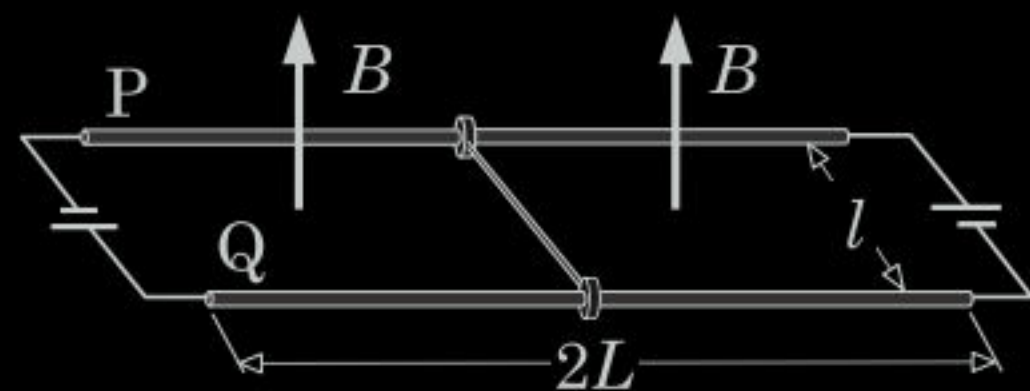
3. The basic principle of magneto-hydrodynamic (MHD) generators used for generating electricity is described here by a simple model that consists of a parallel plate capacitor immersed in a stream of a conducting liquid in presence of a uniform magnetic field of induction B pointing perpendicular to the stream and parallel to the plates. Area of each plates is A , distance between the plates is d and electrical conductivity of the liquid is σ . The liquid flows between the plates with a constant velocity v towards the left. If a resistance R is connected across the capacitor plates how much power will it dissipate? From where does this power come?

$$P_R = \frac{Bvd}{R+r} \cdot \frac{d}{\sigma A} \cdot \frac{Bvd}{R+r} \cdot R = \frac{B^2 v^2 d^2}{\left(R + \frac{d}{\sigma A}\right)^2} \cdot R = \frac{B^2 v^2 d^2 R \sigma^2 A^2}{(R \sigma A + d)^2}$$

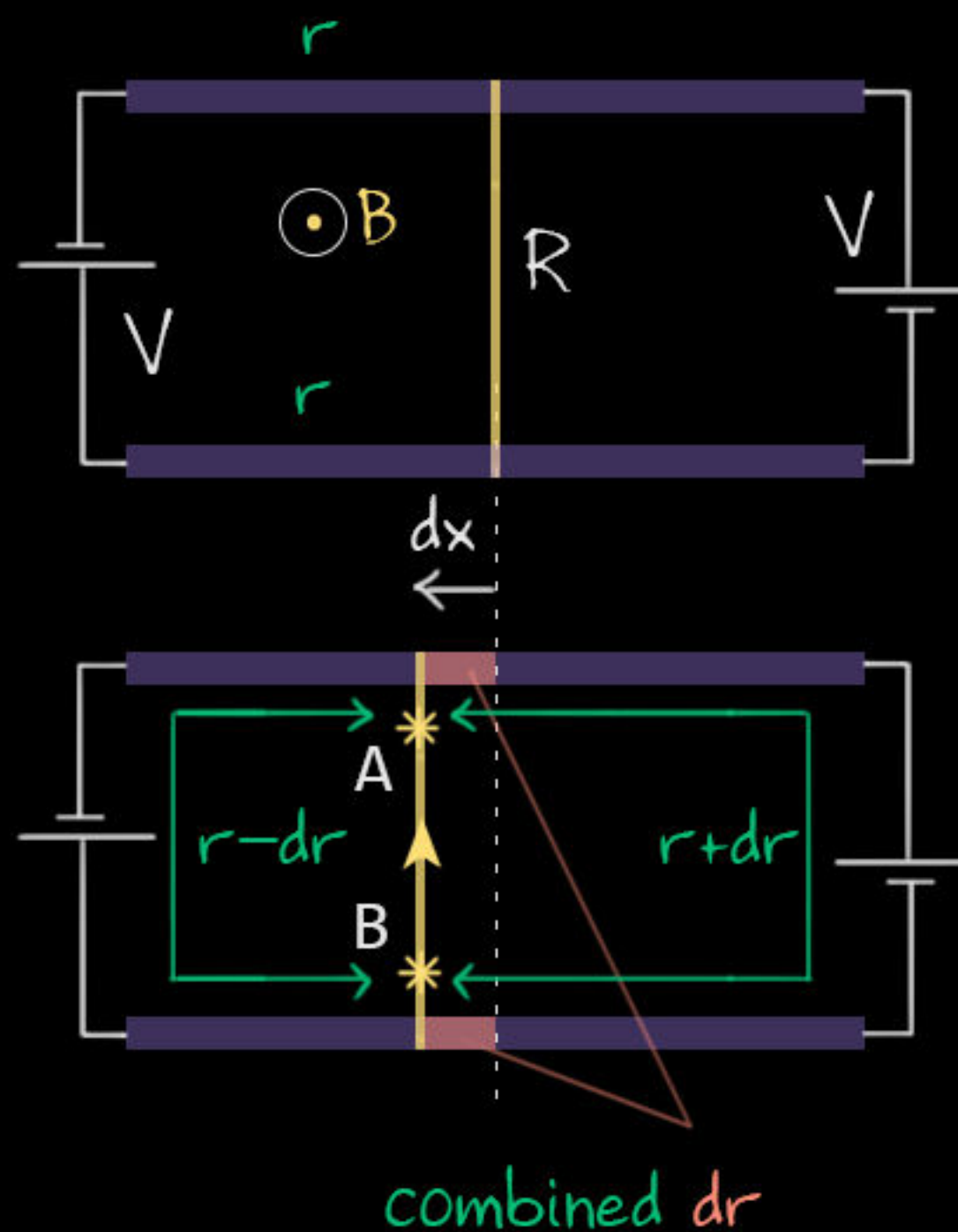
As current flows through stream, due to B it will experience force towards right.

To maintain v therefore someone needs to push the water towards left.

So power comes from whoever is pushing the water towards left.

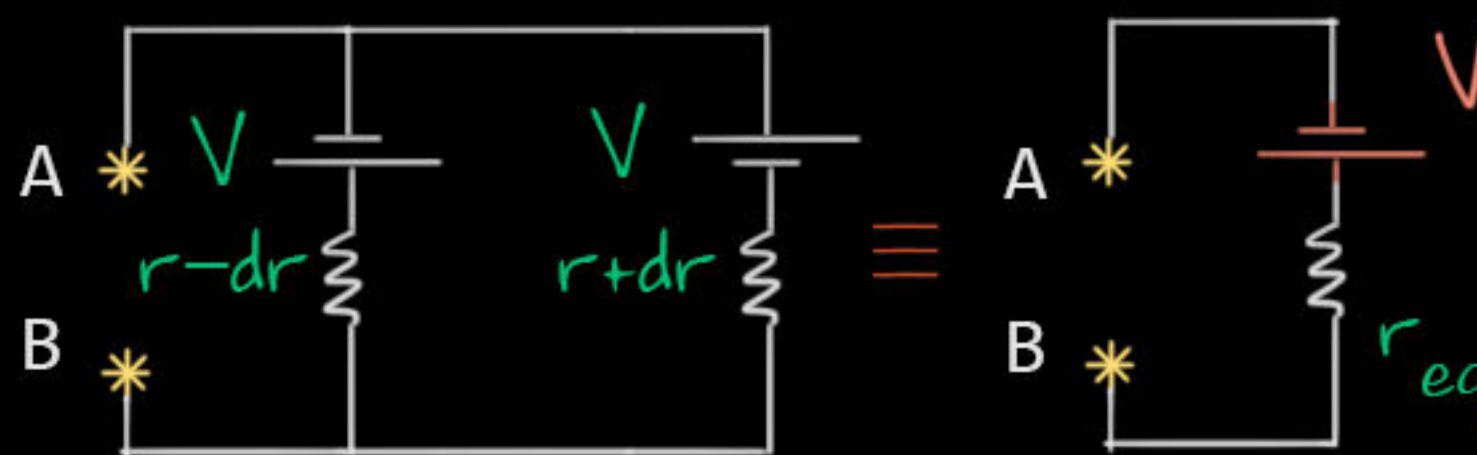


Let the resistance of one rail is r .



4. Two parallel rails P and Q each of length $2L$ and resistance per unit length ρ are fixed in a horizontal plane a distance l apart in presence of a uniform vertical magnetic field of induction B and two ideal batteries each of terminal voltage V_0 are connected between the ends of the rails as shown in the figure. Two light sleeves rigidly connected by a conducting rod of length l and resistance R and mass m can slide without friction on the rails. The rod is slightly displaced from mid position and then released. Find the period of small oscillations of the rod. Neglect potential drop at the contacts and damping.

Using Thevenins



$$V_{eq} = \frac{V}{r-dr} + \frac{(-V)}{r+dr} = \frac{drV}{r}$$

$$r_{eq} = \frac{(r+dr)(r-dr)}{r+dr+r-dr} \approx \frac{r}{2}$$

$$r \rightarrow \rho \cdot 2L$$

$$dr \rightarrow \rho 2dx$$

$$i = \frac{V_{eq}}{R+r_{eq}} = \frac{V dr}{r \left(R + \frac{r}{2} \right)} = \frac{V dx}{L(R+\rho L)} \Rightarrow F = B i l = \frac{B V l}{L(R+\rho L)} dx$$

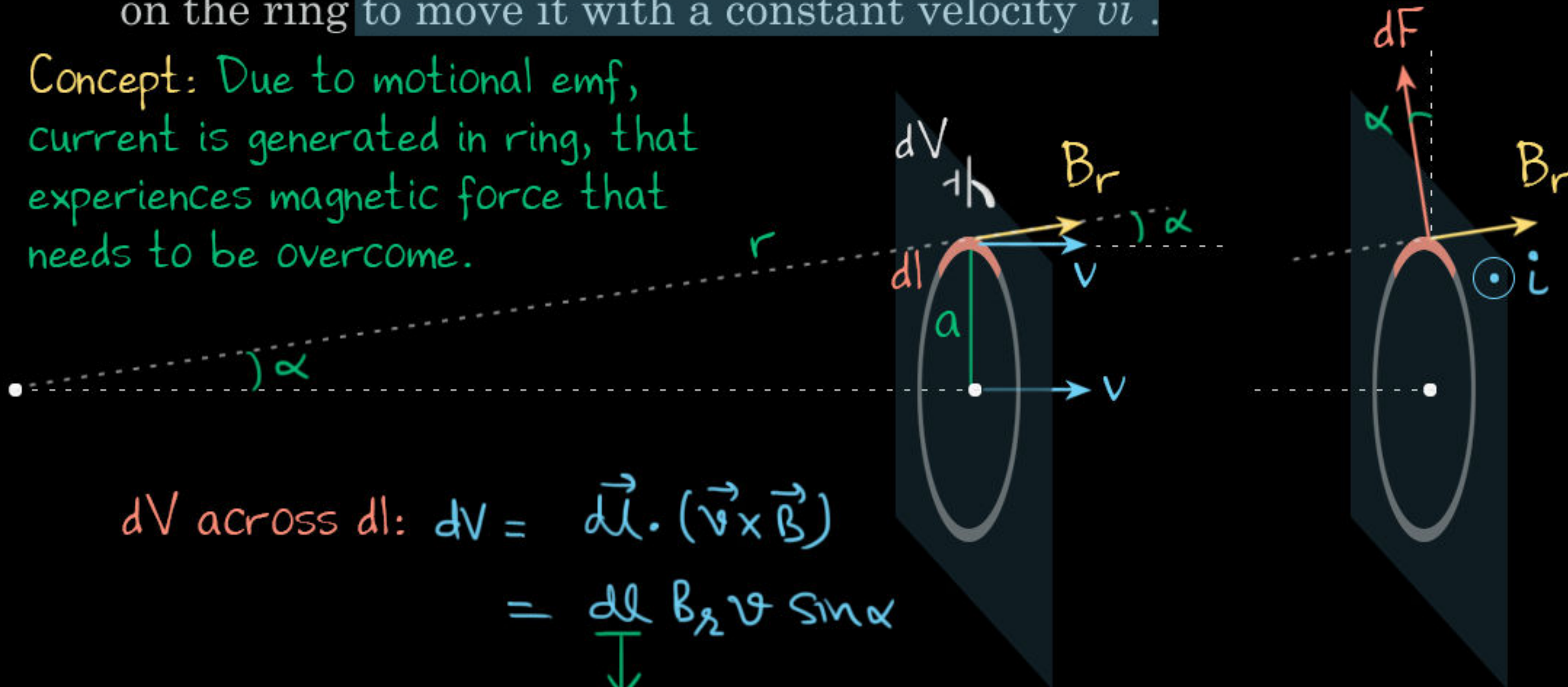
k of SHM

Youtube / Arihant Senior

by Ankit Singhvi

5. In a region of space, a radial magnetic field is established, flux density B of which varies with distance r from the origin of a coordinate system according to the law $B = \beta/r^2$. Here β is a positive constant. A small conducting ring of radius a and resistance R is placed parallel to the y - z plane with its centre at point $(x, 0, 0)$. Find the force that must be applied on the ring to move it with a constant velocity $v\hat{i}$.

Concept: Due to motional emf, current is generated in ring, that experiences magnetic force that needs to be overcome.



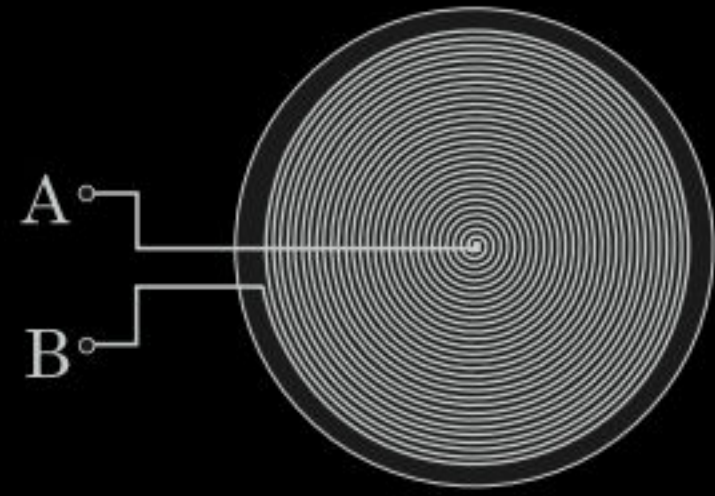
$$\begin{aligned}
 dV \text{ across } dl: dV &= \vec{dl} \cdot (\vec{v} \times \vec{B}) \\
 &= dl B_r v \sin \alpha \\
 \Rightarrow V &= 2\pi a B_r v \sin \alpha \\
 \Rightarrow i &= \frac{V}{R} = \frac{2\pi a B_r v \sin \alpha}{R} \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 d\vec{F} &= i (\vec{dl} \times \vec{B}) \\
 d\vec{F}_{\text{net}} &= i \underbrace{dl B_r}_{\substack{\text{As } Y \text{ component will} \\ \text{cancel out}}} \underbrace{\sin \alpha}_{\substack{\downarrow \\ \frac{a}{r^2}}} (-\hat{i}) \\
 F_{\text{net}} &= i \underbrace{2\pi a}_{(1)} B_r \sin \alpha \quad \left(\frac{\beta}{r^2} \right) \quad \left(\frac{a}{r^2} \right)
 \end{aligned}$$

$$= \frac{4\pi^2 \beta^2 a^4 v}{r^6}$$

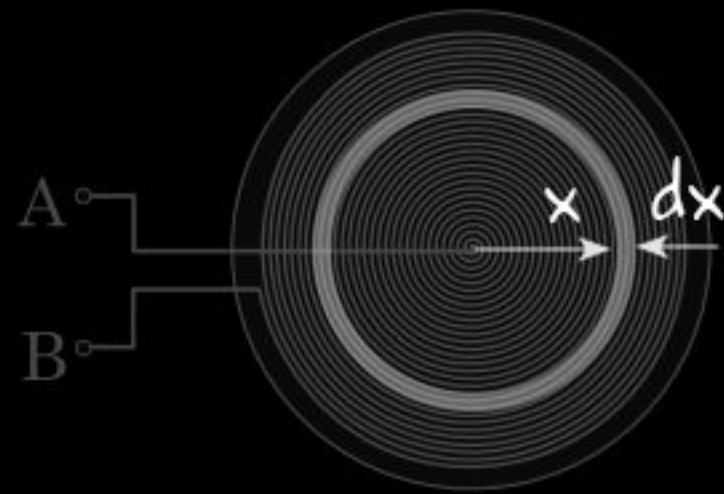
$$r \rightarrow \sqrt{x^2 + a^2}$$

$$F_{\text{net}} = \frac{4\pi^2 \beta^2 a^4 v}{(x^2 + a^2)^3}$$



6. A planar spiral coil of outer radius R consisting of N closely wound turns of a thin conducting wire is placed in a uniform magnetic field with its plane perpendicular to the field. The magnetic field is confined in a circular region shown by shaded area in the figure. If flux density B of the magnetic field varies with time t as $B = B_0 \cos(\omega t)$, find electromotive force induced between the terminals A_1 and A_2 of the coil.

Similar: Irodov 3.306

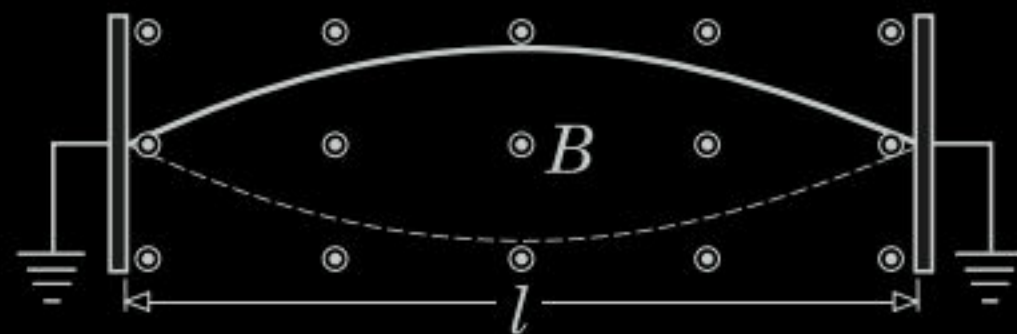


In elemental ring $d\phi = B(\pi x^2) \frac{dx}{R} \cdot N$

Turns in small ring

$$\Rightarrow \phi = \int d\phi = \frac{BN\pi}{R} \int_0^R x^2 dx = \frac{BN\pi R^2}{3}$$

$$\therefore e = \frac{d\phi}{dt} = \frac{N\pi R^2}{3} \frac{dB}{dt} = \frac{N\pi R^2}{3} B_0 \omega \sin \omega t //$$

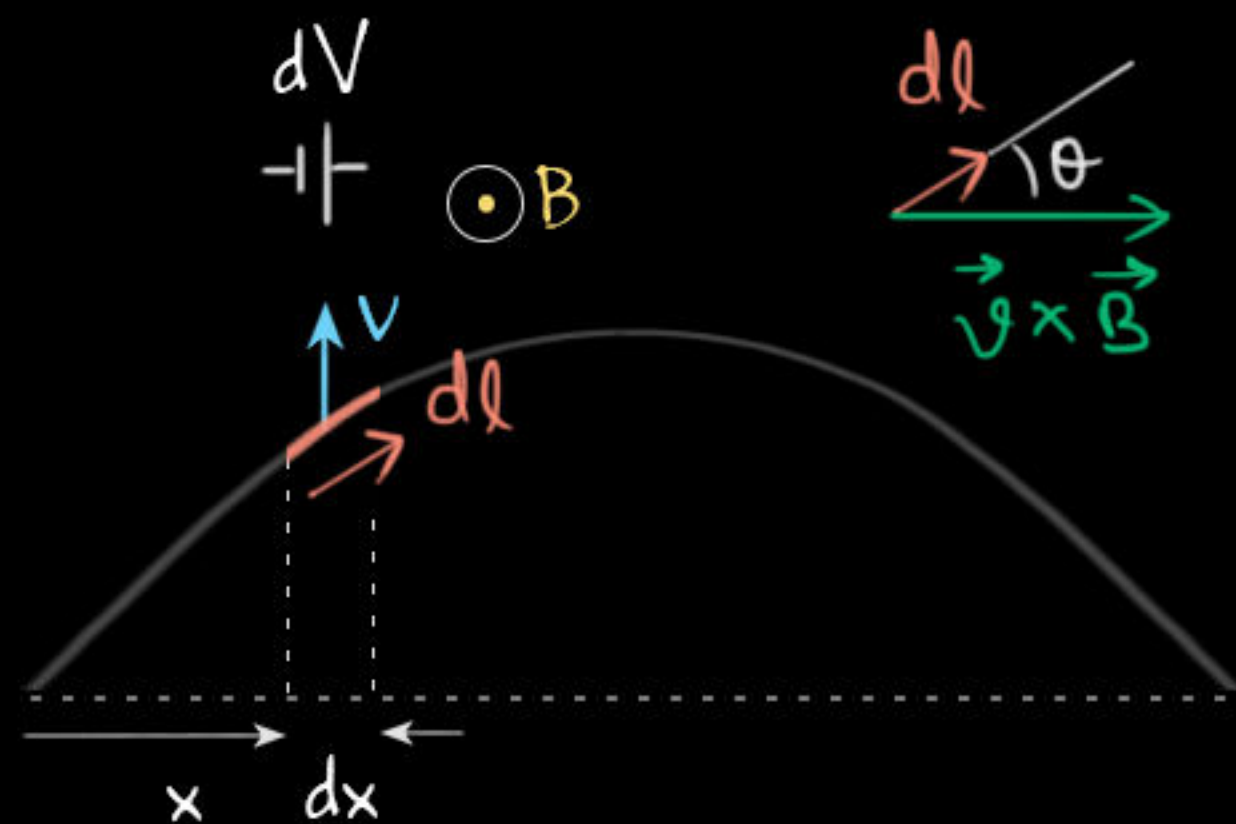


7. A conducting wire of length l and cross-section area A taut between two fixed grounded metallic supports is vibrating in its fundamental mode with a frequency ν and amplitude a in presence of a uniform magnetic field of induction B pointing normal to the plane of vibration of the string. Find an expression for electromotive force induced in the wire as a function of time t assuming the wire straight at $t = 0$.

Equation of standing wave will be:

$$y = A \sin kx \sin \omega t$$

$$\Rightarrow v = a \omega \sin kx \cos \omega t$$



At a fixed time t :

$$\begin{aligned} dV &= d\vec{l} \cdot (\vec{v} \times \vec{B}) \\ &= dl (vB) \cos \theta \\ &= vB dx \end{aligned}$$

$$\begin{aligned} \therefore V &= \int a \omega \sin kx \cos \omega t B dx \\ &= a B \omega \cos \omega t \int_0^{\lambda/2} \sin kx dx \\ &= a B \omega \left(\frac{\lambda}{\pi} \right) \cos \omega t \\ &= 4 \nu l a B \cos(2\pi \nu t) \end{aligned}$$

Concept: Only moving part is the disc.

So motional emf will set across disc alone. Between its center and periphery.

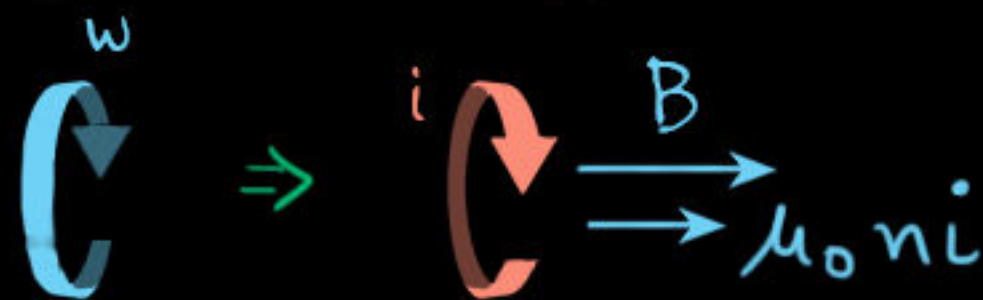
We know its given by:

$$V = \frac{B_{\text{net}} \omega r^2}{2} = iR \quad (1)$$

A B_{net} for ω shown in figure



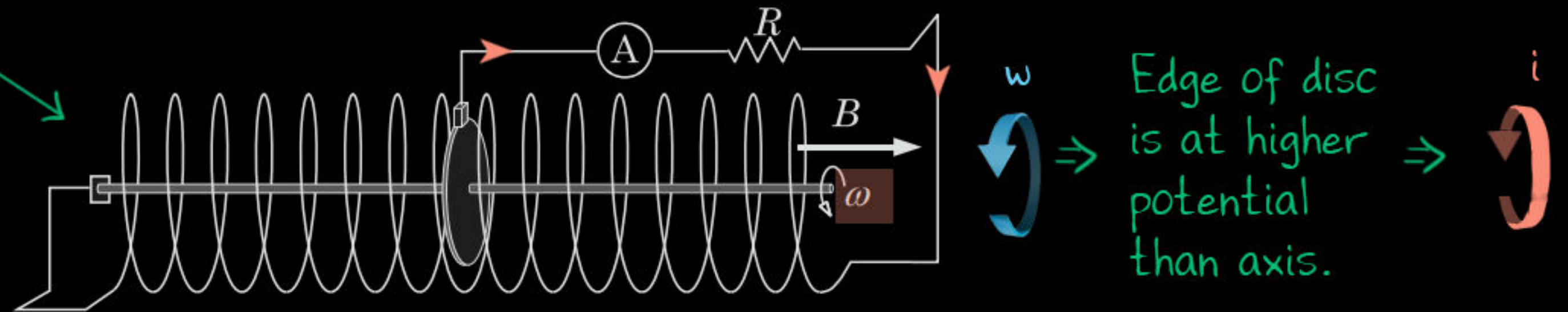
B B_{net} for opposite ω .



$$\therefore B_{\text{net}} = B \mp \mu_0 n i \quad (2)$$

8. A metal disc of radius r is affixed coaxially on a thin conducting shaft that is mounted along the axis of a long solenoid. One end of the solenoid is electrically connected to the periphery of the disc with the help of a graphite brush through an ideal ammeter and a resistance R ; the other end of the solenoid is electrically connected with one end of the shaft. A uniform magnetic field of induction B pointing along the axis of the solenoid towards the right is switched on.

Consider the solenoid an ideal one having n turns per unit of its length, find an expression for the reading of the ammeter in a steady state for the direction of rotation of the disc shown as well as its opposite one.

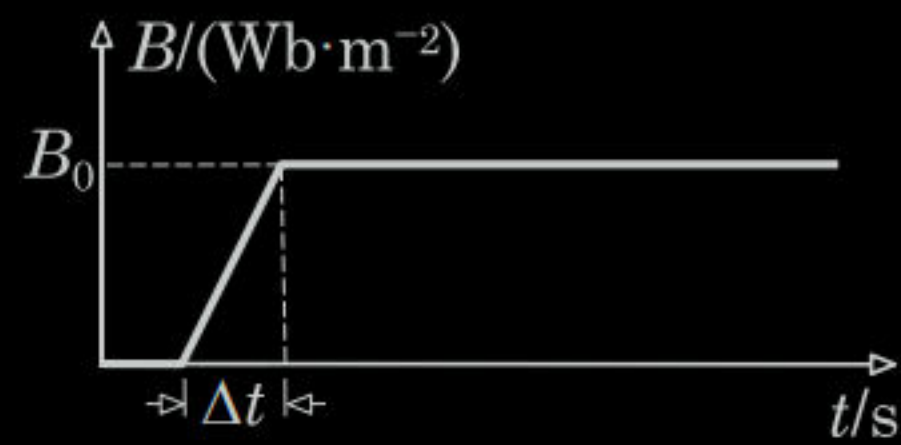


$$\text{From 1,2: } iR = (B \mp \mu_0 n i) \frac{\omega r^2}{2}$$

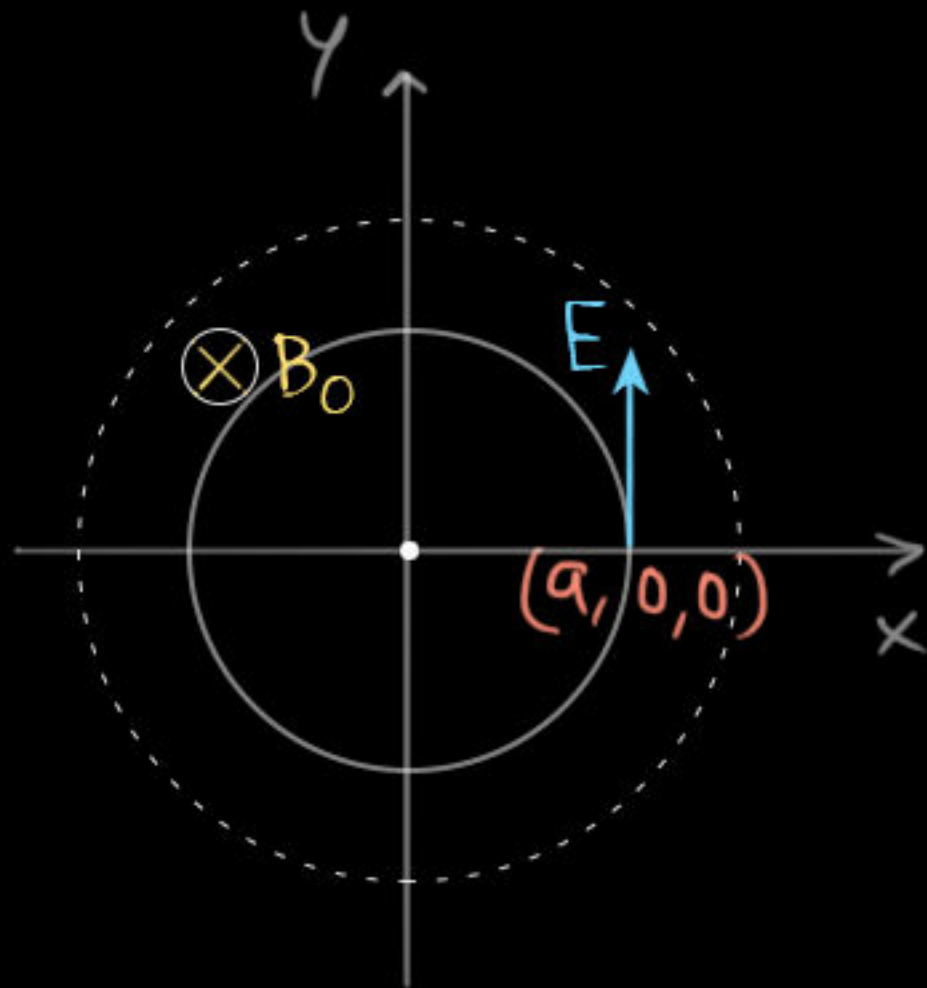
$$\Rightarrow i = \frac{B}{\frac{\omega r^2}{2} \pm \mu_0 n}$$

Youtube / Arihant Senior

by Ankit Singhvi



9. A particle of mass m and charge q is at rest at a point $(a, 0, 0)$ of a coordinate system in free space. A uniform magnetic field that varies with time t according to the given graph and points in the negative z -direction is switched on in a cylindrical region of radius R ($R > a$) coaxial with the z -axis. If the time interval Δt is negligibly small, find equation of path of the particle long time after the magnetic field is switched on.



$$\text{Induced emf} = \frac{d\Phi}{dt}$$

$$\Rightarrow E(2\pi a) = \pi a^2 \left(\frac{B_0 - 0}{\Delta t} \right)$$

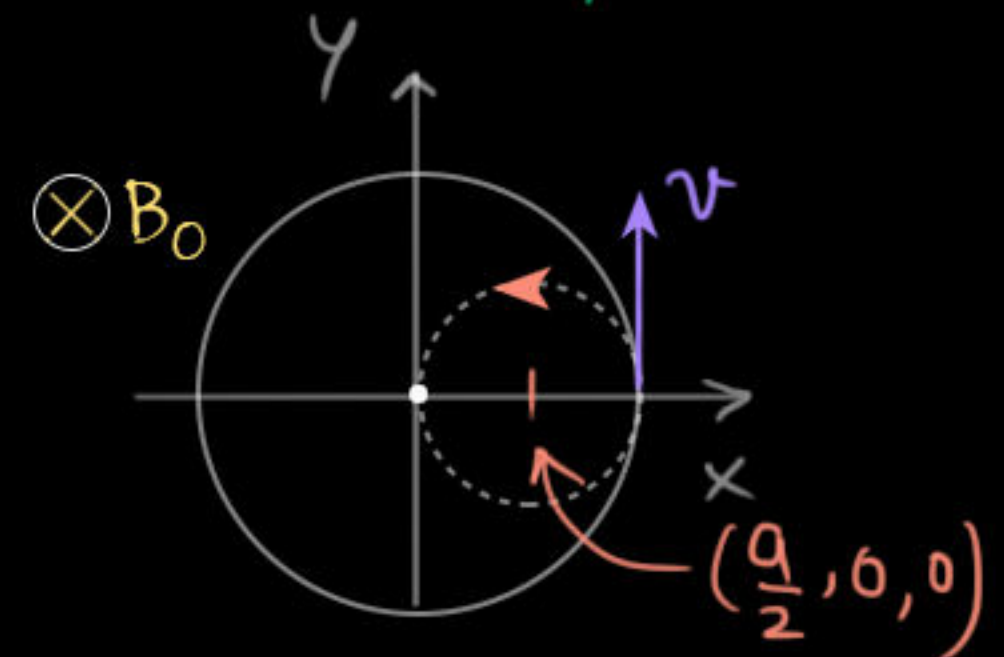
$$\Rightarrow E \Delta t = \frac{B_0 a}{2}$$

Impulse: $J = qE\Delta t = q \frac{B_0 a}{2} = mv$

$$\Rightarrow v = \frac{qB_0 a}{2m}$$

So it kick starts with velocity v in Y direction. And thus will travel in circle with radius:

$$R = \frac{mv}{qB} = \frac{m}{qB_0} \cdot \frac{qB_0 a}{2m} = \frac{a}{2}$$

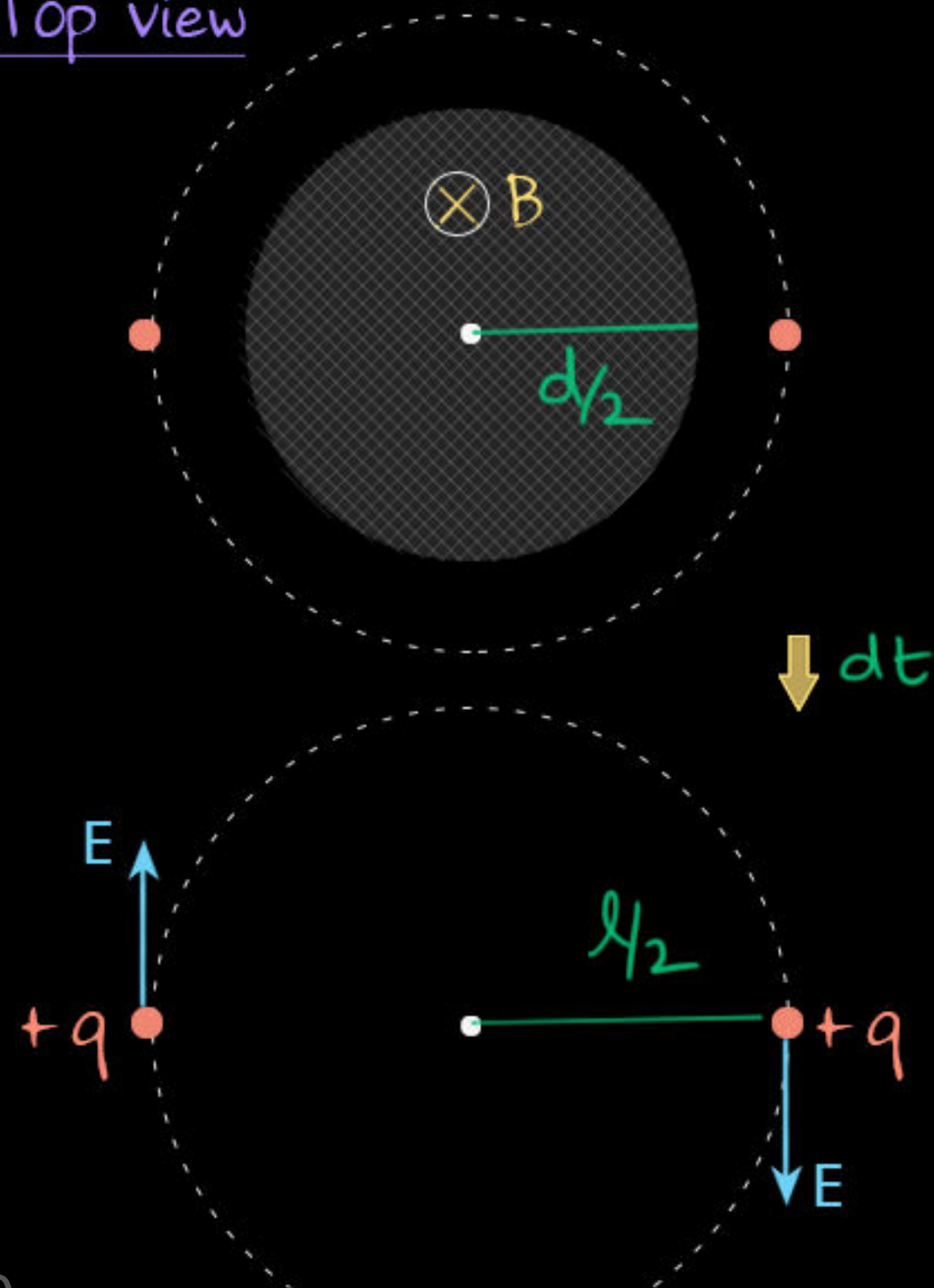


Path equation: $(y-0)^2 + (x-\frac{a}{2})^2 = (\frac{a}{2})^2$

Youtube / Arihant Senior

10. A thin light insulating rod of length l is free to rotate in a horizontal plane about a vertical axle passing through its centre. Two small balls each of mass m and charge q are affixed at the ends of the rod. This assembly is symmetrically placed between cylindrical poles N and S of an electromagnet that have established a uniform vertical magnetic field of induction B_0 . The electric supply to the coils of the electromagnet is switched off. Find the angular velocity acquired by the rod long time after the magnetic field vanishes. Assume that the field was only between the poles of the magnet.

Top view



$$\text{Induced emf} = \frac{d\Phi}{dt}$$

$$\Rightarrow E (2\pi \frac{l}{2}) = \pi (\frac{d}{2})^2 \left(\frac{B_0 - 0}{\Delta t} \right)$$

$$\Rightarrow \frac{E \Delta t}{4l} = \frac{B_0 d^2}{4l}$$

$$\text{Angular impulse: } J = 2 (q E \Delta t) \frac{l}{2} = I \omega$$

$$\Rightarrow q \frac{B_0 d^2}{4l} \cdot 2l = 2m \left(\frac{l}{2} \right)^2 \omega \Rightarrow \omega = \frac{q B_0 d^2}{2ml^2}$$

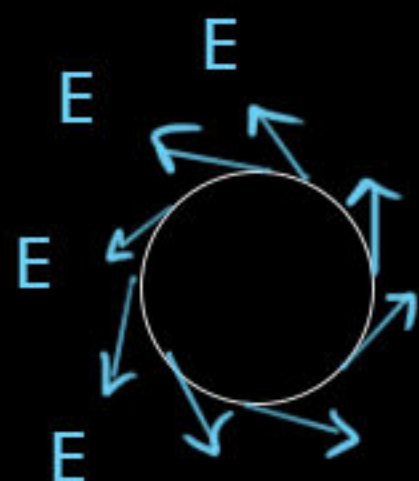
by Ankit Singhvi

Youtube / Arihant Senior



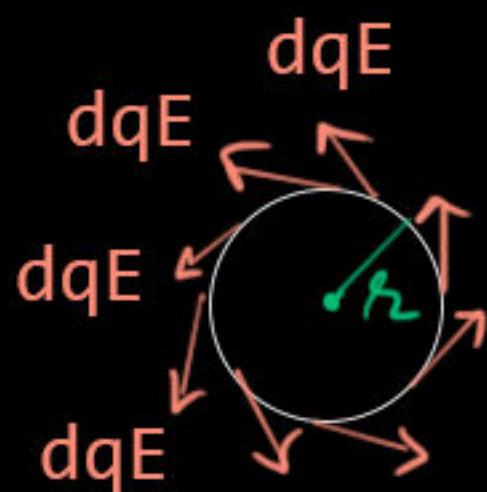
11. Curved surface of a very long hollow cylinder of radius R (much smaller than its length) is uniformly charged with surface charge density σ . At the centre of this cylinder, a small insulating ring of radius r , mass m and charge q is coaxially placed. The arrangement is in a gravity free space. If the cylinder is given an angular velocity ω_0 about its axis, how much maximum angular velocity will the ring acquire and in which direction relative to angular velocity of the cylinder?

B due to 'solenoid' = $\mu_0 j = \mu_0 \frac{q}{2\pi R \omega} \cdot \frac{1}{l} = \frac{\mu_0 \sigma R \omega}{2\pi R \omega}$



Induced electric field around the ring =

$$E = \frac{1}{2\pi R} \pi R^2 \frac{dB}{dt} = \frac{R}{2} \frac{dB}{dt} = \frac{R}{2} \frac{B}{\Delta t} = \frac{\mu_0 \sigma R^2 \omega}{2 \Delta t}$$

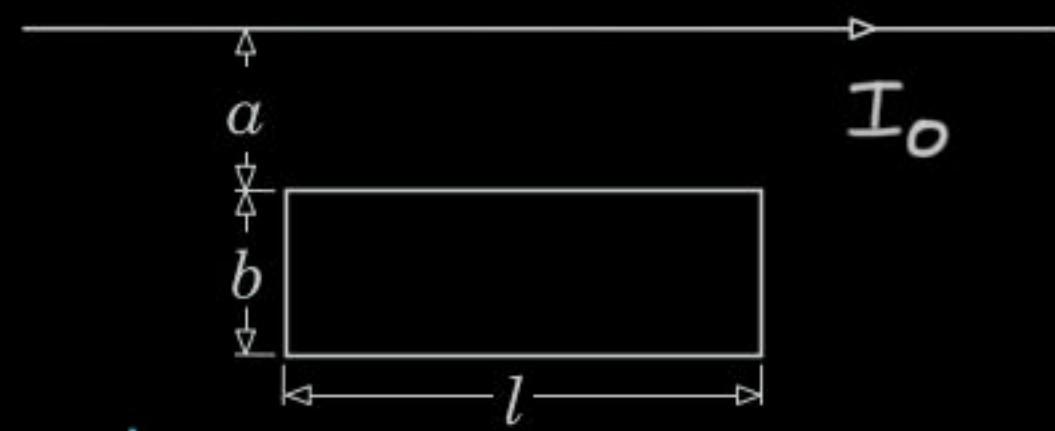


Angular impulse on the ring =

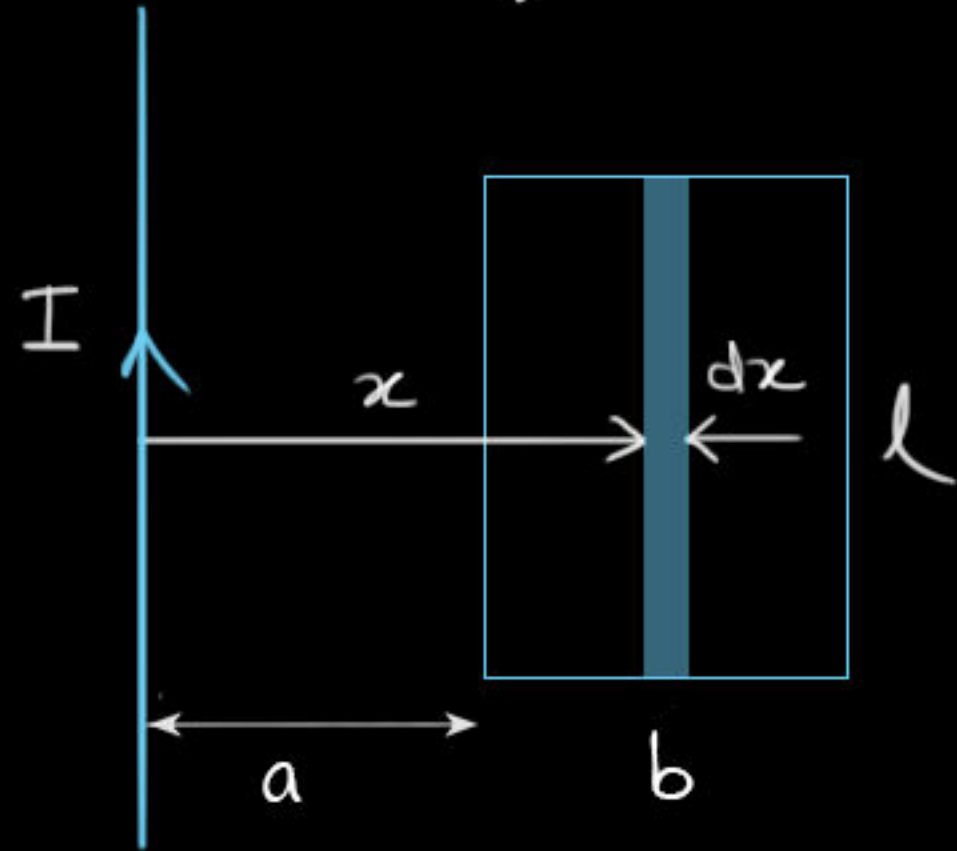
$$J = (qE \Delta t) R = I \omega'$$

$$\Rightarrow q \frac{\mu_0 \sigma R^2 \omega}{2} R = m R^2 \omega' \Rightarrow \omega' = \frac{q \mu_0 \sigma R \omega}{2m}$$

Rotating in opposite sense of cylinder



12. A rectangular loop of width b , length l , mass m and resistance R is placed close to and coplanar with a very long fixed straight conductor carrying current I_0 . One side of the loop is parallel to the straight conductor as shown in the figure. The current in the conductor is suddenly switched off. Neglecting gravity find velocity acquired by the loop.

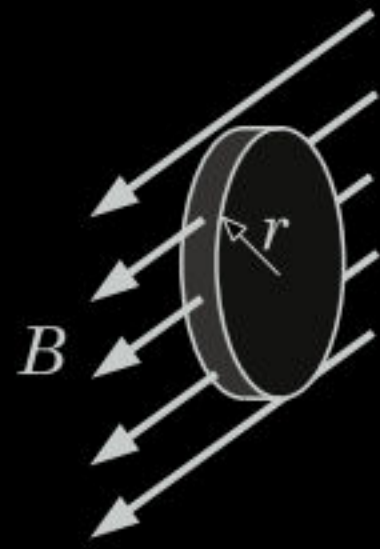


$$\text{Flux in loop: } \phi = \int d\phi = \int_a^{a+b} \left(\frac{\mu_0 I}{2\pi x} \right) l dx = \frac{\mu_0 l}{2\pi} \ln\left(\frac{a+b}{a}\right) I$$

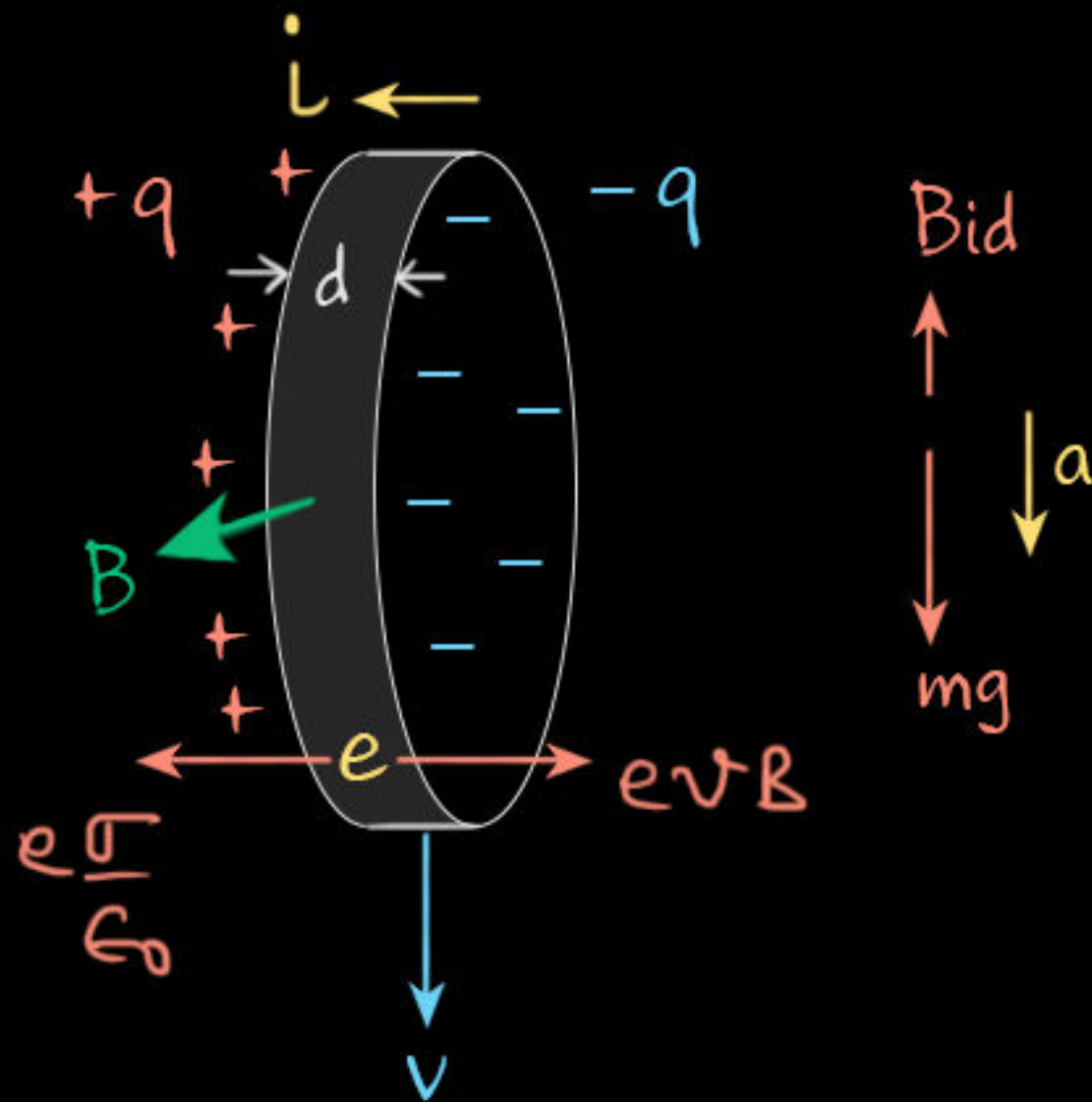
$$\text{Current in loop: } i = \frac{1}{R} \frac{d\phi}{dt} = \frac{\mu_0 l}{2\pi R} \ln\left(\frac{a+b}{a}\right) \frac{dI}{dt}$$

$$\begin{aligned} \text{Impulse on loop: } dJ &= (\vec{F}_1 + \vec{F}_2) dt = (B_1 il - B_2 il) dt \\ &= \frac{\mu_0 I}{2\pi} \left(\frac{1}{a} - \frac{1}{a+b} \right) i l dt = m dv \end{aligned}$$

$$\Rightarrow v = \frac{1}{m} \frac{\mu_0}{2\pi} \frac{b}{a(a+b)} l \frac{\mu_0 l}{2\pi R} \ln\left(\frac{a+b}{a}\right) \int_{I_0}^0 I dI \quad \text{Towards wire}$$



13. A thin conducting disc of mass m , thickness d and radius r ($r \gg d$) is released in a uniform horizontal magnetic field of induction B . The magnetic induction vector is in the plane of the disc as shown in the figure. Denoting acceleration due to gravity by g , find expression for acceleration of the disc.



Force on an electron inside the conductor will be zero.

$$\therefore \frac{e\sigma}{\epsilon_0} = e v B$$

$$\Rightarrow \frac{q}{A \epsilon_0} = v B$$

$$\Rightarrow \frac{dq}{dt} = \epsilon_0 B A \frac{dv}{dt}$$

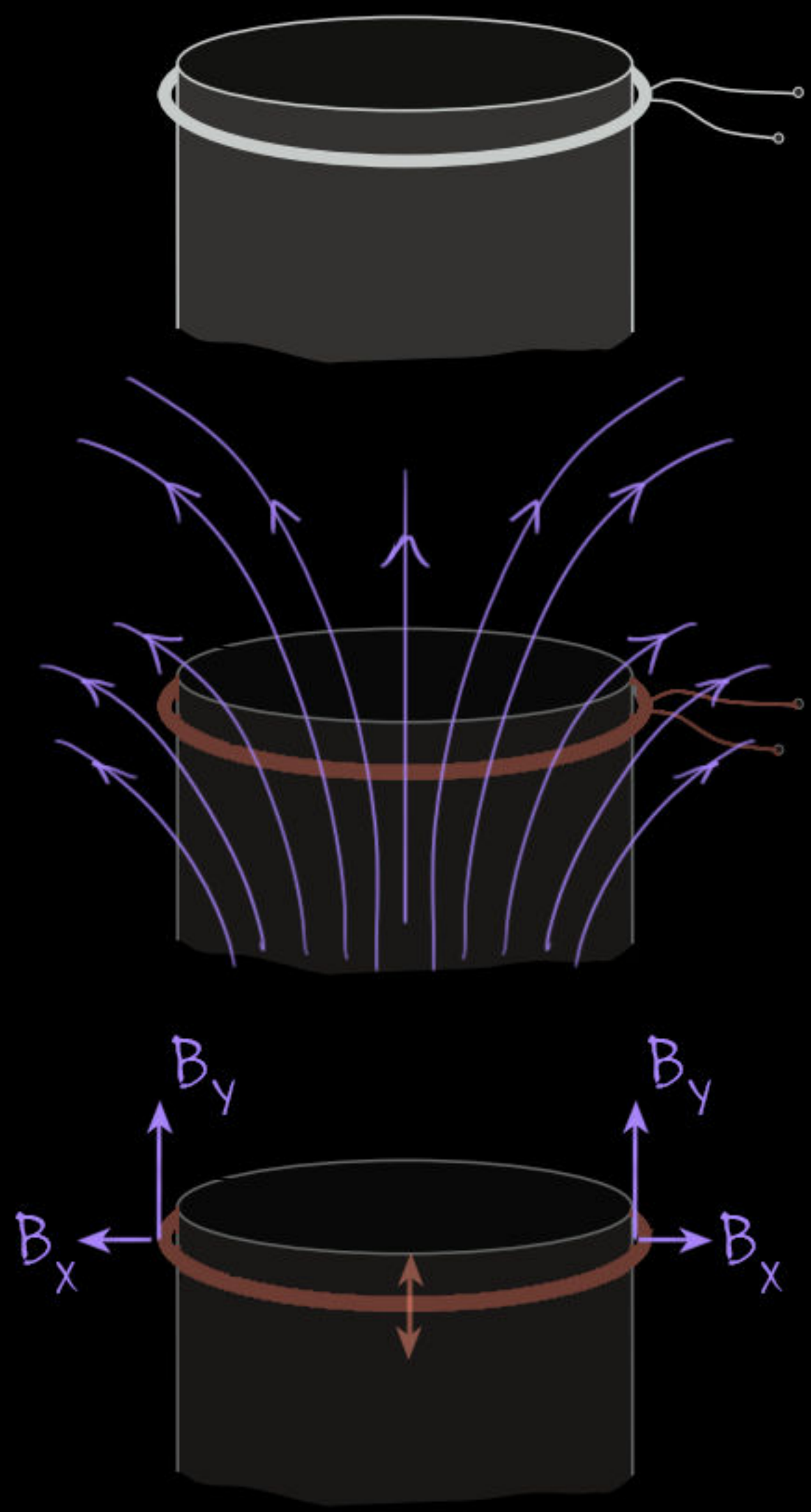
Forces on entire disc in Y direction:

$$F_y: ma = F$$

$$\Rightarrow ma = mg - Bid$$

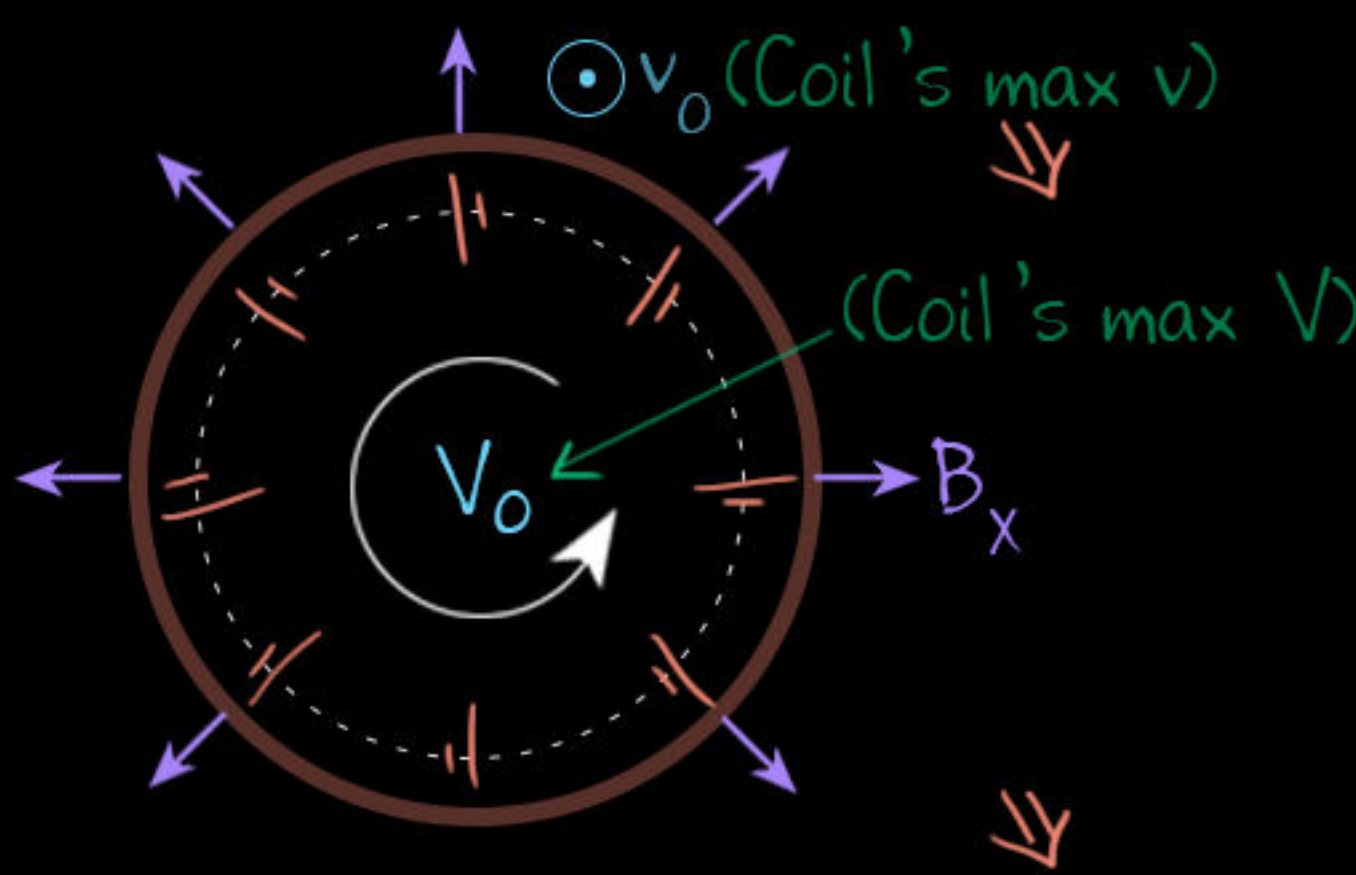
$$\Rightarrow m \frac{dv}{dt} = mg - B \left(\epsilon_0 B A \frac{dv}{dt} \right) d$$

$$\Rightarrow \frac{dv}{dt} = \frac{mg}{m + B^2 (\pi r^2) \epsilon_0 d}$$



14. A conducting coil held coaxially near one pole of a cylindrical permanent magnet as shown in the figure, if made to vibrate up and down simple harmonically with a frequency $f = 100$ Hz and an amplitude $a = 1.0$ mm, much smaller than the linear dimensions of the magnet, the voltage induced in the coil has amplitude $V_0 = 5\pi$ V. If the coil is held stationary at the mean position of the oscillations and a current $I = 200$ mA is made to flow through it, how much force will it experience?

Concept: At the location of ring, its the B_x component, that induces the voltage. And it doesnt change significantly for small vibrations.



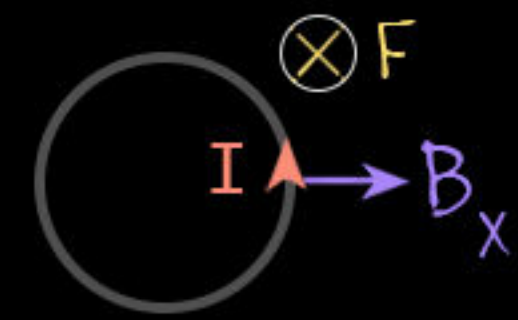
Its motional emf (even though it looks like emf induced due to TVMF)

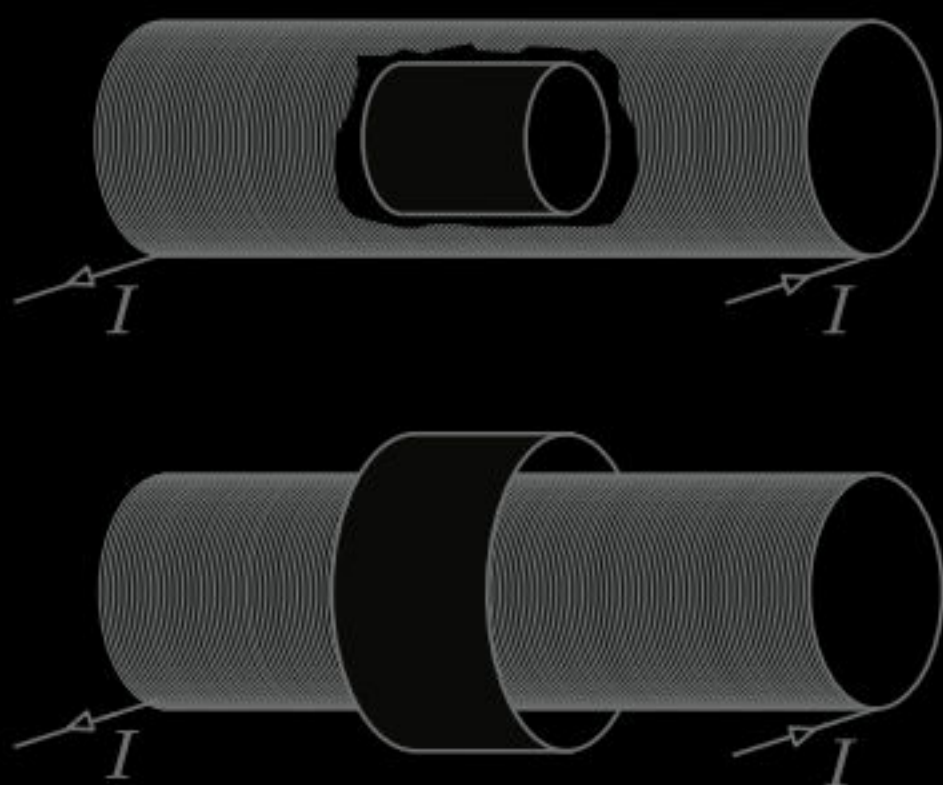
$$V_0 = B_x v_0 l \quad \begin{matrix} \rightarrow \text{Circumference} \\ \rightarrow \omega a = 2\pi f a \end{matrix}$$

$$\Rightarrow B_x = \frac{V_0}{2\pi f a l}$$

If I flows through stationary coil:

$$F = B_x I l = \frac{V_0 I}{2\pi f a}$$





15. A long solenoid of radius r having n turns per unit length carries a constant current I . Consider two situations, in the first an inertia-less paper cylinder of radius r_p ($r_p < r$) and length l is inserted coaxially at the middle of the solenoid and in the second case another inertia-less paper cylinder of radius r_p ($r_p > r$) and length l surrounds coaxially the solenoid in the middle as shown in the figures. The paper cylinders carry uniform charge Q over their curved surfaces. If current in the solenoid is suddenly reduced to I/η , find angular velocity ω acquired by the paper cylinders. Ignore effects of gravity.

Concept: Torque on a massless body is zero.

If flux changes through the area inside the paper, finite Electric field will be induced along the circumference of paper.

As the paper has charge, this field would cause a finite torque. Which is NOT possible.

So flux through the area inside paper won't change. To accomplish that paper will generate a 'current' by rotation. This is similar to a superconducting ring, which generates a current to maintain the flux through it.

A  $\phi_1 = \phi_2 + L \dot{I}$ $\mu_0 n^2 V$

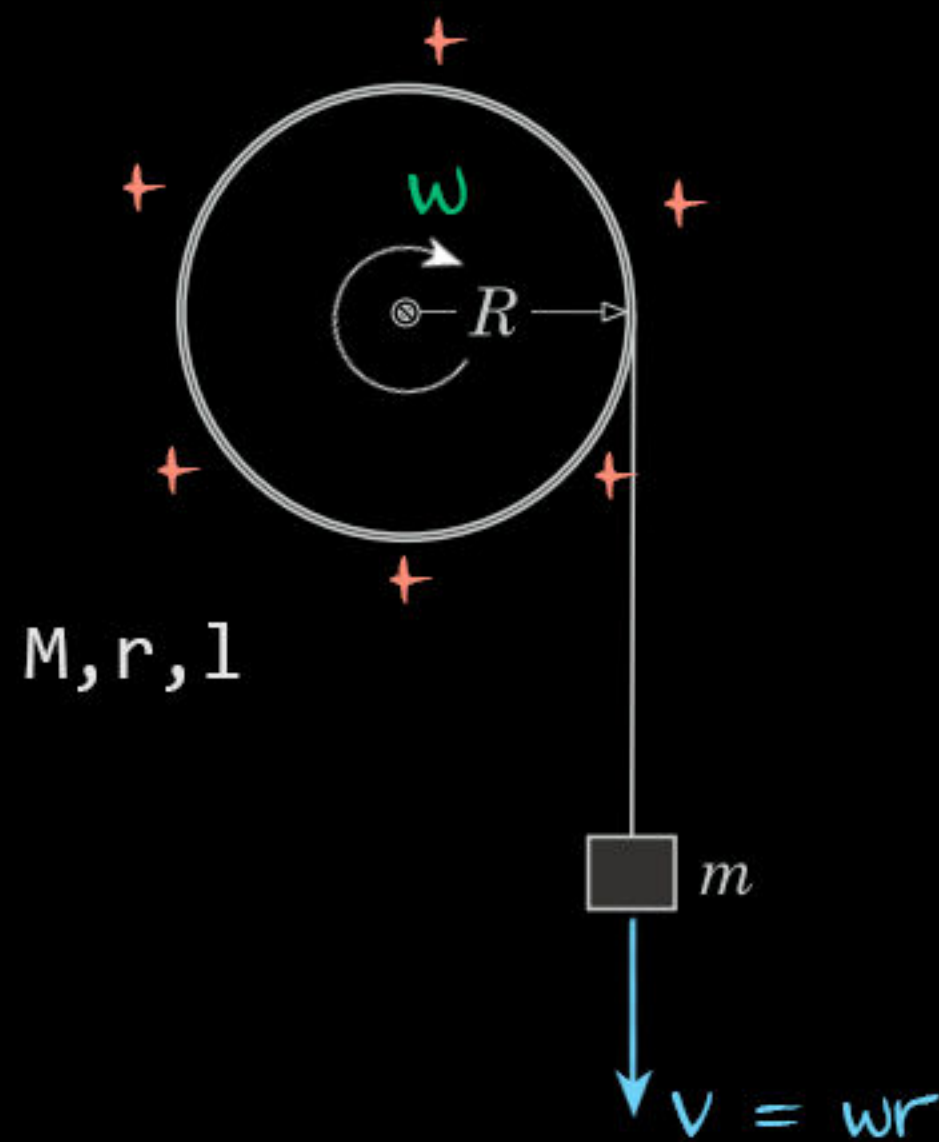
$$\Rightarrow \cancel{\mu_0} n I \left(1 - \frac{1}{\eta}\right) \pi \cancel{r_p^2} = \cancel{\mu_0} \left(\frac{1}{\eta}\right)^2 \pi \cancel{r_p^2} l \left[\frac{Q}{2\pi/\omega} \right]$$

$$\Rightarrow \omega = \frac{2\pi n I l}{Q} \left(\frac{\eta - 1}{\eta} \right) //$$

B  $\phi_1 = \phi_2 + L \dot{I}$

$$\Rightarrow \mu_0 n I \left(1 - \frac{1}{\eta}\right) \pi r^2 = \mu_0 \left(\frac{1}{\eta}\right)^2 \pi r_p^2 l \left[\frac{Q}{2\pi/\omega} \right]$$

$$\Rightarrow \omega = \frac{2\pi n I l}{Q} \left(\frac{\eta - 1}{\eta} \right) \cdot \frac{r^2}{r_p^2} //$$



16. A uniform dielectric hollow cylinder of mass M , radius R , length l carrying uniform charge of surface charge density σ can rotate without friction about a fixed horizontal axle that coincides with the axis of the cylinder. Several turns of a light thin insulating cord are wrapped on the cylinder and a block of mass m is suspended from the free end of the cord. Initially the block is held at rest as shown in the figure. Find acceleration of the block after it is released. Neglect charge transferred to the cord and fringing of magnetic field at the ends of the cylinder. Acceleration due to gravity is g and permeability of the medium inside the cylinder is μ_0 .

Due to mg , there is CW torque and cylinder will rotate.

⇒ Rotating cylinder causes inward flux to increase.

⇒ Increasing inward flux induces CCW electric field.

⇒ CCW electric field causes CCW torque, that opposes the rotation.

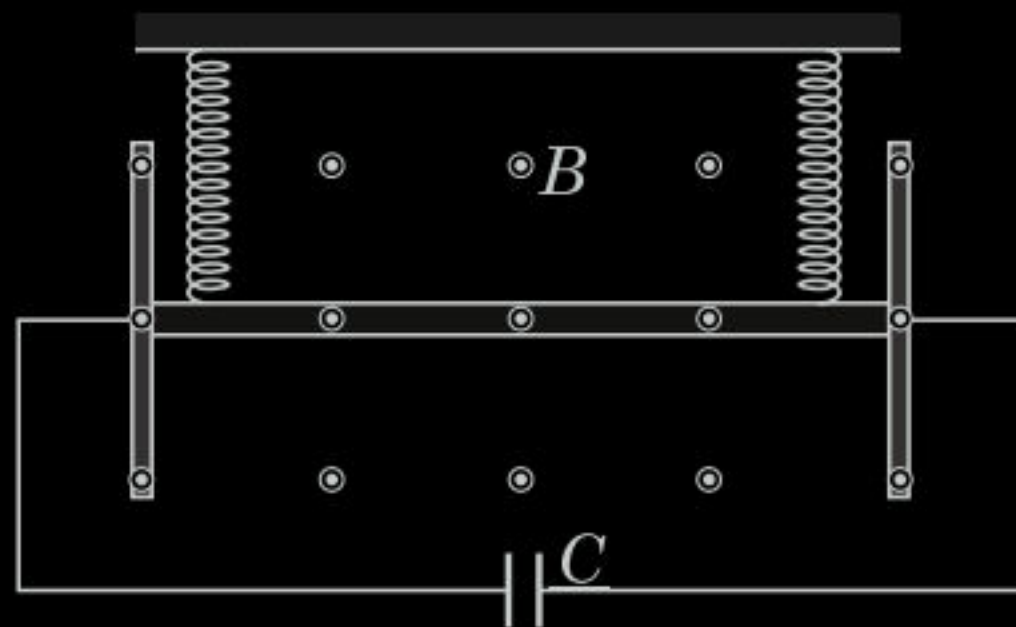
$$mgR - qER = I\alpha$$

$$\sigma \cdot 2\pi Rl \quad \frac{1}{2\pi R} \quad \frac{d\phi}{dt} \quad MR^2 + mR^2$$

$$\mu_0 n^2 V = \mu_0 \frac{1}{l^2} \pi R^2 l \quad L \frac{di}{dt} \quad \frac{d}{dt} \left(\frac{q}{2\pi/\omega} \right) = \frac{q}{2\pi} \alpha$$

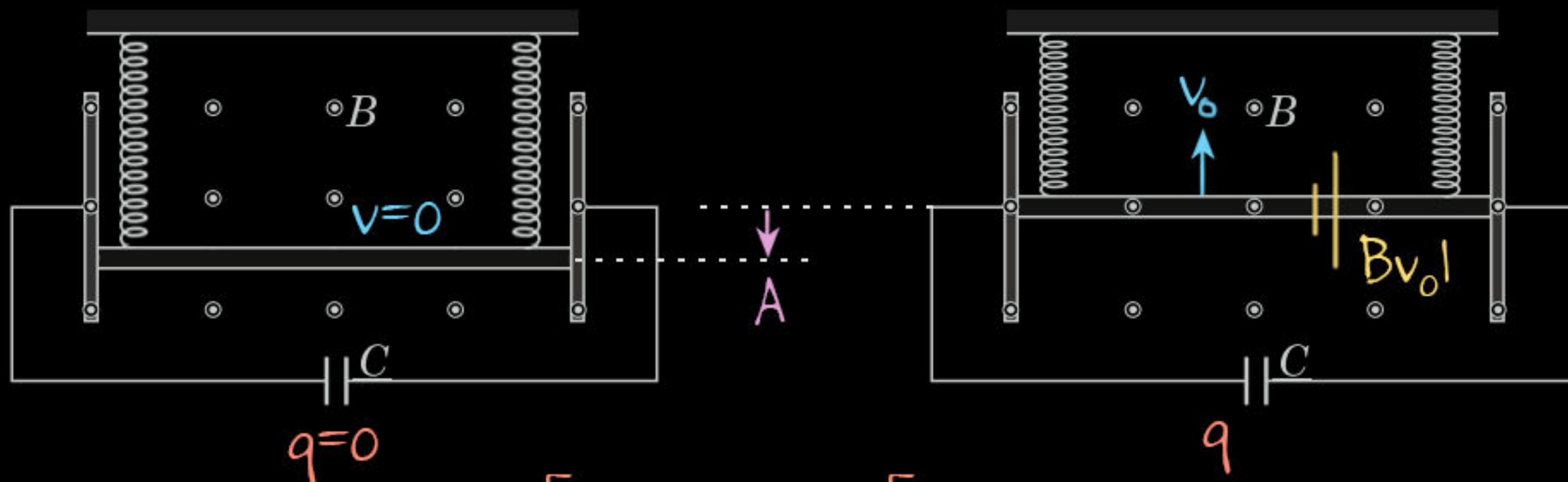
$$\Rightarrow mgR - (\sigma \cdot 2\pi Rl)^2 R \cdot \frac{1}{2\pi R} \frac{\mu_0}{l^2} \pi R^2 l \cdot \frac{1}{2\pi} \alpha = (M+m)R\alpha$$

$$\Rightarrow a = \alpha R = \frac{mg}{M + m + \mu_0 \pi \sigma^2 R^2 l}$$



17. A conducting rod of mass m and length l is suspended horizontally with the help of two identical non-conducting springs each of stiffness k in a uniform horizontal magnetic field B . The rod can slide between two fixed frictionless vertical conducting guides and an uncharged capacitor of capacitance C is connected between the guides as shown in the figure. Find the period of oscillations of the rod. Neglect resistance and inductance in the circuit and assume acceleration of free fall to be g .

Lets pull the rod down by A and release.

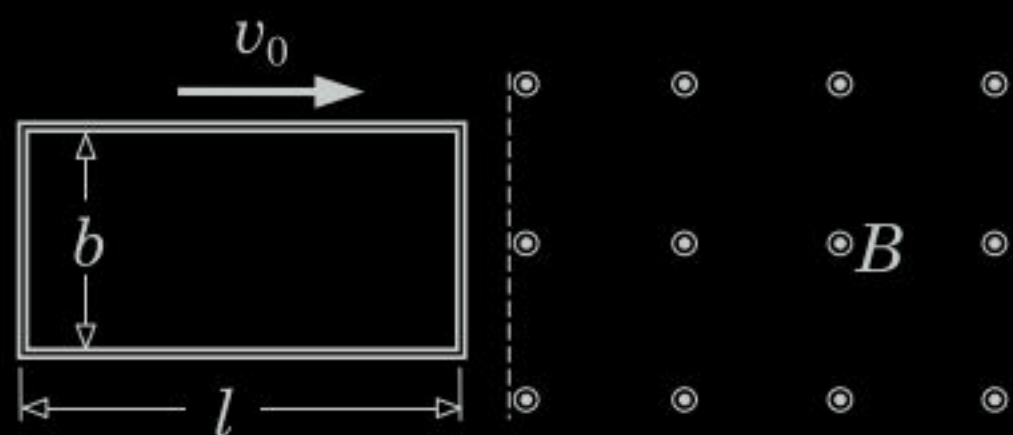


$$E_{\text{extreme}} = E_{\text{mean}}$$

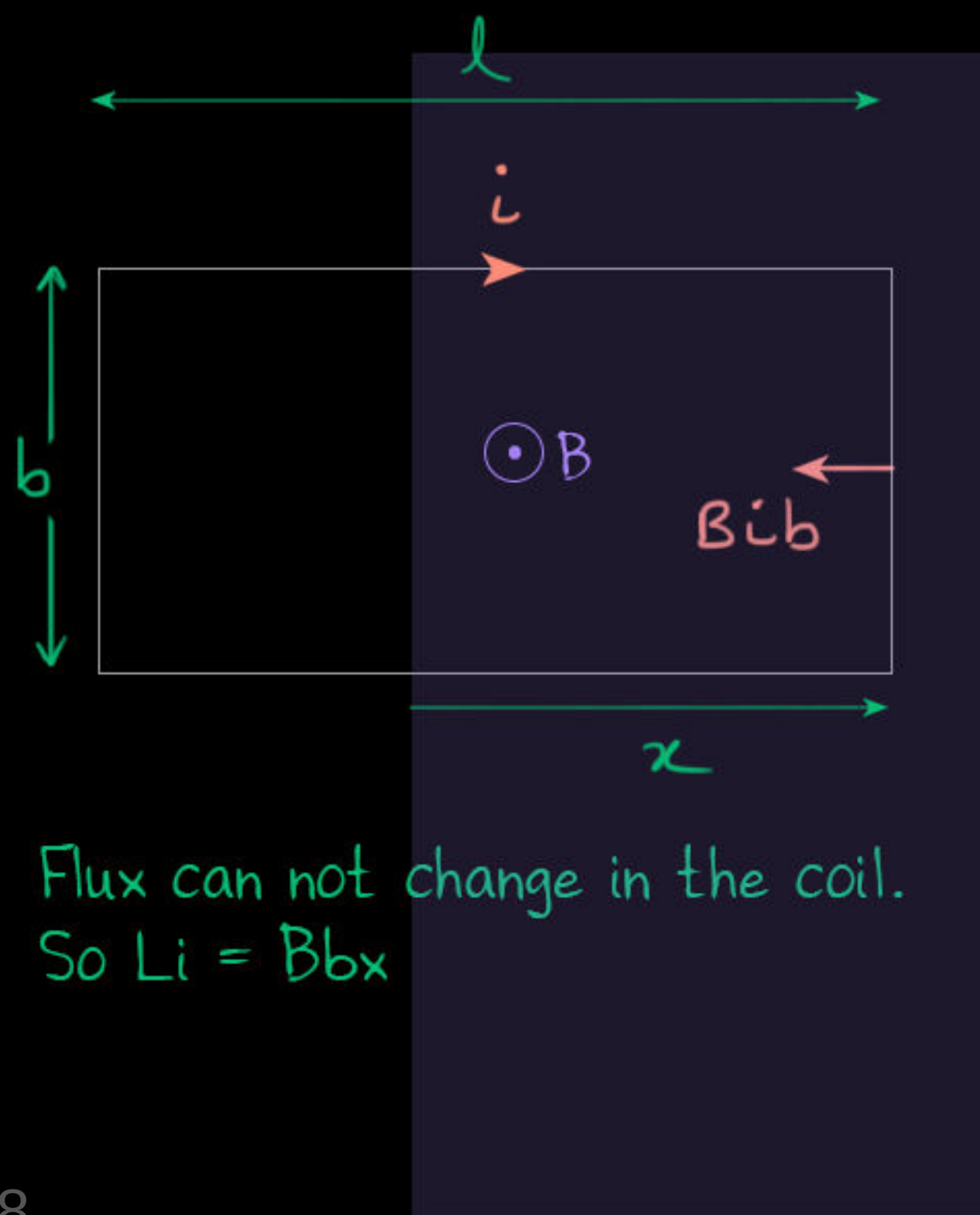
$$\Rightarrow 2 \left(\frac{1}{2} k A^2 \right) = \frac{1}{2} m v_0^2 + \frac{1}{2} C (B v_0 l)^2$$

$$\Rightarrow v_0^2 = \frac{2k}{m + C B^2 l^2} A^2 \quad \xrightarrow{\text{Comparing with } v_0 = \omega A}$$

$$T = 2\pi \sqrt{\frac{m + C B^2 l^2}{2k}}$$



18. A rectangular perfectly conducting coil of mass m , width b , length l and inductance L moving with a velocity v_0 along its length in a region of no magnetic field enters a region of uniform magnetic field B that is everywhere perpendicular to the plane of the coil. How does position x of the leading edge of the coil measured from the entry point into the magnetic field varies with time t .



Restoring force: $ma = B i b$

$$L i = B b x$$

$$\Rightarrow a = \frac{B^2 b^2}{m L} x$$

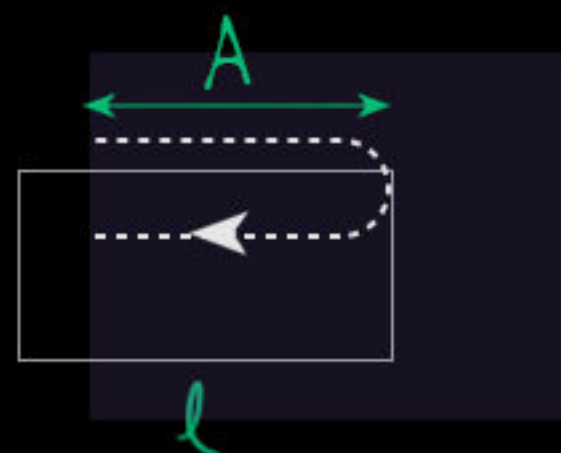
SHM: ω^2

$$\therefore v = v_0 \cos \omega t$$

$$\omega = \frac{B b}{\sqrt{m L}}$$

$$A = \frac{v_0}{\omega}$$

Case A If $A < l$,



$$x = A \sin \omega t$$

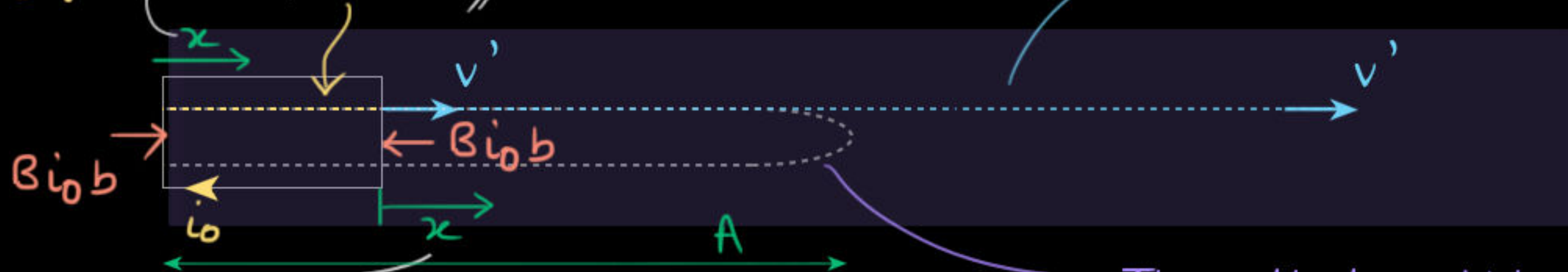
Case B If $A > l$

Once loop is fully inside, flux due to B doesn't change anymore, so current will be maintained, and net force will be zero. So velocity becomes constant.

Let it get completely inside in time t' . And its velocity then be v' .

$$\text{Then: } l = \frac{v_0}{\omega} \sin \omega t' \Rightarrow t' = \frac{1}{\omega} \sin^{-1} \left(\frac{\omega l}{v_0} \right)$$

$$t < t': x = A \sin \omega t //$$

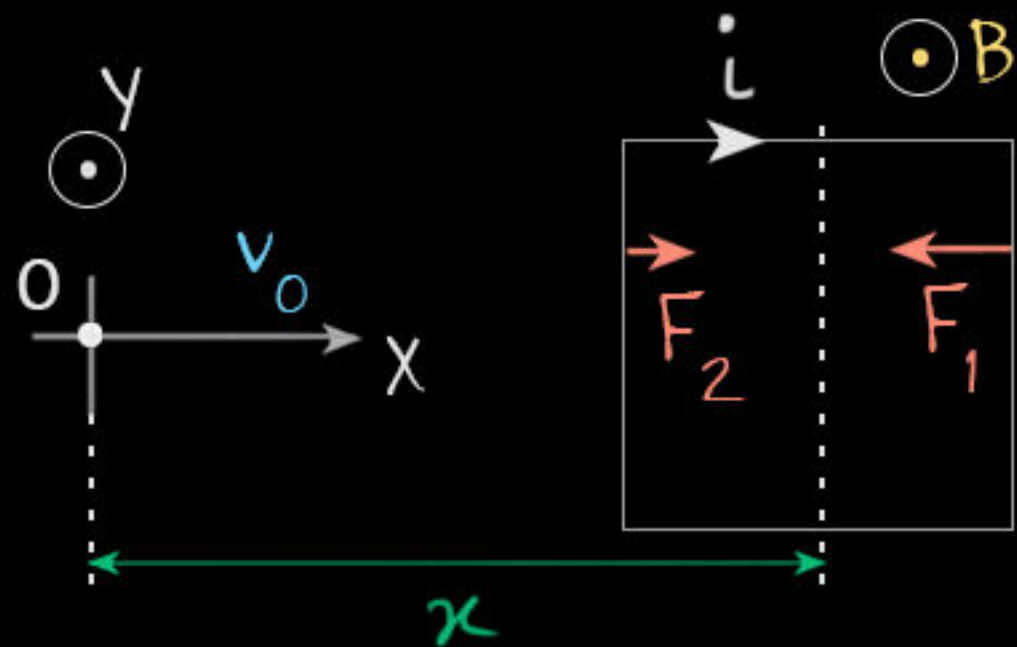


$$\therefore t > t': x = v' t \text{ where } v' = \sqrt{v_0^2 - \omega^2 l^2}$$

$$\omega = \frac{Bb}{\sqrt{mL}}$$

$$A = \frac{v_0}{\omega}$$

As B increases with x , so does flux.
So current will be CW.



$$B_1 = B_0 \left(1 + k\left(x + \frac{l}{2}\right)\right)$$

$$B_2 = B_0 \left(1 + k\left(x - \frac{l}{2}\right)\right)$$

Restoring force: $ma = F_1 - F_2$

$$\Rightarrow a = \frac{(B_1 - B_2)il}{m}$$

$$\Rightarrow a = \frac{B_0 k l}{m} \cdot \frac{B_0 k x l^2}{L} \cdot l = \frac{B_0^2 k^2 l^4}{m L} x$$

ω^2 of SHM

$$\therefore \omega = \frac{B_0 k l^2}{\sqrt{m L}}$$

$$x = \frac{v_0}{\omega} \sin \omega t$$

Youtube / Arihant Senior

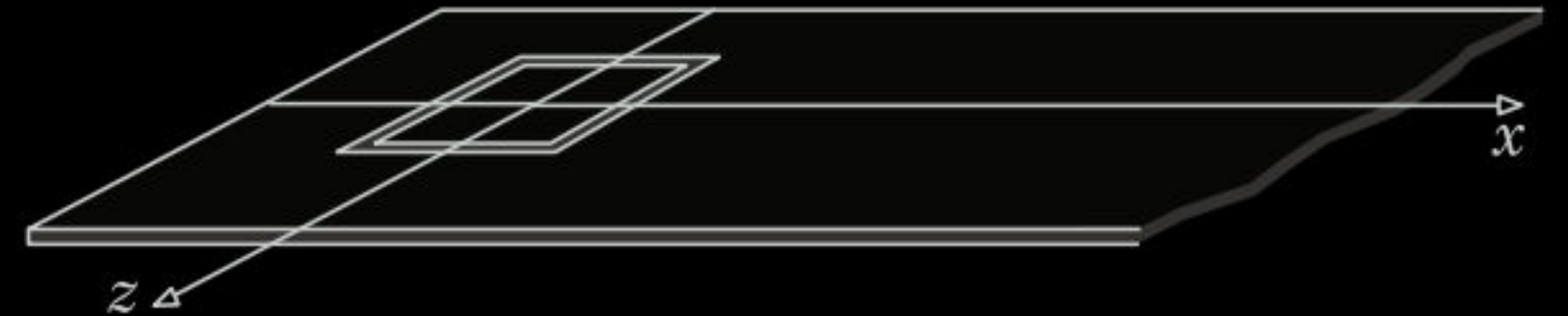
19. A perfectly conducting square frame of mass m , side l and inductance L with a small gap cut in one of its arms is placed on a frictionless horizontal floor where a nonuniform magnetic field that is given by equation $\vec{B} = \hat{j}B_0(1 + kx)$ exists. Here k is a positive constant. Centre of frame initially coincides with the origin of coordinate system attached to the floor as shown in the figure.

If the gap is closed with a perfectly conducting material and then the frame is imparted a velocity v_0 in the positive x -direction without any rotation, how will its displacement x vary with time t .

ϕ_x ϕ_0 $\because B$ is linearly increasing

$$\left[B_0(1 + kx) - B_0 \right] l^2$$

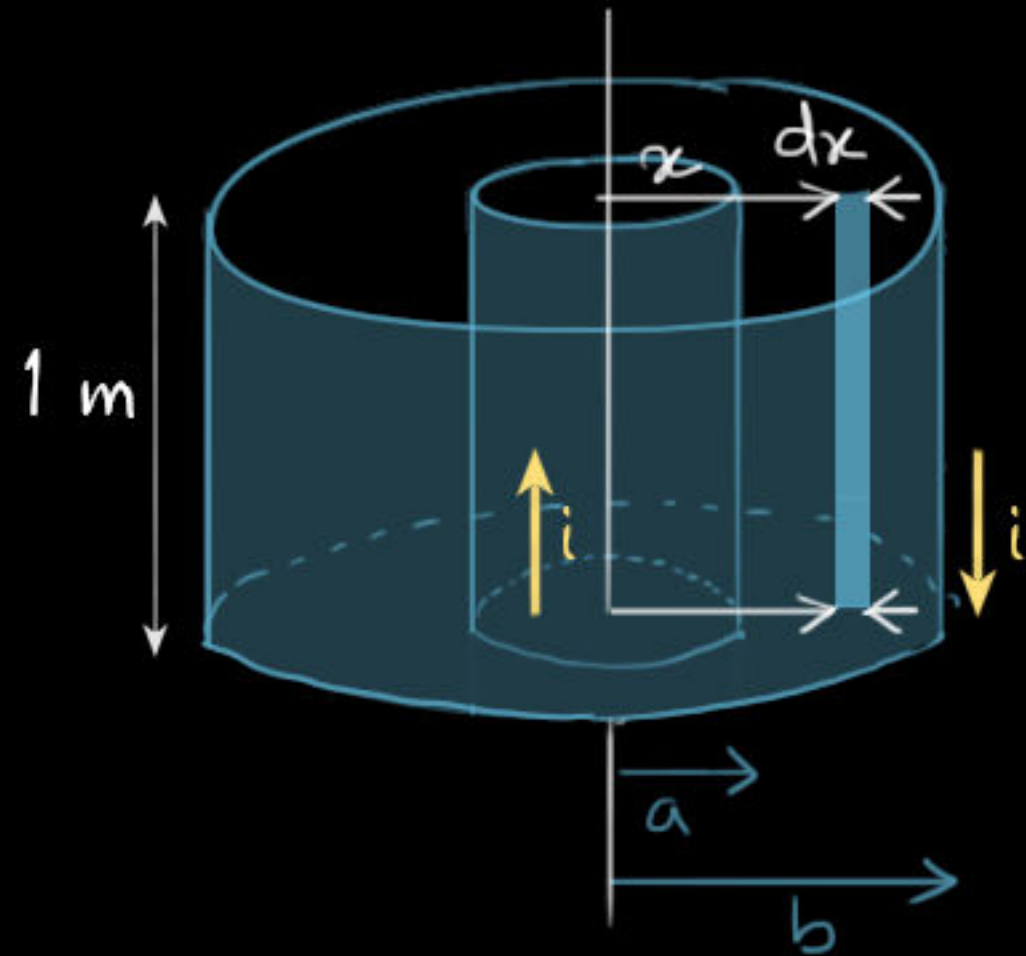
$$Li = \Delta \phi_B$$





20. A cable consists of coaxial conducting cylindrical shells, radii of the inner and the outer shells of which are a and b respectively. Magnetic permeability of insulation between the shells is equal to that of the free space. Find inductance of a unit length of the cable.

Same as Irodov 3.319



For a small area element between the two cables flux passing through is:

$$d\phi = \left(\frac{\mu_0 i}{2\pi x} \right) (1) dx$$

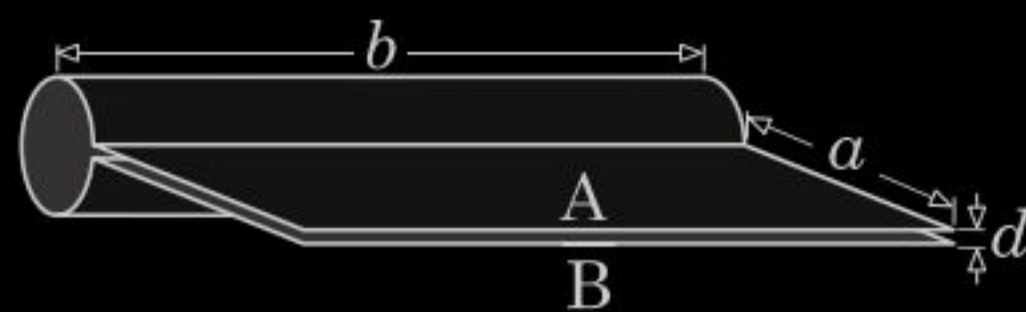
$$\phi = \frac{\mu_0 i}{2\pi} \int_a^b \frac{dx}{x} = \frac{\mu_0 i}{2\pi} \ln \frac{b}{a}$$

$$\phi = Li$$

$$\Rightarrow L = \frac{\mu_0}{2\pi} \ln \frac{b}{a}$$

by Ankit Singhvi

Youtube / Arihant Senior



21. A rectangular copper foil of width b is bent into a shape consisting of a cylindrical portion of radius r ($r \ll b$) and two parallel identical flat portions each of length a as shown in the figure. A current enters uniformly into the edge A, flows uniformly over the whole width and comes uniformly out of the edge B. Find the inductance between edges A and B for a slowly varying current.

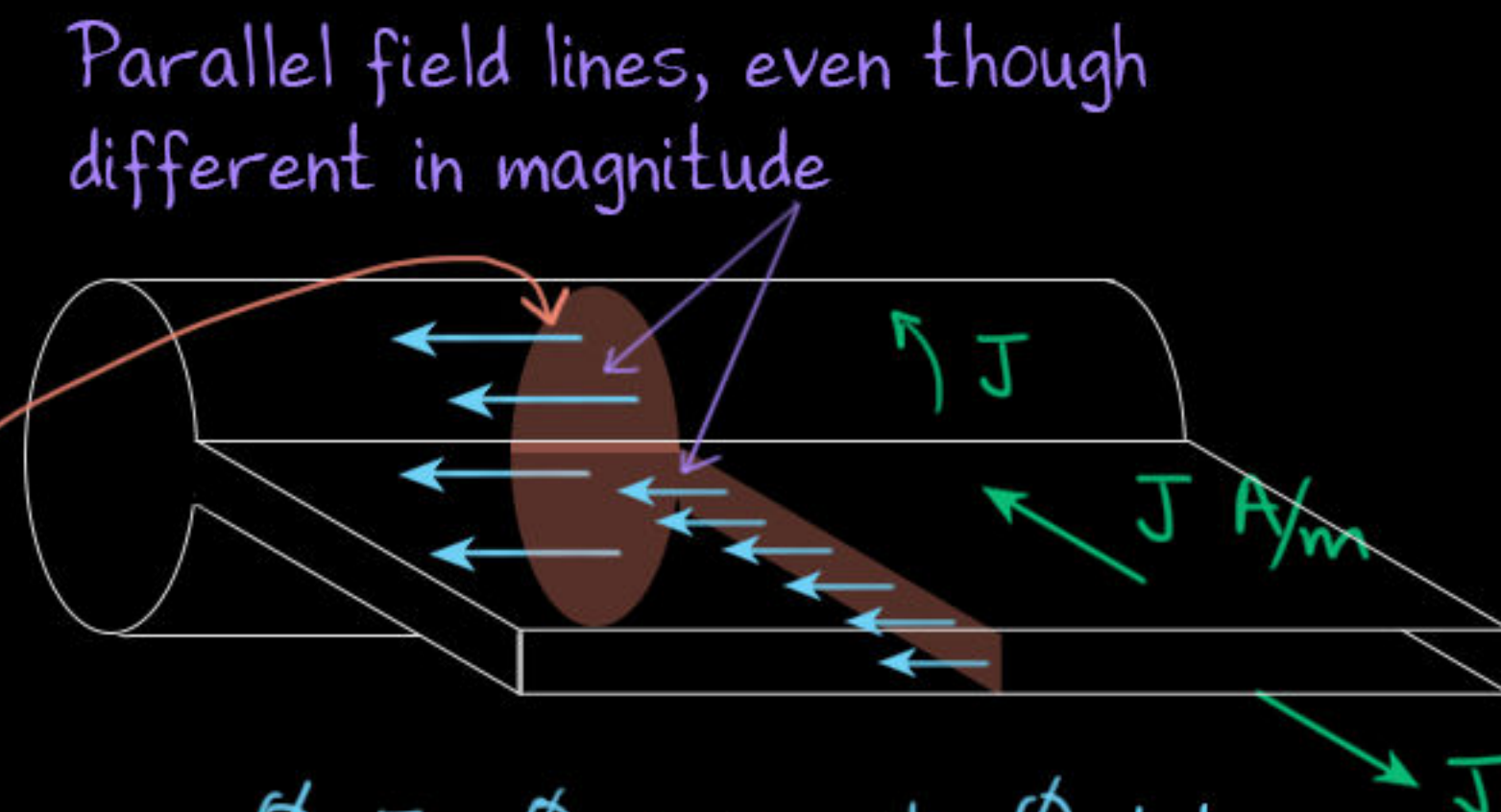
Concepts:

Magnetic flux generated in the "entire region" due to a current carrying loop is found to be proportional to current. Self Inductance is that proportionality constant.

To capture the entire flux, we choose a convenient surface.

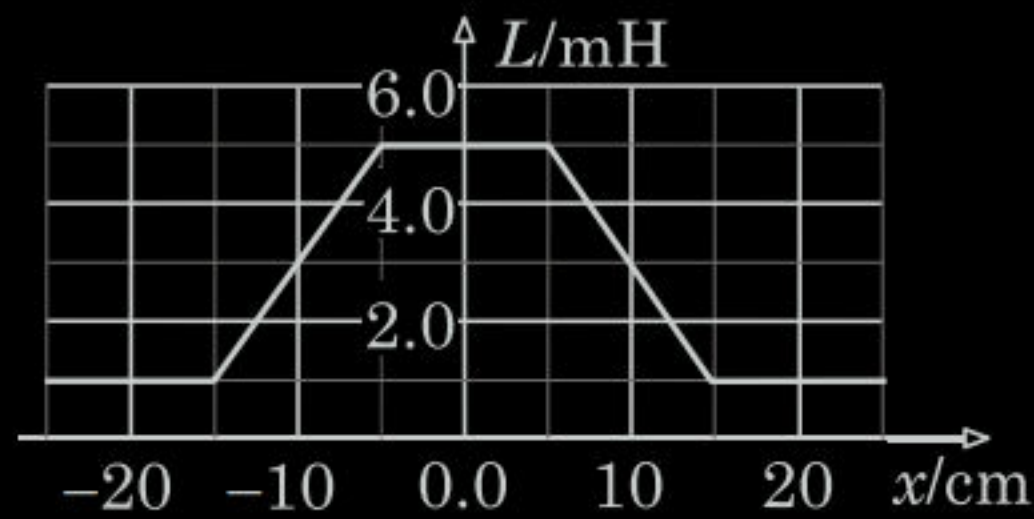
Typically, it's the surface, where

- 1) Field lines are perpendicular, and
- 2) ALL field lines pass through it.



$$\begin{aligned}
 \phi &= \phi_{\text{cylinder}} + \phi_{\text{plates}} \\
 &= (\mu_0 j) \pi r^2 + (\mu_0 j) a d \\
 &= \underbrace{\frac{\mu_0}{b} (\pi r^2 + a d)}_L i
 \end{aligned}$$

$j \rightarrow i/b$



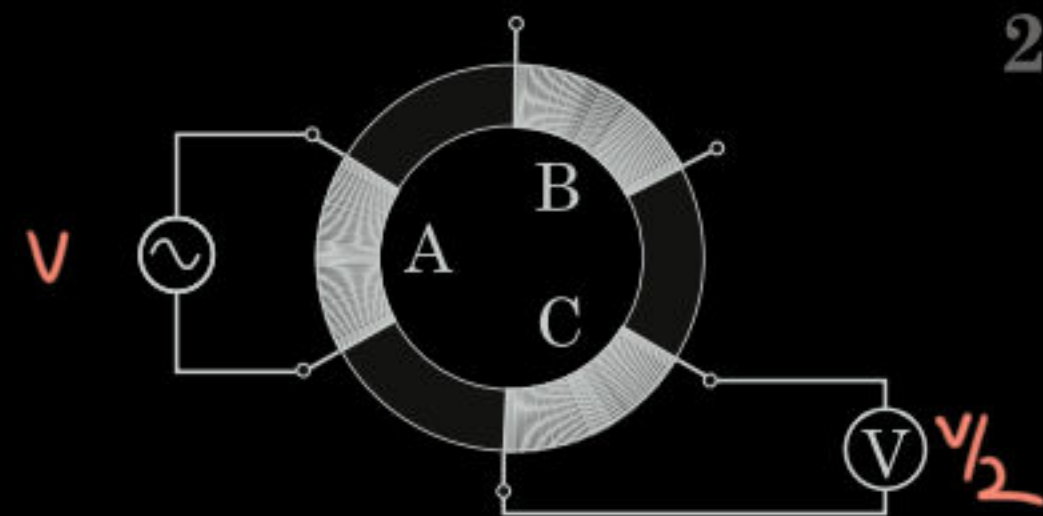
22. A coil is wound on a hollow insulating cylinder, which contains in it a laminated iron core. How the inductance L of the coil varies with the displacement x of the iron core is shown in the figure. In the initial state $x = 0$ (the core is fully inserted into the coil), the current in the coil is 1.0 A. Find current in the coil immediately after the core is quickly taken out of the coil.

$\Downarrow$
 flux is same

$$\Rightarrow L_1 i_1 = L_2 i_2$$

$$\Rightarrow 5(1) = (1) i_2$$

$$\Rightarrow i_2 = 5 \text{ A}$$



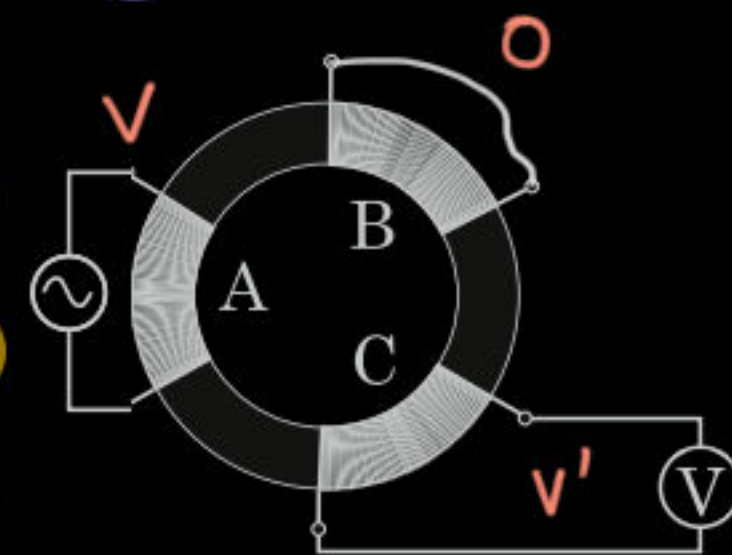
23. Three identical coils A, B and C are wound symmetrically on a toroidal iron core in the same sense. An alternating voltage source is connected across the coil A and an ideal voltmeter is connected across the coil C. If the coil B is left open, the voltmeter reads half of the source voltage. How much will the voltmeter, if the coil B is short-circuited?

Initially, there is no current in B and C.

$$\begin{aligned} \therefore \phi_A &= L I_A \longrightarrow \dot{\phi}_A = L \dot{I}_A = V \\ \text{and } \phi_C &= M I_A \longrightarrow \dot{\phi}_C = M \dot{I}_A = V/2 \end{aligned} \xrightarrow{\text{Dividing}} \underline{L = 2M} \quad \text{and} \quad \begin{aligned} M \dot{I}_A &\rightarrow A \\ M \dot{I}_B &\rightarrow B \end{aligned}$$

Later, B is shorted, but there is still no current in C.

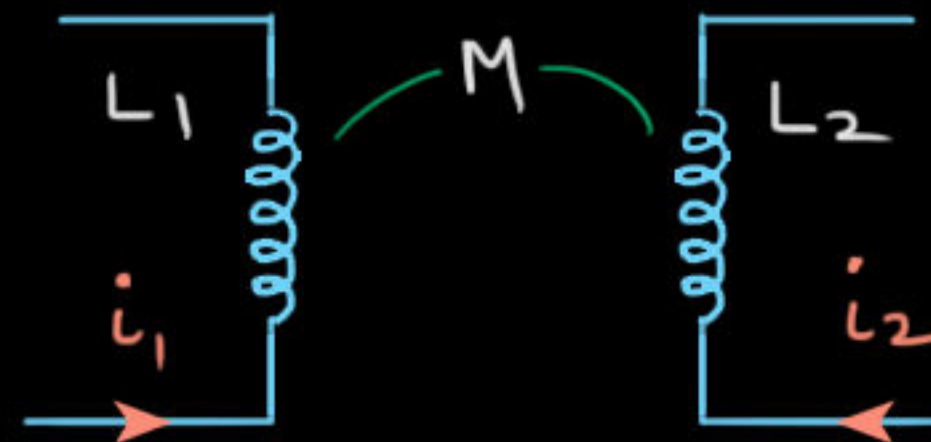
$$\begin{aligned} \therefore \phi_A &= L I_A + M I_B \longrightarrow \dot{\phi}_A = L \dot{I}_A + M \dot{I}_B = V = 2A + B \quad (1) \\ \phi_B &= M I_A + L I_B \longrightarrow \dot{\phi}_B = M \dot{I}_A + L \dot{I}_B = 0 = A + 2B \quad (2) \\ \phi_C &= M I_A + M I_B \longrightarrow \dot{\phi}_C = M \dot{I}_A + M \dot{I}_B = V' = A + B \quad (3) \end{aligned}$$



From 1,2,3: $V' = V/3$
Youtube / Arihant Senior

Energy stored in two coil system is:

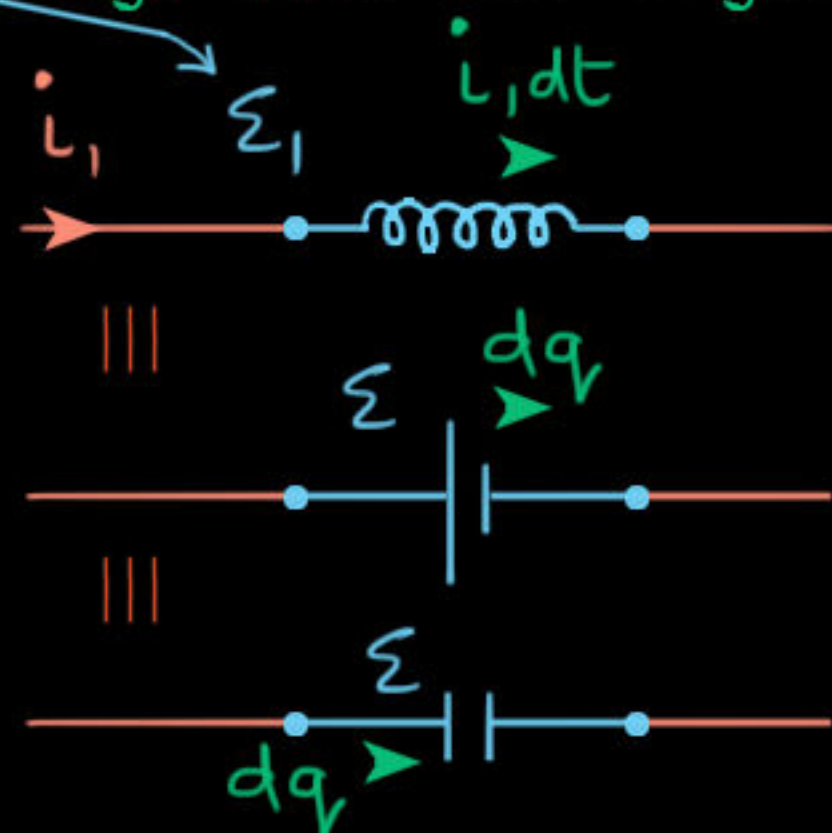
$$U = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$



Proof:

We know emf across coils are: $\Sigma_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ and $\Sigma_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$

∴ Similar to battery, small energy dU stored in coil as small charges $i_1 dt$ and $i_2 dt$ pass through these coils is given by:

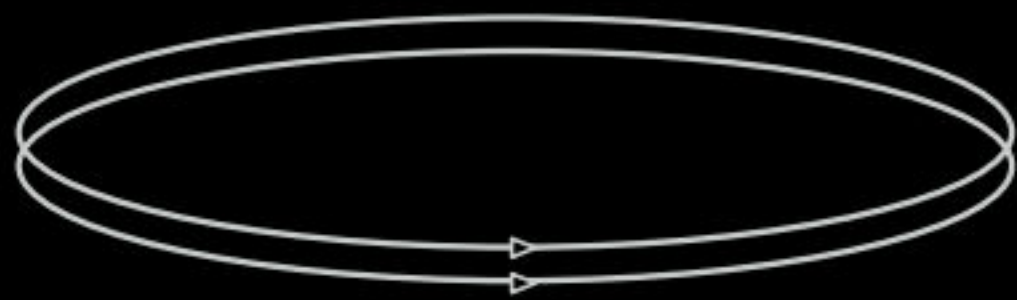


$$\Rightarrow dU = \Sigma_1 i_1 dt + \Sigma_2 i_2 dt$$

$$= (L_1 i_1 di_1 + M i_1 di_2) + (L_2 i_2 di_2 + M i_2 di_1)$$

$$\Rightarrow U = L_1 \int i_1 di_1 + L_2 \int i_2 di_2 + M \int d(i_1 i_2)$$

$$= \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2 //$$



24. Two identical perfectly conducting rings are coaxially placed very close (infinitesimally small separation) to each other. Each of the coils has of inductance L and carries a current I_0 in the same direction. How much minimum amount of work has to be done in making their separation infinitely large?

On any one coil: $\phi_{\text{total}} = \phi_{\text{self}} + \phi_{\text{mutual}}$

$$Li_0 + Mi_0$$

They are equal as the coils are identical, close to each other and have same current i_0 .

$\therefore L = M$ ①

As coils are super conducting, flux through them won't change.

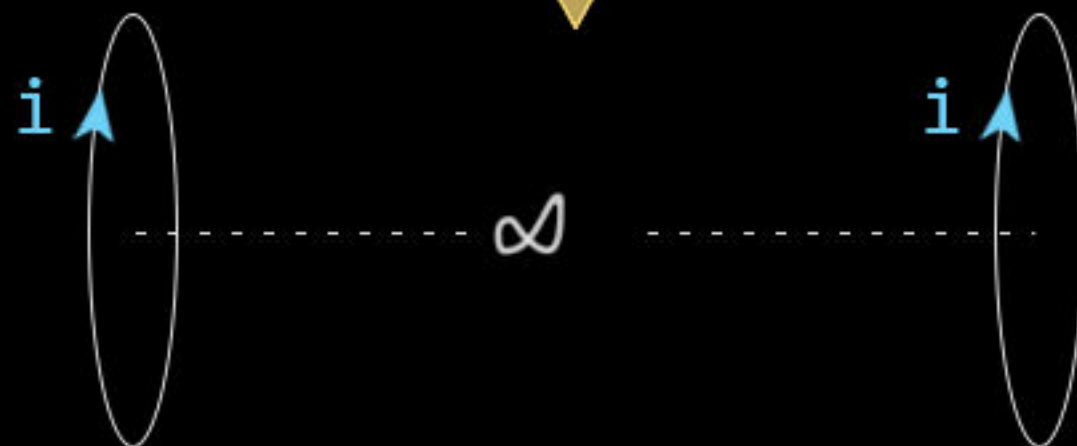
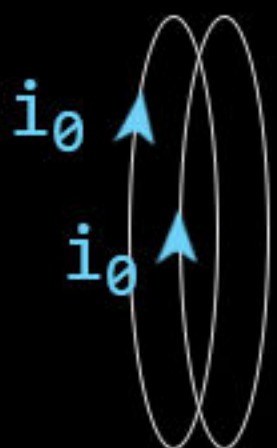
$\therefore$ For any one coil: $\phi_i = \phi_f \Rightarrow Li_0 + Mi_0 = Li$ ②

From 1,2: $i = 2i_0$

Using: $U = \frac{1}{2} Li_1^2 + \frac{1}{2} Li_2^2 + Mi_1i_2$

$W_{\text{ext}} = U_f - U_i$

$$= 2\left(\frac{1}{2} L \underbrace{i^2}_{2i_0}\right) - \left\{ 2\left(\frac{1}{2} Li_0^2\right) + Li_0^2 \right\} = 2Li_0^2 //$$





25. A uniform magnetic field of induction B_0 exists everywhere in a long hollow thin walled metallic cylinder along its axis as shown in the figure. Length l of the cylinder is much larger than its radius r that is much larger than its wall thickness d ($l \gg r \gg d$). Resistivity of the material of the cylinder is ρ . Neglecting flux linkage with the material of the cylinder, find how the magnetic induction B inside the cylinder will decay with time t after the magnetic field is switched off.

Initially flux in cylinder is due to uniform B_0 inside.

If that B_0 is switched off suddenly, current will be induced in the cylinder to retain the flux.

Since field due to current is uniform inside a cylinder, induced current has to generate B_0 to maintain the same flux as before.

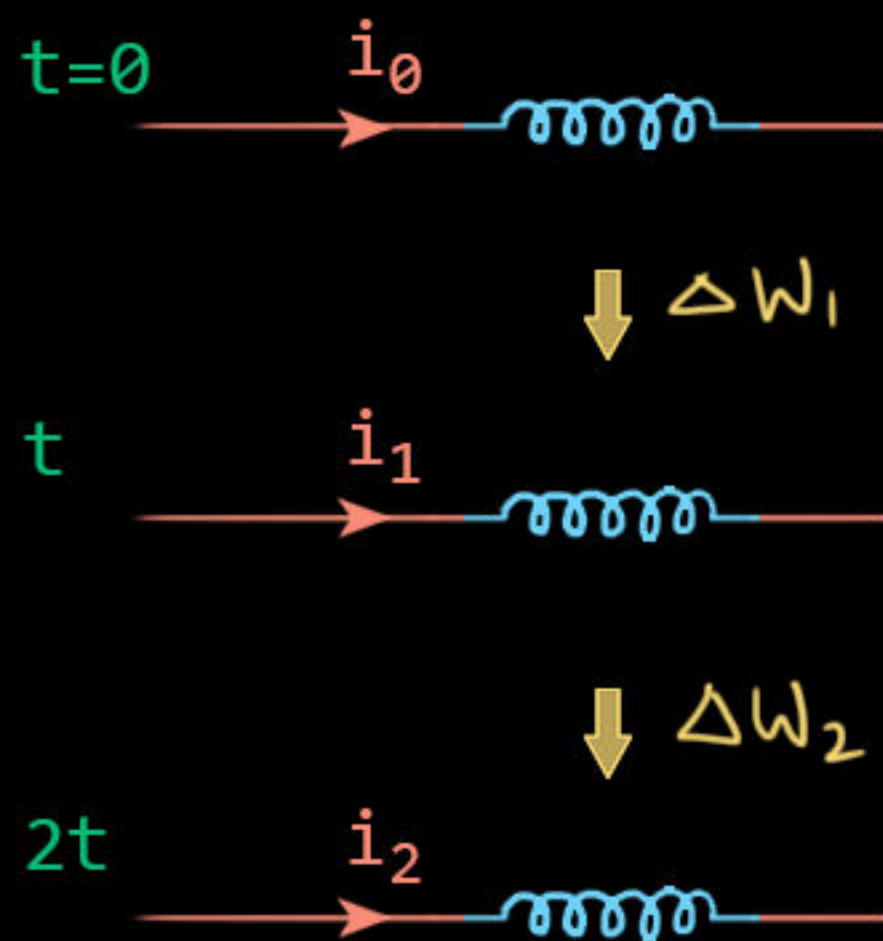
That current will then decay as in a RL circuit.

Now, similar to solenoid, Magnetic field inside is proportional to current in this cylinder. So field will decay similar to current.

$$B = B_0 e^{-Rt/L} = B_0 e^{-\frac{\frac{82\pi r}{ld} \cdot l}{\mu_0 \pi r^2} t} = B_0 e^{-\frac{2\mu_0 r t}{ld}}$$

$$L = \mu_0 n^2 V = \mu_0 \left(\frac{1}{l}\right)^2 \cdot \pi r^2 l = \frac{\mu_0 \pi r^2}{l}$$

$$R = \frac{\rho l}{A} = \frac{\rho \cdot 2\pi r}{(ld)}$$



26. A coil is made by winding a large number of turns of a thin wire on a hollow plastic cylinder. A magnet is inserted in the coil and its ends are short-circuited. The magnet is suddenly removed. Immediately after removal of the magnet, if in first two consecutive time intervals of 0.1 s each, amounts heat dissipated in the coil are $\Delta W_1 = 0.01$ J and $\Delta W_2 = 0.006$ J respectively, find total amount of heat dissipated in the coil.

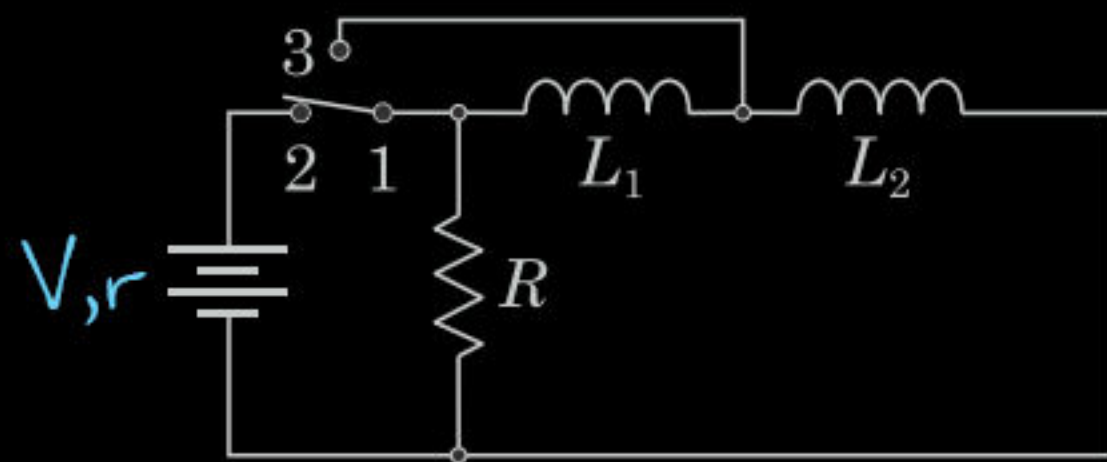
As magnet is removed suddenly, current will be induced in the coil to retain the flux. That current will then decay as in a RL circuit.

We know, heat dissipated = $U_i - U_f$ and Current $i = i_0 e^{-Rt/L}$

$$\therefore \Delta W_1 = \frac{1}{2} L (i_0^2 - i_1^2) = \frac{1}{2} L i_0^2 (1 - e^{-2Rt/L}) = U(1 - x) \quad (1)$$

$$\Delta W_1 + \Delta W_2 = \frac{1}{2} L (i_0^2 - i_2^2) = \frac{1}{2} L i_0^2 (1 - e^{-2R(2t)/L}) = U(1 - x^2) \quad (2)$$

From 1,2: $U = \frac{\Delta W_1^2}{\Delta W_1 - \Delta W_2} = \text{Total heat dissipated eventually.}$

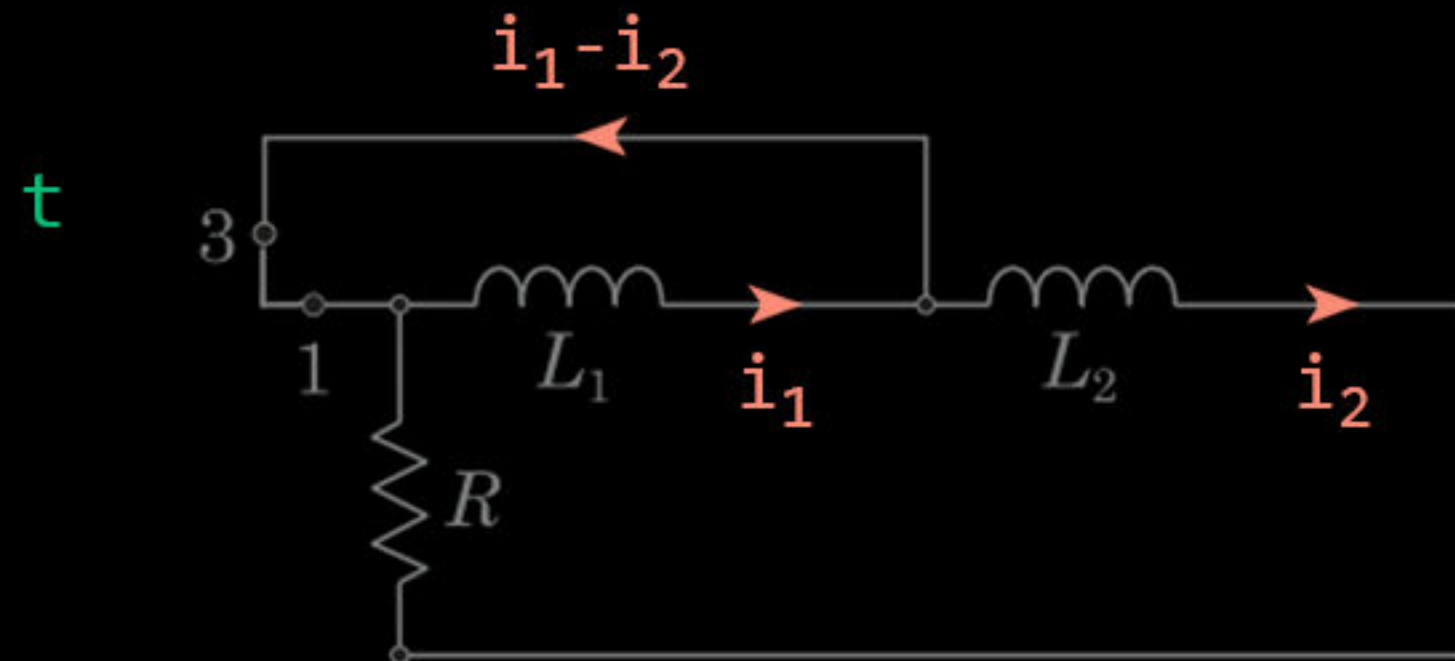
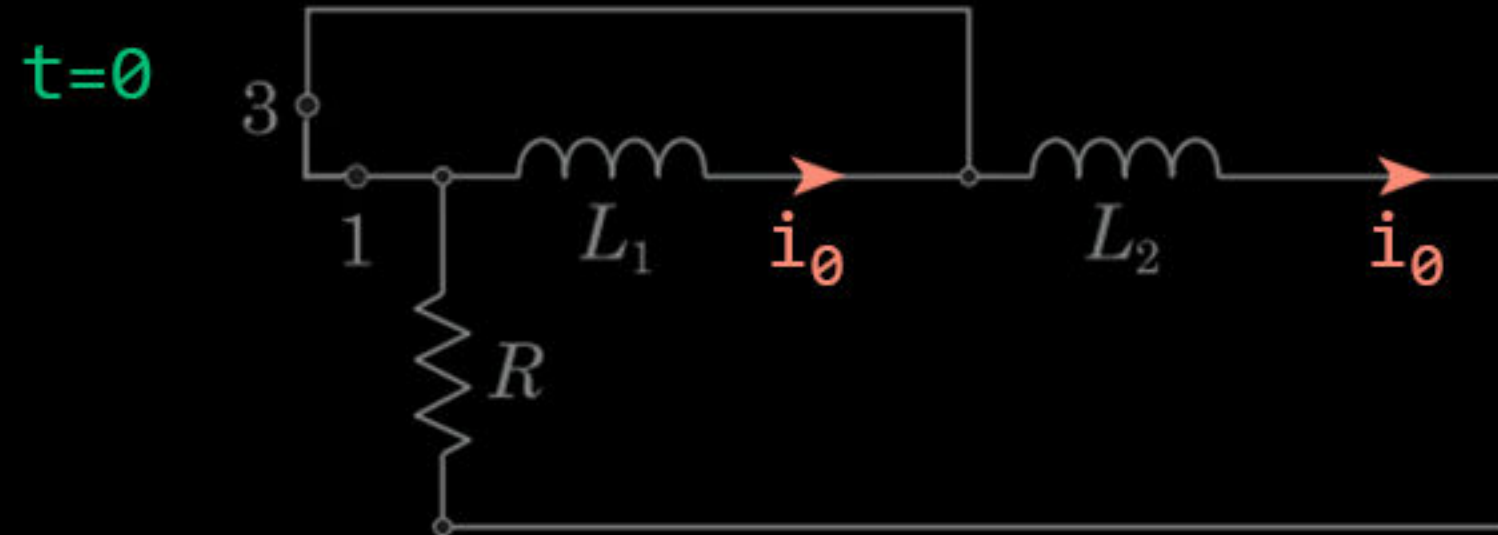


27. A circuit consists of two inductances L_1 and L_2 , a resistance R , a two-way switch, a battery of electromotive force $\mathcal{E}$ and internal resistance r . The switch was in position 2 for a long time and at the instant $t = 0$, it is thrown to position 3.

Just after switching: $i_0 = \mathcal{E}/R$

(a) Find current through the switch as a function of time t after the switching.

(b) Find total heat dissipated in resistance R after the switching.

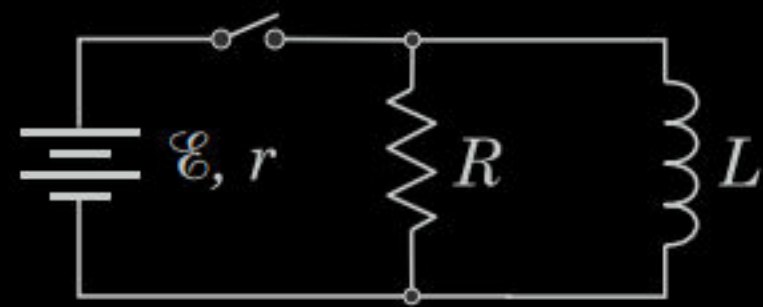


In top loop: $L_1 \frac{di_1}{dt} = 0 \Rightarrow \int_{i_0}^{i_1} di_1 = 0 \Rightarrow i_1 = i_0$

In big loop: $L_2 \frac{di_2}{dt} = -i_2 R \Rightarrow i_2 = i_0 e^{-\frac{Rt}{L_2}}$

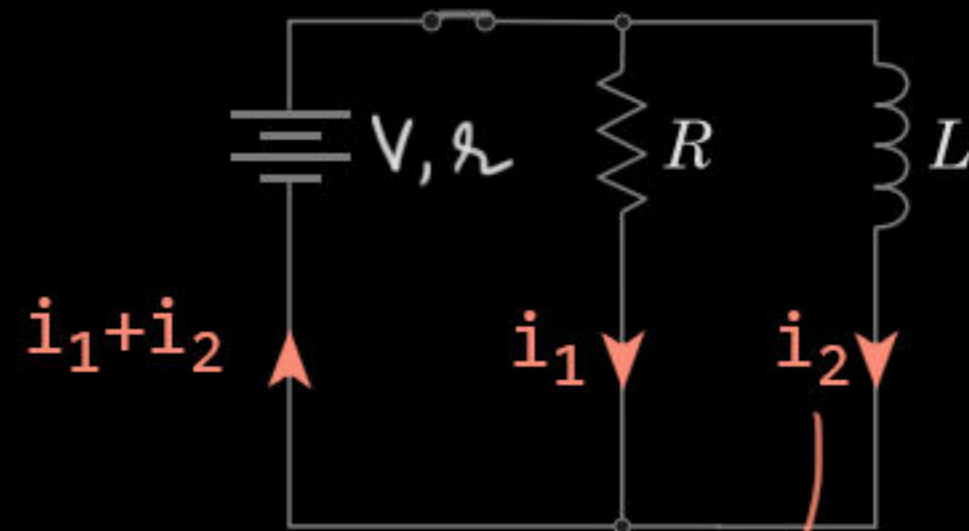
A $\therefore$ Current in switch from 3 - 1: $i_1 - i_2 = i_0 (1 - e^{-Rt/L_2}) //$

B And heat in $R = U_i - U_f$
 $= \frac{1}{2} (L_1 + L_2) i_0^2 - \frac{1}{2} L_1 i_0^2 = \frac{1}{2} L_2 i_0^2 //$



28. In the circuit shown, a parallel combination of a coil of inductance L and a resistance R can be connected across a battery of electromotive force $\mathcal{E}$ and internal resistance r through a switch. Find total heat dissipated in the resistance R after the switch is closed.

At time t



From 0 to V/r

$$V - (i_1 + i_2)r = \overset{1}{i_1 R} = \overset{2}{L \frac{di_2}{dt}}$$

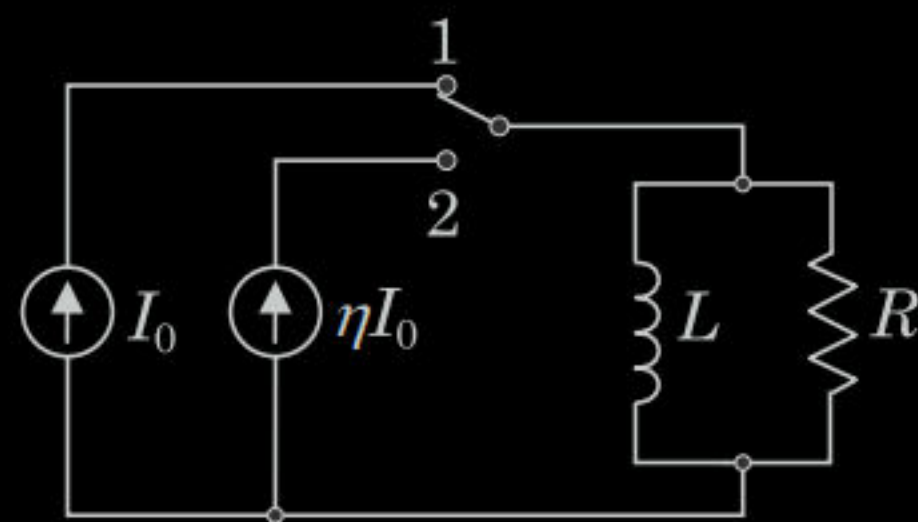
Let dH be Heat dissipated in R in dt time:

$$dH = i_1^2 R dt$$

$$= i_1 L di_2 \quad (\text{From 2})$$

$$= \frac{(V - i_2 r)}{(r + R)} L di_2 \quad (\text{From 1})$$

$$\Rightarrow H = \frac{L}{r + R} \int_0^{V/r} (V - i_2 r) di_2 = \frac{V^2 L}{2r(r + R)}$$



29. A parallel combination of an inductance L and a resistance R can be connected to either of the two ideal current sources of current I_0 and ηI_0 through a switch as shown. Initially the switch is in position 1 for a long time. Now the switch is thrown to position 2. Find amount of charge that will pass through the resistance and heat dissipated in it after the switching until a steady state is established.

Similar: MCQ 9

At time t

$$L \frac{di}{dt} = (\eta i_0 - i) R \quad (1)$$

$$dq = (\eta i_0 - i) dt$$

$$= \frac{L}{R} di \quad (\text{From 1})$$

$$\Rightarrow q = \frac{L}{R} \int_{i_0}^{\eta i_0} di$$

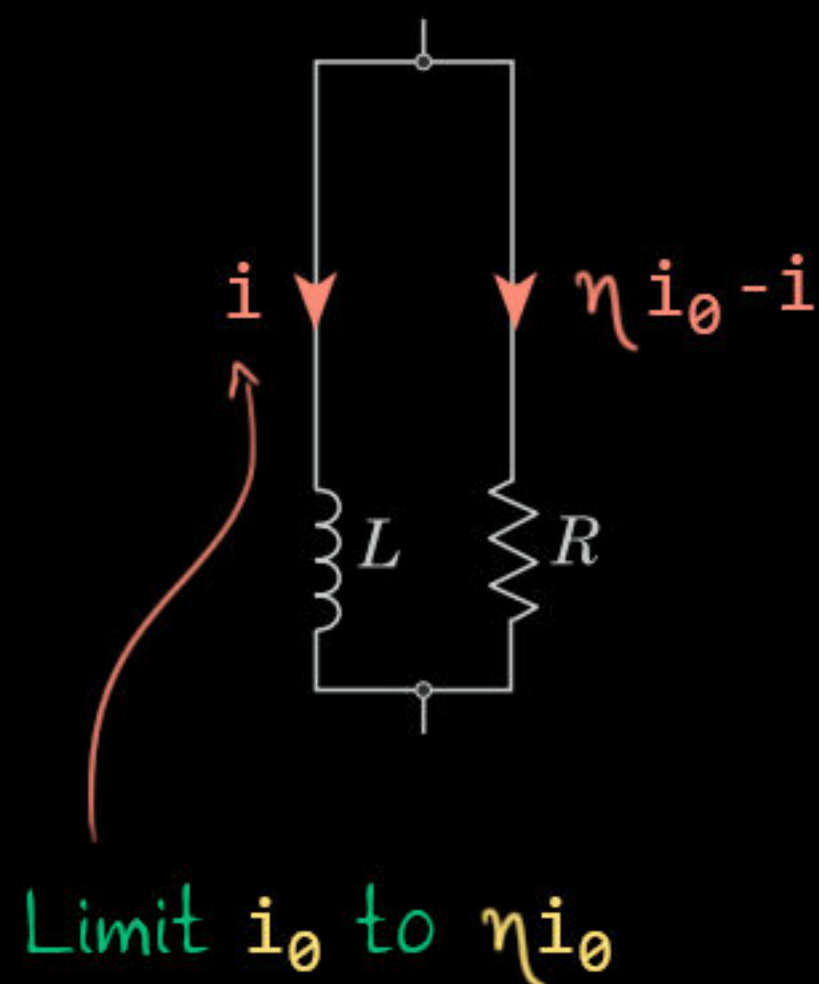
$$= \frac{(\eta - 1) i_0 L}{R} //$$

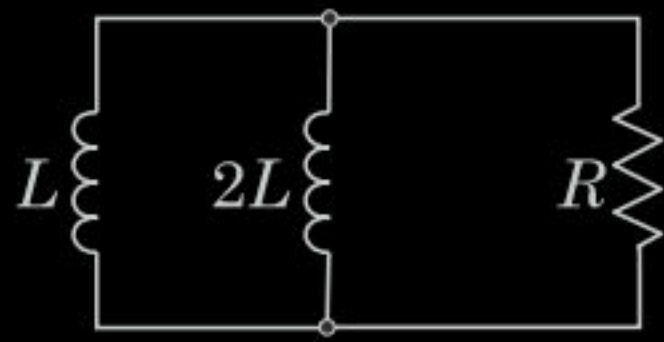
$$dH = (\eta i_0 - i)^2 R dt$$

$$= (\eta i_0 - i) L di \quad (\text{From 1})$$

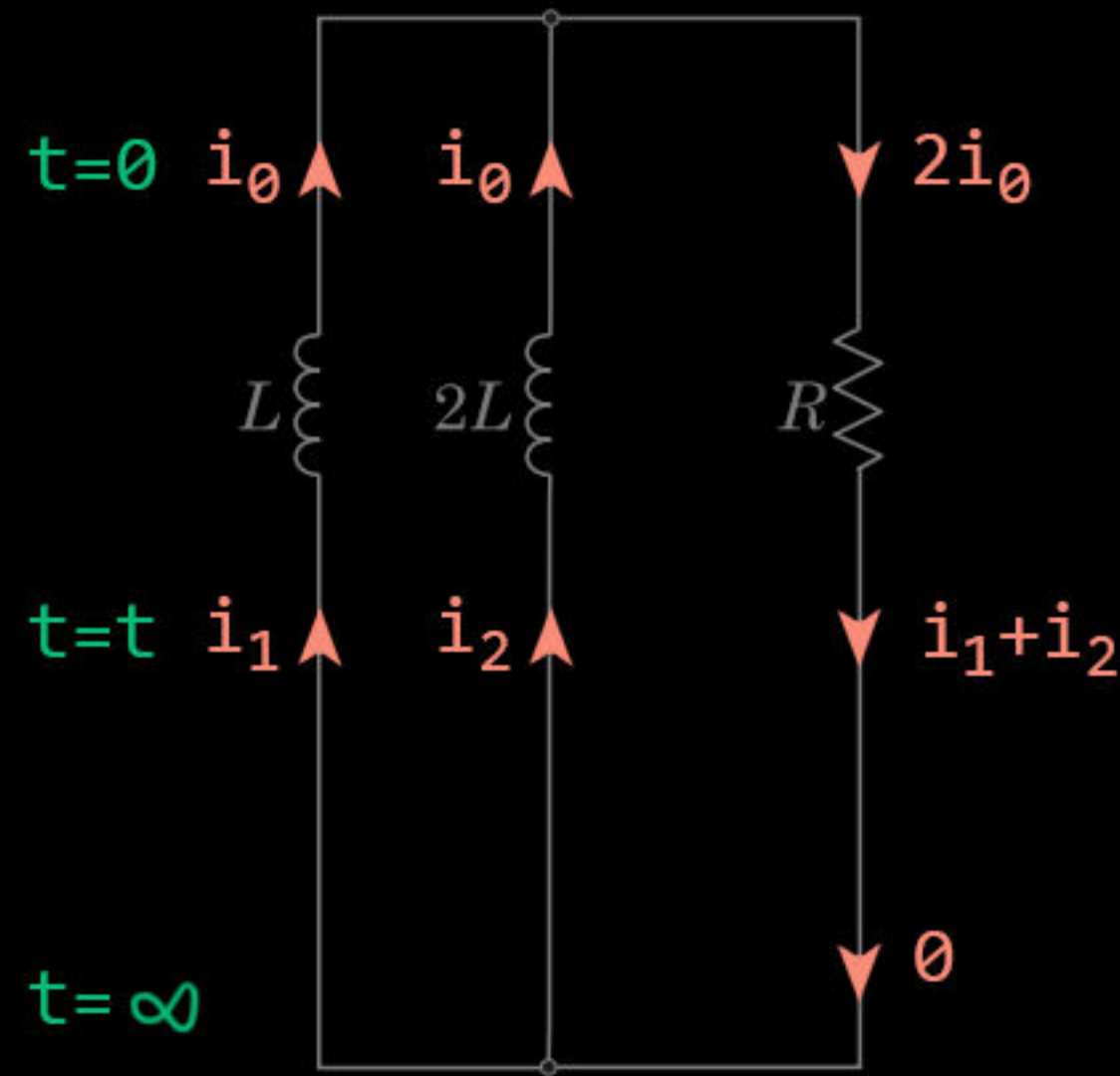
$$\Rightarrow H = L \int_{i_0}^{\eta i_0} (\eta i_0 - i) di$$

$$= \frac{(\eta - 1)^2 L i_0^2}{2} //$$





30. A parallel circuit consists two ideal inductances L and $2L$ and a resistance R . At an instant the same current I_0 is flowing in both the inductances in the same direction. How much charge will flow and how much heat will be dissipated in the resistance after this instant.



From 1 @ t $di_1 = 2 di_2$

$$\Rightarrow \int_{i_0}^{i_1} di_1 = 2 \int_{i_0}^{i_2} di_2$$

$$\Rightarrow i_1 = 2i_2 - i_0$$

$$\therefore i_1 + i_2 = 3i_2 - i_0 \quad (3)$$

At steady state, current in R will be zero.

$$\therefore @ t = \infty \rightarrow i_1 + i_2 = 0$$

$$\Rightarrow i_2 = i_0/3$$

$$\therefore i_2 \text{ limits: } i_0 \rightarrow i_0/3$$

$$L \frac{di_1}{dt} = 2L \frac{di_2}{dt} = -(i_1 + i_2)R$$

Approach: Solve in terms of i_2 .

$$dq = \int (i_1 + i_2) dt$$

$$\Rightarrow q = -\frac{2L}{R} \int_{i_0}^{i_0/3} di_2 = \frac{4Li_0}{3R} //$$

(From 2)
by Ankit Singhvi

$$dH = \int (i_1 + i_2)^2 R dt$$

$$\Rightarrow H = \int_{i_0}^{i_0/3} (3i_2 - i_0)(-2L di_2) \quad (\text{From 2 and 3})$$

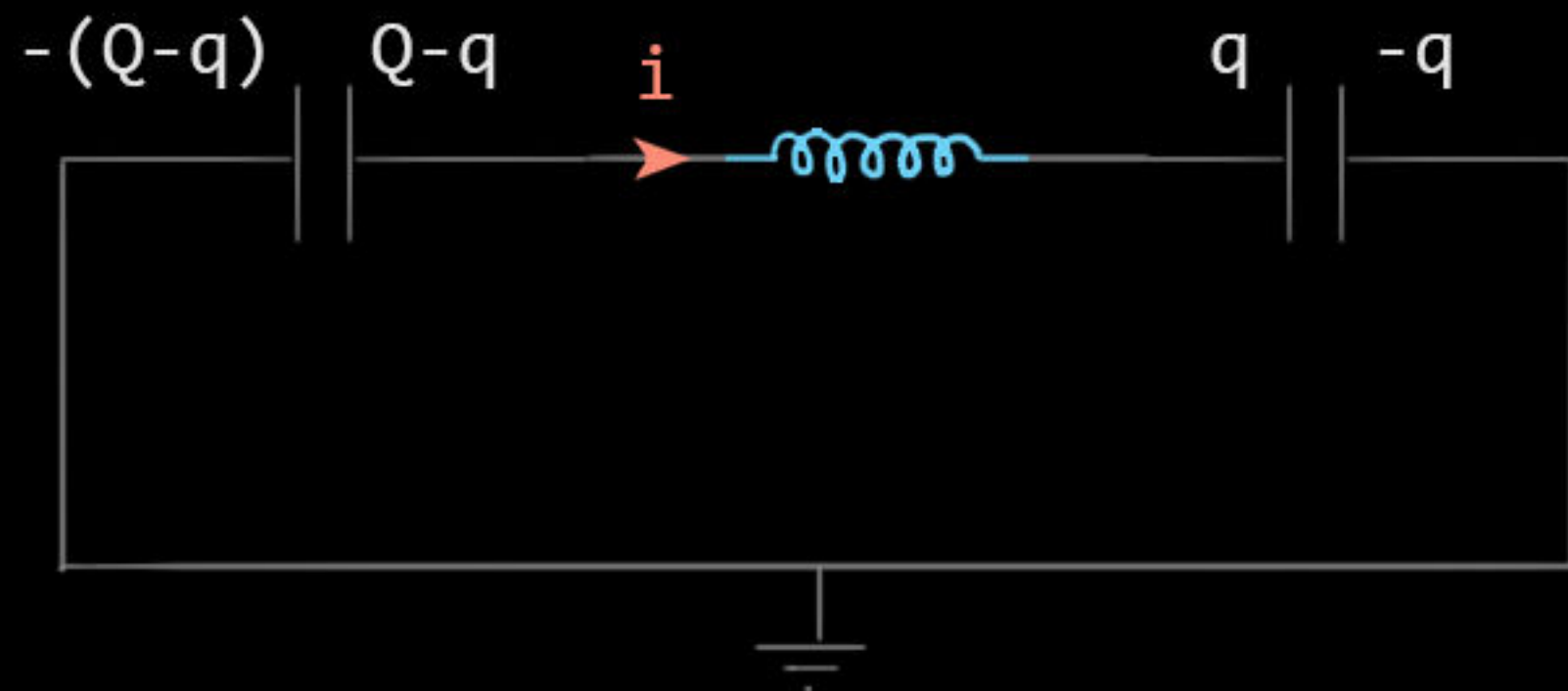
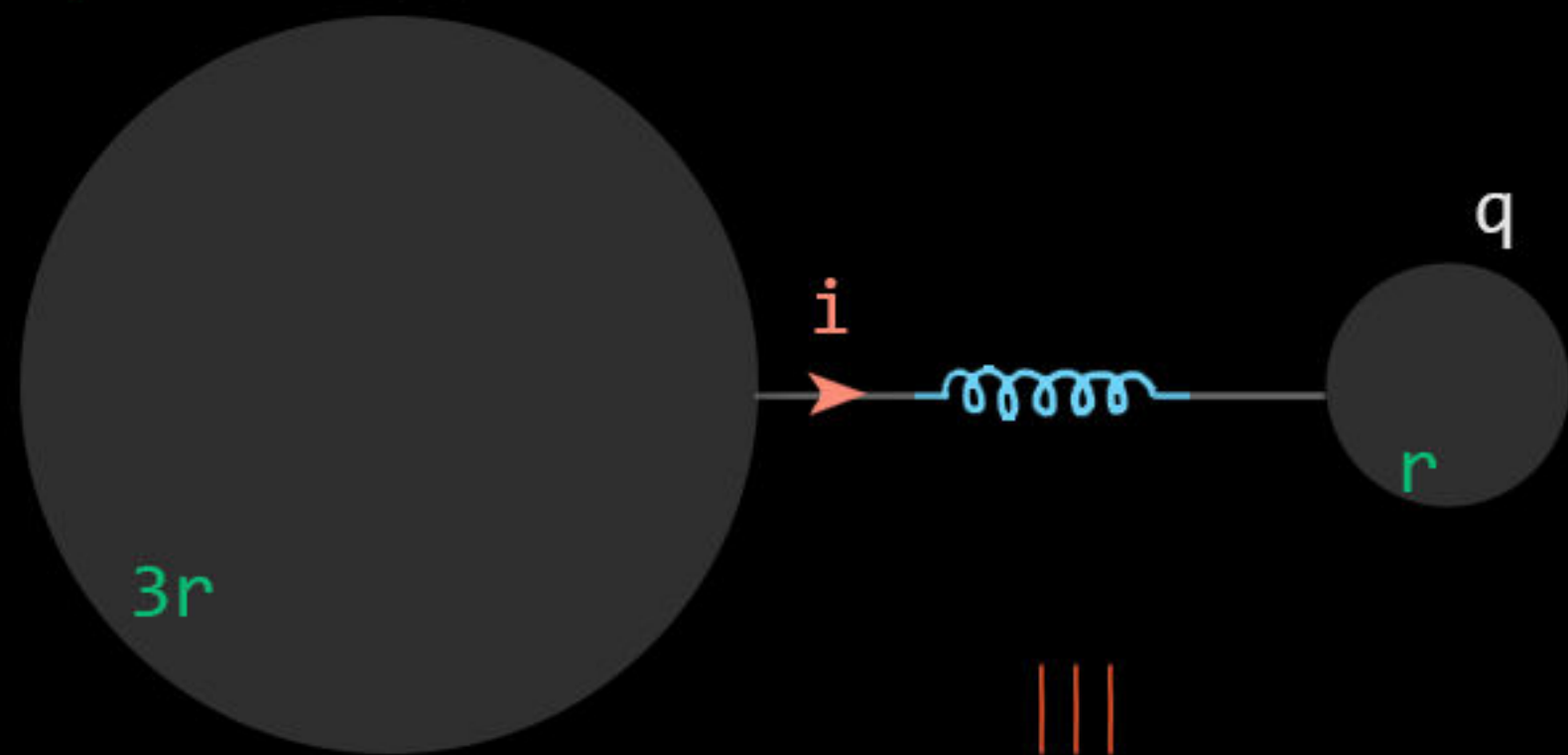
$$= \frac{4Li_0^2}{3} //$$

Youtube / Arihant Senior

We know: $C = 4\pi\epsilon_0 R$

31. A conducting ball of radius $3r$ having charge Q is at a great distance from a neutral conducting ball of radius r . They are connected with a thin perfectly conducting wire of inductance L through a switch. What will the maximum current in the wire be after the switch is closed?

time t : $Q-q$



Method 1

By equation

$$\frac{Q-q}{4\pi\epsilon_0 3r} - L \frac{di}{dt} - \frac{q}{4\pi\epsilon_0 r} = 0$$

$$\Rightarrow \frac{d^2 q}{dt^2} + \underbrace{\frac{1}{4\pi\epsilon_0 r L} \frac{4}{3}}_{\omega^2 \text{ of SHM}} (q - \frac{Q}{4}) = 0$$

Solution

$$q - \frac{Q}{4} = q_0 \sin(\omega t + \phi)$$

$$i = q_0 \omega \cos(\omega t + \phi)$$

$$t=0, i=0 \Rightarrow \phi = \pi/2$$

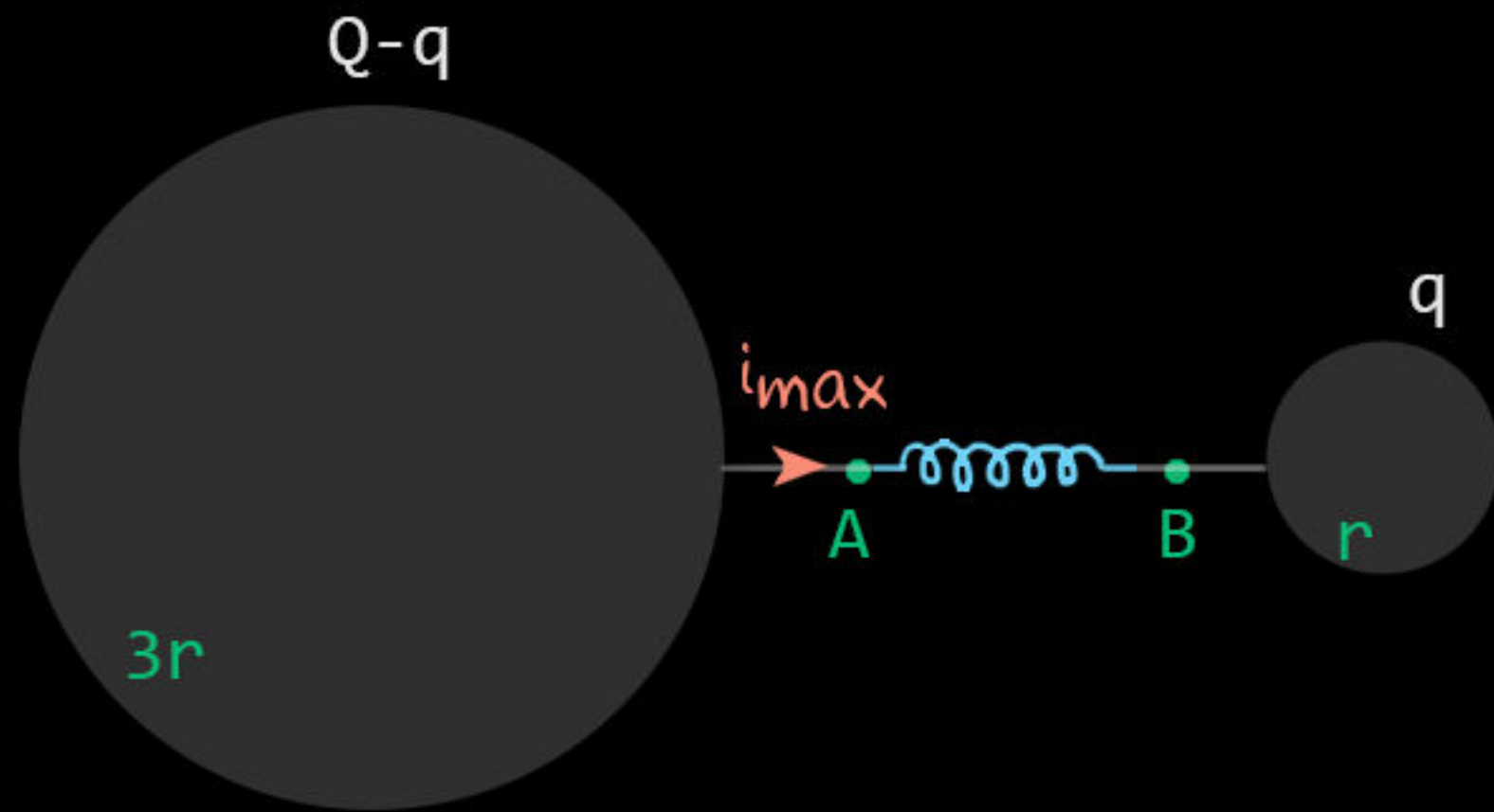
$$\phi = \pi/2 \rightarrow t=0, q=0 \Rightarrow q_0 = -Q/4$$

$$\therefore i_{\max} = q_0 \omega = \frac{Q}{4} \sqrt{\frac{1}{3\pi\epsilon_0 r L}}$$

by Ankit Singhvi

Youtube / Arihant Senior

Method 2 By energy



At i_{max} , $di/dt = 0$

$$\Rightarrow L di/dt = 0$$

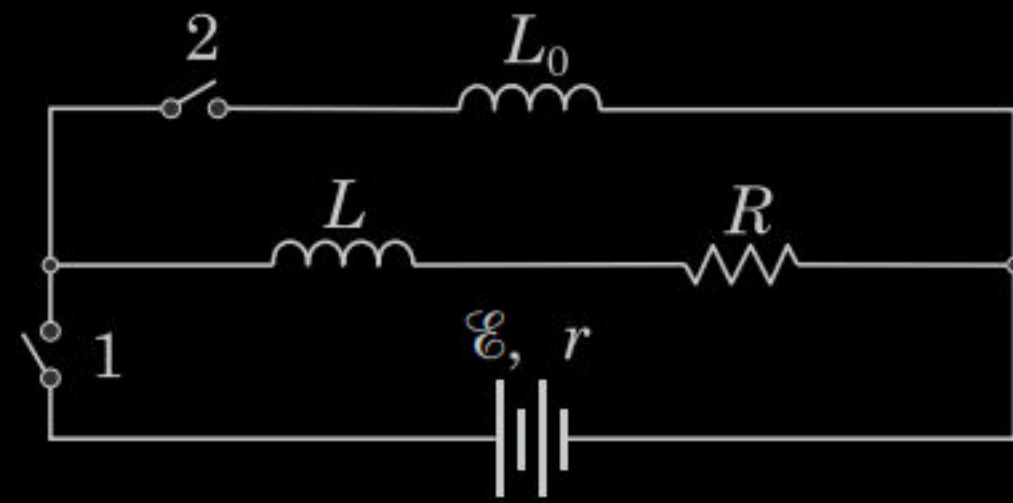
$\Rightarrow$ A and B are equipotential.

$\Rightarrow$ Both spheres/capacitors are at same potential.

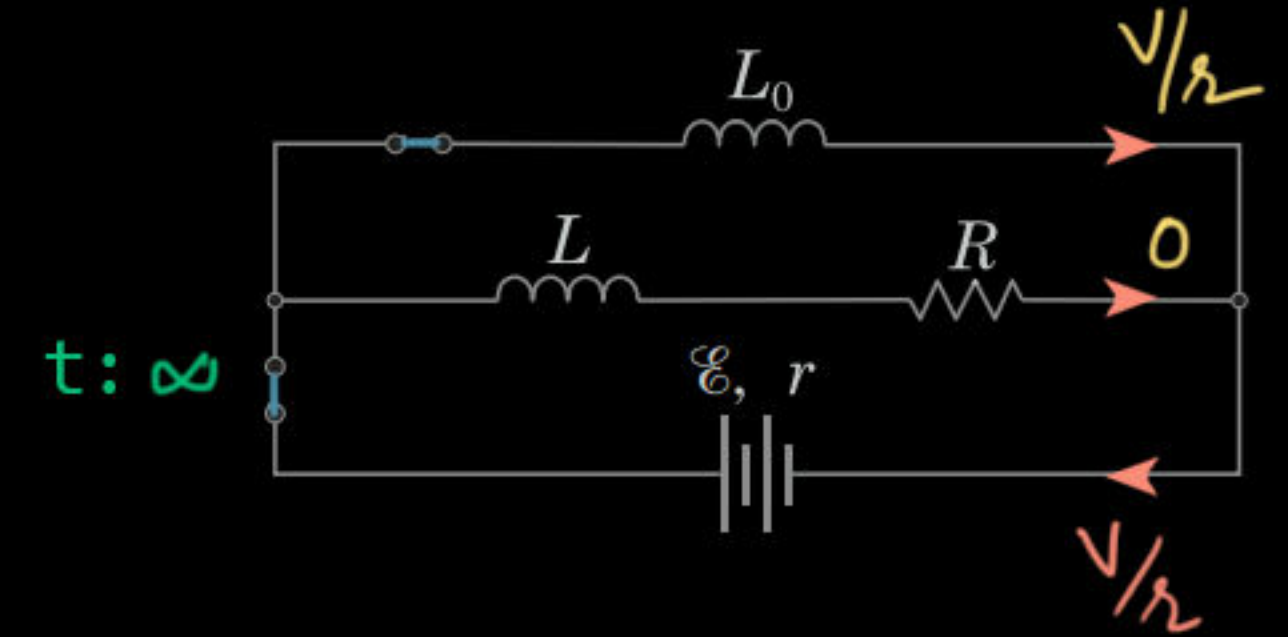
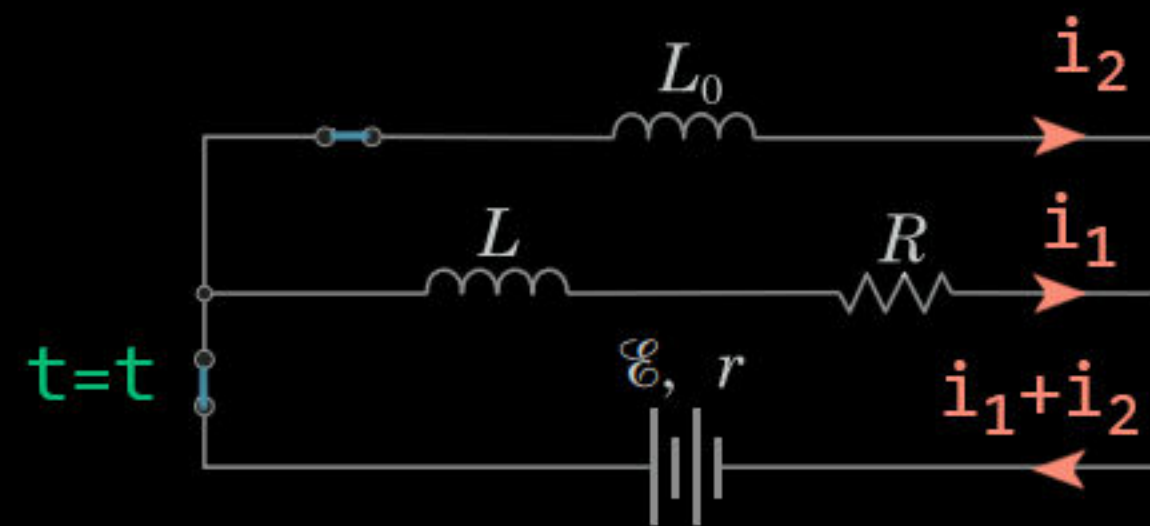
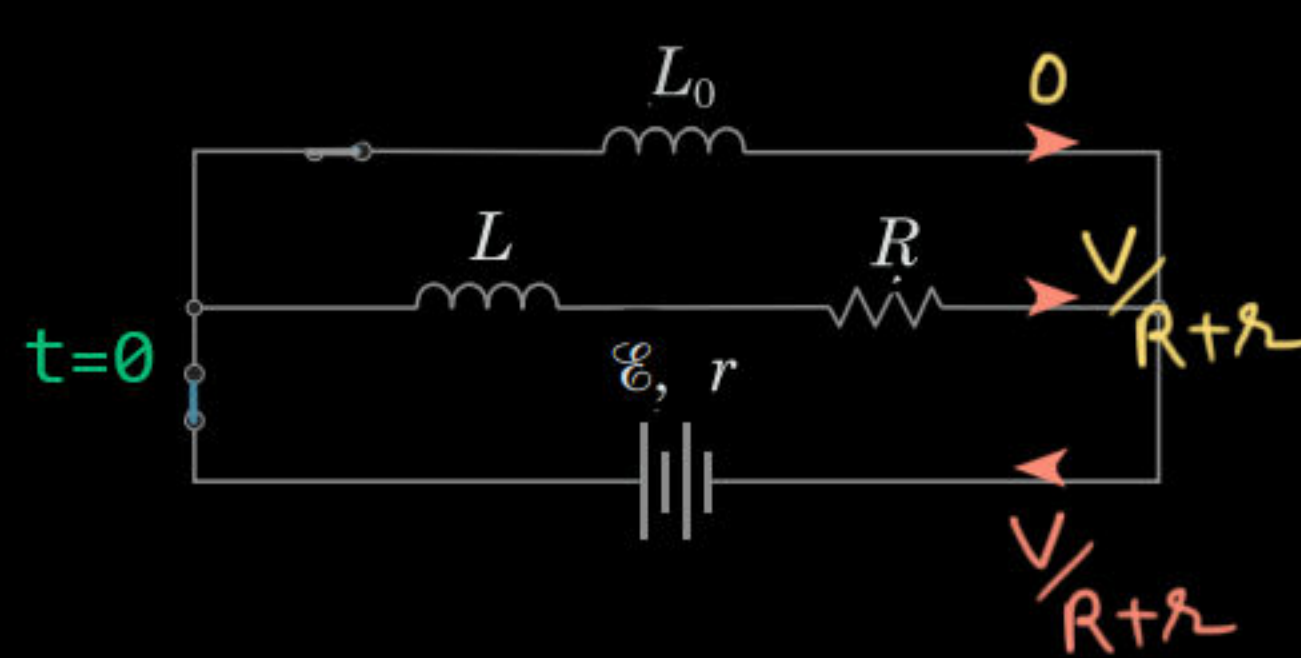
$$\Rightarrow \frac{Q-q}{4\pi\epsilon_0(3r)} = \frac{q}{4\pi\epsilon_0 r} \Rightarrow q = Q/4$$

E conservation: $U_i = U_f$

$$\Rightarrow \frac{Q^2}{2(4\pi\epsilon_0 3r)} = \frac{(Q-q)^2}{2(4\pi\epsilon_0 3r)} + \frac{q^2}{2(4\pi\epsilon_0 r)} + \frac{1}{2} L i_{max}^2 \xrightarrow{q=Q/4} i_{max} = \frac{Q}{4} \sqrt{\frac{1}{3\pi\epsilon_0 r L}}$$



32. A circuit shown consists of two inductances L_0 and L , a resistance R , a battery of electromotive force $\mathcal{E}$ and internal resistance r and two switches 1 and 2. Initially switch 1 is closed for a long time. Find total charge that will flow through the resistance after the switch 2 is closed.

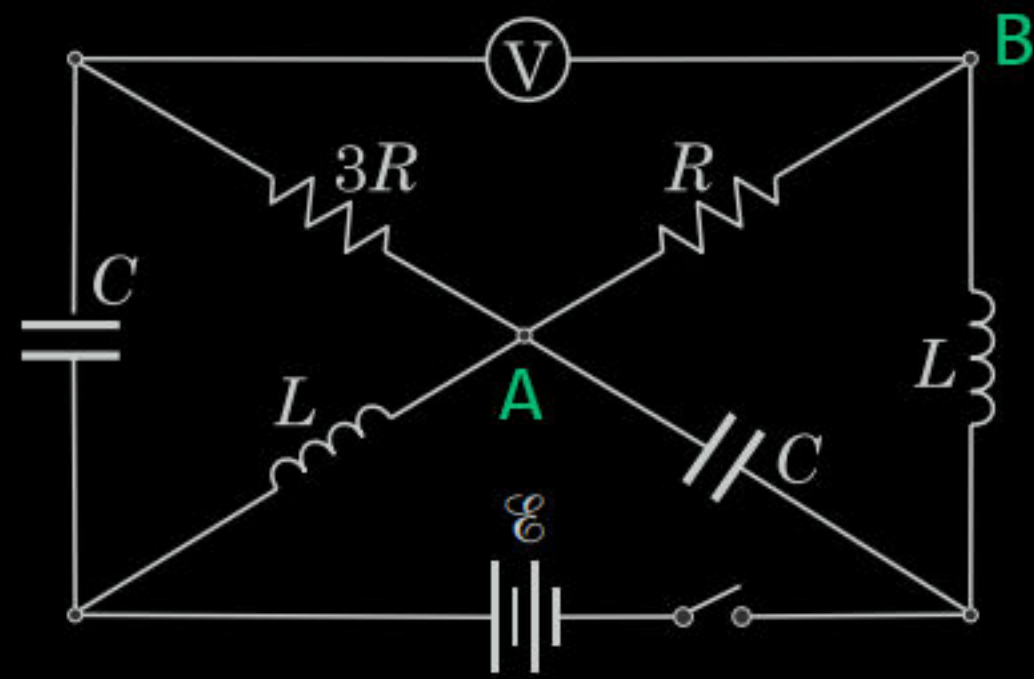


$$L_0 \frac{di_2}{dt} = L \frac{di_1}{dt} + i_1 R = V - (i_1 + i_2)r \quad (\text{Not needed})$$

$$\Rightarrow L_0 di_2 = L di_1 + i_1 R dt$$

$$\Rightarrow L_0 \int_0^{V/R} di_2 = L \int_0^0 di_1 + \underbrace{\int_{V/(R+r)}^0 i_1 R dt}_{RQ_0}$$

$$\Rightarrow Q_0 = \frac{V}{R} \left(\frac{L_0}{r} + \frac{L}{R+r} \right)$$



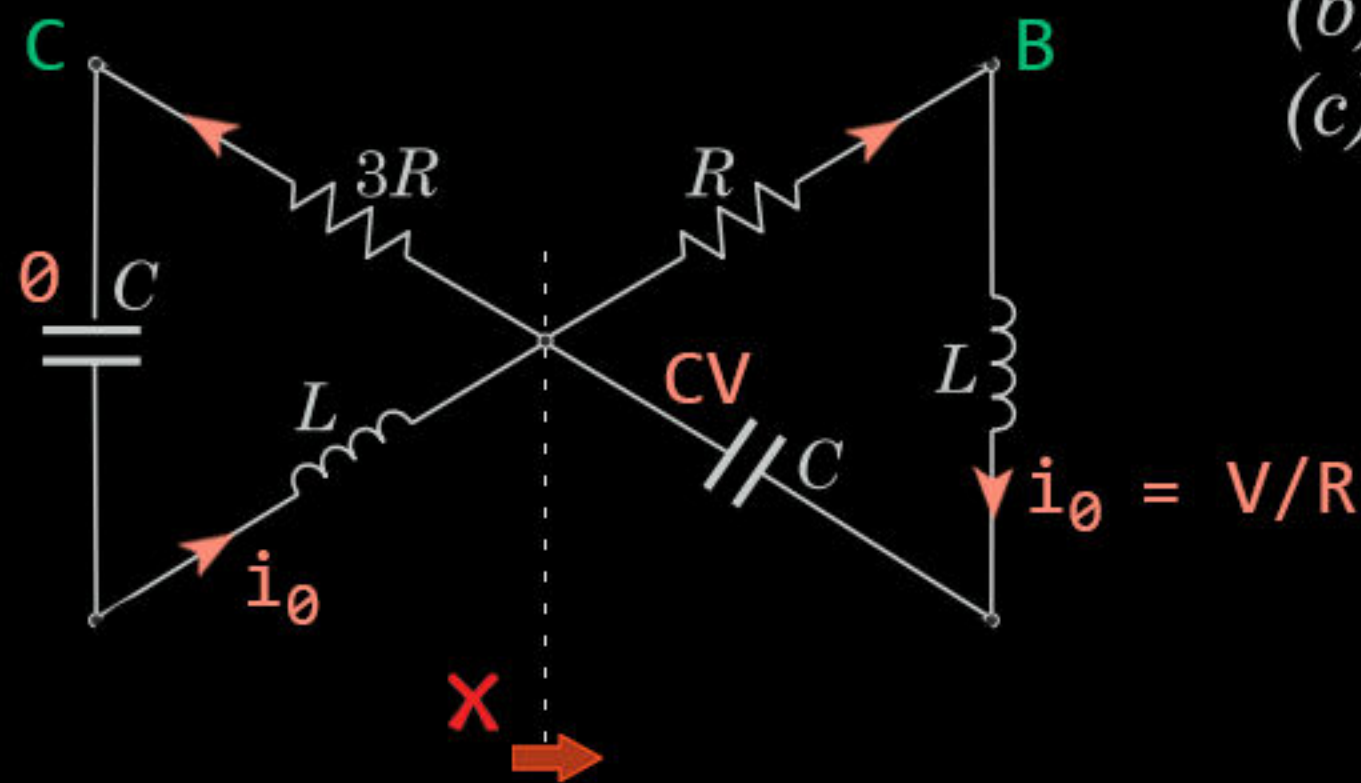
33. A circuit shown consists of two resistances R and $3R$, two capacitors each of capacitance C , two inductors each of inductance L , an ideal battery of electromotive force $\mathcal{E}$, an ideal voltmeter and a switch. Initially the switch is closed and the circuit is in steady state.

(a) Find the voltmeter reading. $= V_{AB} = V_{//}$

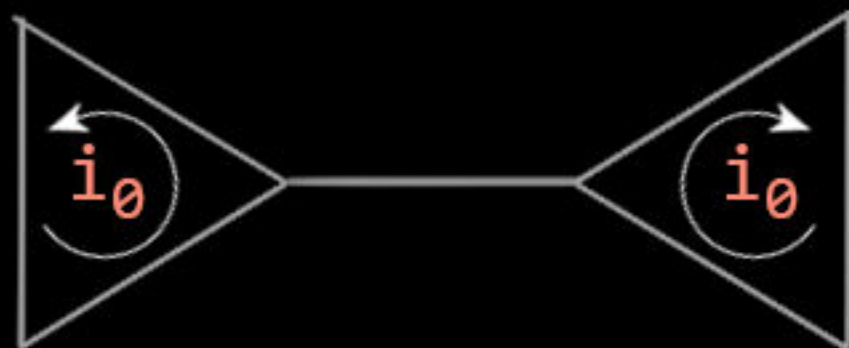
Now, the switch is opened.

(b) Find the voltmeter reading immediately after the switching.

(c) Find the total heat dissipated in the resistors after the switching.



(c) Current can NOT cross this junction. As is evident by extending the junction as shown:

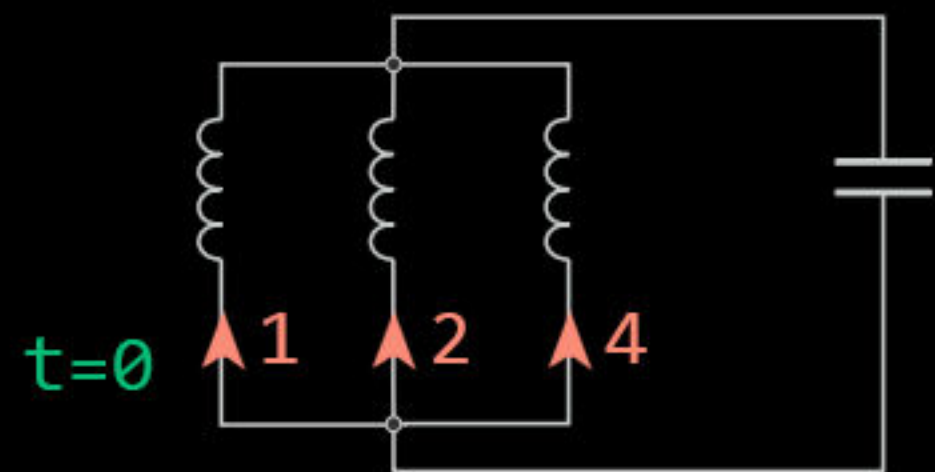


$$\begin{aligned} \text{B Voltmeter reading} &= V_{BC} = -i_0 R + i_0 3R \\ &= 2i_0 R \\ &= 2V_{//} \end{aligned}$$

→ Resistors will dissipate energy initially stored in their respective loops.

$$\therefore \text{Heat}(R) = \frac{1}{2} L i_0^2 + \frac{1}{2} C V^2 = \frac{L V^2}{2R^2} + \frac{1}{2} C V_{//}^2$$

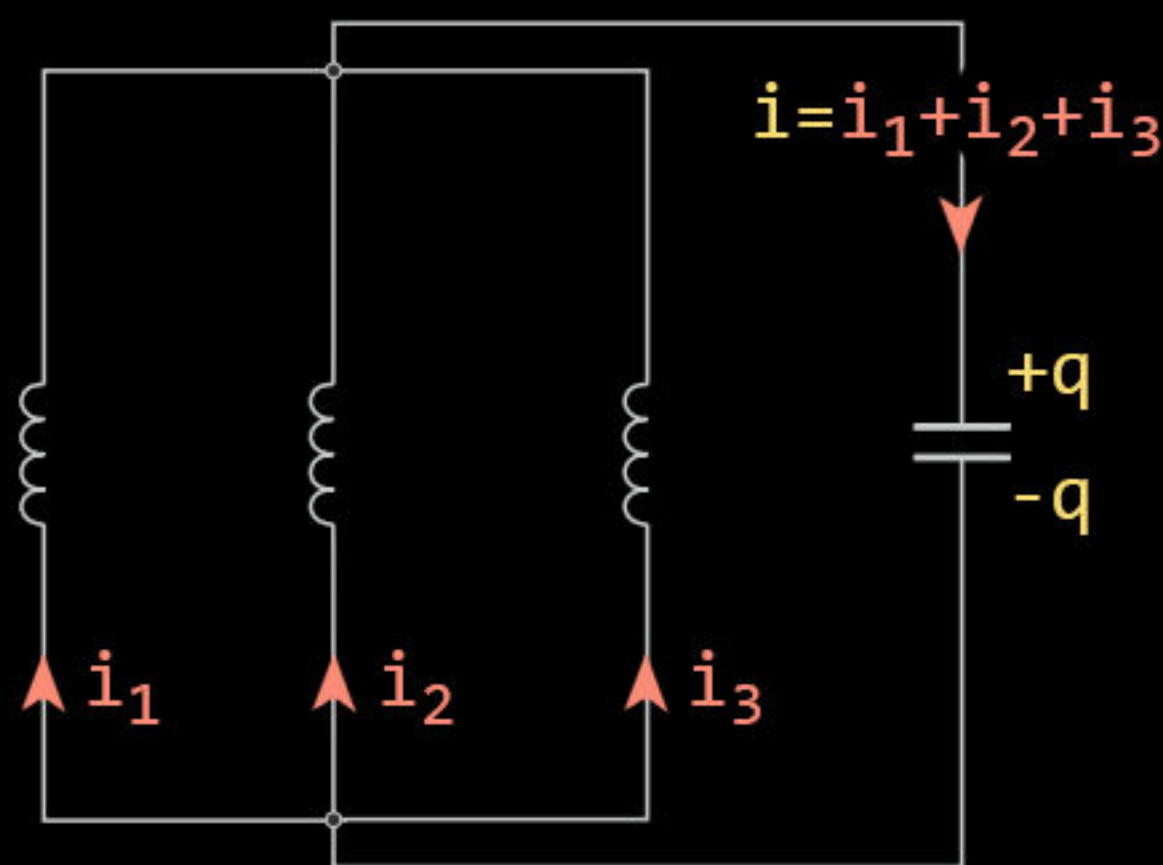
$$\text{And Heat}(3R) = \frac{1}{2} L i_0^2 //$$



34. A circuit shown consists of three coils of inductances 1.0 H each and a $1.0 \mu\text{F}$ capacitance. At an instant, when charge on the capacitor is zero, currents of 1.0 A , 2.0 A and 4.0 A are flowing through the coils in the same direction.

- (a) Find the maximum charge on the capacitor.
 (b) Find the maximum currents through each of the coils.

$t=t$



$$L \frac{di_1}{dt} = L \frac{di_2}{dt} = L \frac{di_3}{dt} = -\frac{q}{C}$$

adding

$$L \frac{d(i_1 + i_2 + i_3)}{dt} = -\frac{3q}{C}$$

$$\Rightarrow L \frac{d^2 q}{dt^2} = -\frac{3q}{C}$$

We can write this LC oscillation equation directly too, as initial conditions do not matter in general equation.

$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{3}{LC} q = 0$$

$$\omega^2 = \frac{1}{L_{eq} C}$$

$$\therefore \text{Let } q = q_0 \sin(\omega t + \phi)$$

$$\downarrow t=0, q=0$$

$$\Rightarrow q = q_0 \sin \omega t$$

$\downarrow$ diff

$$i = q_0 \omega \cos \omega t$$

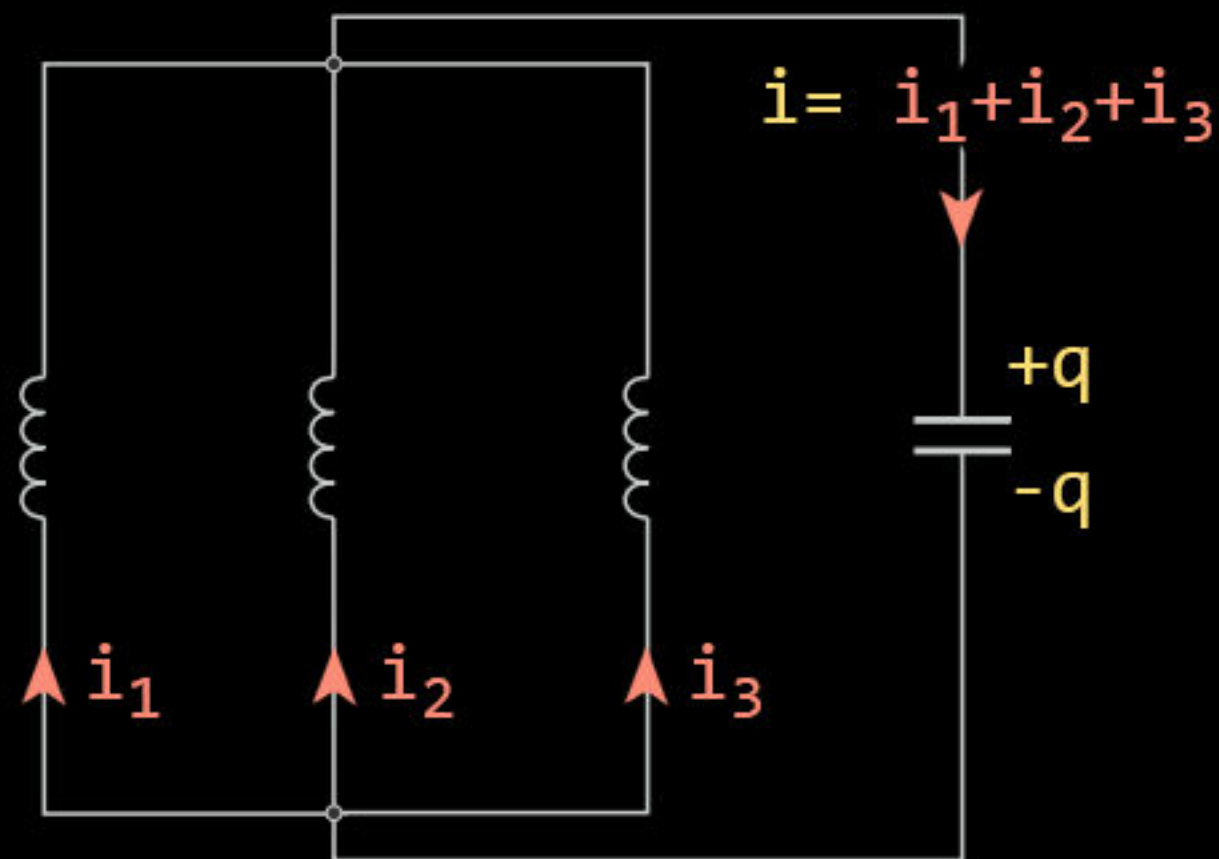
$$\downarrow t=0, i=7$$

$$\Rightarrow 7 = q_0 \omega$$

$$\text{A } q_0 = \frac{7}{\omega} = 7 \sqrt{\frac{LC}{3}}$$

(b) Find the maximum currents through each of the coils.

$t=t$



$$L \frac{di_1}{dt} = L \frac{di_2}{dt} = L \frac{di_3}{dt} = -\frac{q}{C}$$

$$\int_1^i di_1 = \int_2^i di_2 = \int_4^i di_3$$

$$\Rightarrow i_1 - 1 = i_2 - 2 = i_3 - 4 \quad (1)$$

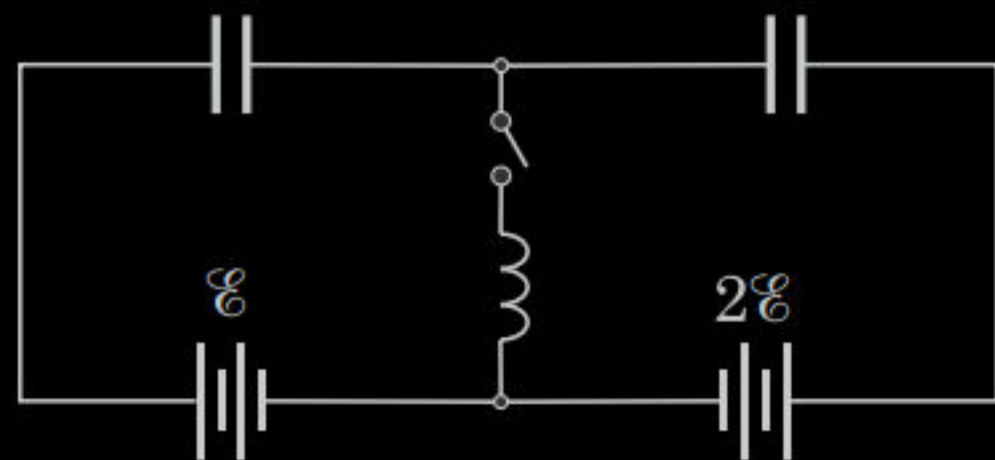
∴ We shall choose either 7 or -7 to maximise currents in individual L's.

We have proven: $i = 7 \cos \omega t \xrightarrow{\therefore} i_1 + i_2 + i_3 = -7 \text{ to } +7$

Using (1) $\left\{ \begin{array}{l} \text{For } i_{1\max} \quad i_1 + (i_1 + 1) + (i_1 + 3) = -7 \Rightarrow i_{1\max} = -11/3 \\ \text{For } i_{2\max} \quad (i_2 - 1) + i_2 + (i_2 + 2) = -7 \Rightarrow i_{2\max} = -8/3 \\ \text{For } i_{3\max} \quad (i_3 - 3) + (i_3 - 2) + i_3 = 7 \Rightarrow i_{3\max} = 4 \end{array} \right.$

by Ankit Singhvi

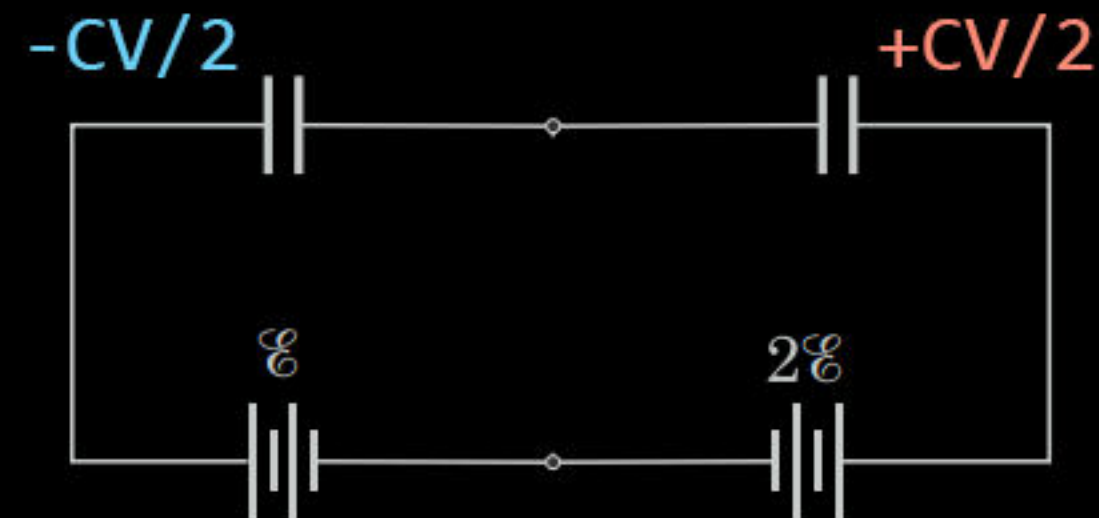
Youtube / Arihant Senior



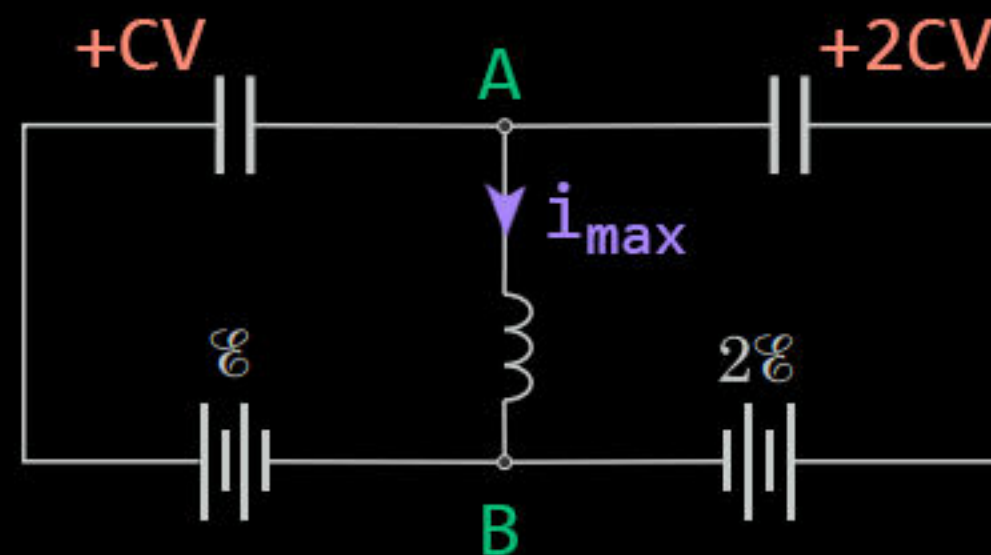
35. A circuit shown consists of two identical capacitors each of capacitance C , a coil of inductance L , two ideal batteries of electromotive force $\mathcal{E}$ and $2\mathcal{E}$ and a switch. The switch is open for a long time. Find maximum current through the coil after the switch is closed.

Method 1 By energy

$t=0$



At $i_{\max}$



At $i_{\max}$, $di/dt = 0$

$\Rightarrow L di/dt = 0$

$\Rightarrow A$ and B are equipotential.

$\Rightarrow$ Charges will develop on capacitors as shown.

$$W_{\text{Battery}} = U_f - U_i$$

$$\Rightarrow V \left[CV - \left(-\frac{CV}{2} \right) \right] + 2V \left(2CV - \frac{CV}{2} \right) = \left[\frac{(CV)^2}{2C} + \frac{(2CV)^2}{2C} + \frac{1}{2} L i_{\max}^2 \right] - \left[\frac{(CV/2)^2}{2C} + \frac{(CV/2)^2}{2C} \right]$$

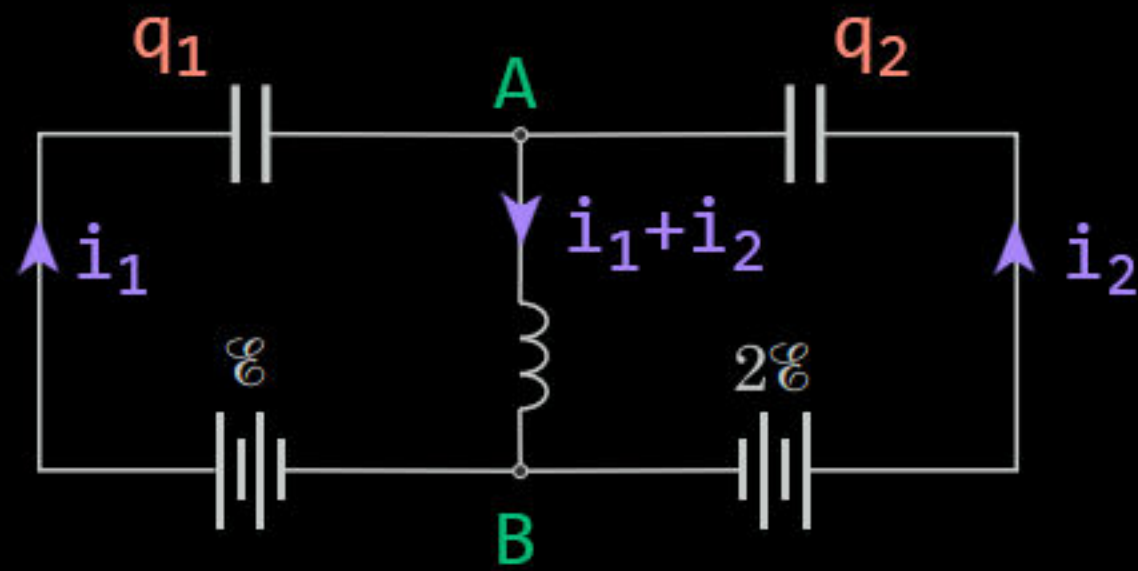
$$\Rightarrow i_{\max}^2 = \frac{2CV^2}{L} \left(1 + \frac{1}{2} + 4 - \frac{1}{2} - 2 + \frac{1}{8} + \frac{1}{8} \right) \Rightarrow i_{\max} = 3V \sqrt{\frac{C}{2L}}$$

by Ankit Singhvi

Youtube / Arihant Senior

Method 2

By equation



$$V - \frac{q_1}{C} = 2V - \frac{q_2}{C} = L \frac{d(i_1 + i_2)}{dt}$$

adding

$$\Rightarrow 3V - \frac{(q_1 + q_2)}{C} = 2L \frac{d(i_1 + i_2)}{dt}$$

$$\downarrow q_1 + q_2 \rightarrow q, i_1 + i_2 \rightarrow i$$

$$\Rightarrow 2L \frac{d^2 q}{dt^2} + \frac{q}{C} - 3V = 0$$

$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{1}{2LC} (q - 3CV)$$

ω^2

$$\therefore \text{Let } q - 3CV = q_0 \sin(\omega t + \phi)$$

↓ diff

$$i = q_0 \omega \cos(\omega t + \phi)$$

$$\downarrow t=0, i=0$$

$$\phi = \pi/2$$

↓

$$\rightarrow q - 3CV = q_0 \cos \omega t$$

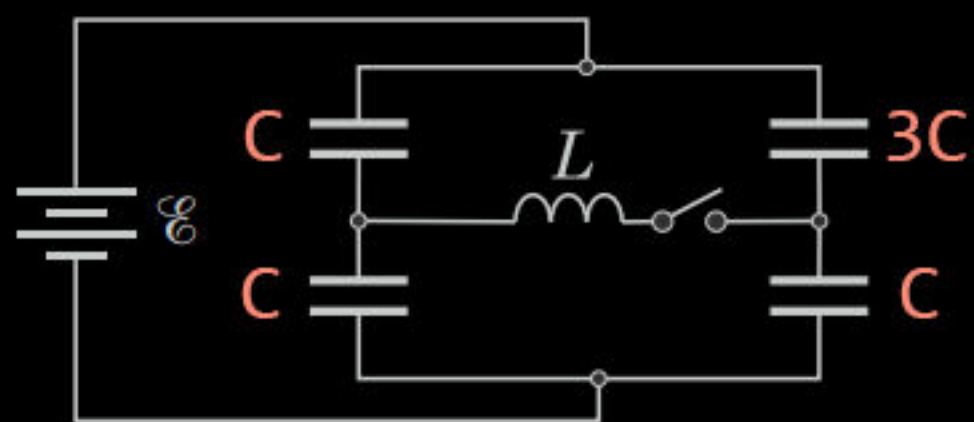
$$\downarrow t=0, q=0$$

$$q_0 = -3CV$$

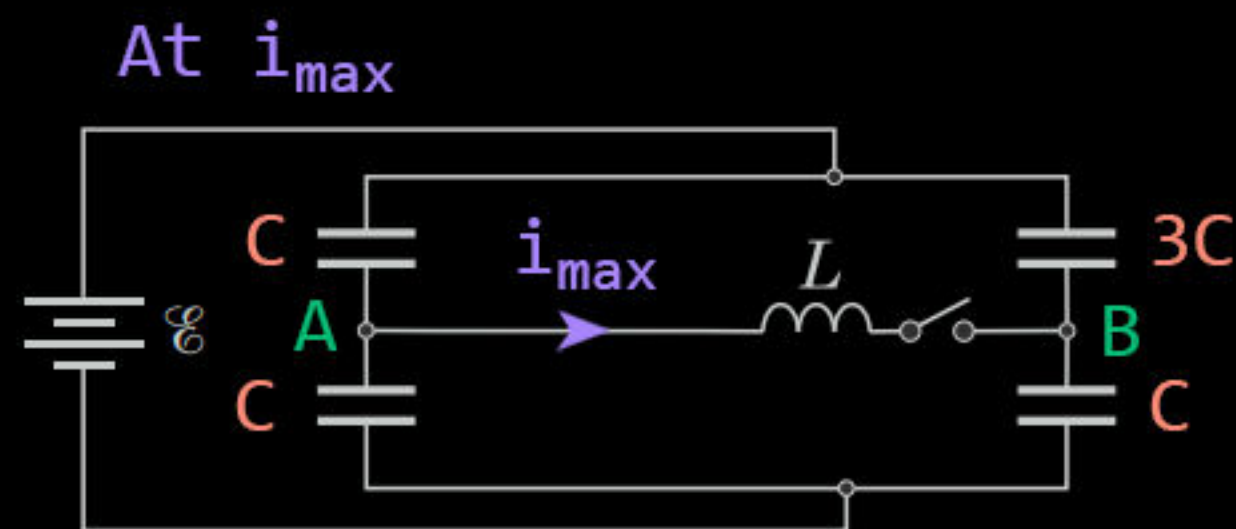
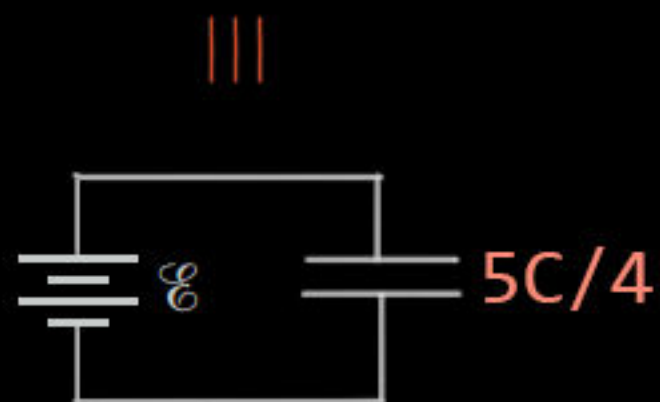
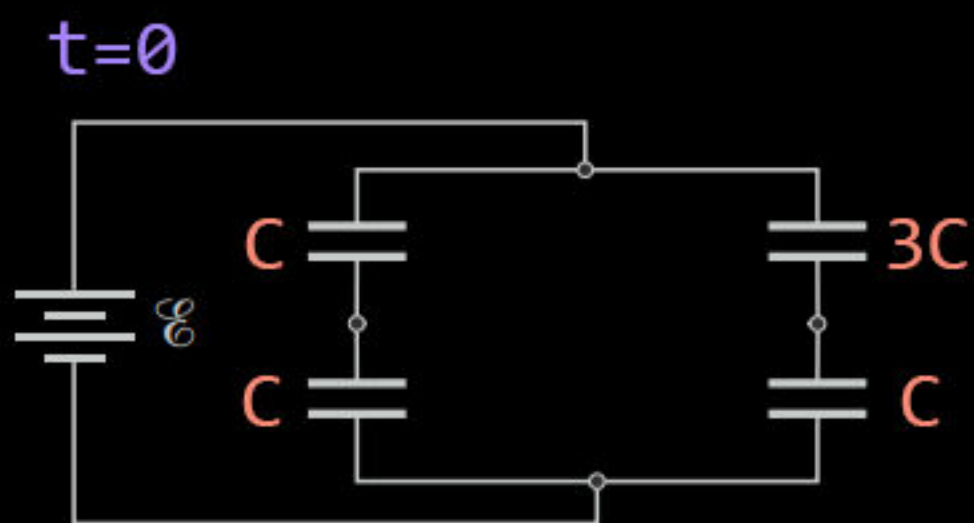
↓

$$i = -3CV \omega \cos \omega t$$

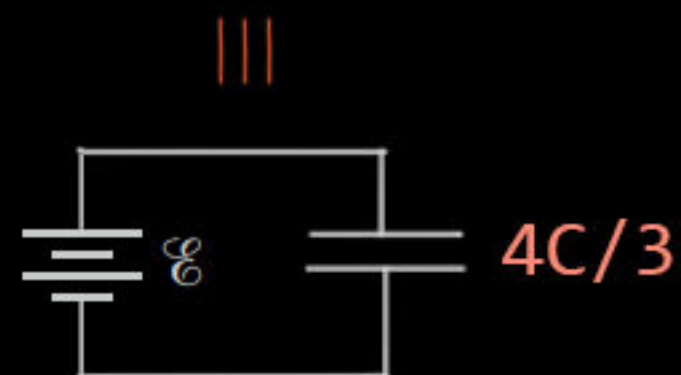
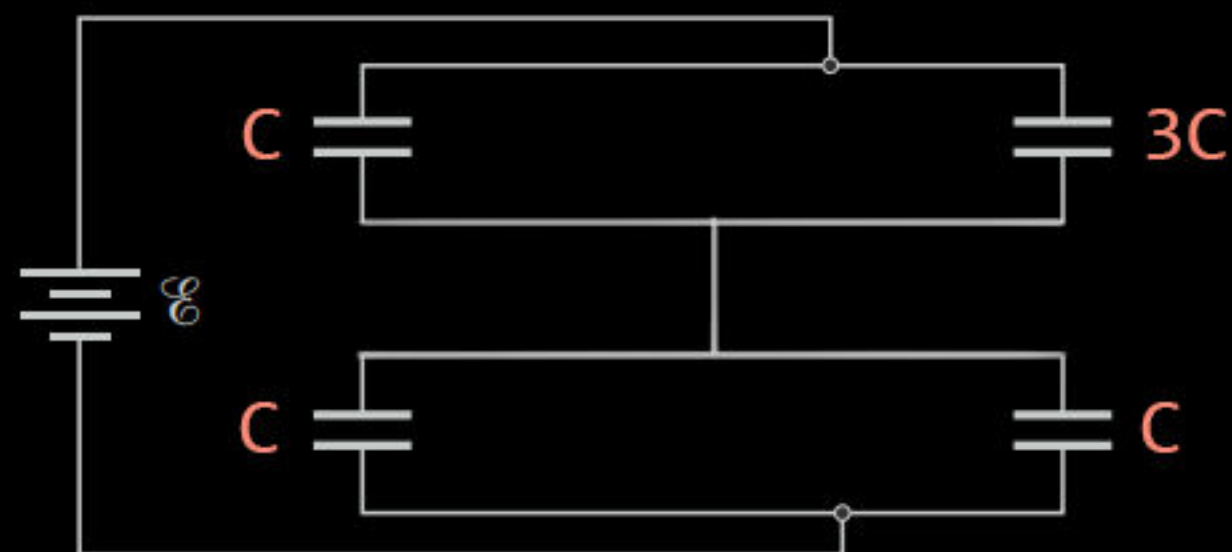
$$i_{\max} = 3CV \sqrt{\frac{1}{2LC}} = 3V \sqrt{\frac{C}{2L}}$$



36. The circuit shown consists of four capacitors, an ideal battery of electromotive force $\mathcal{E}$, a switch and an inductance L . Capacitance of three of these capacitors is C and that of the fourth is $3C$. Find maximum current through the coil after the switch is closed. *Similar: BYU 35*



||| $\because L \frac{di}{dt} = 0 \Rightarrow V_A = V_B$



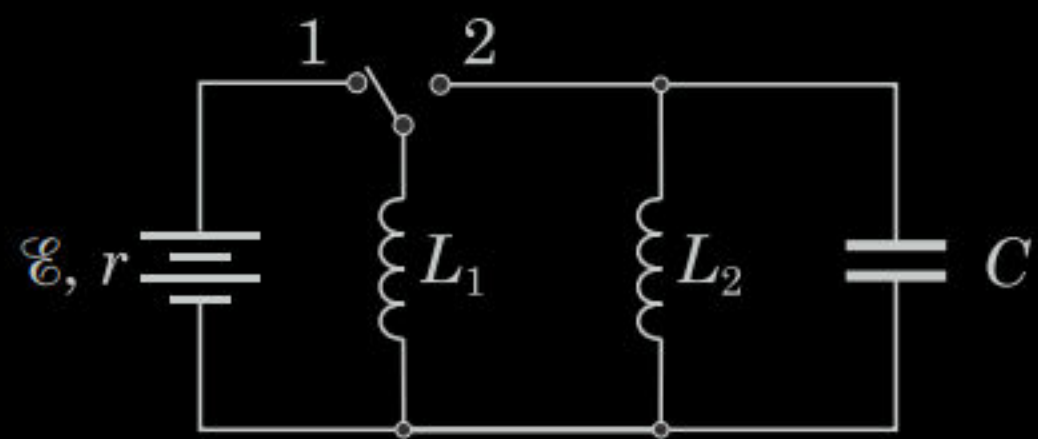
$$W_{\text{Battery}} = U_f - U_i$$

$$\Rightarrow V \left[4 \frac{CV}{3} - \frac{5CV}{4} \right]$$

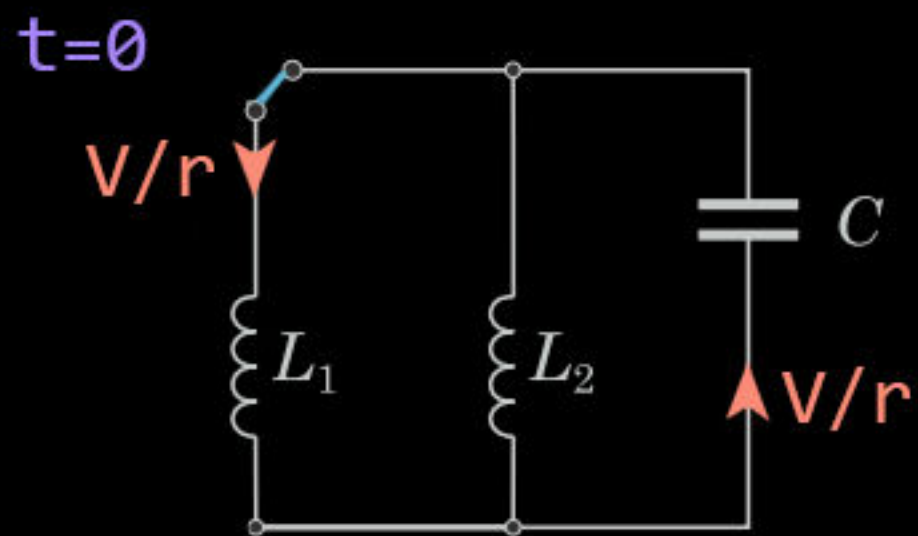
$$= \left[\frac{1}{2} \left(\frac{4C}{3} \right) V^2 + \frac{1}{2} L i_{\max}^2 \right] - \frac{1}{2} \left(\frac{5C}{4} \right) V^2$$

$$\Rightarrow i_{\max} = \sqrt{\frac{2CV^2}{L} \left(\frac{4}{3} - \frac{5}{4} - \frac{2}{3} + \frac{5}{8} \right)}$$

$$= \frac{V}{2} \sqrt{\frac{C}{3L}}$$



37. A circuit shown consists of two inductors of inductances L_1 and L_2 , a capacitor of capacitance C , a battery of electromotive force $\mathcal{E}$ and internal resistance r and a switch. Initially the switch was in position 1 for a long time. Find the maximum charge on the capacitor and maximum current in the inductor L_2 after the switch is thrown to position 2.



$$L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = -\frac{q}{C}$$

$$\downarrow \times L_2 \quad \downarrow \times L_1$$

$$L_1 L_2 \frac{di_1}{dt} + L_2 L_1 \frac{di_2}{dt} = -\frac{q L_2}{C} - \frac{q L_1}{C}$$

$$\Rightarrow L_1 L_2 \frac{di}{dt} = -\left(\frac{L_1 + L_2}{C}\right) q$$

$$\Rightarrow \frac{d^2 q}{dt^2} + \frac{(L_1 + L_2)}{C L_1 L_2} q = 0$$

$$\omega^2 = \frac{1}{L_{eq} C}$$

We can write this LC oscillation equation directly too, as initial conditions don't matter in general equation.

$$\text{Let } q = q_0 \sin(\omega t + \phi)$$

$$\downarrow t=0, q=0$$

$$\phi = 0$$

$$\downarrow q = q_0 \sin \omega t$$

$$\downarrow \text{diff}$$

$$i = q_0 \omega \cos \omega t$$

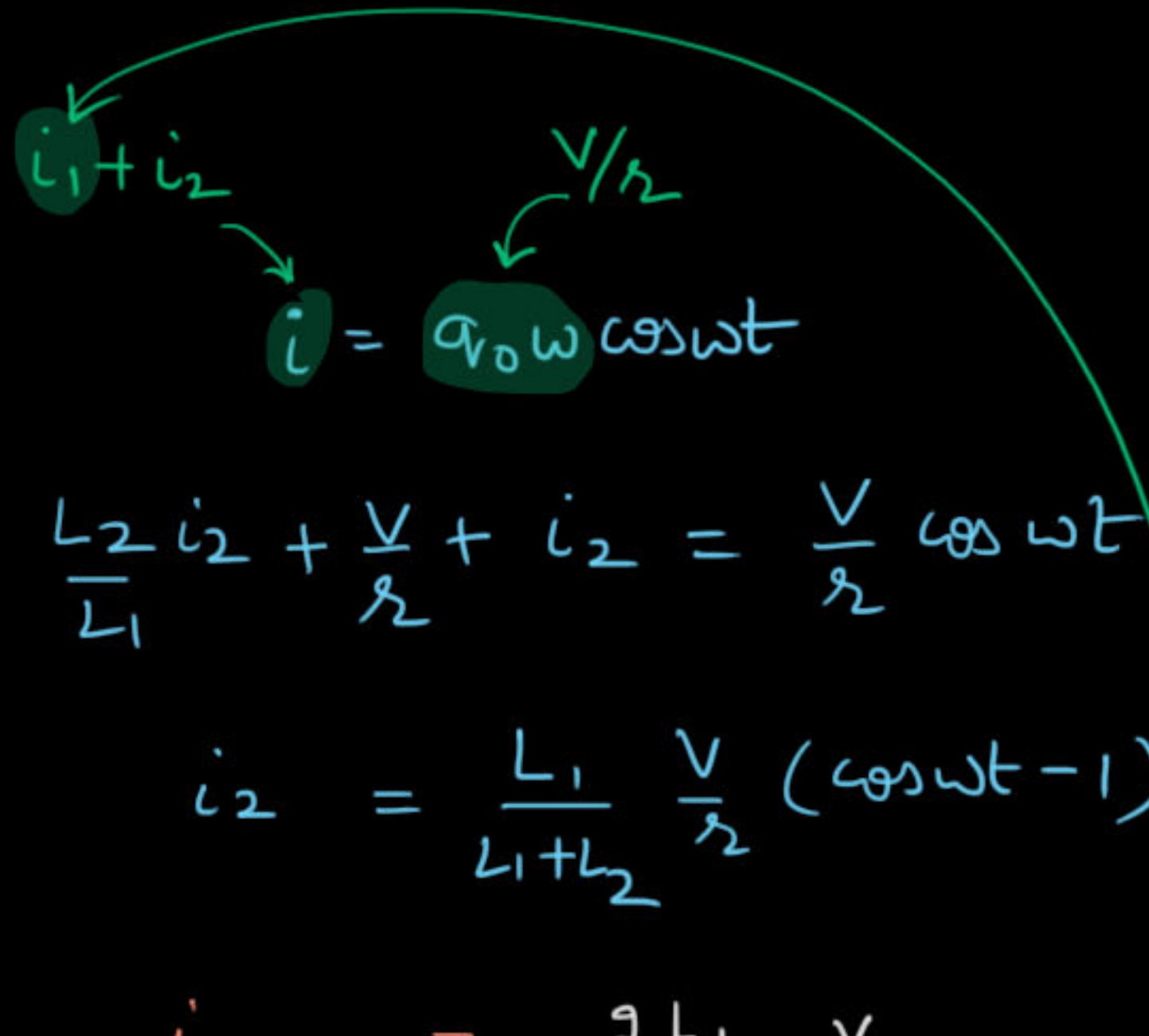
$$\downarrow t=0, i = V/r$$

$$q_0 \omega = V/r$$

$$\Rightarrow q_0 = \frac{V}{\omega r} = \frac{V}{r} \sqrt{\frac{C L_1 L_2}{L_1 + L_2}}$$

Youtube / Arihant Senior

maximum current in the inductor L_2



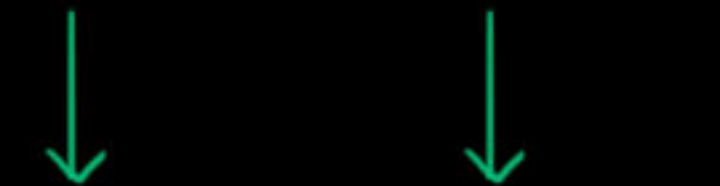
$i_1 + i_2$
 $i = q_0 \omega \cos \omega t$

$$\Rightarrow \frac{L_2 i_2}{L_1} + \frac{V}{R} + i_2 = \frac{V}{R} \cos \omega t$$

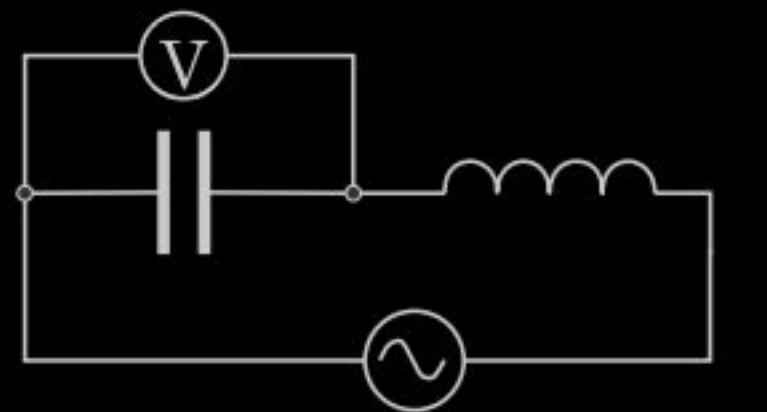
$$\Rightarrow i_2 = \frac{L_1}{L_1 + L_2} \frac{V}{R} (\cos \omega t - 1)$$

$$\Rightarrow i_{2\max} = \frac{2L_1}{L_1 + L_2} \frac{V}{R} //$$

$$L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} = -\frac{q}{C}$$


$$L_1 \int_{V/R} di_1 = L_2 \int_0 di_2$$

$$\Rightarrow i_1 = \frac{L_2 i_2}{L_1} + \frac{V}{R}$$



$$V_0, \omega = 2\pi f$$

38. A capacitor of unknown capacitance and an induction coil of inductance 1.0 H are connected in series and then the series combination is connected across a source of alternating voltage 220 V and frequency 50 Hz. Now an ideal voltmeter is connected across the capacitor. For what value of capacitance, the voltmeter will read 220 V. What value of capacitance can never be used in this circuit? Use $\pi^2 = 10$.

Its a series circuit $V_C = \frac{X_C}{Z} \cdot V_0$

∴ Voltmeter reads V_0 : $\frac{X_C}{Z} = 1$

$$\Rightarrow X_C = \pm (X_L - X_C)$$

+

-

$$X_L = 2X_C$$

$$\omega L = \frac{2}{\omega C}$$

$$\Rightarrow C = \frac{2}{\omega^2 L} = \frac{2}{(2\pi f)^2 L} = 20 \mu F \quad \text{A}$$

$$X_C = -X_L + X_C$$

Possible, if X_C is pretty large, and capacitor acts as an open circuit.

$$\Rightarrow \frac{1}{\omega C} \rightarrow \infty \Rightarrow C = 0 \quad \text{A}$$

by Ankit Singhvi

Net impedance of the circuit should never be zero.

This is to prevent the voltage source being shorted and start fire :).

$$\therefore Z \neq 0$$

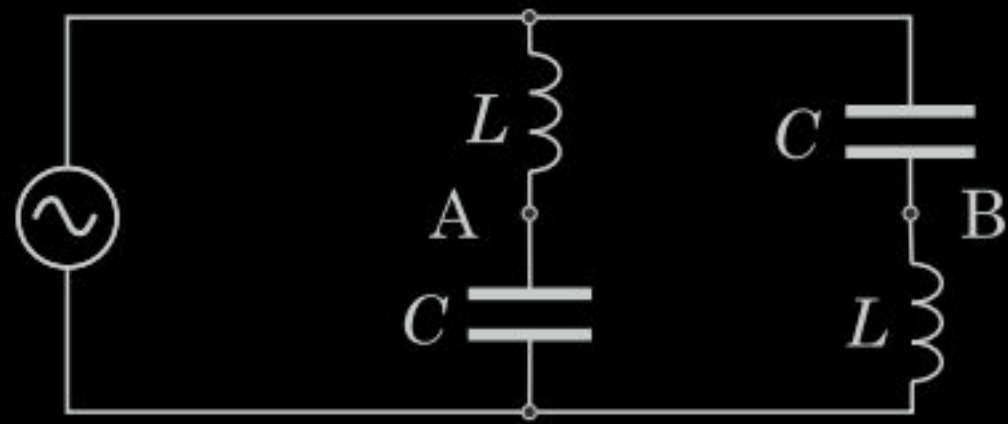
$$\Rightarrow X_L \neq X_C$$

$$\Rightarrow \omega L \neq \frac{1}{\omega C}$$

$$\Rightarrow C \neq \frac{1}{\omega^2 L}$$

$$\Rightarrow C \neq 20/2 \mu F \quad \text{From A}$$

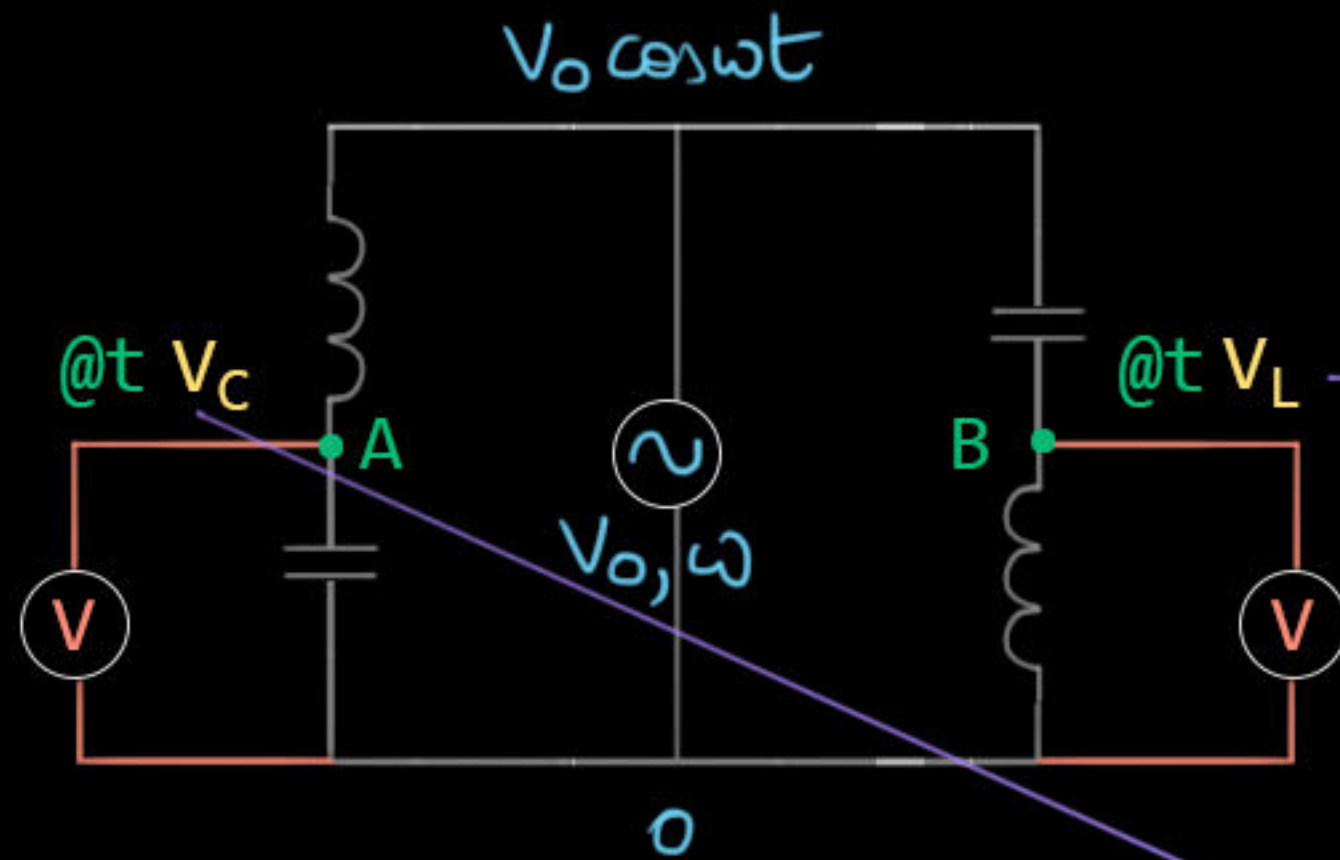
Youtube / Arihant Senior



39. A circuit shown consists of two identical coils each of inductance L , two identical capacitors each of capacitance C and a variable frequency alternating voltage source. Find expression for the angular frequency of the source at which the peak voltage between the terminals A and B becomes η ($\eta \geq 1$) times of the peak voltage of the source.

|||

Let the source voltage be $V_0 \cos \omega t$

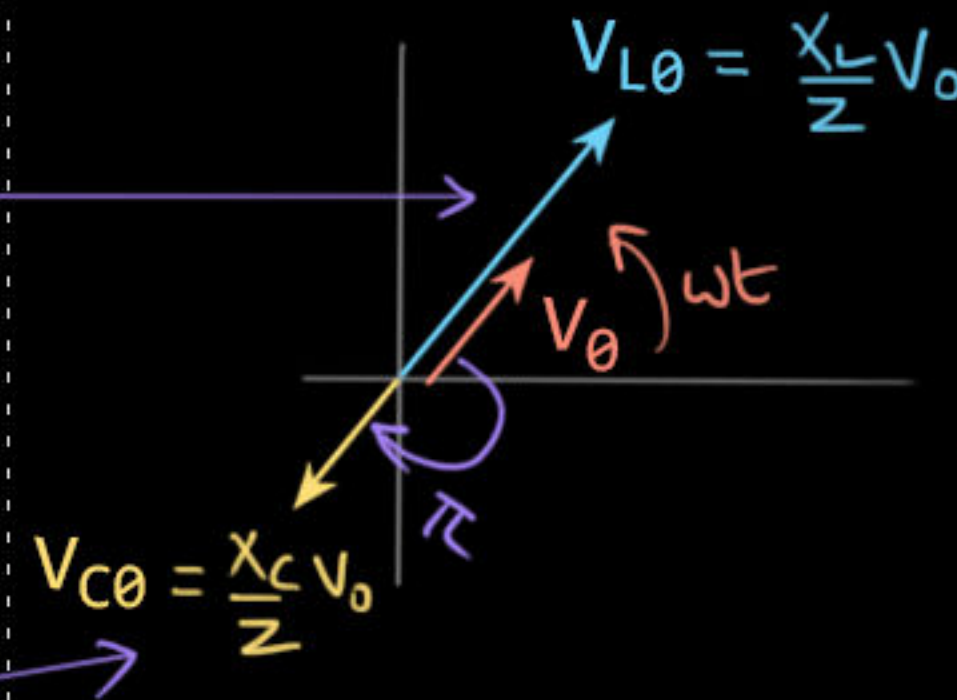


Let's connect voltmeters as shown.

Peak voltage between A and B is simply peak of $V_C - V_L$

We need to find ω . And ω itself determines if X_L is greater or smaller than X_C . So let's take both cases individually.

Case A $X_L > X_C$



$$\therefore V_L = \frac{X_L}{Z} V_0 \cos \omega t$$

$$\text{and } V_C = \frac{X_C}{Z} V_0 \cos(\omega t - \pi)$$

$$\Rightarrow V_C - V_L = -\frac{(X_L + X_C) V_0}{Z} \cos \omega t$$

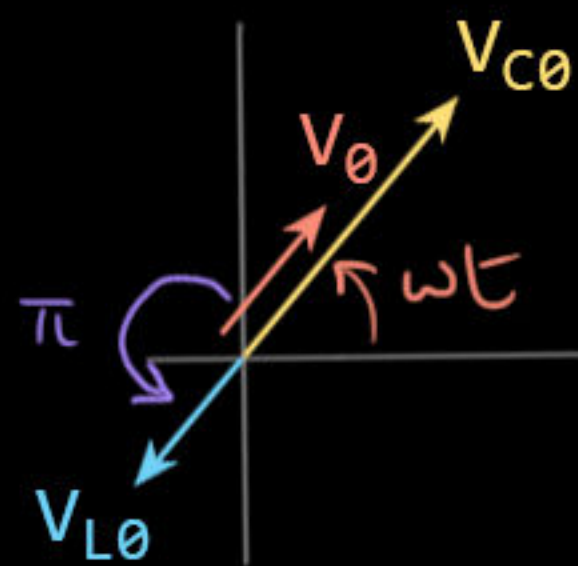
Peak

Given: $\frac{X_L + X_C}{Z} V_0 = \eta V_0$ $\frac{X_L = \omega L}{X_C = \frac{1}{\omega C}} \rightarrow \omega = \sqrt{\frac{(\eta + 1)}{(\eta - 1)} \frac{1}{LC}}$

Case B $X_C > X_L$

$$\therefore V_L = \frac{X_L}{Z} V_0 \cos(\omega t + \pi)$$

$$\text{and } V_C = \frac{X_C}{Z} V_0 \cos \omega t$$

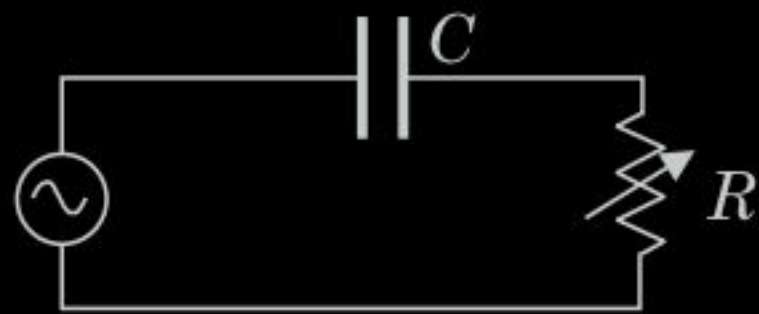


$$\Rightarrow V_C - V_L = \frac{(X_L + X_C) V_0}{Z} \cos \omega t$$

Peak

Given: $\frac{X_L + X_C}{Z} V_0 = \eta V_0$ $\xrightarrow[X_C = \frac{1}{\omega C}]{X_L = \omega L}$ $\omega = \sqrt{\frac{(\eta - 1)}{(\eta + 1)} \frac{1}{LC}}$ //

$\nwarrow$
 $(X_C - X_L)$



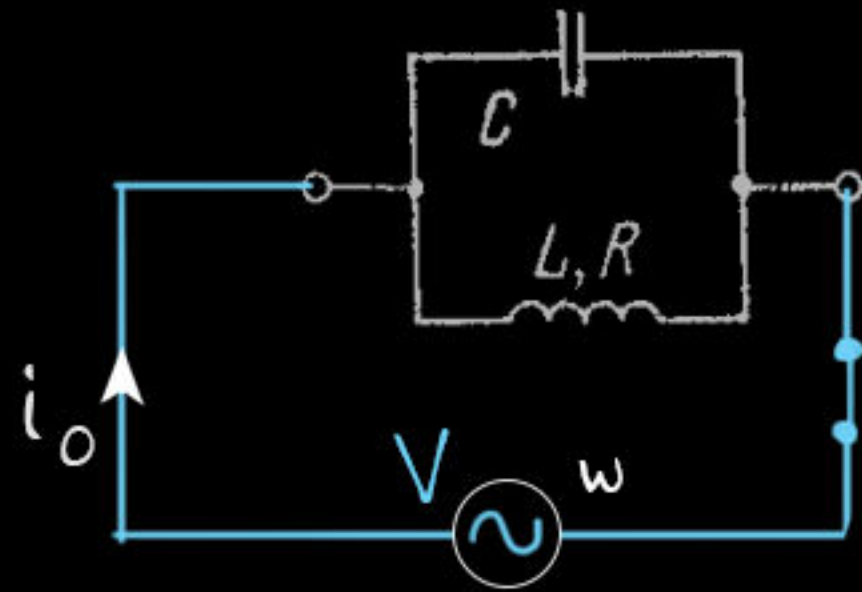
40. An alternating voltage source of peak voltage V_0 and angular frequency ω is connected across a series combination of a capacitance C and a variable resistance R as shown in the figure. Find the maximum power that can be obtained on the load resistance in this circuit.

$$\begin{aligned}
 P &= i_{\text{rms}}^2 R \\
 &= \frac{i_0^2}{2} R \quad \left. \begin{array}{l} i_{\text{rms}} = \frac{i_0}{\sqrt{2}} \\ i_0 = \frac{V_0}{Z} \\ Z = \sqrt{R^2 + X_C^2} \end{array} \right\} \\
 &= \frac{1}{2} \left(\frac{V_0}{Z} \right)^2 R \\
 &= \frac{1}{2} V_0^2 \cdot \frac{R}{R^2 + X_C^2} \\
 &= \frac{V_0^2}{2} \cdot \frac{1}{R + \frac{X_C^2}{R}} \rightarrow \text{minimum @ } R = X_C
 \end{aligned}$$

$$\therefore P_{\text{max}} = \frac{V_0^2}{2} \cdot \frac{1}{2X_C} = \frac{V_0^2 \omega C}{4}$$

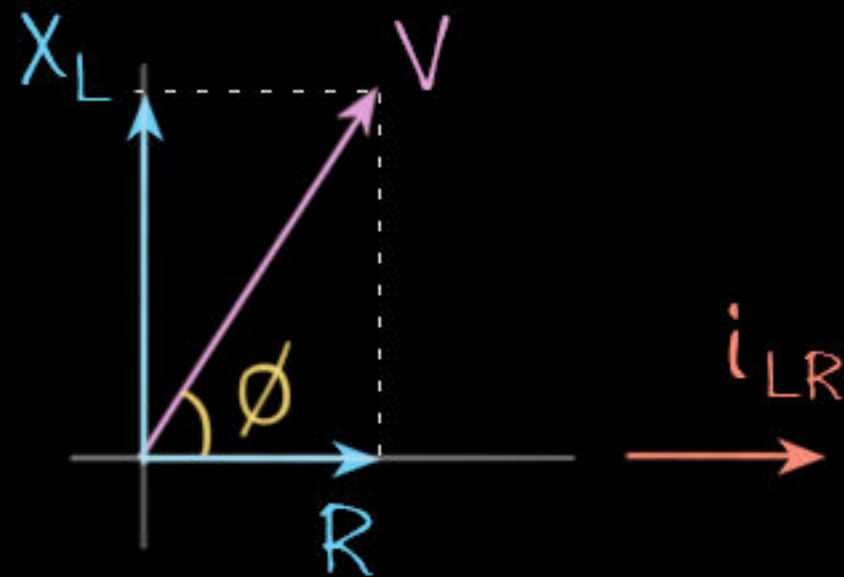
Similar:

Irodov 4.144 (b), 4.146



41. A fluorescent lamp (commonly known as tube light) employs a choke coil (an inductor) and a starter. In normal operating mode, the choke coil is in series with the tube that behaves as a resistance and the starter as an open switch in parallel with the tube. In a particular fluorescent lamp, the voltage drop on the choke coil is η times of that on the tube. If the choke coil is an ideal inductor of inductance L , how much capacitance should you connect in parallel with lamp assembly to make power factor unity? Angular frequency of the supply mains is ω .

In LR branch:

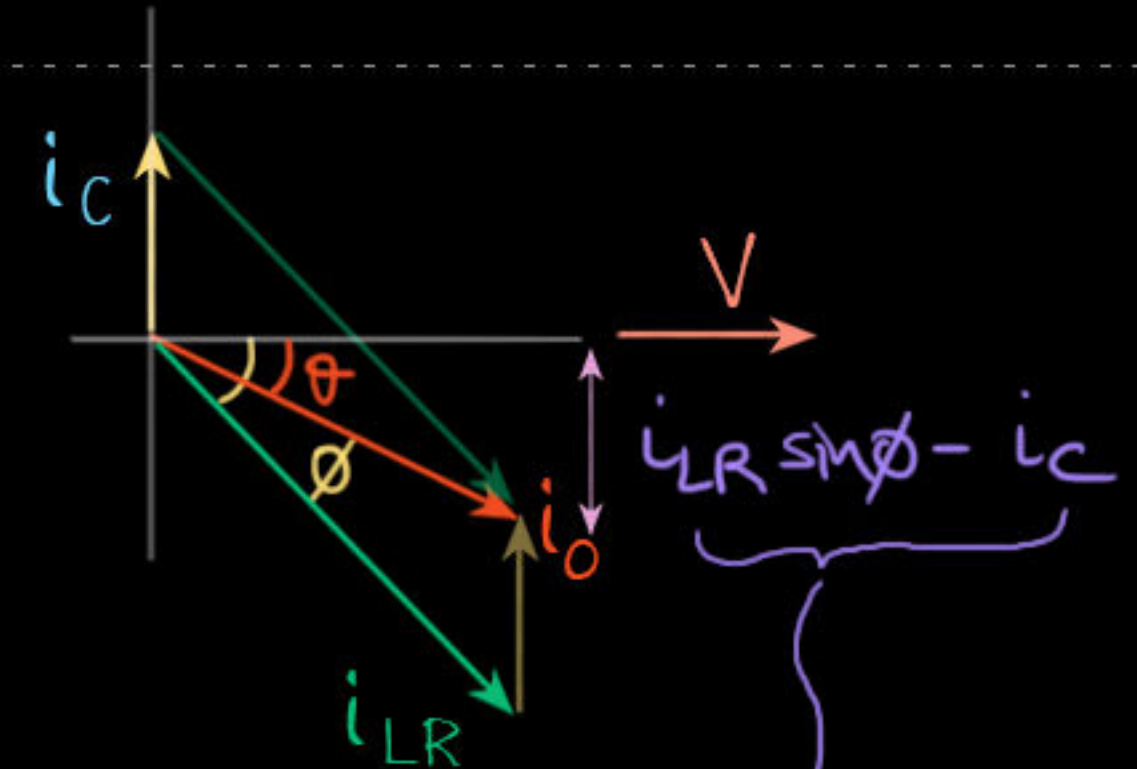


$$i_{LR} = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\tan \phi = \frac{X_L}{R} = \frac{\omega L}{R} \Rightarrow \begin{array}{c} \text{triangle with hypotenuse } \sqrt{R^2 + \omega^2 L^2} \\ \text{adjacent side } R \\ \text{opposite side } \omega L \end{array}$$

• In complete circuit:

$$i_C = \frac{V}{X_C} = V\omega C$$



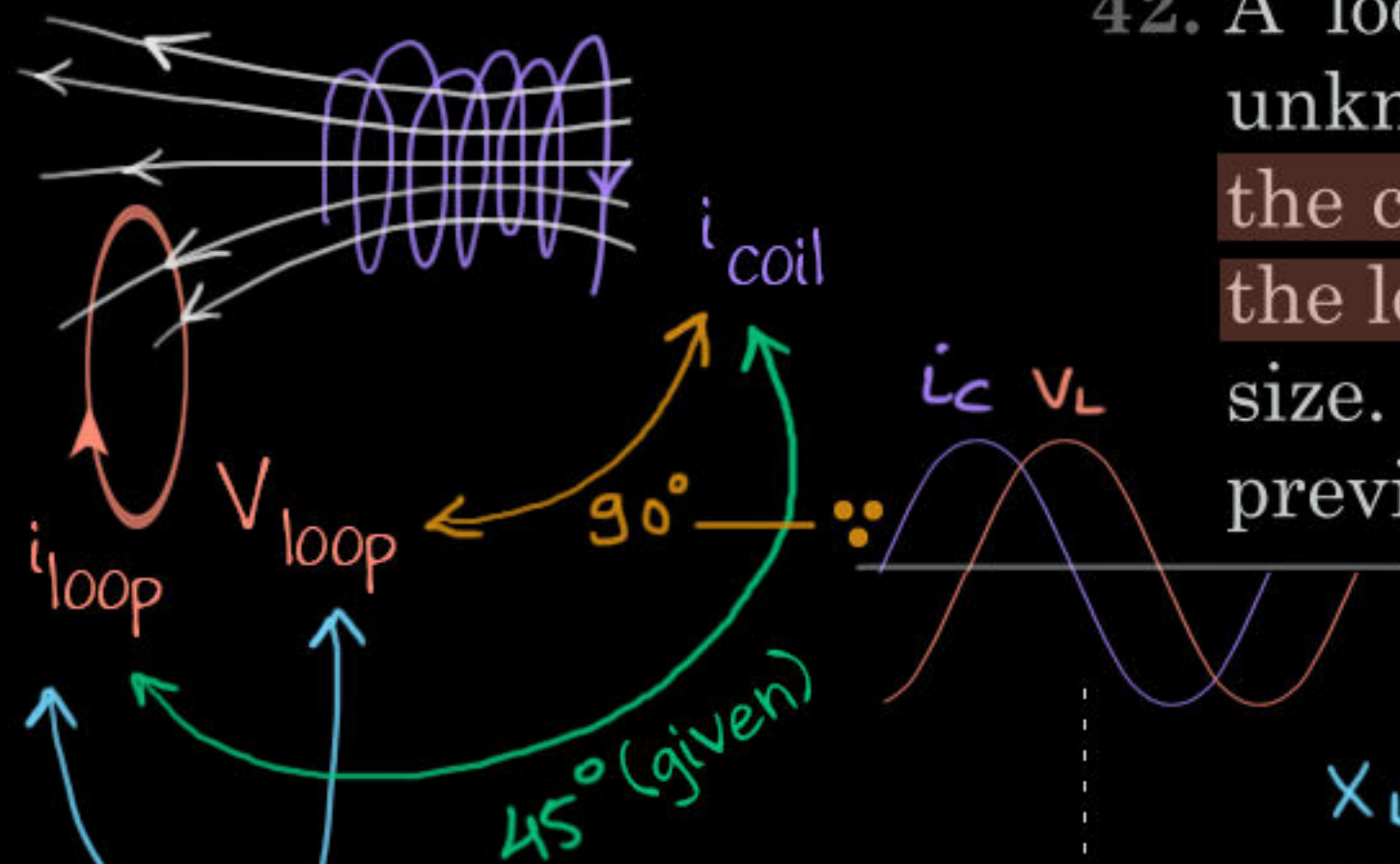
We want power factor unity. i.e. $\theta = 0 \Rightarrow 0$

$$\Rightarrow i_{LR} \sin \phi = i_C$$

$$\Rightarrow \frac{\cancel{V} \omega L}{R^2 + \omega^2 L^2} = \cancel{V} \omega C \xrightarrow{\text{Given: } \omega L = \eta R} C = \frac{\eta^2}{\omega^2 L (1 + \eta^2)}$$

by Ankit Singhvi

Youtube / Arihant Senior



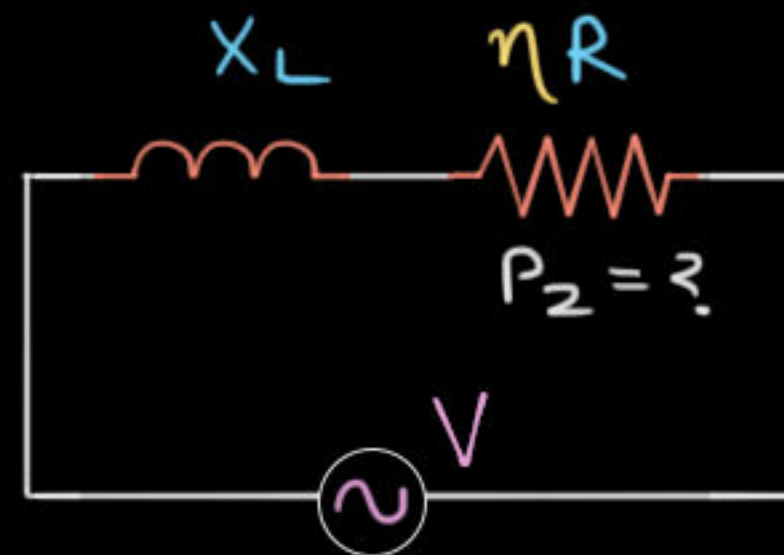
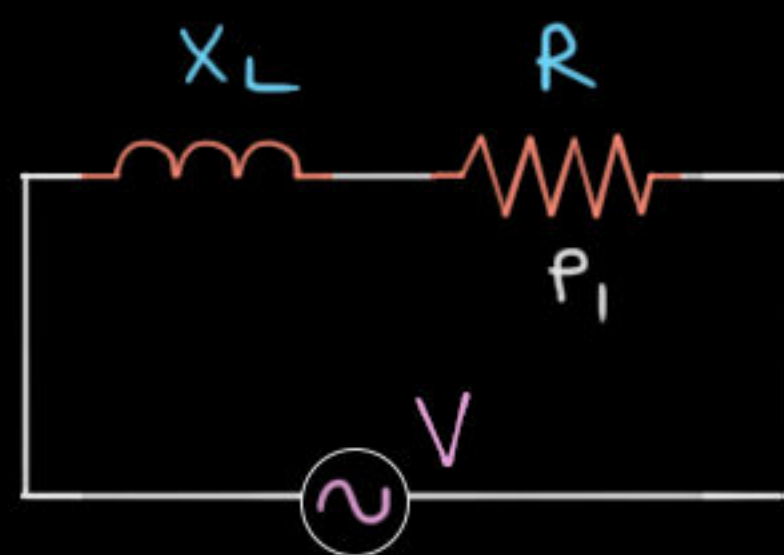
42. A loop of resistance R is placed near a coil excited by a source of unknown alternating voltage. The phase shift between the current in the coil and induced current in the loop is 45° and power dissipated in the loop is p . The loop is replaced by another loop of identical shape and size. If resistivity of material of the new loop is η times of that of the previous loop, how much power will be dissipated in the new loop?

A $\therefore 45^\circ$

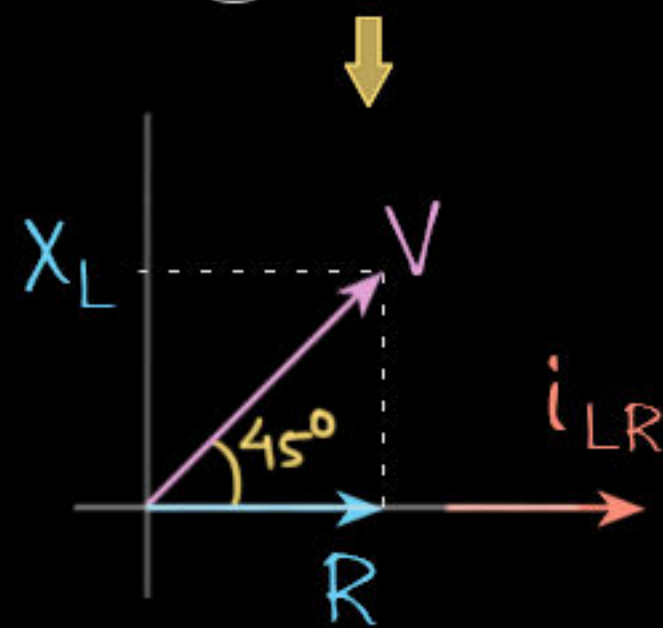
B As the second loop is identical except the material, same flux passes through it and hence induced emf too is same.

C
$$P = i_{rms}^2 R$$

$$= \left(\frac{V_{rms}}{Z} \right)^2 R \propto \frac{R}{Z^2}$$



$$\therefore \frac{P_1 Z_1^2}{R_1} = \frac{P_2 Z_2^2}{R_2}$$

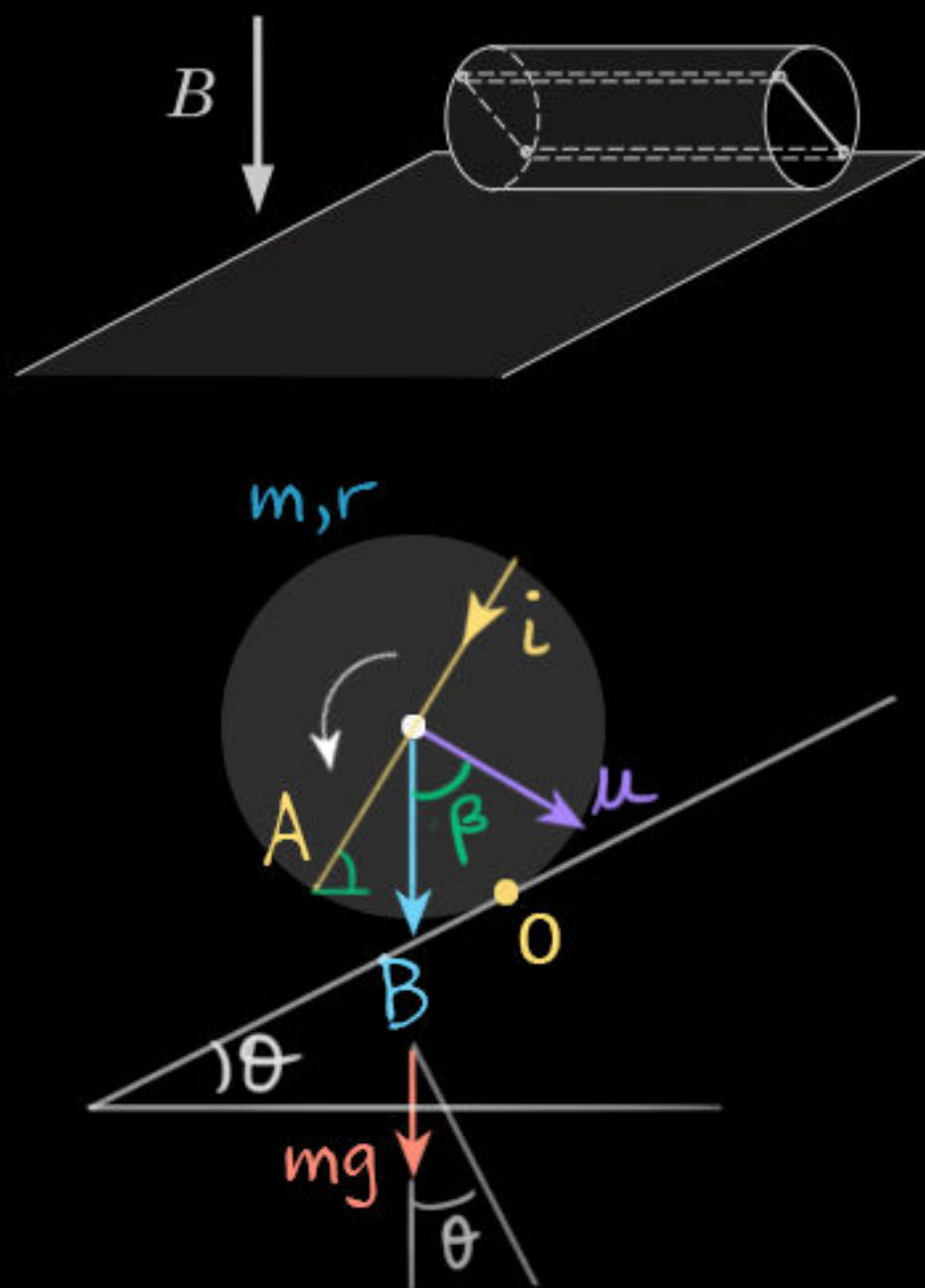


$$\therefore X_L = R$$

$$\Rightarrow \frac{P_1 (R^2 + X_L^2)}{R} = \frac{P_2 [(\eta R)^2 + X_L^2]}{\eta R}$$

$$\downarrow X_L = R$$

$$\Rightarrow P_2 = \frac{2\eta P_1}{\eta^2 + 1}$$



The two torques about O are:

$$\otimes \vec{\mu} \times \vec{B} = \mu B \sin \beta$$

$$\odot (mg \sin \theta) r$$

ω is given constant.

But magnetic torque varies with β , and hence time. So we must assume average net torque to be zero.

43. Two straight parallel diametrically opposite thin grooves are cut on an insulating cylinder parallel to its axis. A metal of density same as that of material of the cylinder is filled in the groove making two parallel conducting rods. The ends of these rods are short circuited by light wires to make a coil. Mechanically the assembly is a uniform cylinder of radius r , length l and mass m and electrically it is a coil of length l , width $2r$ and resistance R . This assembly is released on a slope of uniform inclination θ with the horizontal in a vertical magnetic field B . If the cylinder rolls down the slope without slipping, it acquires a constant velocity soon after it has been released. Neglect inductance and capacitance of the coil and find an expression for this constant velocity.

τ_0 is zero once it rolls with constant velocity

$$\therefore mg \sin \theta r = \mu B \sin \beta \Big|_{\text{average}}$$

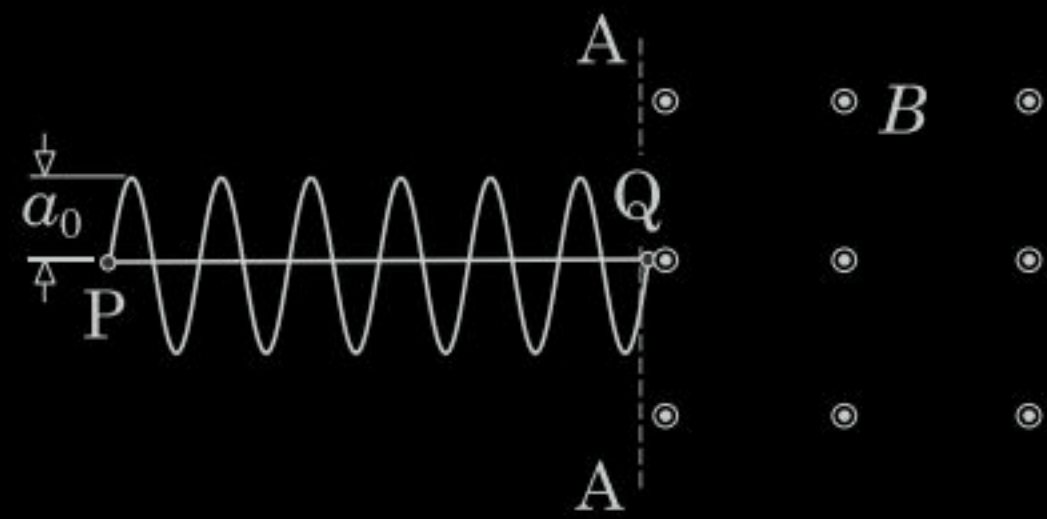
$$\frac{1}{R} \frac{d\phi}{dt} \Big|_{\text{average}} = \frac{d(BA \cos \beta)}{dt} \rightarrow BA \sin \beta \frac{d\beta}{dt}$$

$\omega = \frac{v}{r}$

$$\Rightarrow mg \sin \theta r = \frac{B^2 4r^2 l^2 \sin^2 \beta}{R l} v \Rightarrow v = \frac{mgR \sin \theta}{2B^2 l^2}$$

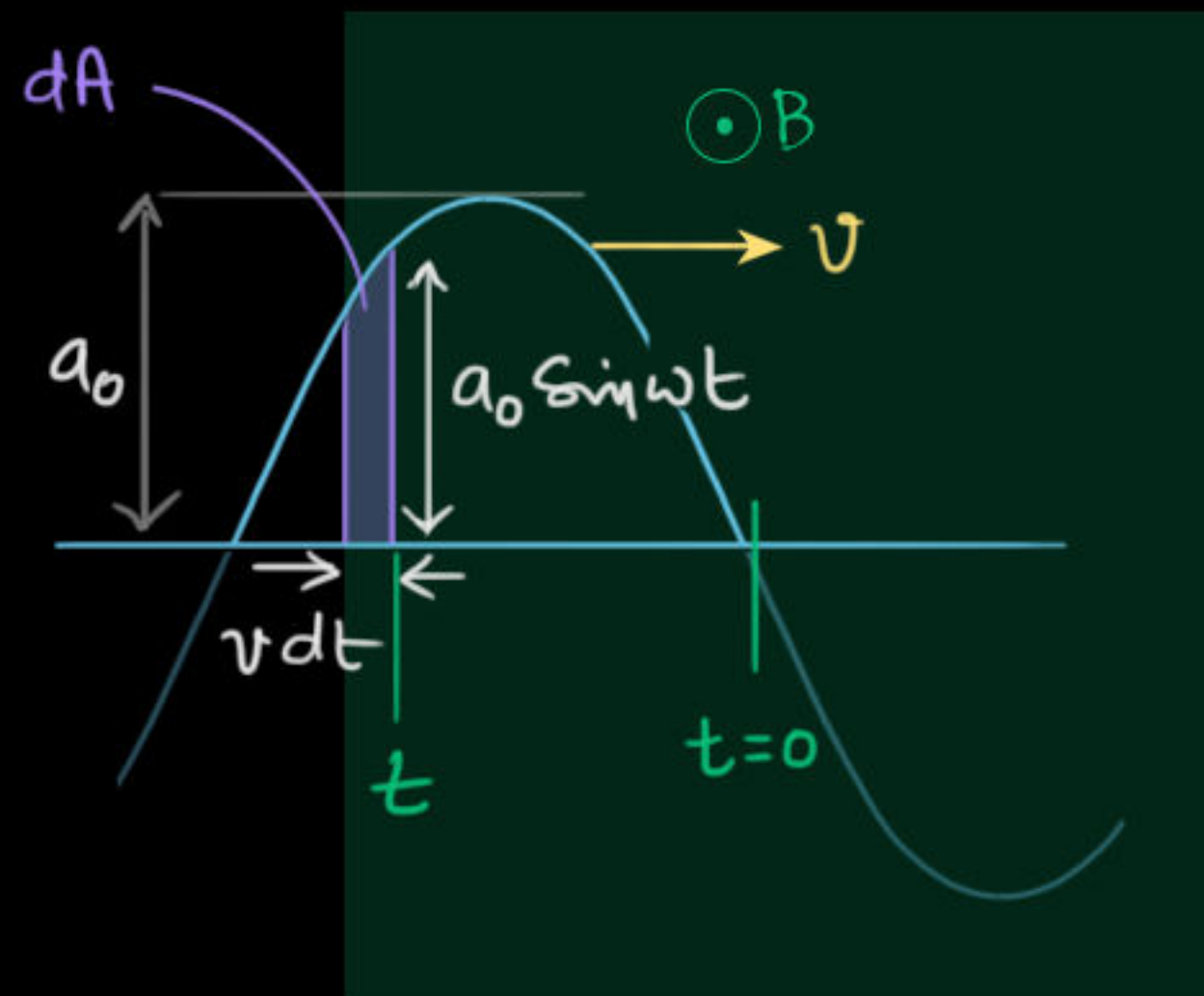
by Ankit Singhvi

Youtube / Arihant Senior



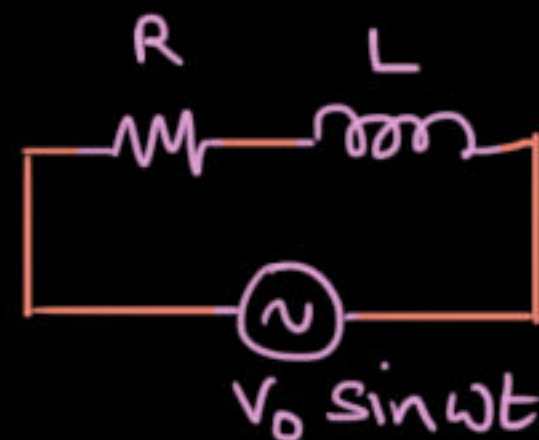
As its average power we need, let's find it for half cycle.

Given: $\omega = \frac{2\pi}{T}$



44. A rubber-coated wire is bent in the shape of a rigid sinusoid and then its ends P and Q are electrically connected with the help of a straight conducting wire making a closed loop of total resistance R and inductance L . The amplitude of the sinusoid is a_0 . On the right of the line AA is a region of uniform magnetic field of induction B pointing perpendicular to the plane of the wire loop. The loop is made to move along the line PQ with a constant velocity v to enter the magnetic field. If one cycle of the sinusoid enters the region of magnetic field in time T , find average thermal power generated in the loop while it is entering the region of magnetic field.

Emf induced in loop = $\mathcal{V} = B \frac{dA}{dt} = \frac{B(v dt) a_0 \sin \omega t}{dt} = Bv a_0 \sin \omega t$



$P_{av} = i_{rms}^2 R = \frac{V_{rms}^2}{Z^2} R = \left(\frac{V_0}{2} \right)^2 \frac{R}{Z^2}$

$Z^2 = R^2 + X_L^2$

$= \frac{(Bv a_0)^2}{2} \frac{R}{R^2 + \omega^2 L^2} = \frac{(Bv a_0)^2 R T^2}{2(R^2 T^2 + 4\pi^2 L^2)}$

$\omega = \frac{2\pi}{T}$

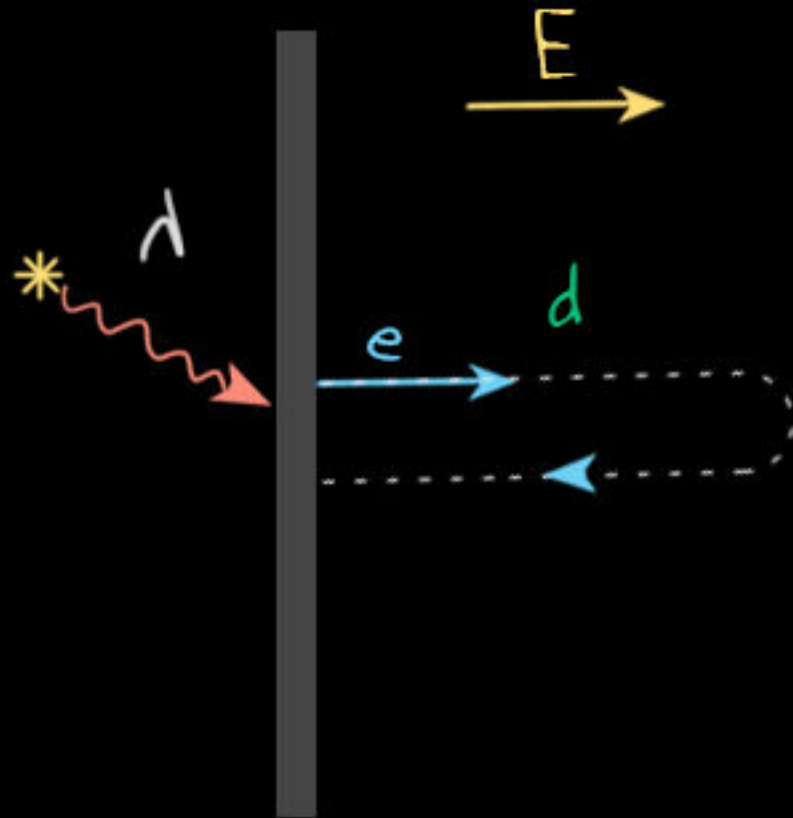
1. If photoelectrons emitted by a metal on irradiating it by a light of wavelength λ cannot move away farther than a distance d in presence of a retarding electric field E , find threshold wavelength of the metal.

(a) $\frac{hc}{eEd}$

(b) $\lambda - \frac{hc}{eEd}$

(c) $\lambda + \frac{hc}{eEd}$

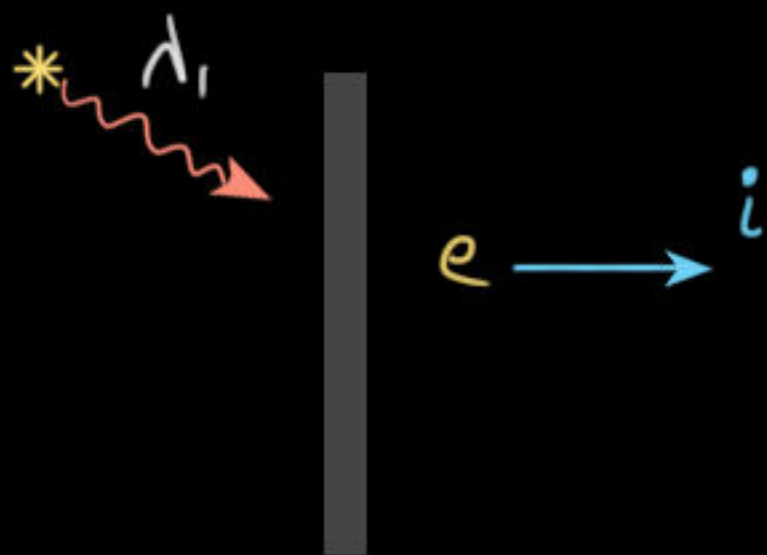
✓ (d) $\left(\frac{1}{\lambda} - \frac{eEd}{hc} \right)^{-1}$



$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + KE$$

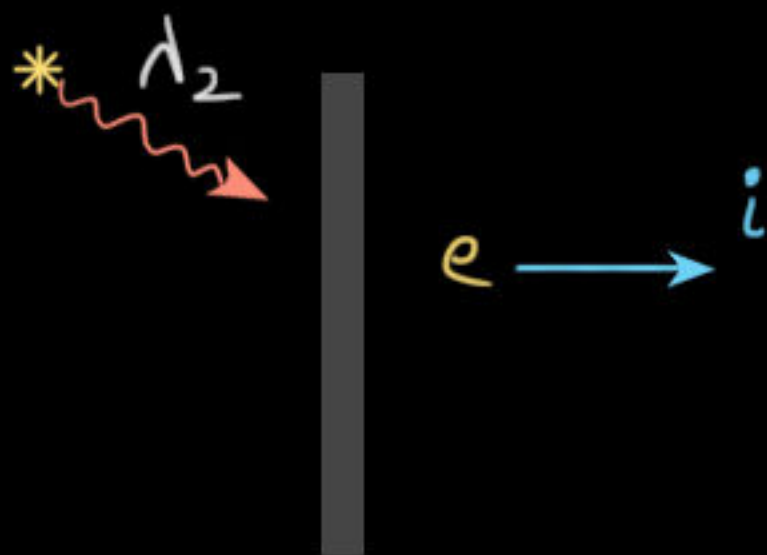
eEd

$$\Rightarrow \frac{1}{\lambda_0} = \frac{1}{\lambda} - \frac{eEd}{hc}$$



Yield: η_1

Power: W_1



Yield: η_2

Power: ?

2. How much power must a light beam of wavelength λ_2 have to produce the same saturation current on irradiating a metal plate as produced by a light beam of wavelength λ_1 and power W_1 ? Quantum yield of photoelectric effect for these wavelengths are η_1 and η_2 respectively.

✓(d) $W_1 \frac{\eta_1 \lambda_1}{\eta_2 \lambda_2}$

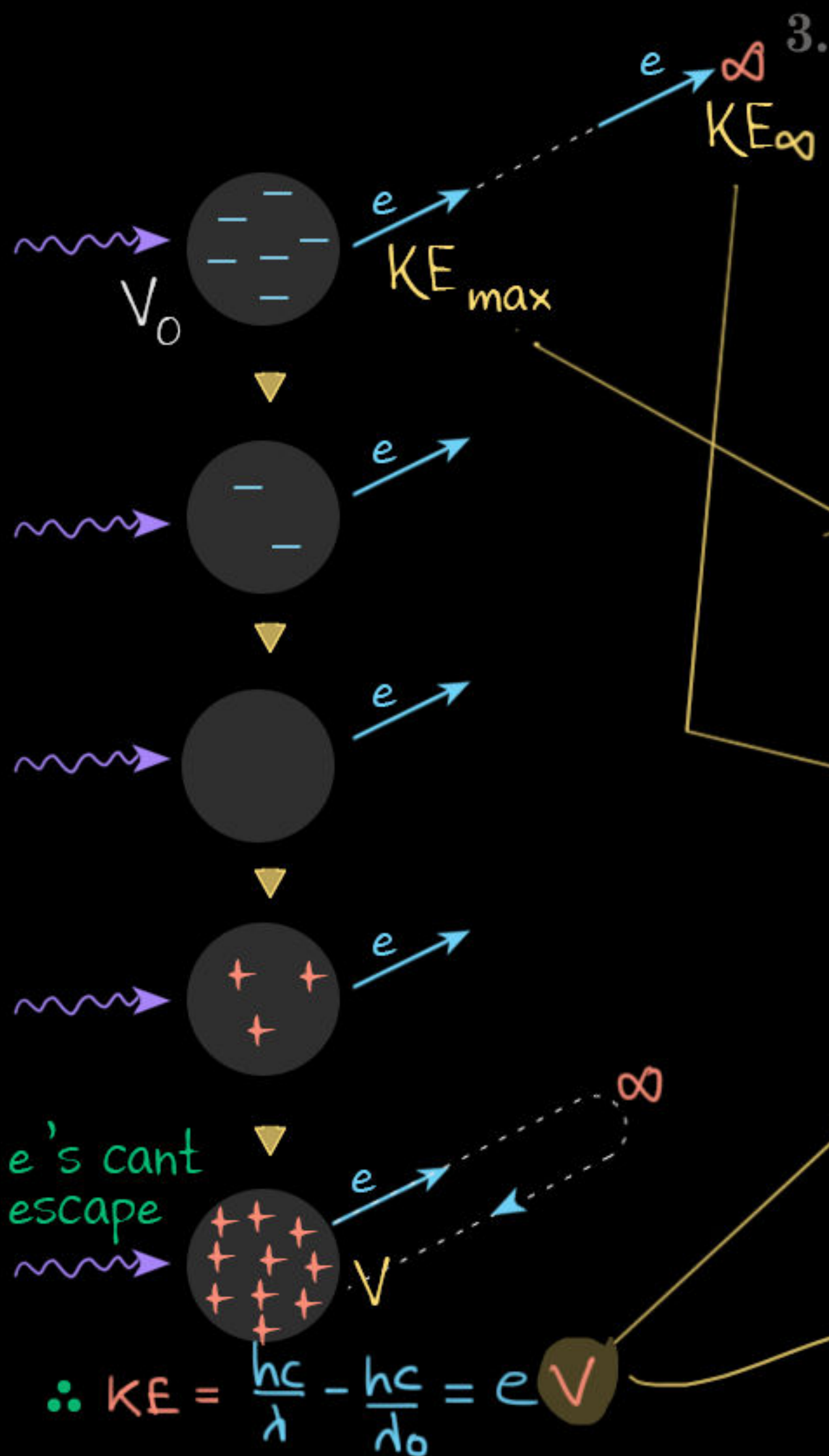
Let n be number of incident photons per second.

$$\eta = \frac{W}{\left(\frac{hc}{\lambda}\right)}$$

Current: $i = (\eta n) e = \frac{\eta W \lambda e}{hc}$

∴ Current is same $\frac{\eta_1 W_1 \lambda_1 \cancel{e}}{\cancel{hc}} = \frac{\eta_2 W_2 \lambda_2 \cancel{e}}{\cancel{hc}}$

$$\Rightarrow W_2 = W_1 \frac{\eta_1 \lambda_1}{\eta_2 \lambda_2} //$$



3. An isolated zinc bead of radius r charged to a negative potential V_0 (assuming potential at infinitely distant points to be zero) placed in free space is being continuously irradiated by ultraviolet light of wavelength λ . Photoelectric threshold wavelength for zinc is λ_0 , speed of light is c , Denoting Planck's constant by h , permittivity of free space by ϵ_0 , charge and mass of an electron by $-e$ and m , select correct statements from the following?

i.e., $-V_0$ ←

(a) Maximum speed of a photoelectron immediately after emission is

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \frac{1}{2}mv_{max}^2 \Rightarrow v_{max} = \sqrt{\frac{2hc}{m} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)} \quad \checkmark$$

(b) Max speed of a photoelectron at a great distance from the bead is

$$\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0} \right) + eV_0 = \frac{1}{2}mv_{\infty}^2 \Rightarrow v_{\infty} = \sqrt{\frac{2hc}{m} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) + \frac{2eV_0}{m}} \quad \checkmark$$

(c) Potential of the bead after prolonged irradiation is $-\frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$. ✗

(d) Total number of photoelectrons emitted from the bead is (lets say n)

We know, $\Delta V = k \Delta q / r$

$$\Rightarrow V - (-V_0) = \frac{1}{4\pi\epsilon_0} \frac{ne}{r} \Rightarrow n = \frac{4\pi\epsilon_0 r}{e} \left\{ \frac{hc}{e} \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right) + V_0 \right\} \quad \checkmark$$

$$\therefore KE = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = eV$$



$$\cancel{m}u = \cancel{5m}v$$

$$\Rightarrow v = \frac{u}{5}$$

5. A moving neutron collides with a singly ionized stationary helium atom that is in ground state. What should be the minimum speed of the moving neutron for the collision to be perfectly inelastic?

(a) $2.50 \times 10^4 \text{ m/s}$

(b) $4.25 \times 10^4 \text{ m/s}$

(c) $6.25 \times 10^4 \text{ m/s}$

✓ (d) $9.89 \times 10^4 \text{ m/s}$

Energy lost in inelastic collision makes the electron jump to higher orbit (lets say n)

$$\therefore \frac{1}{2}mu^2 - \frac{1}{2}(5m)v^2 = 13.6 Z^2 \left(1 - \frac{1}{n^2}\right) \text{ eV}$$

$\swarrow \quad \nwarrow$
 $u/5 \quad 2$

$$\Rightarrow \frac{2m}{5}u^2 = 13.6 \cdot 4 \left(1 - \frac{1}{n^2}\right) \text{ eV}$$

$\nwarrow$
 $2 \text{ (for min } u)$

$$\therefore 2mu_{\min}^2 = 5 \cdot (13.6) \cdot 4 \left(1 - \frac{1}{4}\right) \text{ eV}$$

$$\Rightarrow u_{\min} = \sqrt{\frac{5(13.6)(3) \times 1.6 \times 10^{-19}}{2 \cdot 1.67 \times 10^{-27}}} = 9.89 \times 10^4 \text{ m/s}$$

6. A dose containing radioactive ^{18}F is scheduled to be prepared at 6.10 AM and 1.00 mL of it is to be given to a patient at 8.00 AM. If the patient reaches clinic late and takes the dose at 8.55 AM, how much quantity of the dose should be given to the patient? Half-life of ^{18}F is 110 min.

- ✓ (a) 1.41 ml (b) 2.00 ml
(c) 2.82 ml (d) None of these

Lets say N_0 nuclei of ^{18}F are to be given, whenever any patient arrives.

Required dose to be given at 8:55 AM be x ml.

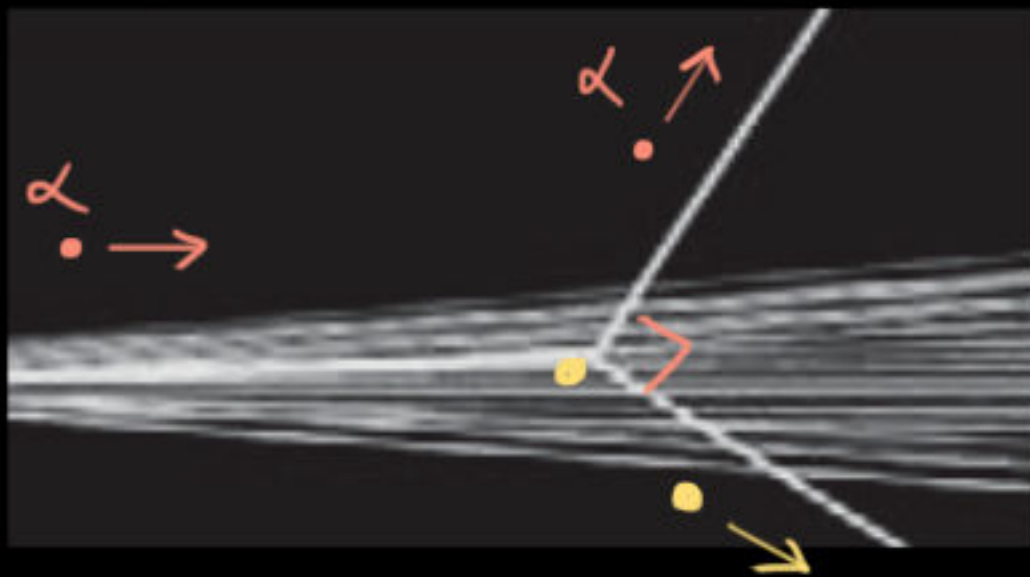
8:00 AM: 1 ml $\longrightarrow$ N_0 nuclei

8:55 AM: x ml $\longrightarrow$ N_0 nuclei

$\therefore$ these, originally at 8:00AM were $N_0 x$ nuclei
 $\downarrow$ which in 55 min became N_0 nuclei

$$\therefore N_0 = \frac{N_0 x}{2^{t/t_{1/2}}}$$

$$\Rightarrow x = 2^{t/t_{1/2}} = 2^{1/2} = \sqrt{2} //$$

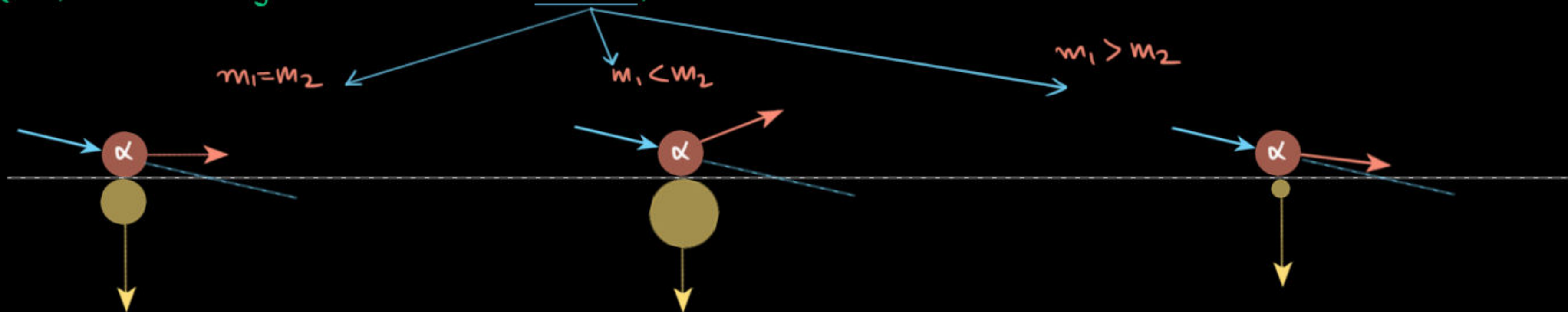


7. A stream of α -particles encounters a stationary unknown nucleus in a cloud chamber. After a collision, the two nuclei are scattered in mutually perpendicular directions. If some kinetic energy is lost in the collision, the unknown nucleus must be

- (a) a proton
(c) an α -particle

- (b) a nucleus lighter than an α -particle
✓ (d) a nucleus heavier than an α -particle

We know, scatter angle is maximum in elastic, and minimum in inelastic collision.



Scatter angle = 90°
 $\therefore < 90^\circ$
if KE is lost

Resultant angle $> 90^\circ$
 $\therefore$ may be 90°
if KE is lost

Resultant angle $< 90^\circ$
 $\therefore < 90^\circ$
if KE is lost

8. In a protonium atom, a proton and an antiproton orbit each other. What is the energy of a photon emitted in transition of a protonium atom from the first excited state to the ground state?

(a) 5.1 eV

✓ (b) 9.37 keV

(c) 12.5 keV

(d) 18.74 keV

For fixed nucleus (as electron is light):

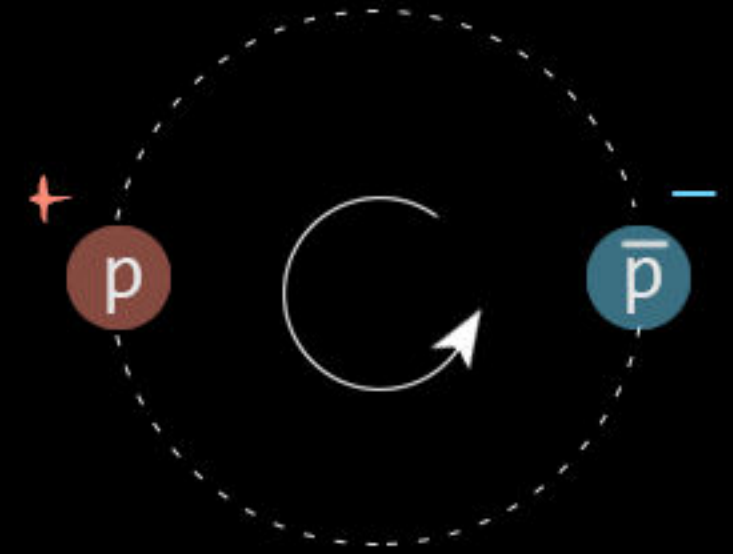
We know, $E_n = -\frac{m_e e^4}{8\epsilon_0^2 h^2} \frac{Z^2}{n^2}$ and $\therefore E_n \propto m_e$

13.6 eV

For flexible nucleus:

$$E_n = 13.6 \frac{Z^2}{n^2} \cdot \frac{\mu}{m_e}$$

Reduced mass



Sol: $Z = 1$

$$\mu = \frac{m_p m_{\bar{p}}}{m_p + m_{\bar{p}}} = \frac{m_p}{2}$$

$n : 2 \rightarrow 1$

$$E \text{ of photon} = \Delta E = 13.6 (1)^2 \frac{1836}{2 m_e} \left(1 - \frac{1}{4}\right)$$

$$= 9.36 \text{ keV}_{//}$$

Refer Irodov 6.271
for proof of

$$KE_{\min} = \left(\frac{m}{m+M} \right) |Q|$$



9. A proton collides with a stationary deuteron and a ${}^3\text{He}$ nucleus is formed. For this reaction to take place, the proton must have minimum kinetic energy 1.4 MeV. If instead, a deuteron collides with a stationary proton to make a ${}^3\text{He}$ nucleus, how much minimum kinetic energy must it possess?

(a) 0.7 MeV

(b) 1.4 MeV

(c) 2.1 MeV

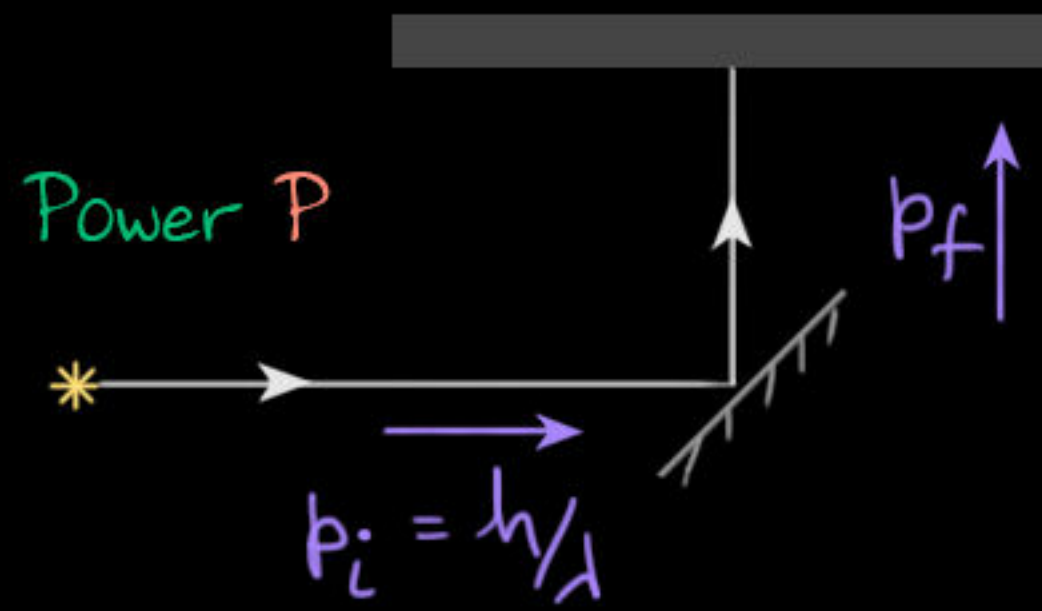
✓ (d) 2.8 MeV

Sol: Q value for both given reactions is same.

$$\frac{m_p}{m_p + m_d} Q = 1.4$$

$$\frac{m_d}{m_p + m_d} Q = x$$

Divide: $\frac{\cancel{m_p}}{\cancel{m_d} \cdot 2} = \frac{1.4}{x} \Rightarrow x = 2.8 //$



A $\Delta \vec{p} = \vec{p}_f - \vec{p}_i$

$$= \frac{h}{\lambda} (-\hat{i} + \hat{j}) //$$

A narrow beam of monochromatic light of wavelength λ emitted from a source of power P is propagating in the positive x -direction. After being reflected from a perfectly reflecting plane mirror of area vector $\vec{A} = A(-\hat{i} + \hat{j})$, the beam falls on a metal plate of surface area A placed parallel to the x - z plane. Work function of the metal is ϕ and photoelectric efficiency is η . Photoelectrons emitted from the metal plate are immediately removed from the vicinity of the plate.

10. Reflection from the mirror changes momentum of photons and exerts force on the mirror. Which of the following statements is/are correct?

✓ (a) Change in momentum of each photon is $\frac{h}{\lambda}(-\hat{i} + \hat{j})$.

✓ (c) Force exerted by light beam on the mirror is $\frac{P}{c}(\hat{i} - \hat{j})$.

C $F_{\text{mirror}} = -F_{\text{beam}} = -\frac{dp}{dt}|_{\text{beam}} = -\Delta p|_{\text{photon}} \times \text{photons per second}$

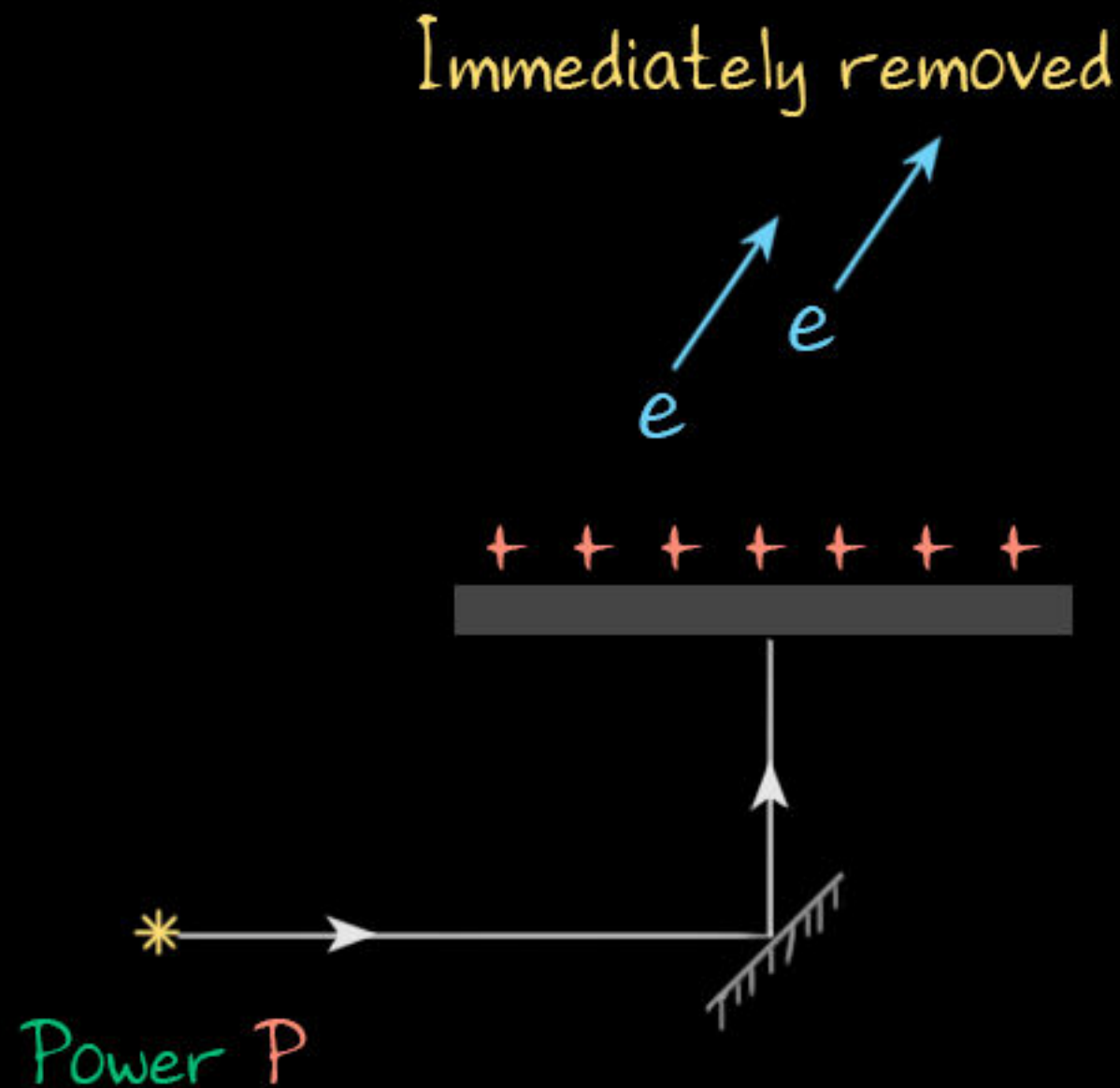
$$= -\frac{h}{\lambda}(-\hat{i} + \hat{j}) \frac{P}{hc/\lambda}$$

$$= \frac{P}{c}(\hat{i} - \hat{j}) //$$

11. After a time interval Δt from the instant light falls on the metal plate, electric field at a point very close to the metal plate and far away from its edges is explored. Which of the following statements is/are true

✓ (b) Modulus of electric field is $\frac{\eta P \lambda e \Delta t}{2 \epsilon_0 h c A}$.

(d) Electric field increases linearly with time in starting and reaches saturation after a short while.



Photons per second, $r = \frac{P}{(hc/\lambda)} = \frac{P\lambda}{hc}$

(B) $E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2A\epsilon_0} = \frac{\eta \left(\frac{P\lambda}{hc} \right) \Delta t e}{2A\epsilon_0} = \frac{\eta P \lambda e \Delta t}{hc 2A\epsilon_0} //$

(D) It will never saturate, as emitted electrons are immediately removed and therefore won't fall back.

A $KE_{\max} = \frac{hc}{\lambda} - \phi$
 $\therefore$ doesn't depend on η

B $KE_{\max} = \frac{hc}{\lambda} - \phi$
 $= \frac{12420}{6000} - 1.9$
 $= 2.07 - 1.9$
 $= 0.17 \text{ eV}$

C $\frac{hc}{\lambda} = \frac{12420}{6000} = 2.07 < 2.1 \text{ eV } (\phi)$

$\therefore$ Incident photons don't have enough energy to dislodge electrons.

12. If power of the source, wavelength of light, surface area and photoelectric efficiency of the metal of the plate are adjusted to 60 W, 6000 Å, 0.04 m² and 10⁻⁴ respectively. Which of the following statements are correct? Use $hc = 12420 \text{ eV} \cdot \text{Å}$.

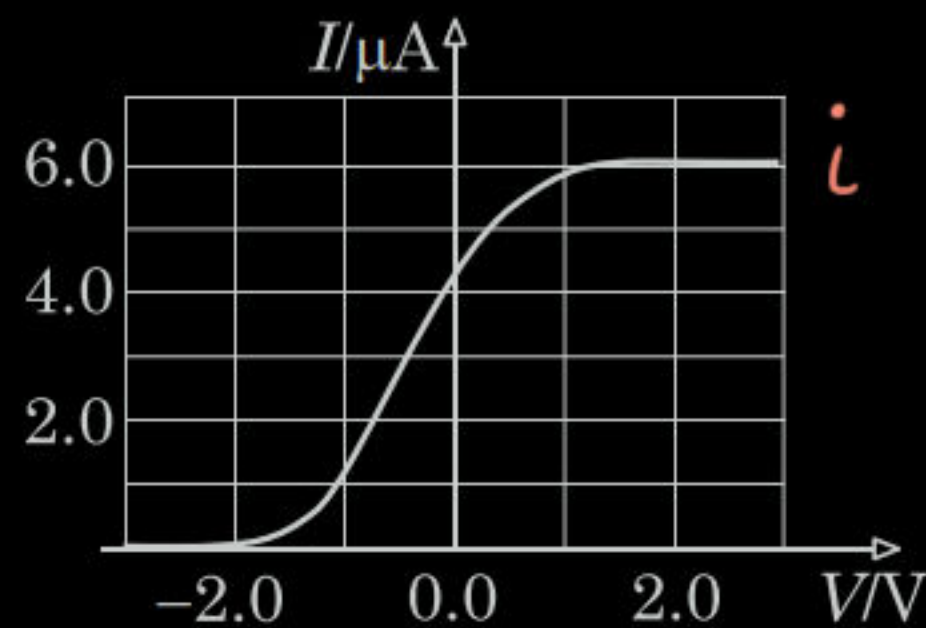
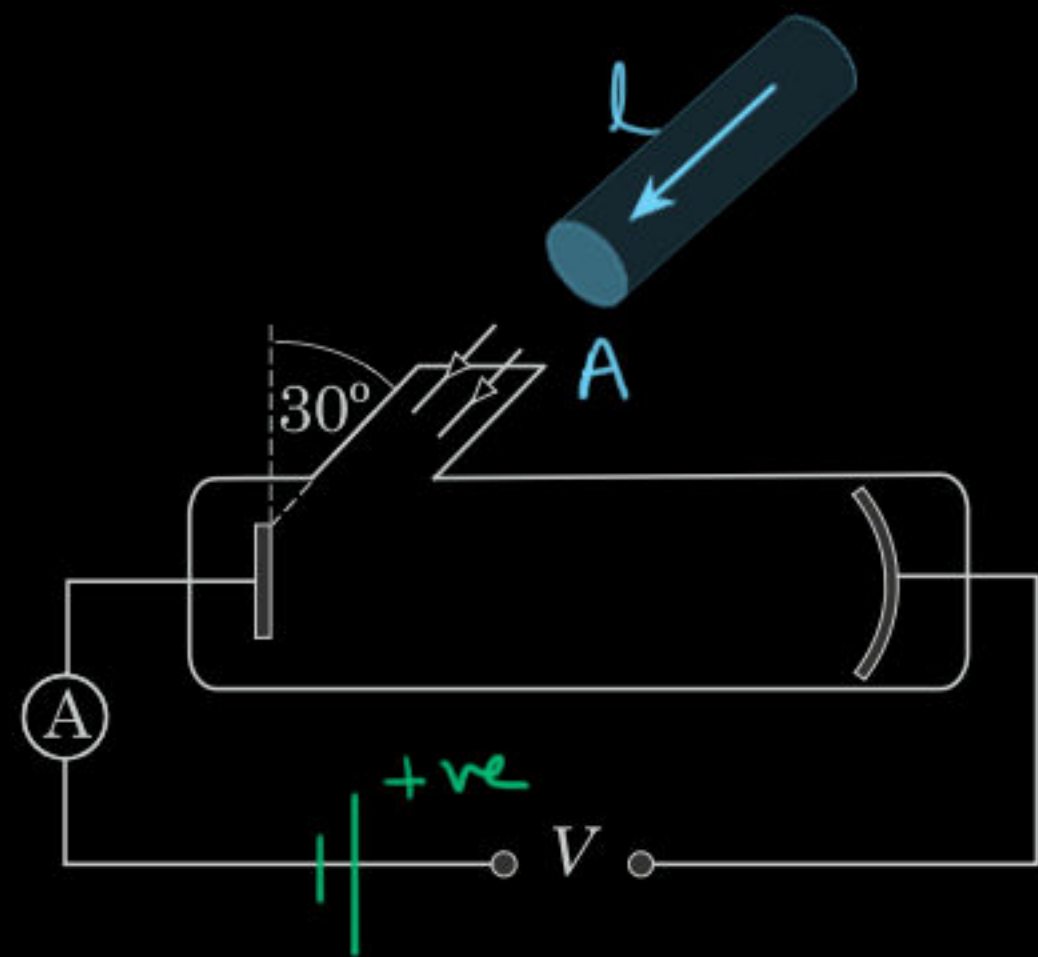
- ✓ (a) Range of kinetic energy of photoelectrons does not depend on photoelectric efficiency of the metal used.
- ✓ (b) If work function of the metal of the ball is 1.9 eV, the maximum kinetic energy of the photoelectrons is 0.17 eV.
- ✓ (c) If material of metal plate has work function 2.1 eV, no photoelectrons are observed.
- ✓ (d) Number of photoelectrons emitted per second is 1.82×10^{16} .

D Electrons emitted per second $= \eta \left[\frac{P}{hc/\lambda} \right]$

$$= 10^{-4} \left[\frac{60}{2.07 \times 1.6 \times 10^{-19}} \right]$$

$$= 1.8 \times 10^{16}$$

Youtube / Arihant Senior



Consider a classic experiment on the photoelectric effect in which a monochromatic plane wave of light falls on a metal plate of area 66.3 cm^2 at an angle 30° as shown in the figure. Electric field in the light wave varies according to equation $E = E_0 \cos(\omega t + \phi_0)$, where amplitude is $E_0 = 15 \text{ V/m}$, angular frequency is $\omega = 9.5 \times 10^{15} \text{ rad/s}$ and ϕ_0 is some initial phase angle. For a plane wave, the density u of electromagnetic energy is given by the equation $u = \frac{1}{2} \epsilon_0 E_0^2$. Dependence of photoelectric current I on voltage V applied between the electrodes is shown by a graph.

Speed of light is $c = 3.0 \times 10^8 \text{ m/s}$, Planck's constant is $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$, electrical permittivity of free space is $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ and charge on an electron is $e = -1.6 \times 10^{-19} \text{ C}$.

13. What is quantum yield in this experiment? ✓ (b) 0.038

$$\eta = \frac{n_e}{n_p} = \frac{i/e}{P/h\nu} = \frac{6 \times 10^{-6} / 1.6 \times 10^{-19}}{\left\{ \frac{\frac{1}{2} (8.85 \times 10^{-12}) 15^2 \cdot [66.3 \times 10^{-4} \cos 60^\circ] 3 \times 10^8}{[6.63 \times 10^{-34} \cdot \frac{9.5 \times 10^{15}}{2\pi}]} \right\}}$$

$$= 0.038 //$$

$$P = \frac{\frac{1}{2} \epsilon_0 E_0^2 \cdot A}{\lambda/c}$$

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

14. If amplitude of electric field in the incident light is made 25 V/m, how much will the saturation current become?

- (a) 8.00 μA (b) 10.00 μA
✓ (c) 16.67 μA (d) None of these

$$i \propto n_e \propto n_p \propto P \propto E^2$$

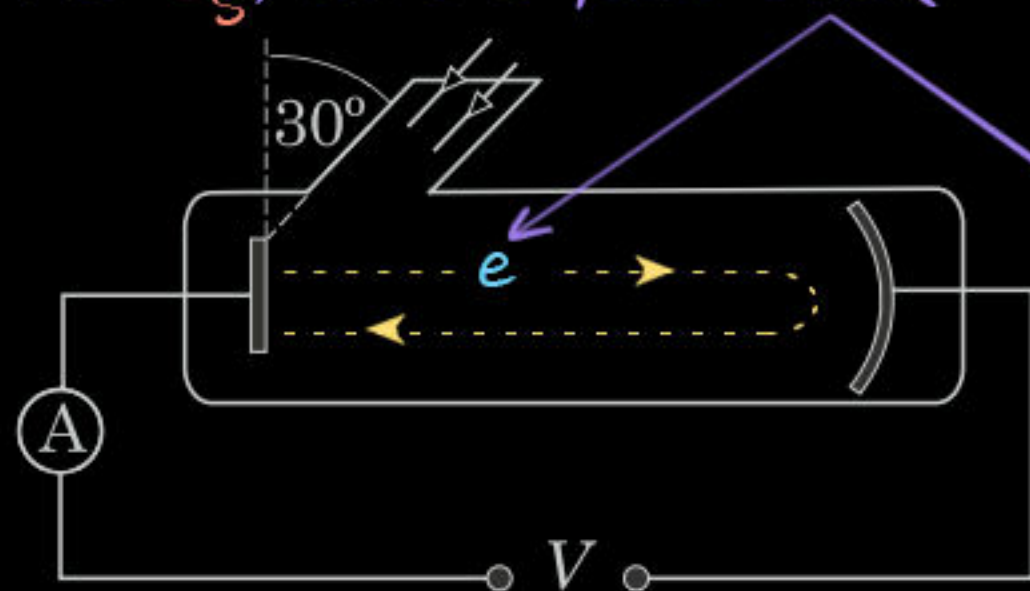
(η is constant, as its a metal surface property.)

$$\therefore \frac{i_1}{E_1^2} = \frac{i_2}{E_2^2}$$

$$\Rightarrow \frac{6}{15^2} = \frac{i_2}{25^2}$$

$$\Rightarrow i_2 = 16.67 \mu\text{A} //$$

At V_s , all e^- 's falls back



15. Instead of a simple plane wave, a complex plane wave of light governed by the following equation falls on the metal plate.

$$E = E_0 \{1 + \cos(\omega t + \phi_0)\} \cos(\omega t + \phi_0)$$

Stopping potential and saturation current respectively are

- (a) 6.27 V and 6.75 μ A
 ✓ (c) 8.27 V and 6.75 μ A
 (b) 6.27 V and 7.50 μ A
 (d) 8.27 V and 7.50 μ A

$$E = E_0 \left[\cos(\omega t + \phi_0) + \frac{1 + \cos 2(\omega t + \phi_0)}{2} \right]$$

$$= E_0 \cos(\omega t + \phi_0) + \frac{E_0}{2} + \frac{E_0}{2} \cos(2\omega t + 2\phi_0)$$

$$\frac{E_0}{2} = \frac{E_0}{2} \cos(\omega t)$$

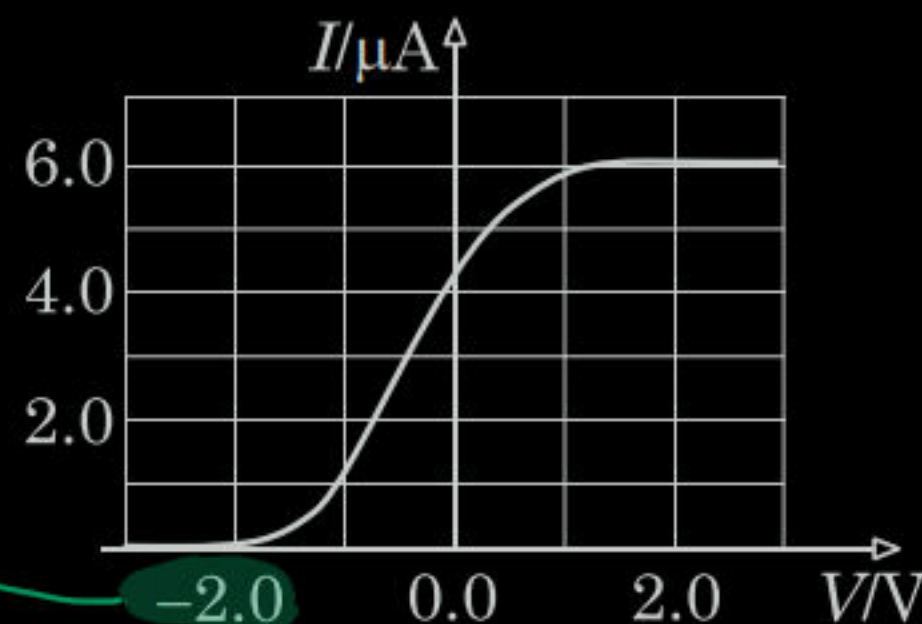
$$\text{Energy} = h\nu = 0$$

Stopping potential:

$$h\nu - \phi = K E_{\max} = eV_s$$

Initially: $h\nu - \phi = e(2)$

Now: $h(2\nu) - \phi = e(V_s)$



Subtract: $V_s = \frac{h\nu}{e} + 2 = \left[(6.63 \times 10^{-34}) \frac{9.5 \times 10^{15}}{2\pi \times 1.6 \times 10^{-19}} + 2 \right] \text{V} = 8.27 \text{V}$

Saturation current:

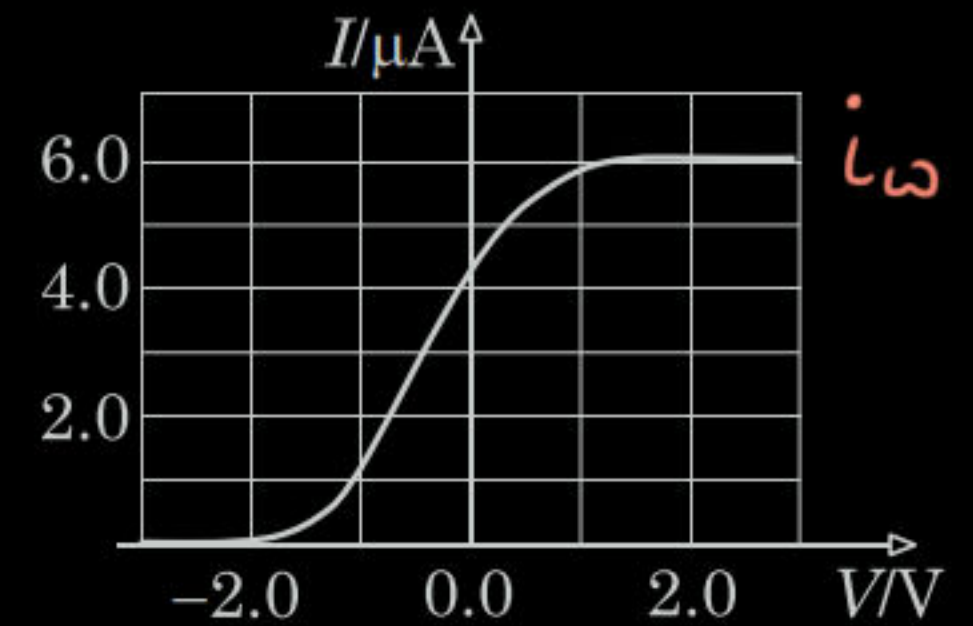
$$E_0 \cos(\omega t + \phi_0) + \frac{E_0}{2} + \frac{E_0}{2} \cos(2\omega t + 2\phi_0)$$

$$i \propto n_e \propto n_p \propto \frac{P}{h\nu} \propto \frac{E^2}{\nu} \propto \frac{E^2}{\omega}$$

$$\therefore \frac{\bar{i}_\omega}{i_{2\omega}} = \frac{E_0^2/\omega}{(E_0/2)^2/2\omega}$$

$$\Rightarrow \frac{6}{i_{2\omega}} = 8$$

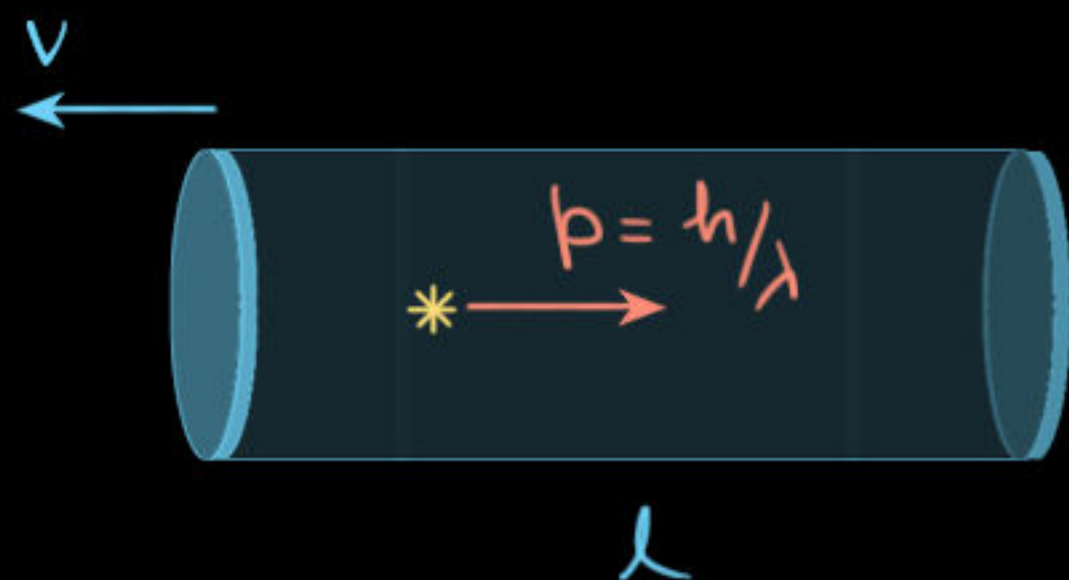
$$\Rightarrow i_{2\omega} = \frac{6}{8} = 0.75$$



$$\begin{aligned} \therefore i &= i_\omega + 0 + i_{2\omega} \\ &= 6 + 0 + 0.75 \\ &= 6.75 \mu A \end{aligned}$$

Youtube / Arihant Senior

1. In a thought experiment, an evacuated tube of mass m and length l closed at its ends by two discs of masses m_1 and m_2 stays at rest in free space. If a photon of frequency ν is emitted from one disk and absorbed at the other, find the displacement of the tube in this process. Denote the Planck's constant by h and the speed of light by c .



Let the tubes total mass be M .

$$Mv = h/\lambda$$

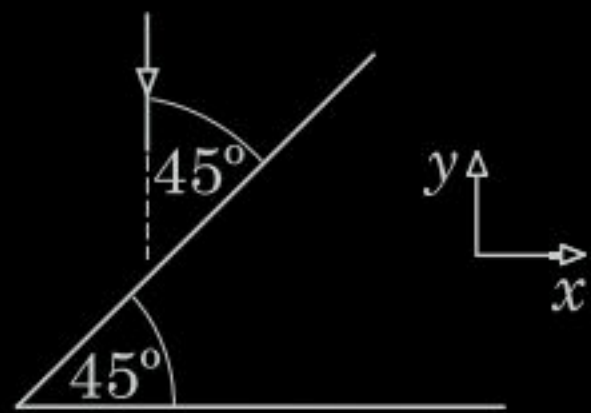
Displacement of tube = $v \Delta t$

$$\Delta t = \frac{l}{v+c} \approx 0$$

$$= \frac{h}{M\lambda} \cdot \frac{l}{c}$$

$$\lambda = \frac{c}{\nu}$$

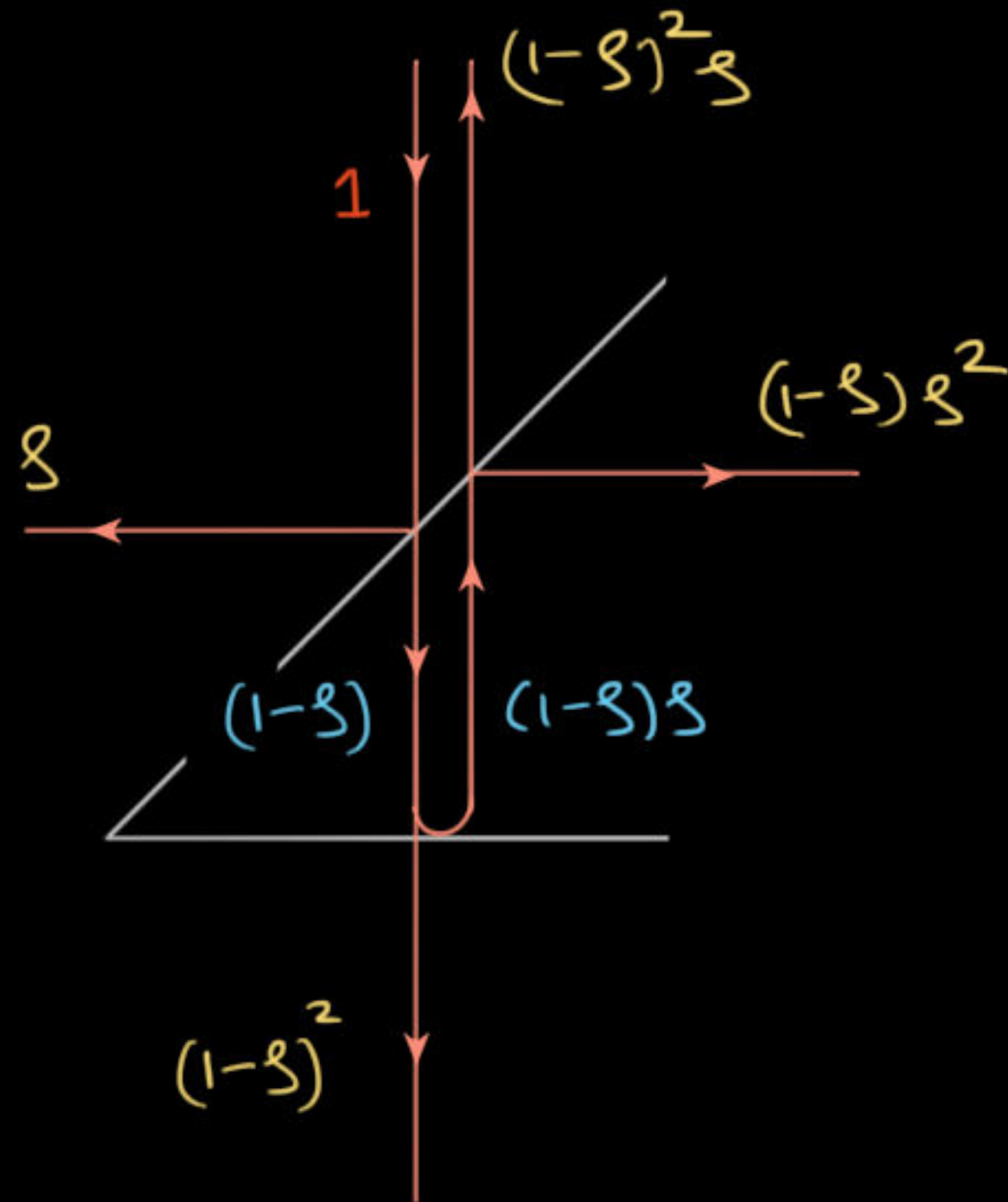
$$= \frac{h\nu l}{Mc^2} //$$



2. Two thin partially silvered glass plates are rigidly joined at an angle 45° . The structure is placed in gravity free region and a short light pulse is made to incident on the structure as shown. Coefficient of reflection of light from each plate is ρ for all angles of incidence. Find the direction of the total force of light pressure on the plate.

■ Let momentum of incoming pulse be 1 unit.

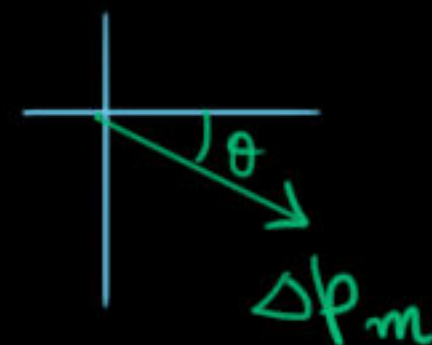
■ It eventually gets broken up in four different pulses as shown.



$$\vec{\Delta p}|_{\text{mirrors}} = -\vec{\Delta p}|_{\text{pulse}} = \vec{p}_i - \vec{p}_f$$

$$y: 1 - \{ (1-rho)^2 - (1-rho)^2 rho \} \downarrow = 1 - (1-rho)^3 \downarrow$$

$$x: 0 - \{ (1-rho)rho^2 - rho \} \rightarrow = rho - rho^2(1-rho) \rightarrow$$



$$\therefore \tan \theta = \frac{1 - (1-rho)^3}{rho - rho^2(1-rho)}$$

by Ankit Singhvi

Youtube / Arihant Senior

We know, from KTG,

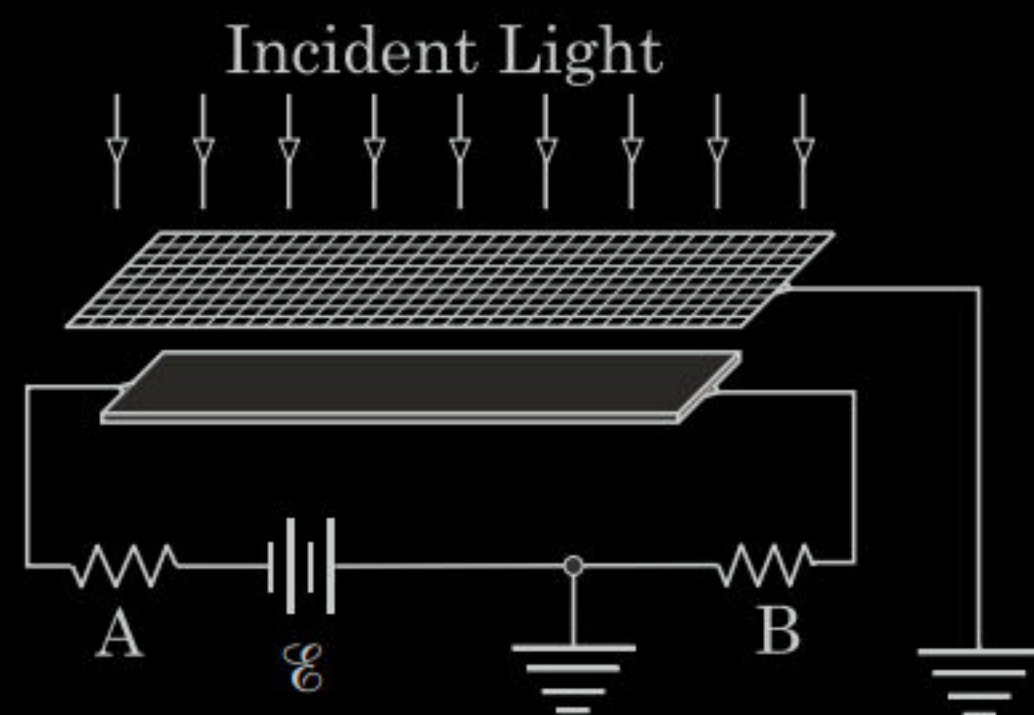
$$P = \frac{1}{3} \rho v_{rms}^2$$

$$v \rightarrow c$$

$$P = \frac{1}{3} \rho c^2 = \frac{1}{3} \left(\frac{m}{V} \right) c^2 = \frac{1}{3} \frac{(mc) c}{V} = \frac{1}{3} \frac{(n h / \lambda) c}{l^3}$$

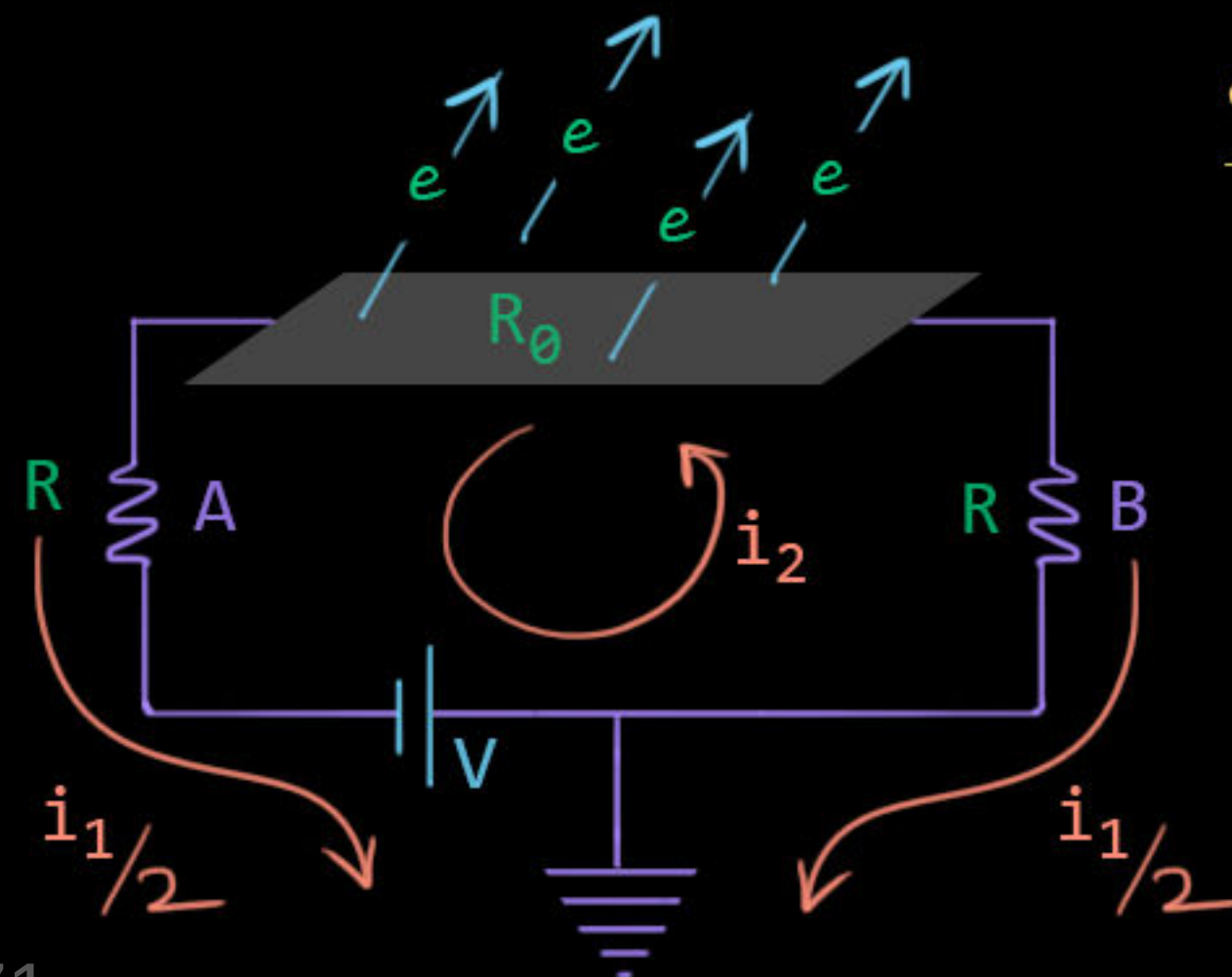
$$\Rightarrow n = \frac{3 P l^3 \lambda}{h c} //$$

3. How many photons of wavelength $\lambda = 450 \text{ nm}$ does it take to achieve pressure $p = 10^5 \text{ Pa}$ in an empty cubical box of edge length $l = 10 \text{ cm}$? Inner surfaces of all the faces of the box are perfectly reflecting. Planck's constant is $h = 6.67 \times 10^{-34} \text{ J}\cdot\text{s}$ and speed of light is $c = 3 \times 10^8 \text{ m/s}$.



4. A plate of area S connected to a battery of electromotive force $\mathcal{E}$ through two identical resistors A and B each of resistance R is uniformly illuminated by a monochromatic light of wavelength λ . The photoelectrons emitted from the plate are collected by an earthed wire grid as shown in the figure. Intensity of the light that reaches the plate after passing through the grid is I_0 . Resistance of the plate to current flow along its length is R_0 . Quantum efficiency of the photoelectric emission is η . Find the strength of currents through each of the resistors A and B. Denote modulus of charge on an electron by e and the Planck's constant by h .

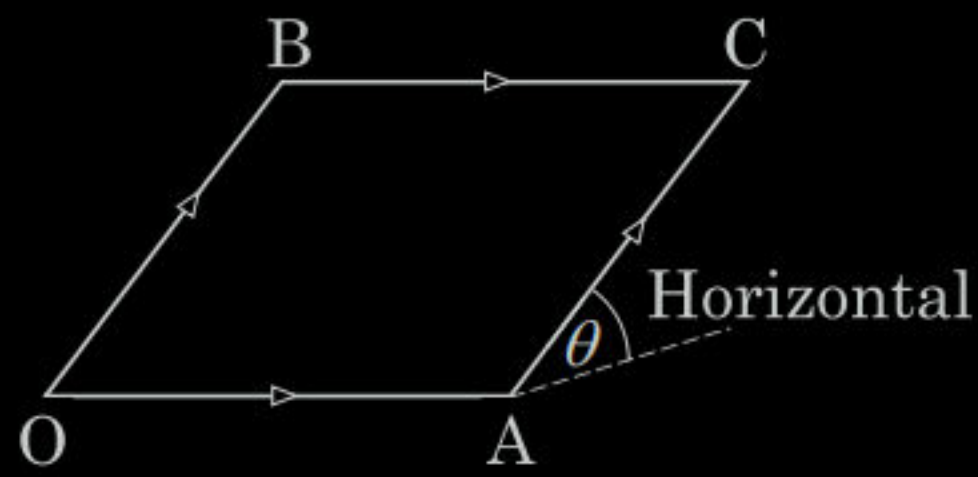
Concept: We shall determine final current by superimposing photocurrent i_1 and battery current i_2 . Also, photocurrent will be half in each branch as $R_A = R_B$.



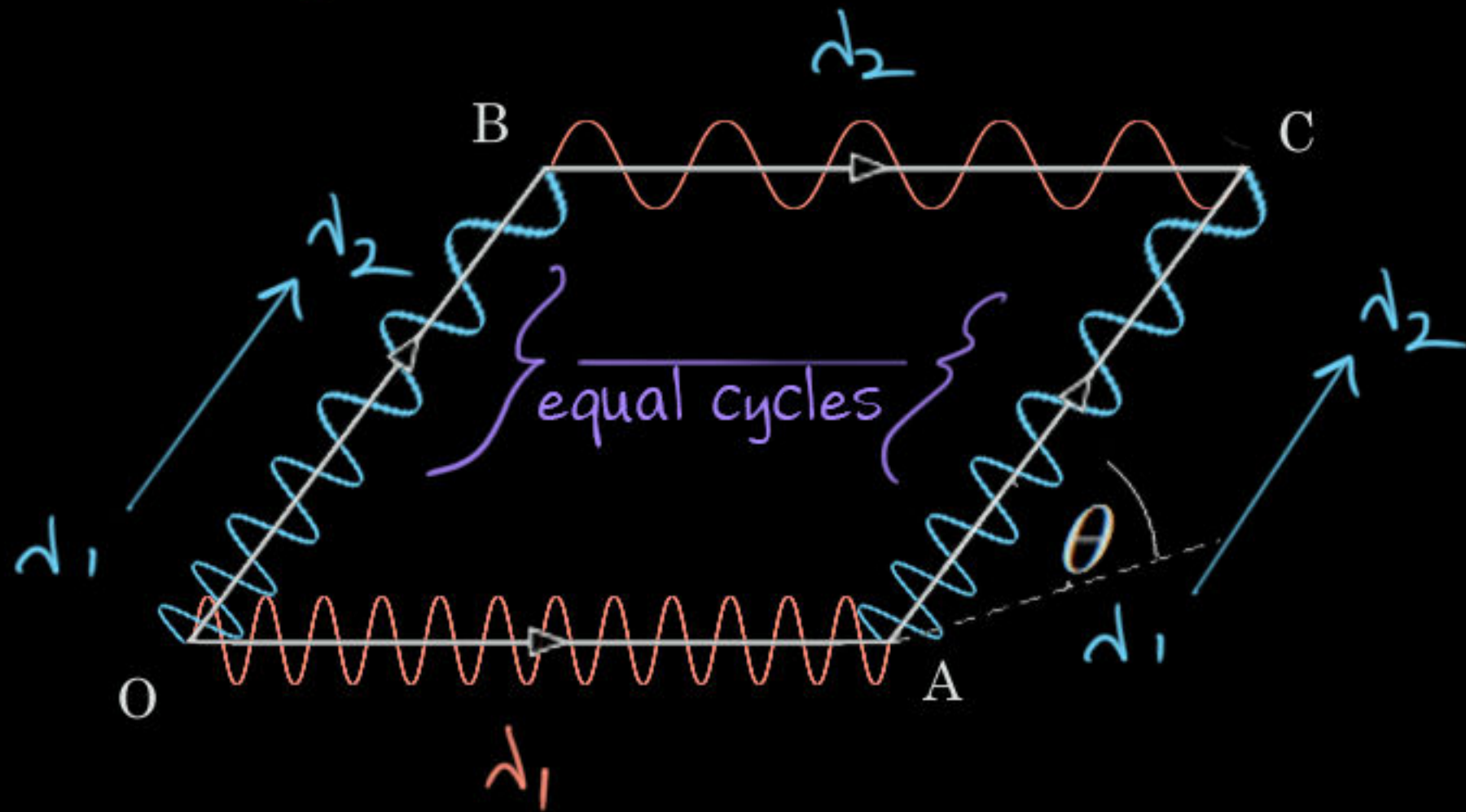
Sol: $i_2 = \frac{V}{2R + R_0}$

$$i_1 = e n_e |_{\text{per sec}} = \eta e n_p |_{\text{per sec}} = \eta e \frac{P}{hc/\lambda} = \frac{\eta e I_0 A \lambda}{hc}$$

$$\therefore i_A = i_2 + \frac{i_1}{2} \quad i_B = i_2 - \frac{i_1}{2}$$



Concept: With height, kinetic energy and thus momentum of photon decreases, hence its wavelength will increase.



5. A neutron beam of initial kinetic energy E_0 is divided into two beams at point O by some mechanism. Thereafter the beams proceed on the path OAC and OBC without a phase change at the corners A and B to interfere with each other at point C. Here OACB is a square of side l tilted at an angle θ above the horizontal. Mass of a neutron is m , Planck's constant is h and acceleration of free fall is g . Find an expression for the angle θ at which a maxima is detected at point C.

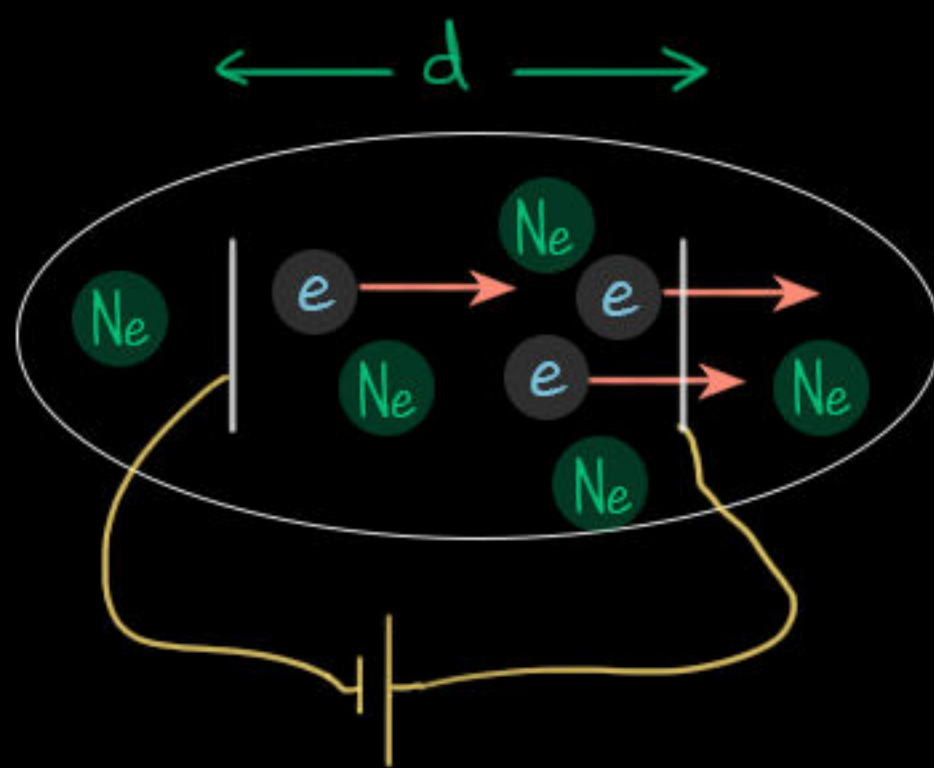
For maximas: $\text{Cycles}_{|OAC} - \text{Cycles}_{|OBC} = \text{Cycles}_{|OA} - \text{Cycles}_{|BC} = n$

$$\frac{h}{\lambda} = \sqrt{2mE} \Rightarrow \frac{l}{\lambda_1} - \frac{l}{\lambda_2} = n$$

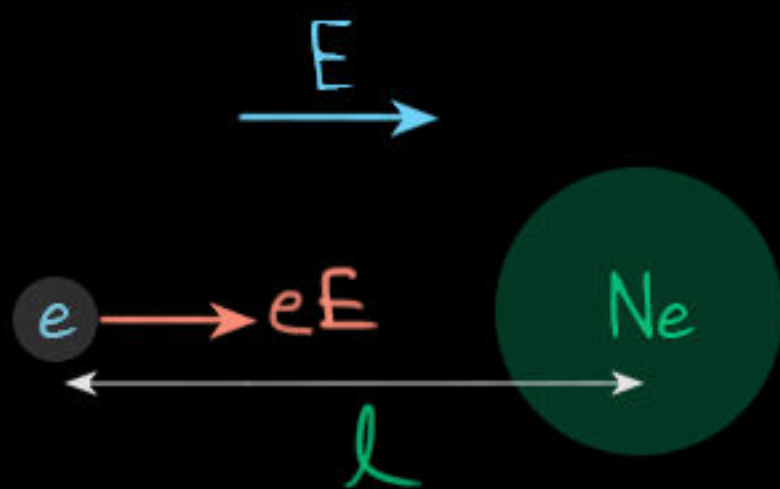
$$\Rightarrow \frac{l \sqrt{2mE_0}}{h} - \frac{l \sqrt{2m(E_0 - mgl \sin \theta)}}{h} = n$$

$$\Rightarrow \sin \theta = \frac{1}{mgl} \left[E_0 - \frac{1}{2m} \left(\frac{nh}{l} - \sqrt{2mE_0} \right)^2 \right]$$

$$= \frac{1}{2m^2gl^3} \left(2nhl\sqrt{2mE_0} - n^2h^2 \right), n=1,2,3...$$



|||



6. In a glass bulb filled with gaseous neon two metal plates are used as electrodes separated by a distance $d = 4$ mm that is much smaller than the linear dimensions of the plates. Ionization energy for neon atom is $E_0 = 21.5$ eV, average distance travelled by an electron between two consecutive collisions with neon atoms is $l = 0.4$ mm and charge on an electron is $e = -1.6 \times 10^{-19}$ C. How much potential difference between the electrodes should be applied to initiate a discharge process?

Concept: An electron must gain enough energy in between two collisions to ionize a Neon atom.

$$\therefore eEl > E_0$$

$\swarrow$
 v/d

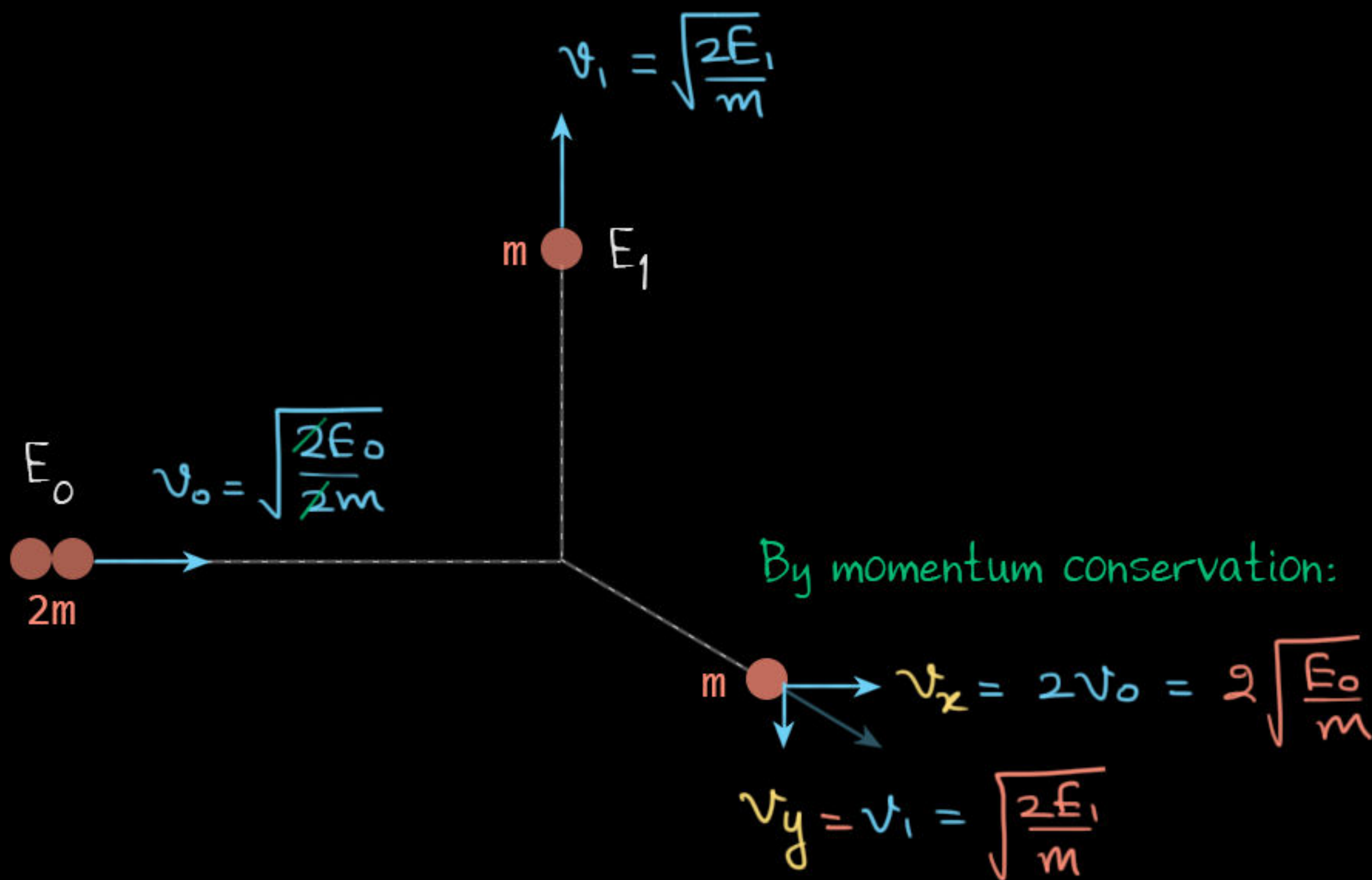
$$\Rightarrow \frac{evl}{d} > E_0$$

$$\Rightarrow v > \frac{dE_0}{el}$$

$$E = \frac{1}{2}mv^2$$

$$\Rightarrow v = \sqrt{\frac{2E}{m}}$$

7. A Hydrogen molecule in excited state travelling in the x -direction with kinetic energy of 1.0 eV dissociate into two hydrogen atoms. If one of them moves perpendicular to the x -direction with a kinetic energy 0.8 eV, calculate the energy released in the dissociation reaction.



$$E_{\text{released}} = KE_f - KE_i$$

$$= E_1 + \frac{1}{2}m(v_x^2 + v_y^2) - E_0$$

$$= E_1 + \frac{1}{2}m \left[\frac{4E_0}{m} + \frac{2E_1}{m} \right] - E_0$$

$$= 2E_1 + E_0$$

$$= 2(0.8) + 1$$

$$= 2.6 \text{ eV} //$$

B Let the highest state after photon excitation be n .

Given, $nC_2 = 10$

$\Rightarrow n = 5 //$

i.e, 4th excited state.

8. If a gaseous sample consisting of identically excited hydrogen like ions is irradiated by a stream of photons each of energy $144E_0/225$, ten distinct lines are observed in the emission spectrum. Some of the lines correspond to energy lower than, some higher than and some equal to the energy of the incident photons. Here E_0 is the ionization energy of a hydrogen atom in ground state.

- (a) In which excited state were the ions in the gaseous sample before interaction with the incident photons? *my answer is different.*
 (b) To which highest state were the ions in the gaseous sample excited?
 (c) Identify the element in the gaseous sample.

C Given, $E_0 = 13.6$

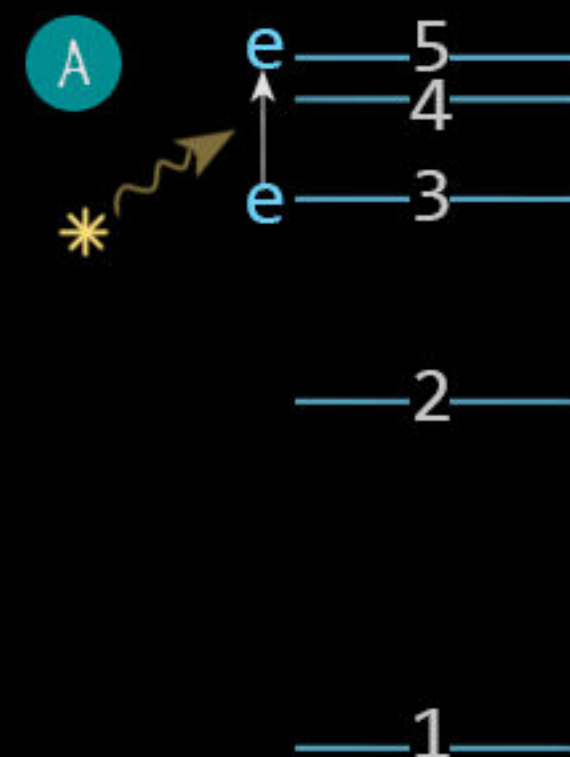
Lets say line corresponding to energy equal to photon is formed as electron jumps from n_2 to n_1 .

$$\frac{144}{225} = \left(\frac{12}{15}\right)^2 \rightarrow \left(\frac{4}{5}\right)^2 E_0 = -E_0 Z^2 \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

In denominator we have 5^2 , so n_2 has to be 5.

n_1 can now take 1, 2, 3 or 4.

Only $n_1 = 3$, and $Z=3$ satisfies it. $\therefore$ Its Lithium

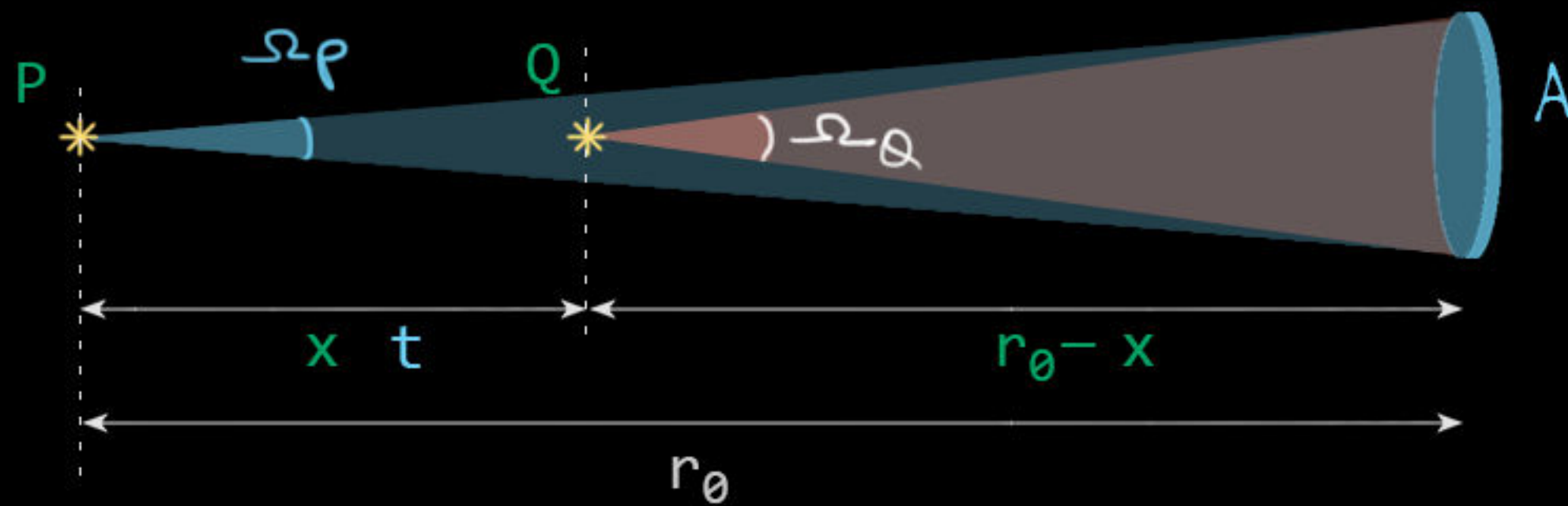


And originally electron was in $n=3$ i.e, 2nd excited state.

It couldn't have been in $n=2$ or less, as it won't reach $n=5$ then. And it couldn't have been in $n=4$, as it would reach $n=6$ then.

10. A point-like radioactive sample of decay constant λ initially at a distance r_0 from a radiation counter starts moving towards the counter. If the count number recorded by the counter remains constant with time, express speed of the radioactive sample as function of time.

Let radiation counter be a small disc of area A , that counts number of product particles that hit it per second.



Lets say sample travels x in t time.
and solid angles subtended by A be Ω_P and Ω_Q

Hit rates, when source is at P and Q are equal.

$$\therefore \frac{\Omega_P}{4\pi} \cdot \frac{dN}{dt}\bigg|_P = \frac{\Omega_Q}{4\pi} \frac{dN}{dt}\bigg|_Q$$

$$\frac{A}{r_0^2} \lambda N_0 = \frac{A}{(r_0 - x)^2} \lambda N_0 e^{-\lambda t}$$

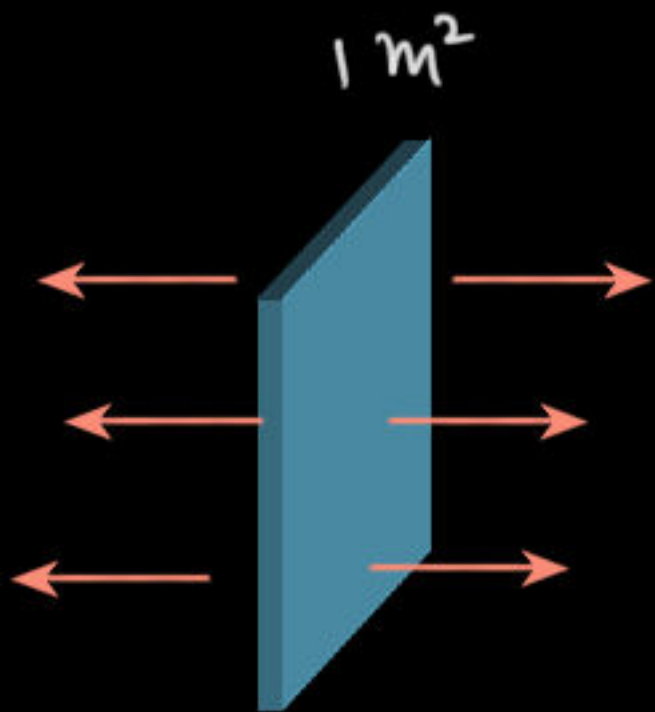
$$\Rightarrow 1 - \frac{x}{r_0} = e^{-\lambda t/2}$$

$$\text{differentiate: } -\frac{1}{r_0} dx = -\frac{\lambda}{2} e^{-\lambda t/2} dt$$

$$\Rightarrow \frac{dx}{dt} = \frac{\lambda r_0}{2} e^{-\lambda t/2}$$

Youtube / Arihant Senior

11. Plutonium ^{239}Pu is an unstable nuclei and decays with a half-life $\tau = 2.4 \times 10^4$ years by producing smaller nuclei including an alpha particle. Find the alpha particle flux density (number of the alpha particles per unit area per unit time) near the surface of a large plate of ^{239}Pu having thickness $d = 1$ mm. Density of plutonium is $\rho = 19800 \text{ kg/m}^3$ and mass of an atom of ^{239}Pu is $m_0 = 3.84 \times 10^{-25} \text{ kg}$.

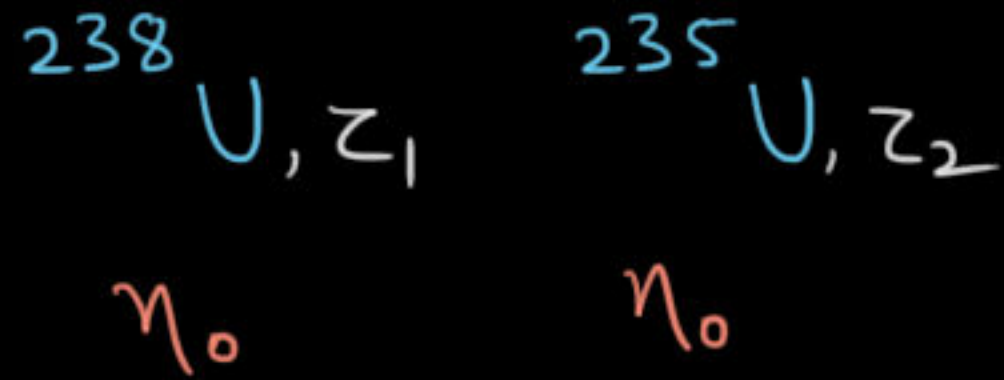


Lets consider 1m^2 plate.

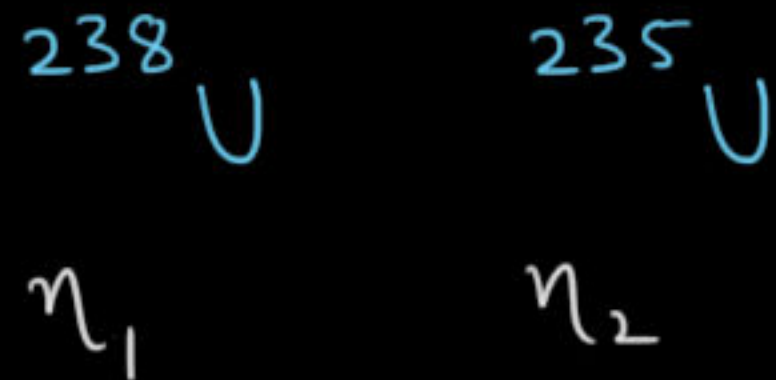
$$\therefore \text{Flux density} = \frac{dN/dt}{2} = \frac{\lambda N}{2} = \frac{\left(\frac{\ln 2}{\tau}\right) \left[\frac{\rho d(1)}{m_0}\right]}{2} = \frac{\ln 2 \rho d}{2 m_0 \tau} //$$

Let initial concentration of each be η_0 .

Initially



↓ $t = ?$



12. At present, concentrations of ^{238}U and ^{235}U in natural uranium are η_1 and η_2 and their half-lives are τ_1 and τ_2 respectively. Assuming concentrations of each of these isotopes were identical at the time of the birth of the earth; find the approximate age of the earth.

$$\therefore \eta_1 = \frac{\eta_0}{2^{t/\tau_1}} \quad (1)$$

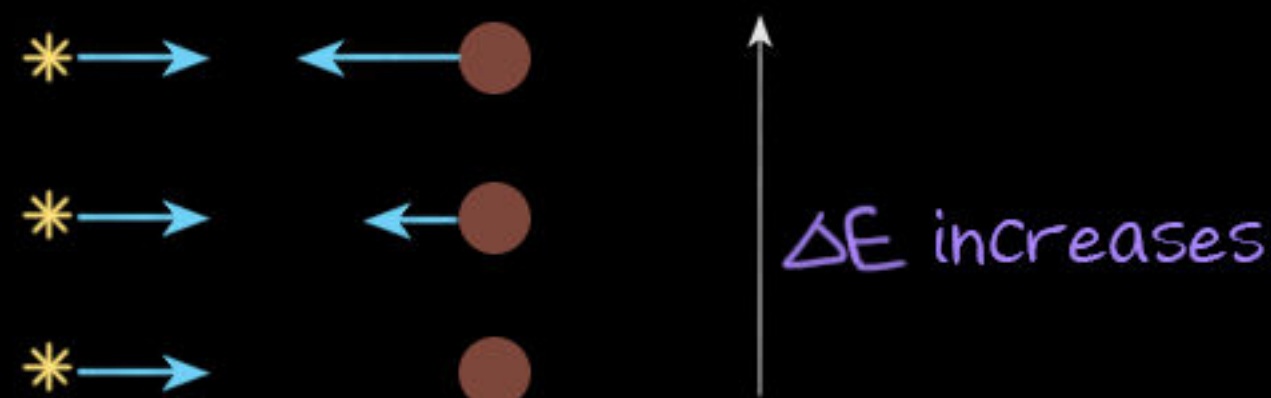
$$\text{and } \eta_2 = \frac{\eta_0}{2^{t/\tau_2}} \quad (2)$$

Divide: $\frac{\eta_1}{\eta_2} = 2^{t(\frac{1}{\tau_2} - \frac{1}{\tau_1})} \Rightarrow t = \frac{\ln \eta_1 / \eta_2 (\tau_1 \tau_2)}{\ln 2 (\tau_1 - \tau_2)} //$

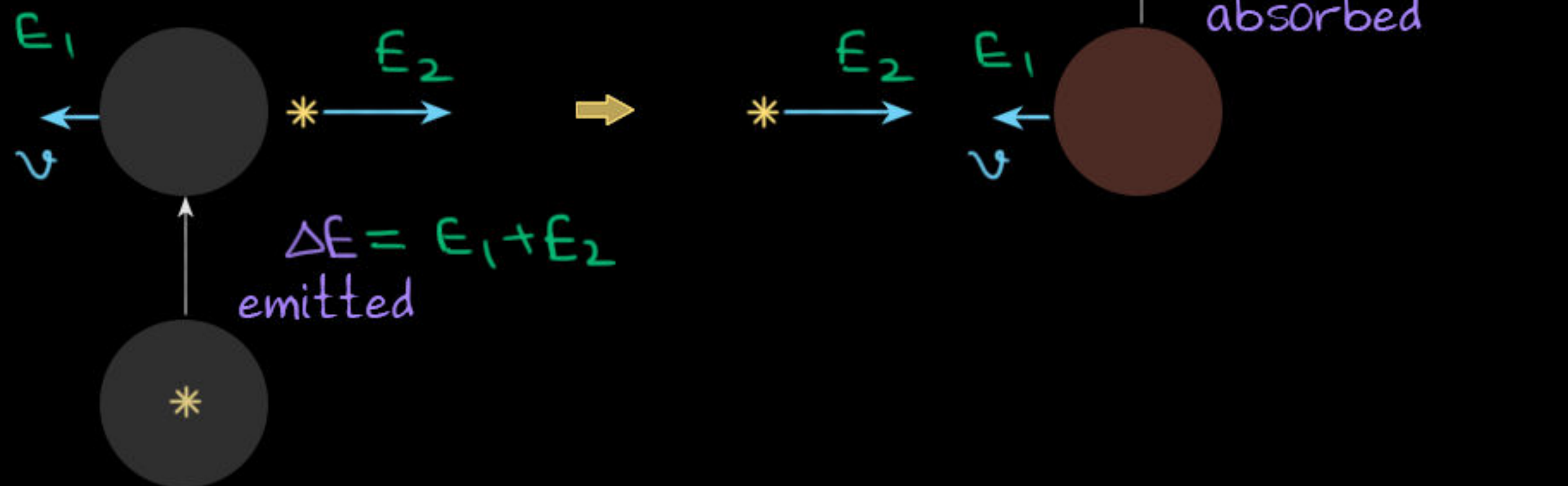
Concept: Velocity of second nucleus before absorption will be same as velocity of first nucleus after emission.

13. A free motionless nucleus of tin [^{119}Sn] emits a γ -photon of energy 22.6 keV, which is absorbed by another tin nucleus approaching the γ -photon. Find velocity of the moving nucleus, if in the emission and absorption of γ -photon, transitions of the nuclei take place between the same energy states. The rest energy of the tin nucleus is 113 GeV.

∴ In an inelastic collision, for a fixed momenta of gamma *, and fixed ΔE , v of nucleus will be fixed too.



∴ ΔE is same



$$mv = \frac{h}{\lambda}$$

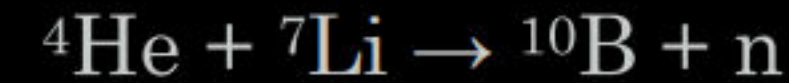
$$\Rightarrow v = \frac{h}{\lambda m} \times \frac{c^2}{c^2}$$

$$= \left(\frac{hc}{\lambda} \right) \frac{1}{mc^2} \cdot c$$

$$= \frac{22.6 \text{ keV}}{113 \cdot 10^6 \text{ keV}} \cdot 3 \times 10^8$$

$$= 60 \text{ m/s} //$$

14. When an α -particle collides with a stationary lithium nucleus, the following nuclear reaction may take place.



If Q-value of this reaction is $E = -2.87 \text{ MeV}$, find the minimum value of kinetic energy of incoming α -particles required for the reaction.

For a general endoergic reaction:



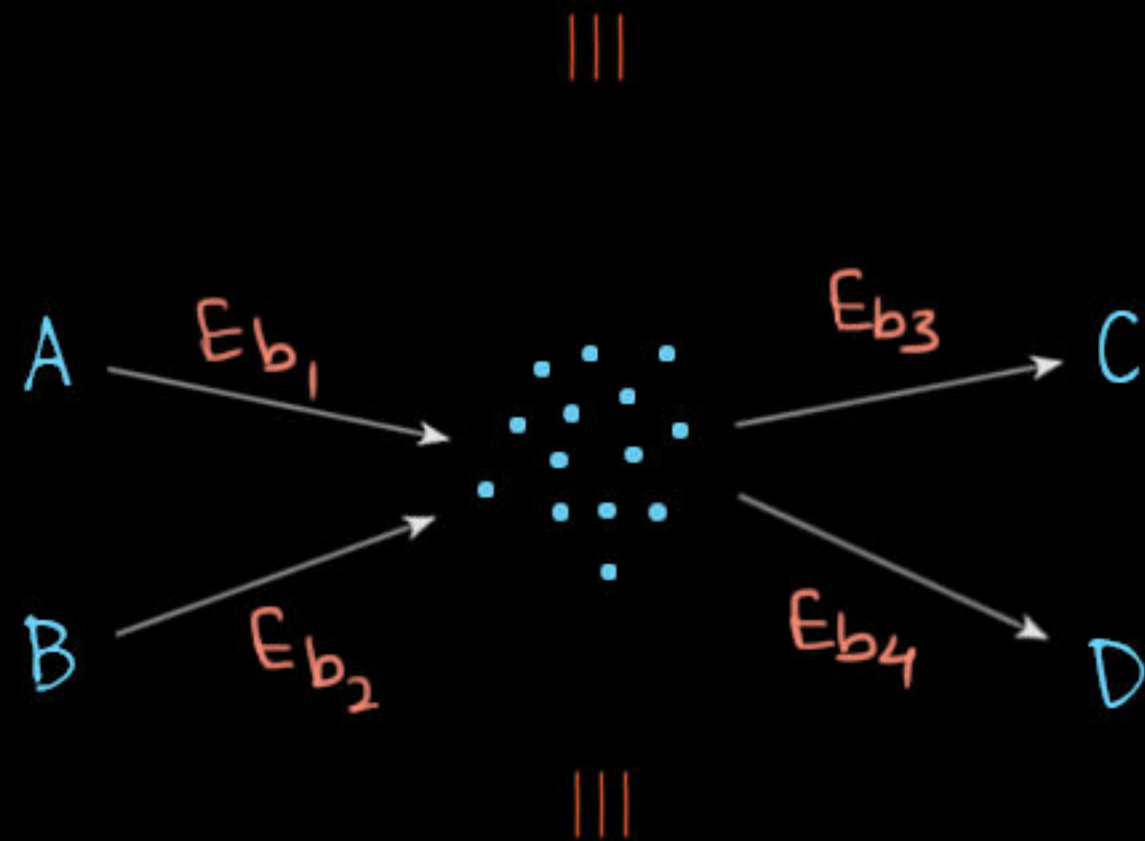
$$\text{Minimum } KE_m = \left(\frac{m+M}{m} \right) |Q|$$

Proof: Irodov 6.271

$$\begin{aligned} \text{Sol: Minimum } KE_\alpha &= \left(\frac{m_\alpha + m_{\text{Li}}}{m_{\text{Li}}} \right) |Q| \\ &= \left(\frac{4+7}{7} \right) 2.87 \text{ MeV} \\ &= 4.51 \text{ MeV} // \end{aligned}$$

Concept:

For a generic nuclear reaction:



From 1,2: $Q = E_{b3} + E_{b4} - (E_{b1} + E_{b2})$

$$\Rightarrow Q = E_{b|products} - E_{b|reactants}$$

Similar: Irodov 6.264

15. Nuclei of deuterium D (^2H) and tritium T (^3H) can fuse according to the following reaction to produce a neutron and an alpha particle.



Calculate energies carried away by the neutron and the alpha particle in the above nuclear reaction.

Assume binding energies per nucleon of deuterium, tritium and helium to be 1.00 MeV, 2.80 MeV and 7.00 MeV respectively and neglect kinetic energy of the nuclei before the fusion.

Sol:



$$Q = KE_{\text{He}} + KE_n = E_{b|products} - E_{b|reactants}$$
$$\Rightarrow KE_{\text{He}} + 4KE_n = 4(7) - [2(1) + 3(2.8)]$$

$$\Rightarrow KE_{\text{He}} = 3.52 \text{ MeV}$$

$$\& KE_n = 4KE_{\text{He}} = 14.08 \text{ MeV}$$

Youtube / Arihant Senior